Open Market Operations*

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Abstract

In open market operations, a central bank swaps currency for bonds. We show how injecting money in this way is different from transfers, as policy is usually formulated in similar models. For this we capture liquidity explicitly by modeling the roles of assets in frictional exchange. Under various specifications for market structure and the acceptability or pledgeability of assets, we discuss implications for the Fisher and quantity equations, the possibility of negative nominal yields, liquidity traps, and market segmentation. When liquidity is endogenized using information theory, multiple equilibria emerge with different policy predictions. We also analyze interest on reserves.

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1 Introduction

In an open market operation, or OMO, the central bank swaps currency for bonds. That this policy is important is clear from the substantial discussion in textbooks on monetary economics, yet there is little formal analysis in monetary theory, and hence the effects are not completely understood. This paper uses a microfounded New Monetarist model to systematically analyze OMO's. We find that injecting money in this way is *very different* from the lump-sum transfers – the proverbial "helicopter drops" – previously studied in this framework.[1]

This involves extending the standard framework in several ways. First, we introduce interest-bearing government bonds, A_b, in addition to money, A_m. In the interest of robustness, short- and long-term bonds, plus real and nominal bonds, are considered. Second, both assets can be used in decentralized trade, as media of exchange or collateral, but to induce differential liquidity premia we let A_b and A_m differ in either their acceptability (the fraction of trades in which the asset is accepted) or pledgeability (the amount of the asset that is accepted). Third, the focus is squarely on asset swaps and their use in targeting interest rates, not transfers. Finally, and importantly, rather than taking acceptability and pledgeability as primitives, we endogenize them using information frictions. While there are previous information-theoretic analyses of liquidity, they do not pursue the implications for OMO's, which are interesting because endogenous information naturally leads to multiple equilibria with very different policy implications.

To summarize, a typical OMO involves an increase in A_m engineered by central bank bond purchases of A_b, which means a decrease in private holdings of A_b. This is only the same as increases in A_m engineered by transfers in special cases. The reason is simple: with OMO's, not only does A_m go up, A_b also goes down, and in fact, the latter is more important. Another reason they may not be the same

[1] That lump-sum transfers are the usual way to inject cash is clear from recent surveys of this literature by Lagos et al. (2017) and Rocheteau and Nosal (2017).

concerns their fiscal implications, but in the interest of controlled experiments, we sterilize these by having fiscal policy passively accommodate monetary policy.[2]

To put the contribution in perspective, consider Wallace (1981), who has a model where swapping A_m for A_b has no effect. His results are special, because A_m and A_b must be perfect substitutes in his OLG (overlapping generations) framework. While OLG models allow fiat money to be valued, they do not allow multiple assets to be valued, unless they have the same return, by no-arbitrage conditions that apply in any Walrasian market. Wallace calls his result a Modigliani-Miller Theorem for OMO's because it is similar to noticing that the mix of debt and equity does not matter for corporate finance when they are perfect substitutes. So when Wallace swaps A_b for A_m, it is like swapping two ten-dollar bills for a twenty. Of course, if the assets have different risks, the results change, but no one (at least since Tobin 1958) thinks the key distinction between T-bills and currency is risk. Here the key property is liquidity, something missing in Walrasian theory.

In terms of results, first, we characterize the effects of OMO's on various interest rates, discuss the Fisher equation, and show how bonds can bear negative nominal yields in some cases.[3] Then we characterize the effects on the price level and inflation. The model by design obeys the quantity equation – classical neutrality – in the sense that one-time unanticipated increases in A_m due to transfers raise all nominal variables proportionately with no impact on real variables. The point is not whether money is neutral in reality; the model is that way, by design, to

[2]On this, we cannot improve on the Editor's letter: "One important part of [the experiments] ... is to separate the integration of the monetary and fiscal policy resource constraints. Separate the two to be clearer that there is a fiscal authority that chooses some lump sum taxes, and that issues government bonds in amount A_b. Its behavior can be completely passive, essentially just fiscally backing whatever the monetary authority does. Then there is a monetary authority that buys the bonds, and prints money, rebating profits to the fiscal authority. By picking A_m and buying bonds, the monetary authority essentially is choosing also A_b, now understood not as the total supply of these bonds, but rather as the amount of government bonds held by the private sector. Separating the two makes clearer what an OMO is."

[3]Whether or not our conditions for negative nominal rates constitute the relevant case empirically, the results demonstrate how the phenomena can emerge logically. While we do check these conditions econometrically, our work is a complement to, e.g., Krishnamurthy and Vissing-Jorgensen (2012), who do show empirically that T-bills have a "convenience yield."

examine OMO's impact via liquidity without nominal rigidities, signal-extraction problems, etc. (although Sections 3.1 and 3.2 mention how some times neutrality fails). Yet even in this case, increases in A_m from OMO's raise nominal variables less than proportionately and change the allocation, except when the assets are perfect substitutes (as in a liquidity trap) or agents are satiated in bonds (as in a liquidity glut). Finally, liquidity is endogenized, which as mentioned generates multiple equilibria. Hence, if a policy maker asks about the impact of OMO's, the answer must depend on knowing the type equilibrium we are in.

Given the surveys cited in fn. 1 we do not review the New Monetarist literature.[4] On reduced-form monetary economics – e.g., CIA (cash-in-advance) or MUF (money-in-the-utility function) models – there is too much work to list, but see, e.g., Bansal and Coleman (1996) for a representative example and more references. One branch of this research, e.g., Alvarez et al. (2002) focuses on market segmentation, where not everyone is active in all markets or there is a cost to transferring resources across markets. We also get different assets accepted in different markets, but there are no CIA constraints, and agents can always go to a cashless market. Moreover, heterogeneous portfolios here are choices, not restrictions.

Section 2 describes the environment. Sections 3 and 4 study equilibrium with exogenous and endogenous liquidity. Section 5 sketches some extensions, including a model with interest on reserves. Section 6 concludes.

2 Environment

As in Lagos and Wright (2005) or Rocheteau and Wright (2005), at each date $t = 0, 1, ...$ two markets convene sequentially: a decentralized market, or DM, with frictions discussed below; and a frictionless centralized market, or CM. In the CM,

[4] We mention Williamson (2012,2016), Rocheteau and Rodriguez-Lopez (2014), Shi (2014) and Dong and Xiao (2015), which are similar, but with a big difference – namely, we endogenize liquidity based on information, which requires going beyond take-it-or-leave-it bargaining, as in most of those papers, as sellers do not invest in information if they get no gain from trade.

a large number of agents work, consume and adjust their portfolios. In the DM some agents, called *sellers*, can provide something – a good, service, input or asset – wanted by other agents, called *buyers* (buyer and seller types are permanent, but not much changes if they are random each period). Let μ be the measure of buyers and n the seller/buyer ratio. They meet pairwise in the DM, with α the probability a buyer meets a seller and α/n the probability a seller meets a buyer.

The period payoffs for buyers and sellers are

$$\mathcal{U}(q,x,\ell) = u(q) + U(x) - \ell \quad \text{and} \quad \widetilde{\mathcal{U}}(q,x,\ell) = -c(q) + \widetilde{U}(x) - \ell, \tag{1}$$

where q is traded in the DM, x is the CM numeraire and ℓ is CM labor.[5] In the original models, $c(q)$ is a cost and $u(q)$ a utility function. In other applications, $u(q)$ is production function mapping q into x (e.g., Shi 1999; Silveira and Wright 2010). In other applications, DM traders are financial institutions, like banks trading Fed Funds (e.g., Koeppl et al. 2008; Afonso and Lagos 2015) or investors trading assets (e.g., Lagos and Zhang 2015; Mattesini and Nosal 2016). While in some contexts it is important to be precise about institutional details, here we keep things abstract, so the theory applies to various decentralized markets.

Assume $u(q)$ and $c(q)$ are twice continuously differentiable with the usual monotonicity and curvature properties. Also, let $u(0) = c(0) = 0$, assume there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q}) > 0$, and define the efficient q by $u'(q^*) = c'(q^*)$. There is a discount factor $\beta = 1/(1+r)$, $r > 0$, between the CM and DM, while any discounting between the DM and CM can be subsumed in the notation. We also assume that x and q are nonstorable, to hinder barter, and that agents are to some degree anonymous in the DM, to hinder unsecured credit. As is well understood, these frictions generate a role for assets in the facilitation of exchange.

For now, there are two assets that can serve in this capacity: money in supply

[5] Quasi-linearity in (1) simplifies things by making the distribution of assets for a given type degenerate at the start of each DM. However, quasi-linearity can be relaxed in various ways without changing the results, as discussed in the surveys cited in fn. 1,

A_m; and bonds meant to represent T-bills in supply A_b. Their CM prices are ϕ_m and ϕ_b. As a benchmark we use short-term real bonds issued in one CM that yield a unit of numeraire in the next, but later consider nominal and long-term bonds. The real value of money and bonds per buyer are z_m and z_b. For money $z_m = \phi_m A_m$; for real bonds $z_b = A_b$; for nominal bonds $z_b = \phi_m A_b$; and for long bonds $z_b = (\phi_b + \delta) A_b$ where δ is the coupon. All assets can be used in some transactions up to some limit. A simple way to describe this is to say that a given seller accepts some assets but not others as media of exchange, as in Kiyotaki and Wright (1989,1993).

However, that interpretation is too narrow. Consider instead deferred settlement, as in Kiyotaki and Moore (1997). Thus, a buyer (borrower) in the DM getting q promises the seller payment in numeraire in the next CM, but due to limited commitment he can renege. This leads to a role for assets as collateral in secured credit. The usual interpretation is that if a borrower reneges his assets are seized. This dissuades opportunistic default, and captures the way many assets facilitate intertemporal exchange beyond serving as media of exchange in quid pro quo transactions. We can also describe DM trade as repurchase agreements, where a buyer getting q gives assets to a seller, who gives them back at prearranged terms in the next CM.[6]

Models of secured credit typically allow only a fraction $\chi_j \in [0,1]$ of asset j to be used, and we do the same. Section 4 shows how to endogenize χ_j using private information; for now χ_m and χ_b are exogenous fractions of A_m and A_b that can be used in DM transactions, with $\chi_j > 0$ unless stated otherwise. In deferred settlement, χ_b describes the haircut one takes when using bonds as collateral, often motivated by saying defaulters can abscond with a fraction $1-\chi_b$ of their holdings.

[6] Whether it is important to give back the same assets, or to prearrange the terms, merits discussion, but the idea here is simply to suggest that repos are another realistic way that assets facilitate trade, and T-bills are routinely used in this way by financial institutions. We are proposing merely a flexible mapping between theory and institutions, not a "deep" theory of repos (e.g., Vayanos and Weill 2008; Antinolfi et al. 2015; Gottardi et al. 2015).

For χ_m, equally plausible stories have sellers worried about counterfeiting, or, thinking about money broadly to include demand deposits, bad checks. While $\chi_j = 1$ is a fine special case, there is no reason to impose that at this point.

Let α_m be the probability a random seller in the DM accepts only money, α_b the probability he accepts only bonds, and α_2 the probability he accepts both. Special cases include $\alpha_b = \alpha_2 = 0$ (no one accepts bonds), $\alpha_b = 0$ (no one accepts only bonds), and $\alpha_b = \alpha_m = 0$ (the assets are perfect substitutes). Notice α_j and χ_j capture liquidity on the extensive and intensive margin. As regards $\alpha_b > 0$, we can easily imagine such situations – e.g., bonds are entries in a spreadsheet that can be transferred electronically between spatially-separated counterparties, while cash in your wallet cannot. In any case, while $\alpha_b = 0$ is a fine special case, there is no reason to impose that at this point.

In stationarity equilibrium $z_m = \phi_m A_m$ is constant and so the growth rate of the money supply, π, equals the inflation rate, $\phi_m/\phi_{m,+1} = 1 + \pi$, where $+1$ indicates next period. As usual we assume $\pi > \beta - 1$, but also consider the limit $\pi \to \beta - 1$, which is the Friedman rule. Stationarity also implies z_b is constant, which means A_b is constant for real bonds and $B = A_b/A_m$ is constant for nominal bonds. These policy variables are set by a monetary-fiscal authority subject to a consolidated budget constraint. With one-period real bonds, e.g., this is

$$G + T - \pi\phi_m A_m + A_b(1 - \phi_b) = 0, \tag{2}$$

where G is government consumption, T is a lump-sum transfer, or tax if $T < 0$, the third term is seigniorage, and the fourth is debt service. As discussed in fn. 2, fiscal policy is passive, with T adjusting to satisfy (2) given the other variables.

Let ι_0 be the return on an illiquid nominal asset, defined by the Fisher equation $1 + \iota_0 = (1 + \pi)/\beta$, where $1/\beta = 1 + r$ is the return on an illiquid real asset. An illiquid asset is one that cannot be traded in the DM. Thus, $1 + \iota_0$ denotes the dollars in the next CM that make you willing to give up a dollar today, and $1 + r$

denotes the x in the next CM that makes you willing to give up a unit today (and as usual, these trades can be priced whether or not they occur in equilibrium). For a real liquid bond, the nominal yield ι_b is the amount of cash you can get in the next CM by investing a dollar in the asset today, $1+\iota_b = \phi_m/\phi_b\phi_{m,+1} = (1+\pi)/\phi_b$. Also, define the spread between the nominal yields on illiquid and liquid bonds, $s_b = (\iota_0 - \iota_b)/(1+\iota_b)$; this is the opportunity cost of the liquidity services embodied in bonds. For symmetry, define $s_m = (\iota_0 - \iota_m)/(1+\iota_m)$ as the spread between illiquid assets and currency, where ι_m is interest on currency, which is 0 as a benchmark, but $\iota_m > 0$ is considered in Section 5.2.

Since $1+\iota_0 = (1+\pi)/\beta$, the Friedman rule is equivalent to $\iota_0 = 0$. There is no equilibrium with $\iota_0 < 0$, but $\iota_b < 0$ is possible (see below). The usual policy studied involves changing ι_0. We are more interested in OMO's that swap A_b and A_m to satisfy (2) within a period. Usually, we assume the change in A_b is permanent, with T covering future changes in debt service; later we consider changing A_b for just one period. Also note that we can think of the central bank targeting some interest rate, which here can be the T-bill rate since, as shown below, any ι_b in a range can be implemented with a unique A_b. This captures actual policy well, in a stylized way, and can be understood as (unanticipated) real-time changes: there are no transitional dynamics, so this economy can jump from one stationary equilibrium directly to another.

3 Equilibrium

3.1 Baseline: Short Real Bonds

A buyer's DM state is his portfolio (z_m, z_b), while what matters in the CM is $z = z_m + z_b$. Let the CM and DM value functions be denoted $W(z)$ and $V(z_m, z_b)$. Then the CM problem is

$$W(z) = \max_{x,\ell,\hat{z}_m,\hat{z}_b} \{U(x) - \ell + \beta V(\hat{z}_m, \hat{z}_b)\} \text{ st } x = z + \ell + T - (1+\pi)\hat{z}_m - \phi_b \hat{z}_b \quad (3)$$

where $\hat{z}_j$ is the real value of asset j taken out of the CM, and the real wage is $\omega = 1$ because we assume 1 unit of ℓ produces 1 unit of x (that is easy to relax). Given $x \geq 0$ and $\ell \in [0,1]$ are slack, the key FOC's are $1 + \pi = \beta V_1(\hat{z}_m, \hat{z}_b)$ and $\phi_b = \beta V_2(\hat{z}_m, \hat{z}_b)$. The envelope condition is $W'(z) = 1$, meaning $W(z)$ is linear. Sellers' CM value function (not shown) is similarly linear.

Letting p_j denote payment in type-j meetings, and using $W'(z) = 1$, we write buyers' DM value function as

$$V(\hat{z}_m, \hat{z}_b) = W(\hat{z}_m + \hat{z}_b) + \alpha_m[u(q_m) - p_m] + \alpha_b[u(q_b) - p_b] + \alpha_2[u(q_2) - p_2].$$

The first term on the RHS is the continuation value from not trading; the rest are the surpluses from different types of meetings. Payments are constrained by $p_j \leq \bar{p}_j$, where $\bar{p}_j$ is the buyer's *liquidity position* in a type-j meeting: $\bar{p}_m = \chi_m z_m$, $\bar{p}_b = \chi_b z_b$ and $\bar{p}_2 = \chi_m z_m + \chi_b z_b$. Sellers' DM value function is similar, except their surplus is $p_j - c(q_j)$, and they are not constrained by their asset positions.

The terms of trade are determined by an abstract mechanism: to get q you must pay $p = v(q)$. Kalai's proportional bargaining solution, e.g., implies $v(q) = \theta c(q) + (1-\theta) u(q)$, where θ is the buyer's bargaining power. Nash bargaining is similar, but messier if $p_j \leq \bar{p}_j$ binds. We can even use Walrasian (marginal cost) pricing – e.g., when $c(q) = q$ that is given by $v(q) = q$. However, other than $v(0) = 0$ and $v'(q) > 0$, all we need for now is this: Let $p^* = v(q^*)$ be the payment that gets the efficient q. Then $p^* \leq \bar{p}_j \Rightarrow p_j = p^*$ and $q_j = q^*$, while $p^* > \bar{p}_j \Rightarrow p_j = \bar{p}_j$ and $q_j = v^{-1}(\bar{p}_j)$. This holds the above examples and many others, and can also be derived axiomatically (Gu and Wright 2016).

As usual, $\iota_0 > 0$ implies buyers cash out – i.e., spend all the money they can – in type-m meetings and are still constrained: $p_m = \chi_m z_m < p^*$. Also, they may as well cash out in type-2 meetings before using bonds, since in these meetings both parties are indifferent between z_m and z_b. Buyers use all the bonds they can in type-2 meetings iff $\bar{p}_2 \leq p^*$, and in type-b meetings iff $\bar{p}_b \leq p^*$. It is obvious that

$p_2 \geq p_b$, leaving three possibilities: 1. $\bar{p}_2 < p^*$ and $\bar{p}_b < p^*$ (buyers are constrained in all meetings); 2. $\bar{p}_2 \geq p^*$ and $\bar{p}_b < p^*$ (they are constrained in type-b but not type-2 meetings); or 3. $\bar{p}_2 \geq p^*$ and $\bar{p}_b \geq p^*$ (they are not constrained in type-b or type-2 meetings). We now consider each case in turn.[7]

In Case 1 (buyers are always constrained), $\mathbf{q} = (q_m, q_b, q_2)$ solves

$$v(q_m) = \chi_m \hat{z}_m,\ v(q_b) = \chi_b \hat{z}_b \text{ and } v(q_2) = \chi_m \hat{z}_m + \chi_b \hat{z}_b. \tag{4}$$

Differentiating $V(z_m, z_b)$ using (4) and inserting the results into the FOC's from the CM, we get the Euler equations

$$1 + \pi = \beta \left[1 + \alpha_m \chi_m \lambda(q_m) + \alpha_2 \chi_m \lambda(q_2)\right] \tag{5}$$

$$\phi_b = \beta \left[1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)\right], \tag{6}$$

where $\lambda(q_j) \equiv u'(q_j)/v'(q_j) - 1$ is the *liquidity premium* in a type-j meeting, i.e., the Lagrange multiplier on $p_j \leq \bar{p}_j$. Using s_m and s_b,

$$s_m/\chi_m = \alpha_m \lambda(q_m) + \alpha_2 \lambda(q_2) \tag{7}$$

$$s_b/\chi_b = \alpha_b \lambda(q_b) + \alpha_2 \lambda(q_2), \tag{8}$$

where $s_m = \iota_0$ when $\iota_m = 0$, but we use s_m to emphasize the symmetry.[8]

Recall that $1 + \iota_b = (1+\pi)/\phi_b$. Hence (5)-(6) immediately imply

$$\iota_b = \frac{\alpha_m \chi_m \lambda(q_m) - \alpha_b \chi_b \lambda(q_b) + (\chi_m - \chi_b)\alpha_2 \lambda(q_2)}{1 + \alpha_b \chi_b \lambda(q_b) + \alpha_2 \chi_b \lambda(q_2)}. \tag{9}$$

From (9), even before defining equilibrium, we have this result:

[7] In what follows we assume monetary equilibrium exists. It is standard to show $\alpha_m > 0$ implies it exists iff $\iota_0 < \bar{\iota}_0$, and $\alpha_m = 0$ implies it exists iff $\alpha_2 > 0$, $\chi_b A_b < p^*$ and $\iota_0 < \hat{\iota}_0$. Note $\bar{\iota}_0$ and $\hat{\iota}_0$ may be finite, as with Kalai bargaininig, or infinite, as with Nash bargaining.

[8] Intuitively, the LHS of (7) is the marginal cost of holding cash, adjusted for pledgeability, while the RHS is the benefit: with probability α_m a buyer is in a situation where relaxing the constraint $p_m \leq \bar{p}_m$ is worth $\lambda(q_m)$, and with probability α_2 he is in a situation where relaxing $p_2 \leq \bar{p}_2$ is worth $\lambda(q_2)$. Condition (8) is similar with s_b the marginal cost of bond liquidity.

Proposition 1 *If* $\alpha_b = \alpha_2 = 0$ *or* $\chi_b = 0$ *then* $\iota_b = \alpha_m \chi_m \lambda(q_m) = \iota_0 \geq 0$*; in general we can have* $\iota_b < \iota_0$ *and even* $\iota_b < 0$*. As special cases,* $\alpha_m \lambda(q_m) = \alpha_b \lambda(q_b) \Rightarrow \iota_b < 0$ *iff* $\chi_b > \chi_m$*, and* $\chi_m = \chi_b \Rightarrow \iota_b < 0$ *iff* $\alpha_b \lambda(q_b) > \alpha_m \lambda(q_m)$*.*

Consider the first special case in Proposition 1, where $\alpha_m \lambda(q_m) = \alpha_b \lambda(q_b)$, which includes the case $\alpha_b = \alpha_m = 0$ where if one asset is accepted then so is the other. Then $\iota_b < 0$ if bonds are more pledgeable, $\chi_b > \chi_m$. In the second special case, $\iota_b < 0$ if they are equally pledgeable but bonds have a higher liquidity premium, either because $\alpha_b > \alpha_m$ (they can be used more often) or $\lambda(q_b) > \lambda(q_m)$ (when they can be used they are very valuable). Importantly, we can get negative nominal rates without violating no-arbitrage conditions: while any agent can issue bonds – i.e., borrow in the CM – he cannot exploit $\iota_b < 0$ if his liabilities are not liquid in the DM.[9]

Formally, in Case 1, a stationary monetary equilibrium is a list $(\mathbf{q}, z_m, s_b)$ solving (4), (7) and (8) with $z_m > 0$. Notice the asymmetry between assets: for money, policy sets the spread $s_m = \iota_0$, given $\iota_m = 0$, and the market determines z_m; for bonds, policy sets z_b and the market determines s_b.[10] Also notice that equilibrium is recursive: First use (4) to rewrite (7) as

$$s_m/\chi_m = \alpha_m L\left(\chi_m z_m\right) + \alpha_2 L\left(\chi_m z_m + \chi_b z_b\right), \tag{10}$$

[9] As regards practical relevance, consider *The Economist* (July 14, 2014): "Not all Treasury securities are equal; some are more attractive for repo financing than others. With less liquidity in the market, those desirable Treasuries can be hard to find: some short-term debt can trade on a negative yield because they are so sought after." Or the Swiss National Bank (2013): "The increased importance of these securities is reflected in the trades on the interbank repo market which were concluded at negative repo rates." Our theory does not have all the institutional details, but in an abstract way this is what is going on: agents are willing to accept negative nominal yields on A_b if it has an advantage in some transactions. Relatedly, when cash is subject to theft, nominal rates can be negative without violating no-arbitrage if issuers must incur costs to guarantee their liabilities will be safe, travellers' checks being a leading example (e.g., He et al. 2008). Here liquidity takes over for safety, but they are related: Section 4 shows $\chi_b > \chi_m$ iff bonds are harder to counterfeit than cash.

[10] Saying the market determines s_b is equivalent to saying it determines ϕ_b or ι_b. Also, to be clear, policy determines ι_0 due to the Fisher equation $1 + \iota_0 = (1 + \pi)(1 + r)$, since the central bank controls π (money growth, or inflation, in stationarity equilibrium); market forces are still relevant for r, of course, but here this means $1 + r = 1/\beta$.

where $L(\cdot) \equiv \lambda \circ v^{-1}(\cdot)$. Under standard conditions a solution $z_m > 0$ to (10) exists, is generically unique, and entails $L'(\cdot) < 0$ (e.g., see Gu and Wright 2016). Given z_m, (4) determines $\mathbf{q}$; and finally, (8) determines s_b.

To discuss policy, first note that level increases in A_m reduce ϕ_m to leave $z_m = \phi_m A_m$ the same. This classical neutrality, or quantity theory, result is immediate from (10), which solves for z_m independent of A_m. Next note the usual negative effect of higher nominal interest (or inlfation or money growth) rates on real balances, $\partial z_m/\partial \iota_0 = 1/D_R < 0$, with $D_R \equiv \alpha_m \chi_m^2 L'_m + \alpha_2 \chi_m^2 L'_2 < 0$, where $L'_m = L'(\chi_m z_m)$ and similarly for L'_2 or L'_b. Also,

$$
\begin{aligned}
\frac{\partial q_m}{\partial \iota_0} &= \frac{\chi_m}{v'_m D_R} < 0, \; \frac{\partial q_b}{\partial \iota_0} = 0, \; \frac{\partial q_2}{\partial \iota_0} = \frac{\chi_m}{v'_2 D_R} < 0 \\
\frac{\partial s_b}{\partial \iota_0} &= \frac{\alpha_2 \chi_m \chi_b L'_2}{D_R} > 0, \; \frac{\partial \phi_b}{\partial \iota_0} = \beta \frac{\alpha_2 \chi_m \chi_b L'_2}{D_R} > 0 \\
\frac{\partial \iota_b}{\partial \iota_0} &= \frac{\alpha_m L'_m + \alpha_2 \left[1 - (1+\iota_b)\chi_b/\chi_m\right] L'_2}{(1+s_b)\left(\alpha_m L'_m + \alpha_2 L'_2\right)} \gtrless 0.
\end{aligned}
$$

where $v'_m = v'(q_m)$ and similarly for v'_2 or v'_b. As usual in the paper, these results are for generic parameters; there are special cases where they fail – e.g., $\alpha_2 = 0$ implies $\partial s_b/\partial \iota_0 = \partial \phi_b/\partial \iota_0 = 0$, but any $\alpha_2 > 0$ implies s_b and ϕ_b rise with ι_0 as agents try to substitution out of cash and into bonds. The only ambiguous effect is $\partial \iota_b/\partial \iota_0$, naturally, due to tension between the Fisher and Mundell effects.[11]

Now consider the main policy of interest, OMO's. Suppose A_b rises via central bank bond sales with the cash receipts retired. Given we sterilize future fiscal implications using T, the real effect is the same as raising A_b keeping A_m fixed, because A_m is neutral. Therefore we have

$$
\frac{\partial q_m}{\partial A_b} = -\frac{\alpha_2 \chi_b L'_2}{v'_m D_R} < 0, \; \frac{\partial q_b}{\partial A_b} = \frac{\chi_b}{v'_b} > 0 \text{ and } \frac{\partial q_2}{\partial A_b} = \frac{\alpha_m \chi_b L'_m}{v'_2 D_R} > 0.
$$

[11]The Fisher effect says that, because agents only care about real returns, nominal rates move one-for-one with inflation, but as our results show, this is valid for illiquid and not liquid assets (e.g., it is obviously not valid for currency). The Mundell effect says that, because money and bonds are substitutes in one's portfolio, an increase in ι_0 gives one an incentive to move out of cash and into bonds, which raises bonds' prices and lowers their returns.

Intuitively, higher A_b decreases z_m and q_m because it makes liquidity less scarce in type-2 meetings, so agents economize on cash, but that comes back to haunt them in type-m meetings. Because of this, the net impact of A_b on total DM output is ambiguous. One can check $\partial s_b/\partial A_b < 0$, $\partial \phi_b/\partial A_b < 0$ and $\partial \iota_b/\partial A_b > 0$. This last result, $\partial \iota_b/\partial A_b > 0$, means there is an invertible mapping between the T-bill supply and yield, and so, as mentioned above, the central bank can set A_b to achieve any ι_b within certain bounds.

This completes Case 1. In Case 2, increasing A_b does not affect z_m, q_m or q_2, but increases q_b and ι_b and decreases s_b. For Case 3, with $q_b = q_2 = q^*$, bonds provide no liquidity at the margin, so changing A_b affects nothing of interest. Which case obtains? If bonds are abundant, in the sense that $A_b \geq A_b^* \equiv v(q^*)/\chi_b$, it is Case 3. Otherwise there is an $A_b^o < A_b^*$, that depends on ι_0, such that $A_b^o < A_b < A_b^*$ implies Case 2 and $A_b < A_b^o$ implies Case 1. While we need not take a stand on this, in the sense that the theory can handle all three possibilities, many people argue that in reality there is a scarcity of high-quality liquid assets, which corresponds to Case 1.[12] In any event, we summarize as follows:

Proposition 2 *Consider an OMO that injects A_m. If $A_b < A_b^o$, then q_2 and q_b decrease with ι_b while s_b and q_m increase. If $A_b^o < A_b < A_b^*$ then ι_b and q_b decrease, s_b increases, while q_m and q_2 stay the same. If $A_b > A_b^*$ then these variables all stay the same.*

In Fig. 1, an OMO injecting A_m moves us from right to left, going from Case 3 to Case 2 to Case 1. Again, the real effects are due to decreasing A_b, not increasing A_m, which is neutral. Yet notice something: in Case 1 z_m increases, so ϕ_m does not not go up as much as A_m, giving an *appearance* of stickiness. Heuristically, this is because the demand for z_m increases when bonds get more scarce. One might mistake this for a failure of the quantity equation; that would be wrong,

[12] See, e.g., BIS (2001), Caballero and Krishnamurphy (2006), IMF (2012), Gorton and Ordonez (2013), or Andolfatto and Williamson (2015).

since cash injections by lump-sum transfer keep $\phi_m A_m$ the same. Hence, it is not easy to test neutrality by looking at changes in A_m without conditioning on how they are engineered. One might conjecture that there is a way to resurrect a quantity equation for OMO's by saying that nominal prices are proportional to *some* aggregate of A_m and A_b; that would be wrong, too, because while money and bonds are substitutes, in general, they are not perfect substitutes.

A related idea is that to test the Fisher Equation one should not look at the effect of π on ι_b, because theory actually predicts it is nonmonotone. In examples ι_b increases with π when π is low or high, but decreases when π is in between. This nonmonotonicity arises because inflation tends to raise nominal returns for a given real return, by the Fisher effect, but also tends to lower real returns, by the Mundell effect. To test the Fisher Equation one should not compare π and ι_b, but π and ι_0 where ι_0 is the nominal rate on an illiquid asset, which may be hard to find empirically, as in practice most assets have some degree of liquidity.

This is the New Monetarist anatomy of an OMO.[13] In what follows we check robustness with respect to several details. But, before that, it seems incumbent upon us to acknowledge that one can get similar results by putting assets in utility functions – just take $V(A_m, A_b)$ as a primitive – as if assets were apples. But unlike apples, assets are valued for their liquidity, which is not a primitive like the utility of eating an apple. Now, some assets are somewhat like apples – e.g., apple trees – but if they are also valued for liquidity, that should be modeled explicitly. One reason to do so is that taking $V(A_m, A_b)$ as exogenous imposes no discipline as to when demand is satiated, while here the A_b^o and A_b^* at which $\lambda(q_2)$ and $\lambda(q_b)$ hit 0 are equilibrium outcomes. Another reason is that liquidity depends on policy, but it is hard to know how without deriving actually $V(A_m, A_b)$. For these and other reasons, we say asset valuations should be endogenous.

[13] Some other effects are presented in an Online Appendix, e.g., changes in the α's and χ's, which can be interpreted as financial innovation. While none of these are especially surprising, what might be surprising is that the results are so sharp, with ambiguity only when it makes economic sense, as with the tension between the Fisher and Mundell effects.

3.2 Nominal or Long Bonds

Consider nominal bonds, paying 1 dollar in the next CM. Assume A_b and A_m grow at the same rate, so $B = A_b/A_m$, $z_m = \phi_m A_m$ and $z_b = Bz_m$ are stationary. As in the benchmark model, in Case 1, $\partial z_m/\partial \iota_0 = 1/D_N$ where $D_N < 0$. Also,

$$\frac{\partial q_m}{\partial \iota_0} = \frac{\chi_m}{v_m' D_N} < 0,\ \frac{\partial q_b}{\partial \iota_0} = \frac{B\chi_b}{v_b' D_N} < 0 \text{ and } \frac{\partial q_2}{\partial \iota_0} = \frac{\chi_m + B\chi_b}{v_2' D_N} < 0.$$

The only qualitative difference is that ι_0 now affects q_b. For OMO's that change B, we have $\partial z_m/\partial B = -\alpha_2 \chi_m \chi_b z_m L_2'/D_N < 0$, and

$$\frac{\partial q_m}{\partial B} = -\frac{\alpha_2 C L_2'}{v_m' D_N} < 0,\ \frac{\partial q_b}{\partial B} = \frac{C\left(\alpha_m L_m' + \alpha_2 L_2'\right)}{v_b' D_N} > 0,\ \frac{\partial q_2}{\partial B} = \frac{\alpha_m C L_m'}{v_2' D_N} > 0$$

where $C > 0$. We can also derive effects on ι_b, consider Cases 2 or 3, etc. Since the results are similar to Section 3.1, we revert to real bonds below.[14]

Now consider long-term bonds, say consols paying δ in CM numeraire in perpetuity. Then $z_b = (\phi_b + \delta) A_b$ is endogenous due to the bond's resaleability in the CM. In Case 1, the Euler equations for money and bonds are

$$\iota_0 = \alpha_m \chi_m L(\chi_m z_m) + \alpha_2 \chi_m L(\chi_m z_m + \chi_b z_b) \tag{11}$$

$$r = \frac{\delta(1+r)A_b}{z_b} + \alpha_b \chi_b L(\chi_b z_b) + \alpha_2 \chi_b L(\chi_m z_m + \chi_b z_b). \tag{12}$$

The Online Appendix shows the effects of ι_0 and A_b are qualitatively similar to the benchmark model, and so we revert to short bonds in what follows. However, it is useful to first consider (11)-(12) in (z_m, z_b) space, shown as the EM and EB curves in the upper panels of Fig. 2. One can show they cross uniquely.

For comparison, the bottom panels show the situation with one-period bonds. In the lower right, EB shifts down after an increase in A_m, and since z_m increases, prices rise less than A_m. The upper right, with long bonds, is similar but has

[14]First we clarify a point: An increase in A_m reduces ϕ_m, so if the nominal bond supply is constant $\phi_m A_b$ falls. In this sense money is not neutral, but that's like saying money is not neutral when there are fixed nominal taxes – it's true, but not especially remarkable.

additional multiplier effects.[15] We summarize as follows:

Proposition 3 *With nominal or long bonds the results similar, except $\partial q_b/\partial \iota_0 \neq 0$ with nominal bonds and there are additional multiplier effects with long bonds.*

3.3 Temporary OMO's

In the baseline model OMO's permanently change A_b and A_m. Suppose instead we inject A_m by buying A_b at t, but do not renew the operation at $t+1$, so A_m and A_b revert to their previous paths. Then the rise in A_m at t is not neutral, different from the baseline experiment, because the one-time fall in A_m at $t+1$ is known at t.[16] A one-time OMO in the CM at t changes A_b and A_m in the DM at $t+1$ according to $\phi_{b,t}\Delta A_{b,t+1} = -\phi_{m,t}\Delta A_{m,t+1}$, or

$$\Delta A_b = -(1+\iota_{b,t+1})\phi_{m,t+1}\Delta A_{m,t+1}. \tag{13}$$

This holds for permanent OMO's, too, but now $\phi_{m,t+1}$ is the equilibrium value after ΔA_m has been reversed, so $\partial z_m/\partial A_b = -(1+\iota_b)^{-1}$, and note we do not need to know $\partial \iota_b/\partial A_b$ to evaluate this, due to (13).

Now the effects on DM trade are given by

$$\begin{aligned}
\frac{\partial q_m}{\partial A_b} &= \frac{\chi_m}{\upsilon'_m}\frac{\partial z_m}{\partial A_b} = \frac{-(1+\iota_b)^{-1}\chi_m}{\upsilon'_m} < 0 \\
\frac{\partial q_b}{\partial A_b} &= \frac{\chi_b}{\upsilon'_n} > 0 \\
\frac{\partial q_2}{\partial A_b} &= \frac{\chi_m \partial z_m/\partial A_b + \chi_b}{\upsilon'_2} = \frac{-(1+\iota_b)^{-1}\chi_m + \chi_b}{\upsilon'_2}.
\end{aligned}$$

[15] By multiplier effects we mean this: After A_b falls, ϕ_b rises because bonds are more scarce, which partially offsets the impact, but on net z_b falls. As with short bonds, lower z_b raises z_m as agents try to substitute across assets, but now higher z_m makes lower z_b not as bad, so the demand for and price of bonds fall, and z_b falls further. That leads to an additional rise in z_m, an additional fall in z_b, etc.

[16] This is like Gu et al. (2017), where it is known at t that A_m will change at $t' > t$, implying a complicated transition path where neutrality applies only in the long run. Here $t' = t+1$, so the effects last just one period. The intuition is that buyers have more cash at t, but prices do not rise to neutralize this because sellers evaluate it using ϕ_{t+1}, after A_m goes back down.

Thus, injecting cash with a temporary OMO increases q_m and decreases q_b, while the effect on q_2 is ambiguous. If $\alpha_m = \alpha_b = 0$, e.g., then $\iota_b = 0$ and $\partial q_2/\partial A_b > 0$ iff $\chi_b > \chi_m$. Alternatively, if $\alpha_m, \alpha_b > 0$ and $\chi_m = \chi_b = 1$ then one can check $\partial q_2/\partial A_b < 0$ if $\alpha_m \lambda(q_m) < \alpha_b \lambda(q_b)$, which is true when A_b is small. In any case, the model nicely accommodates temporary OMO's, where money is not neutral due to the 'announcement effect' of reversing the change in A_m next period. Given this is understood, we revert to permanent OMO's in what follows.

3.4 A Liquidity Trap

As Keynes (1936) put it: "after the rate of interest has fallen to a certain level, liquidity-preference may become virtually absolute in the sense that almost everyone prefers cash to holding a debt which yields so low a rate of interest. In this event the monetary authority would have lost effective control over the rate of interest." This is a *liquidity trap*. It does not correspond to $A_b \geq A_b^*$ in Fig. 1, where ι_b is at its upper bound – that's a *liquidity glut*. We now describe a trap, where ι_b and output are at their lower bounds.

For this exercise we add heterogeneity: type-m buyers use only money – i.e., for them $\alpha_m > 0 = \alpha_b = \alpha_2$ – while type-2 buyers use money and bonds as perfect substitutes – i.e., for them $\alpha_2 > 0 = \alpha_m = \alpha_b$. One can think of type-$m$ as households that use only cash, and type-2 as financial institutions that can use cash or bonds. Because of type-m, money will be valued even when A_b is big, which is not true with only type-2. Now, if type-2 choose $\hat{z}_m, \hat{z}_b > 0$, then

$$1 + \pi = \beta \left[1 + \alpha_2 \chi_m \lambda(q_2)\right] \text{ and } \phi_b = \beta \left[1 + \alpha_2 \chi_b \lambda(q_2)\right]. \tag{14}$$

Moreover, given $\alpha_m = \alpha_b = 0$ for type-2, (9) implies as a special case that the lower bound for ι_b is $\underline{\iota_b} = \iota_0 \left(\chi_m - \chi_b\right) / \left(\chi_m + \iota_0 \chi_b\right)$. Thus, when $\hat{z}_m, \hat{z}_b > 0$ for type-2, ι_b is at its lower bound and independent of A_b.

In Fig. 3, if $A_b \geq A_b^*$ then type-2 hold no cash since they can get q^* with bonds, so a marginal change in A_b has no effect. If $\bar{A}_b < A_b < A_b^*$ then type-2 do not get q^* but get close enough that it is not worth topping up bond liquidity with cash, so changes in A_b matter. If $A_b < \bar{A}_b$ then type-2 hold bonds plus cash, but their total liquidity is independent of A_b since, at the margin, it's money that matters. This is a liquidity trap: changes in A_b induce changes in real balances to leave total liquidity the same, with ι_b and $\mathbf{q}$ stuck at their lower bounds. A general lesson is this: asset swaps that raise A_m and lower A_b do not increase liquidity, but are either neutral or make things worse; the way out of the trap is to raise A_b.[17]

Proposition 4 *For $A_b < \bar{A}_b$, changes in A_b crowd out z_m to leave total liquidity, ι_b and $\mathbf{q}$ the same. For $A_b > \bar{A}_b$, changes in A_b matter until we reach A_b^*.*

4 Endogenous Liquidity

We now endogenize α_j and χ_j using information frictions – i.e., *recognizability* – a notion going back at least to Law, Jevons and Menger (see the surveys in fn. 1). One interpretation concerns counterfeiting, which is relevant even if it does not occur in equilibrium, as a *threat* of counterfeiting still impinges on liquidity. With a broad view of money, this may include bad checks or hacked payment cards.

4.1 Acceptability

As in Lester et al. (2012), suppose some sellers cannot distinguish high- from low-quality versions of certain assets, and low-quality assets can be produced on the spot for free. We assume low quality assets have 0 value (Nosal and Wallace 2007), although this can be relaxed (Li and Rocheteau 2011). Then sellers unable to recognize the quality of an asset reject it outright – if they were to accept it buyers would just hand over worthless paper. Here we set $\chi_j = 1$ and use Kalai bargaining,

[17] Note that the above demonstration concerns the special but (for some applications) realistic case $\alpha_b = 0$; if $\alpha_b > 0$ then lowering A_b in a liquidity trap is even worse.

for simplicity, and assume all sellers recognize A_m in the DM, but to recognize A_b they must pay an individual-specific cost with $F(\kappa)$ denoting its CDF.

Let n_2 be the measure of sellers that pay κ and hence accept bonds. The marginal seller is one with $\kappa = \Delta$, where

$$\Delta = \alpha (1-\theta) [u(q_2) - c(q_2) - u(q_m) + c(q_m)] \tag{15}$$

is the increase in profit from being informed. Equilibrium solves $n_2 = F(\Delta)$, with $\Delta = \Delta(z_m)$ because $\mathbf{q}$ depends on z_m. In Fig. 4, $n_2 = F \circ \Delta(z_m)$ defines a curve in (n_2, z_m) space called IA for *information acquisition*. It slopes down and shifts right with A_b. Also, the Euler equation for z_m defines a curve called RB for *real balances*. It slopes down, and shifts down with A_b or ι_0. Equilibrium is where the curves cross. As Fig. 4 shows, RB can cut IA from below or above.

In equilibrium $n_2 = F \circ \Delta \circ z_m (n_2) \equiv \Upsilon (n_2)$.[18] We can have $n_2 = 0$, $n_2 = 1$ or $0 < n_2 < 1$, and it is easy to check that it iis easy to get multiplicity, as one should expect when payments methods are endogenous (Kiyotaki and Wright 1989). Intuitively, higher n_2 decreases z_m, since it makes buyers less likely to encounter sellers that take only cash; then lower z_m raises the profitability of recognizing bonds; and that increases the measure of sellers investing in information.

Despite multiplicity, the model has sharp predictions conditional on the type of equilibrium. Using '$x \backsimeq y$' to mean 'x and y take the same sign,' we have

$$\frac{\partial z_m}{\partial A_b} = -\frac{\alpha v'_m n \left[{}_2\lambda'{}_2 + \alpha (1-\theta) F' (u'_2 - c'{}_2) (\lambda_2 - \lambda_m)\right]}{n D_\alpha} \backsimeq D_\alpha$$
$$\frac{\partial q_m}{\partial A_b} = -\frac{\alpha \left[n_2 \lambda'{}_2 + \alpha (1-\theta) F' (u'{}_2 - c'_2) (\lambda_2 - \lambda_m)\right]}{n D_\alpha} \backsimeq D_\alpha$$
$$\frac{\partial q_2}{\partial A_b} = \frac{\alpha \left[(n-n_2) \lambda'{}_m + \alpha (1-\theta) F' (u'{}_m - c'{}_m) (\lambda_2 - \lambda_m)\right]}{n D_\alpha} \backsimeq -D_\alpha$$
$$\frac{\partial n_2}{\partial A_b} = \frac{\alpha \alpha (1-\theta) F' \left[(n-n_2) (u'{}_2 - c'{}_2) \lambda'_m + n (u'_m - c'{}_m) \lambda'_2\right]}{n D_\alpha} \backsimeq -D_\alpha.$$

[18] One can show $\Upsilon : [0,1] \to [0,1]$ is increasing (this is where Kalai bargaining is helpful). Hence, existence follows by Tarski's theorem even if $F(\cdot)$ is not continuous, as when there is a mass of sellers at the same κ. Having Υ increasing also makes multiplicity natural.

where $D_\alpha = \alpha^2 (1-\theta)(\lambda_2 - \lambda_m)(c'_m u'_2 - c'_2 u'_m) F' + \alpha n_m \lambda'_m v'_2 + \alpha n_2 \lambda'_2 v'_2$. Note $D_\alpha < 0$ iff RB cuts IA from below, so the results alternate across equilibria.

In Fig. 4, if $D_\alpha < 0$, as at point a, an OMO that injects currency shifts RB up and IA left, increasing z_m and decreasing n_2; if $D_\alpha > 0$ the effects are reversed. There is no compelling reason to select one type of equilibrium, and indeed, it is not hard to have a unique equilibrium of one type or the other. So to make policy predictions, we need to know the parameters *plus* the type of equilibrium – difficult in practice, but inescapable in theory when liquidity is endogenous.

Proposition 5 *With endogenous α's, monetary equilibrium is not generally unique. The effects of policy depend on the configuration in Fig. 4, but given that, they are precisely determined.*

4.2 Pledgeability

Now as in Rocheteau (2011) or Li et al. (2012), assume that to produce low-quality assets agents must pay costs proportional to their values: for money the cost is $\gamma_m \phi_m$; and for bonds it is γ_b. Also, all sellers are uninformed, and fraudulent assets are produced in the CM before visiting the DM. As is standard in signaling models, here we use bargaining with $\theta = 1$, and as a first pass let $\alpha_2 > 0 = \alpha_m = \alpha_b$. Hence, for now, there is only one type of meeting, but we still must distinguish payments made in money and bonds, say d_m and d_b, for incentive reasons.

The incentive conditions for d_m and d_b are:

$$(\phi_{m,-1} - \beta\phi_m)\, a_m + \beta\alpha_2\phi_m d_m \le \gamma_m \phi_m a_m \tag{16}$$

$$(\phi_{b,-1} - \beta)\, a_b + \beta\alpha_2 d_b \le \gamma_b a_b. \tag{17}$$

The intuition is clear: The RHS of (16) is the cost of counterfeiting a_m; the LHS is the cost of acquiring a_m genuine dollars $(\phi_{m,-1} - \beta\phi_m)a_m$, plus the cost of trading away d_m with probability α_2. Sellers can believe a_m is genuine since, after all, who would spend \$20 to make a phony \$10 bill? DM trade now has multiple constraints: bargaining implies $c(q_2) = \phi_m d_m + d_b$; feasibility implies $\phi_m d_m \leq z_m$ and $d_b \leq z_b$; and (16)-(17) imply $d_j \leq \chi_j z_j$ where

$$\chi_m = \frac{\gamma_m - \beta\iota_0}{\beta\alpha_2} \text{ and } \chi_b = \frac{\gamma_b - \beta s_b}{\beta\alpha_2}. \tag{18}$$

The outcome, or regime, depends on which constraint binds. Consider first the regime $\chi_m, \chi_b \in (0,1)$. Then (7)-(8) reduce to

$$\beta\iota_0 = (\gamma_m - \beta\iota_0)\,\lambda(q_2) \text{ and } \beta s_b = (\gamma_b - \beta s_b)\,\lambda(q_2). \tag{19}$$

The first condition yields q_2, then $s_b = \iota_0\gamma_b/\gamma_m$ and $\chi_b = \gamma_b\left(\gamma_m - \beta\iota_0\right)/\alpha_2\beta\gamma_m$. This regime is consistent with equilibrium iff $\gamma_m > \beta\iota_0$, $\gamma_m < \beta\left(\iota_0 + \alpha_2\right)$ and $\gamma_b < \beta\alpha_2\gamma_m/\left(\gamma_m - \beta\iota_0\right)$.[19] Similarly, consider next $\chi_m = 1$ and $\chi_b \in [0,1)$. Then $\iota_0 = \alpha_2\lambda(q_2)$ and $s_b = (\gamma_b - \beta s_b)\,\lambda(q_2)$, and this is consistent with equilibrium iff $\gamma_m > \beta\left(\iota_0 + \alpha_2\right) > \gamma_b$. Other regimes are similar, and it is not hard to show where each is an equilibrium in parameter space (Rocheteau et al. 2014).

The above results are easy because A_m and A_b are perfect substitutes, but then OMO's are irrelevant. To change that, let $\alpha_m > 0$, and consider the natural regime where $\chi_m = 1$ and $\chi_b \in (0,1)$. The equilibrium conditions reduce to

$$\iota_0 = \alpha_m L\left(z_m\right) + \alpha_2 L\left(z_m + \chi_b z_b\right) \tag{20}$$

$$\gamma_b/\beta = \alpha_2\chi_b\left[1 + L\left(z_m + \chi_b z_b\right)\right], \tag{21}$$

defining two curves in (χ_b, z_m) space labeled RB and IC in Fig. 5. While RB slopes down, IC can be nonmonotone, since its slope s proportional to $\Theta \equiv 1 + L_2 + \chi_b z_b L_2'$. It is not hard to get multiplicity – intuitively, if χ_b is low then q_2 is low and s_b is high, which gives a big incentive to create fraudulent bonds, and so χ_b is low.

The results now depend on $D_\chi = (1 + L_2)\left(\alpha_m L'{}_m + \alpha_2 L_2'\right) + \alpha_m\chi_b z_b L_2' L_m'$.

[19] In this regime $\iota_b = (\gamma_m - \gamma_b)\,\iota_0/\left(\gamma_m + \gamma_b\iota_0\right)$, and so $\iota_0 < 0$ iff A_m is easier to counterfeit than A_b. This goes a level deeper than Proposition 1, and is arguably realistic.

Notice $\Theta > 0$ implies $D_\chi < 0$, so there are three relevant configurations: (i) $\Theta > 0$ implies IC is upward sloping and cuts RB from below, as at point a in the left panel of Fig. 5; $\Theta < 0$ implies IC is downward sloping and either (ii) cuts RB from below, as at e in the right panel, or (iii) cuts it from above, as at point c or g. An increase in ι_0 shifts RB down. This implies $\partial z_m/\partial \iota_0 = \Theta/D_\chi > 0$ when $\Theta < 0$, as in the move from e to f, or can imply $\partial z_m/\partial \iota_0 < 0$, as in the other cases. Similarly, $\partial \chi_b/\partial \iota_0 \simeq D_\chi$ depends on the configuration of RB and IC.

In terms of OMO's, we have these results:

$$\begin{aligned}
\frac{\partial z_m}{\partial A_b} &= -\frac{\alpha_2^2 \chi_b (1+\lambda_2) \upsilon_m' \lambda_2'}{D_o} \simeq D_o, \ \frac{\partial q_m}{\partial A_b} = -\frac{\alpha_2^2 \chi_b (1+\lambda_2) \lambda_2'}{D_o} \simeq D_o \\
\frac{\partial q_2}{\partial A_b} &= \frac{\alpha_m \alpha_2 \chi_b (1+\lambda_2) \lambda_m'}{D_o} \simeq -D_o, \ \frac{\partial \chi_b}{\partial A_b} = -\frac{\alpha_m \alpha_2 \chi_b \lambda_m' \lambda_2'}{D_o} \simeq -D_o.
\end{aligned}$$

Also, $\partial s_b/\partial A_b \simeq D_o$, $\partial \phi_b/\partial A_b \simeq D_o$ and $\partial \iota_b/\partial A_b \simeq -D_o$. So OMO's affect pledgeability endogenously, but the sign depends on the equilibrium. As in Section 4.1, this may be unfortunate in practice, but it's hard to avoid in theory.

Proposition 6 *With endogenous χ's, monetary equilibrium is not generally unique. The effects of policy depend on the configuration of IC and RB in Fig. 5, but given that, they are precisely determined.*

5 Extensions

5.1 Directed Search

Directed search allows buyers to choose to trade with sellers that accept different payment methods.[20] Suppose there are two types of sellers: a measure n_m accept A_m; a measure n_2 accept A_m and A_b. For now, fix n_m, $n_2 = 1 - n_m$ and $\chi_j = 1$. Define submarket j as the set of type-j sellers and the set of buyers searching for

[20] The presentation here is brief, but Wright et al. (2017) provide a survey of the relevant directed search theory. Note that in many models directed search is only interesting if prices are posted, rather than bargained; that is not true here since buyers can direct their search to sellers accepting different payment methods, not only to those posting different prices.

them, with measure μ_j, where *SM* denotes the submarket where A_m is accepted and *S2* the one where A_m and A_b are accepted. Assume $\mu_m + \mu_2 = \mu$ is not too large, so all buyers participate, and let $\sigma_j = n_j/\mu_j$. As usual, the probability a buyer meets a seller in submarket j is $\alpha(\sigma_j)$, and the probability a seller meets a buyer is $\alpha(\sigma_j)/\sigma_j$, with $\alpha(n)$ satisfying standard conditions. We first consider (Kalai) bargaining; then consider posting.

Buyers going to *SM* take $\hat{z}_m^m > 0$ and $\hat{z}_b^m = 0$, and those going to *S2* take $\hat{z}_b^2 = A_b/\mu_2 > 0$ and $\hat{z}_m^2 \geq 0$. Then $\hat{z}_m^m = v(q_m)$ and $\hat{z}_b^2 + \hat{z}_m^2 \geq v(q_2)$, where the latter holds with equality iff $q_2 < q^*$. Given $q_2 < q^*$, we can have $\hat{z}_m^2 = 0$ or $\hat{z}_m^2 > 0$, with $\iota_0 > s_b$ in the former case and $\iota_0 = s_b$ in the latter. Since buyers now meet only one type of seller, as opposed to meeting a type at random,

$$\iota_0 = \alpha(\sigma_m)\lambda(q_m) \text{ and } s_b = \alpha(\sigma_2)\lambda(q_2). \tag{22}$$

If *SM* and *S2* are both open, buyers must be indifferent between them,

$$\alpha(\sigma_m)\left[u(q_m) - v(q_m)\right] - \iota_0 z_m^m = \alpha(\sigma_2)\left[u(q_2) - v(q_2)\right] - s_b z_b^2. \tag{23}$$

Therefore, since the total measure of buyers is μ,

$$n_m/\sigma_m + n_2/\sigma_2 = \mu. \tag{24}$$

In equilibrium (q_j, σ_j, s_b) solves (22)-(24). Again there are three regimes: bonds are scarce, so type-2 carry cash; bonds are less scarce, so type-2 carry no cash even though $q_2 < q^*$; bonds are plentiful, so type-2 need no cash since $q_2 = q^*$. Fig. 6 shows the results, where again OMO's are neutral when A_b is below $\bar{A}_b$ or above A_b^*, but not in between. A difference from random search that we consider important is this: now σ_2 and σ_m depend on A_b, since tightness is endogenous, so if A_b increases buyers in *SM* get better terms and a higher probability of trade, even though A_b is not actually used in *SM*. Otherwise, the results are similar.

Now supposes sellers post the terms of trade. As is standard, the same results emerge if market makers set up submarkets with posted terms to attract traders, who then meet bilaterally, as above. Using this solution method, we have market makers in the CM post $(q_j, \hat{z}^j_m, \hat{z}^j_b, \sigma_j)$ for the next DM. The problem for a market maker considering a submarket of type SM is

$$U^b(\iota_0, \Pi_m) = \max_{q,\hat{z}_m,\sigma} \{\alpha(\sigma)[u(q) - \hat{z}_m] - \iota_0 \hat{z}_m\} \text{ st } \frac{\alpha(\sigma)}{\sigma}[\hat{z}_m - c(q)] = \Pi_m, \quad (25)$$

which maximizes buyers' surplus given sellers get Π_m, which is determined in equilibrium but taken as given in this problem. Generically (25) has a unique solution, so all type-m submarkets are the same.

To solve (25), eliminate $\hat{z}_m$ and take FOC's wrt q_m and σ_m to get

$$\frac{u'(q)}{c'(q)} - 1 = \frac{\iota_0}{\alpha(\sigma)} \quad (26)$$

$$\alpha'(\sigma)[u(q) - c(q)] = \Pi_m \left\{1 + \frac{\iota_0[1 - \varepsilon(\sigma)]}{\alpha(\sigma)}\right\}, \quad (27)$$

where $\varepsilon(\sigma) \equiv \sigma\alpha'(\sigma)/\alpha(\sigma) \in (0, 1)$. *S2* is similar, with ι_0 replaced by s_b and Π_m by Π_2. Here, to ease the exposition, consider the matching function $\alpha(\sigma) = \min\{1, \sigma\}$ (the Online Appendix proceeds more generally). Without going into detail, the outcome looks like Fig. 3 instead of Fig. 6, due to the special matching technology, but again agents choose which submarket to visit and which assets to bring. As in Section 4.1, we can also make sellers pay a cost κ to recognize bonds and participate in S2. Importantly, all these choices depend on policy, which is not true in models with exogenous market segmentation or CIA constraints.

5.2 Interest on Money or Reserves

We now revert to random search and bargaining for an application suggested by the editor, where money pays interest, in dollars, at rate ι_m. The simplest case follows Andolfatto (2010) or Bajaj et al. (2017), but later we generalize this by splitting money into currency plus reserves. If $z_m = \phi_m A_m(1+\iota_m)$, $z_b = A_b$ and $z = z_m + z_b$, the CM problem is

$$W(z) = \max_{x,\ell,\hat{z}_m,\hat{z}_b} \{U(x) - \ell + \beta V(\hat{z}_m, \hat{z}_b)\} \text{ st } x = z + \ell + T - \frac{1+\pi}{1+\iota_m}\hat{z}_m - \phi_b \hat{z}_b,$$

where the cost in numeraire of having $\hat{z}_m$ in the next DM is $(1+\pi)/(1+\iota_m)$. The key FOC's are $(1+\pi)/(1+\iota_m) = \beta V_1(\hat{z}_m, \hat{z}_b)$ and $\phi_b = \beta V_2(\hat{z}, \hat{z}_b)$. The DM value function and the bargaining conditions are the same Section 3.1.

In the case where $p_j \leq \bar{p}_j$ always binds, the usual methods lead to

$$s_m/\chi_m = \alpha_m \lambda(q_m) + \alpha_2 \lambda(q_2) \tag{28}$$

$$s_b/\chi_b = \alpha_b \lambda(q_b) + \alpha_2 \lambda(q_2), \tag{29}$$

which are like (7)-(8), except now $s_m = (\iota_0 - \iota_m)/(1+\iota_m)$ is determined by two policy instruments, ι_0 and ι_m, while again $s_b = (\iota_0 - \iota_b)/(1+\iota_b)$ is determined by the market given that policy sets A_b. As in the baseline model, (29) reduces to

$$s_m/\chi_m = \alpha_m L(\chi_m z_m) + \alpha_2 L(\chi_m z_m + \chi_b z_b). \tag{30}$$

As usual, this determines z_m; bargaining determines $\mathbf{q}$; and (29) determines s_b.

Note that (30), and hence all real variables, are again independent of A_m, which merely determines the nominal price level from $z_m = \phi_m A_m(1+\iota_m)$. This is classical neutrality generalized to $\iota_m \neq 0$. However, ϕ_m depends on the policy variable ι_m, and a change in ι_m holding ι_0 fixed has real effects because it changes s_m; still, a change in ι_m with an offsetting change in ι_0 that keeps s_m the same is neutral. Moreover, as A_m is neutral, again OMO's are effectively described by changing A_b, and the results are identical to the baseline model. This may be counterintuitive because interest-bearing money seems similar to bonds. The difference is this: for A_m, the real supply z_m is endogenous and while ι_m is exogenous; for A_b, the real supply z_b is exogenous and ι_b is endogenous.

Now decompose the monetary base A_m into currency plus reserves, where the latter pays interest ι_r and the former pays interest ι_c, which is typically 0 but we do not need that yet. In the CM, the central bank stands ready to convert currency into reserves, and vice versa, at par, so they have the same price ϕ_m. There are three spreads, $s_c = (\iota_0 - \iota_c) / (1 + \iota_c)$, $s_r = (\iota_0 - \iota_r) / (1 + \iota_r)$ and $s_b = (\iota_0 - \iota_b) / (1 + \iota_b)$, where the first two are determined by policy, while the third is determined by the market, as in the baseline model. Let α_a be the probability of meeting a seller that takes only asset a, let $\alpha_{aa'}$ be probability of meeting one that takes a and a', and let α_3 be the probability of meeting one that takes all assets. If the constraints always bind, $\mathbf{q}$ solves

$$v(q_c) = \chi_c z_c,\ v(q_{cr}) = \chi_c z_c + \chi_r z_r,\ ...\ v(q_3) = \chi_c z_c + \chi_r z_r + + \chi_b z_b, \tag{31}$$

We again emulate the approach from the baseline to get the Euler equations,

$$s_c/\chi_c = \alpha_c \lambda(q_c) + \alpha_{cb}\lambda(q_{cb}) + \alpha_{cr}\lambda(q_{cr}) + \alpha_3\lambda(q_3) \tag{32}$$

$$s_r/\chi_r = \alpha_r \lambda(q_r) + \alpha_{rc}\lambda(q_{rc}) + \alpha_{rb}\lambda(q_{rb}) + \alpha_3\lambda(q_3) \tag{33}$$

$$s_b/\chi_b = \alpha_b \lambda(q_b) + \alpha_{cb}\lambda(q_{cb}) + \alpha_{rb}\lambda(q_{rb}) + \alpha_3\lambda(q_3). \tag{34}$$

Equilibrium is a list $(\mathbf{q}, z_c, z_r, s_b)$ solving (31)-(34). The usual method implies

$$s_c/\chi_c = \alpha_c L_c + \alpha_{cb} L_{cb} + \alpha_{cr} L_{cr} + \alpha_3 L_3 \tag{35}$$

$$s_r/\chi_r = \alpha_r L_r + \alpha_{rc} L_{rc} + \alpha_{rb} L_{rb} + \alpha_3 L_3, \tag{36}$$

where $L_c = L(\chi_c z_c)$, $L_{cb} = L(\chi_c z_c + \chi_b z_b)$, etc. While we now have two endogenous balances (z_c, z_r), instead of the single z_m, the economics is similar: (35)-(36) jointly determine (z_c, z_r); then (31) yields $\mathbf{q}$; and (34) yields s_b. Notice (z_c, z_r) is independent of $A_{\mathbf{m}}$, which determines only the nominal price level by $\phi_m A_m = z_c / (1 + \iota_c) + z_r / (1 + \iota_r)$, which is another generalization of classical neutrality. Again, OMO's are the same as changing A_b.

The Online Appendix gives more detail, but let us highlight a few results. From (35)-(36) we have

$$\frac{\partial z_c}{\partial A_b} = \frac{-\chi_b}{\chi_c D_g}[\alpha_{cr}L'_{cr}(\alpha_{cb}L'_{cb} - \alpha_{rb}L'_{rb}) + \alpha_{cb}L'_{cb}(\alpha_r L'_r + \alpha_3 L'_3 + \alpha_{rb}L'_{rb}) + \alpha_r\alpha_3 L'_r L'_3]$$
$$\frac{\partial z_r}{\partial A_b} = \frac{-\chi_b}{\chi_r D_g}[\alpha_{cr}L'_{cr}(\alpha_{rb}L'_{rb} - \alpha_{cb}L'_{cb}) + \alpha_{rb}L'_{rb}(\alpha_c L'_c + \alpha_3 L'_3 + \alpha_{cb}L'_{cb}) + \alpha_c\alpha_3 L'_c L'_3]$$

where $D_g > 0$. These are unambiguous iff we add some restrictions. If $\alpha_{rb}L'_{rb} = 0$, e.g., which means that no one accepts reserves and bonds but not currency, then $\partial z_c/\partial A_b < 0$. Similarly, if $\alpha_{cb}L'_{cb} = 0$, e.g., then $\partial z_r/\partial A_b < 0$. Hence, under reasonable restrictions, increasing A_b lowers the endogenous liquidity embodied in both currency and reserves.[21] In general, if $\chi_c \geq \chi_r$, e.g., in the natural specification $\chi_c = 1$, increasing A_b must lower at least one of them, since then

$$\begin{aligned}\frac{\partial z_c}{\partial A_b} + \frac{\partial z_r}{\partial A_b} \leq & \frac{-\chi_b}{\chi_c D_g}[\alpha_{cb}L'_{cb}(\alpha_r L'_r + \alpha_3 L'_3 + \alpha_{rb}L'_{rb}) + \alpha_r\alpha_3 L'_r L'_3] \\ & + \frac{-\chi_b}{\chi_c D_g}[\alpha_{rb}L'_{rb}(\alpha_c L'_c + \alpha_3 L'_3 + \alpha_{cb}L'_{cb}) + \alpha_c\alpha_3 L'_c L'_3] < 0.\end{aligned}$$

As regards the effects of A_b on $\mathbf{q}$, we have $\partial q_c/\partial A_b \simeq \partial z_c/\partial A_b$ and $\partial q_r/\partial A_b \simeq \partial z_r/\partial A_b$. For the rest, obviously $\partial q_b/\partial A_b > 0$, and the Online Appendix shows

$$\frac{\partial q_{cr}}{\partial A_b} < 0, \frac{\partial q_{cb}}{\partial A_b} > 0,\ \frac{\partial q_{rb}}{\partial A_b} > 0,\ \frac{\partial q_3}{\partial A_{b\,3}} \gtrless 0.$$

Only the last is ambiguous, but the above restrictions that deliver $\partial z_c/\partial A_b < 0$ and $\partial z_r/\partial A_b < 0$ also imply $\partial q_3/\partial A_b > 0$.

The Online Appendix also derives the impact of ι_r,

$$\frac{\partial z_c}{\partial \iota_r} = \frac{(1+\iota_0)(\alpha_{cr}L'_{cr} + \alpha_3 L'_3)}{\chi_c\chi_r D_g(1+\iota_r)^2} < 0$$
$$\frac{\partial z_r}{\partial \iota_r} = -\frac{(1+\iota_0)(\alpha_{cr}L'_{cr} + \alpha_{cb}L'_{cb} + \alpha_c L'_c + \alpha_3 L'_3)}{\chi_r^2 D_g(1+\iota_r)^2} > 0.$$

[21] A special case of these restrictions is this: for each asset a, there is a type-a meeting where only it is accepted, plus type-3 meetings, and no other meetings.

Hence, higher interest on reserves naturally implies the monetary base shifts to more z_r and less z_c. The effects on $\mathbf{q}$ are:

$$\frac{\partial q_c}{\partial \iota_r} < 0,\ \frac{\partial q_r}{\partial \iota_r} > 0,\ \frac{\partial q_b}{\partial \iota_r} = 0,\ \frac{\partial q_{cr}}{\partial \iota_r} > 0,\ \frac{\partial q_{cb}}{\partial \iota_r} < 0,\ \frac{\partial q_{rb}}{\partial \iota_r} > 0,\ \frac{\partial q_3}{\partial \iota_r} > 0.$$

Remarkably, these are all unambiguous. However, since higher ι_r raises q in some trades and lowers q in others, the net effect is unclear.[22]

One can also expand the set of assets to include physical capital k, which is relevant because some central banks these days are holding stocks, corporate bonds or mortgage-backed securities, including the ECB and Bank of Japan. Each unit of k accumulated in the CM yields $F(k)$ in numeraire next CM, and the rental price of capital is $R_k = F'(k)$. Assume for the sake of illustration that $F'(k)k$ is increasing in k, and it fully depreciates each period. Then $z_k = R_k(k - k^c)$ where k^c is capital held by the central bank. The real return on capital is $r_k = R_k - 1$, and the spread is $s_k = (\iota_0 - r_k)/(1 + r_k)$. Under reasonable assumptions, a central bank purchase of k using cash increases k, s_k, s_b, q_r and q_m while it decreases r_k, q_k and q_4. This is only meant to show the flexibility of the approach, but future work could push it further.

6 Conclusion

This project has analyzed monetary policy in economies with frictions where assets facilitate trade. The main finding is that injections of currency by open market operations are generally quite different from lump sum transfers. There are many predictions, some of which are consistent with conventional wisdom, although perhaps for different reasons – e.g., injecting cash by OMO lowers ι_b, but due to lower A_b, not higher A_m. Other predictions contrast with conventional wisdom – e.g.,

[22] There are exceptions to the above results for extreme parameters. Suppose, e.g., $\chi_r = \chi_b$ and if z_r is accepted then so is z_b. Then they are perfect substitutes, so if both are held then $s_r = s_b$. In this special case, an OMO that swaps bonds for reserves is neutral – it changes the composition but not the value of $\chi_a (z_r + z_b)$ – although ι_r still has real effects.

injecting cash by OMO is not a good idea in a liquidity trap. Many results are robust – e.g., ι_b does not move one-for-one with π, and is in fact nonmonotone, due to the Fisher and Mundell effects. Different specifications were considered, including random vs directed search, bargaining vs posting, short vs long bonds, and nominal vs real bonds. We used information frictions to endogenize liquidity, which led to interesting multiplicities from a policy perspective. We also analyzed interest on currency or reserves. While more can be done, this is a useful step in reducing the gap between monetary theory and policy.

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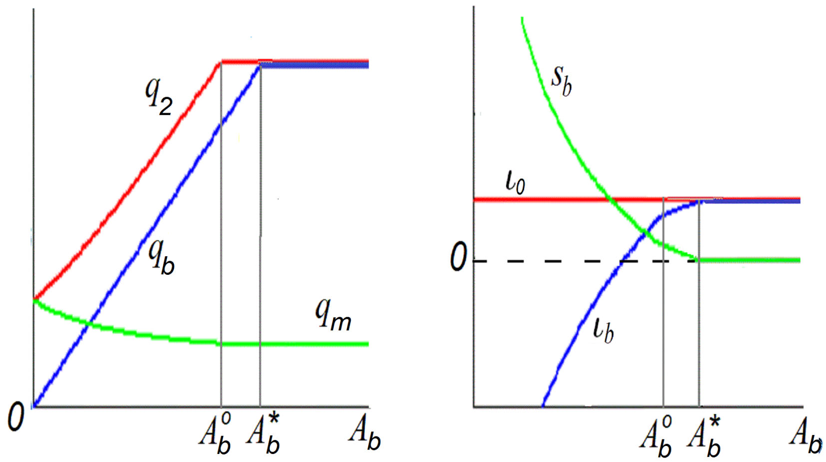

Figure 1: The effects on increasing A_b.

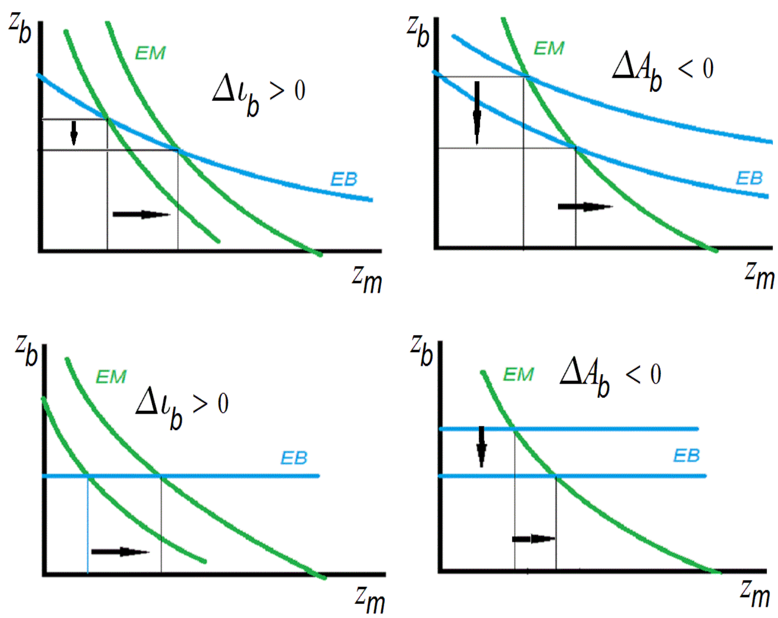

Figure 2: Increase in ι_b and decrease in A_b with long and short bonds,

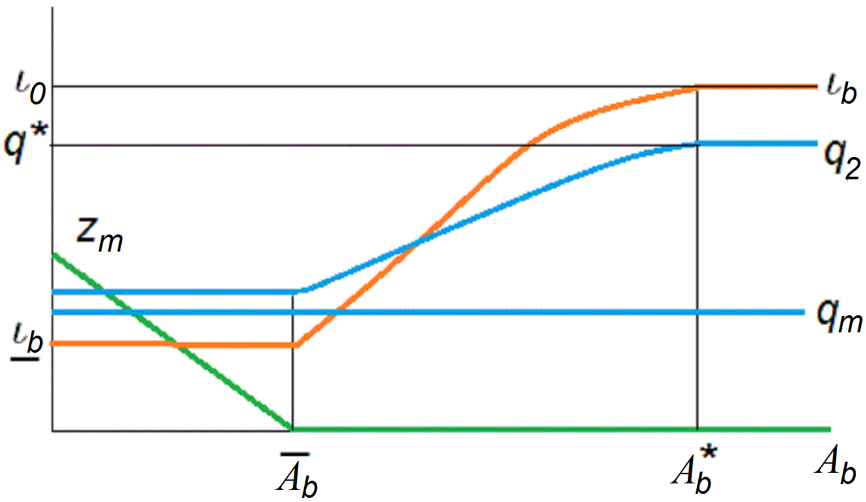

Figure 3: The effects of A_b with a liquidity trap in $(0, \bar{A}_b)$.

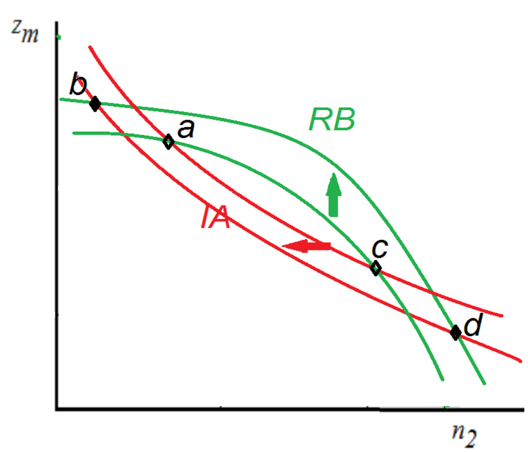

Figure 4: Different configurations with endogenous α's.

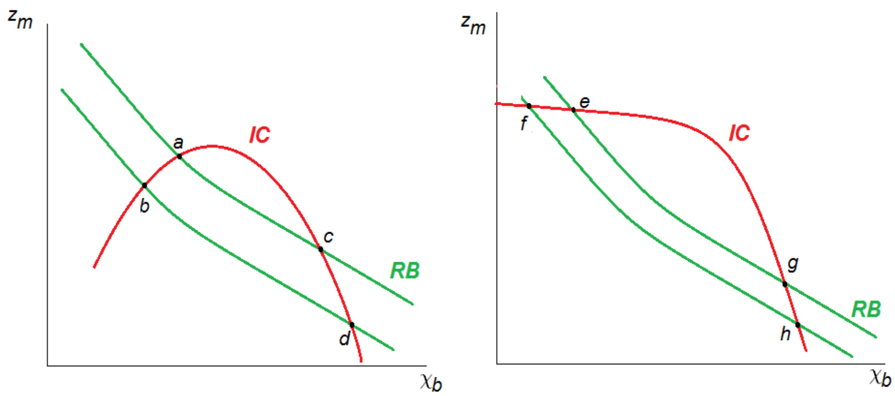

Figure 5: Different configurations with endogenous χ's.

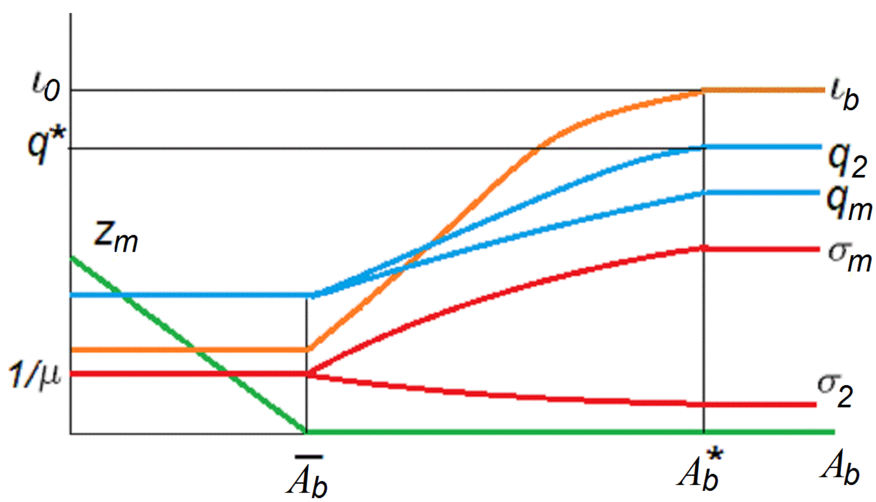

Figure 6: The effects of A_b with directed search.

Supplemental Appendix for Open Market Operations

by Guillaume Rocheteau, Randall Wright and Sylvia Xiaolin Xiao

Intended for Online Publication

A: Additional Effects in Baseline Model

Here we consider the effects of the α's and χ's in the baseline model, in Case 1, where the constraints bind in all meetings (Cases 2 and 3 are similar but easier). First, to reduce notation, let $D_a = \alpha_m L'_m + \alpha_2 L'_2$. The effects of acceptability on $\mathbf{q}$ are

$$\begin{aligned}
\frac{\partial q_m}{\partial \alpha_m} &= \frac{-L_m}{D_a v'_m} > 0 \\
\frac{\partial q_m}{\partial \alpha_2} &= \frac{-L_2}{D_a v'_m} > 0 \\
\frac{\partial q_2}{\partial \alpha_m} &= \frac{-L_m}{D_a v'_2} > 0 \\
\frac{\partial q_2}{\partial \alpha_2} &= \frac{-L_2}{D_a v'_2} > 0,
\end{aligned}$$

plus $\partial q_2/\partial \alpha_b = \partial q_m/\partial \alpha_b = \partial q_b/\partial \alpha_m = \partial q_b/\partial \alpha_b = \partial q_b/\partial \alpha_2 = 0$. The effects on the other variables are

$$\begin{aligned}
\frac{\partial z_m}{\partial \alpha_m} &= \frac{-L_m}{D_a \chi_m} > 0 \\
\frac{\partial z_m}{\partial \alpha_2} &= \frac{-L_2}{D_a \chi_m} > 0 \\
\frac{\partial s_b}{\partial \alpha_m} &= \frac{-\chi_b \alpha_2 L'_2 L_m}{D_a} < 0 \\
\frac{\partial s_b}{\partial \alpha_b} &= \chi_b L_b > 0 \\
\frac{\partial s_b}{\partial \alpha_2} &= \frac{\chi_b \alpha_m L'_m L_2}{D_a} > 0 \\
\frac{\partial \iota_b}{\partial \alpha_m} &= \frac{(1+\iota_b)\chi_b \alpha_2 L'_2 L_m}{D_a \left(1 + \chi_b \alpha_b L_b + \chi_b \alpha_2 L_2\right)} > 0 \\
\frac{\partial \iota_b}{\partial \alpha_b} &= \frac{-(1+\iota_b)\chi_b L_b}{1 + \chi_b \alpha_b L_b + \chi_b \alpha_2 L_2} < 0 \\
\frac{\partial \iota_b}{\partial \alpha_2} &= \frac{-\alpha_m L'_m (1+\iota_b)\chi_b L_2}{D_a \left(1 + \chi_b \alpha_b L_b + \chi_b \alpha_2 L_2\right)} < 0
\end{aligned}$$

plus $\partial z_m/\partial \alpha_b = 0$. The effects on ϕ_b are similar to ι_b with the opposite sign.

The effects of pledgeability on $\mathbf{q}$ are

$$\begin{aligned}
\frac{\partial q_m}{\partial \chi_m} &= \frac{-\left(\alpha_m L_m + \alpha_2 L_2\right)}{D_a \chi_m v_m'} > 0 \\
\frac{\partial q_2}{\partial \chi_m} &= \frac{-\left(\alpha_m L_m + \alpha_2 L_2\right)}{D_a \chi_m v_2'} > 0 \\
\frac{\partial q_m}{\partial \chi_b} &= \frac{-A_b \alpha_2 L_2'}{D_a} < 0 \\
\frac{\partial q_b}{\partial \chi_b} &= \frac{A_b}{v_b'} > 0 \\
\frac{\partial q_2}{\partial \chi_b} &= \frac{A_b \alpha_m L_m'}{D_a v_2'} > 0
\end{aligned}$$

plus $\partial q_b / \partial \chi_m = 0$. For the other variables,

$$\begin{aligned}
\frac{\partial z_m}{\partial \chi_m} &= -\frac{z_m}{\chi_m} - \frac{\iota_0}{D_a \chi_m^3} \gtrless 0 \\
\frac{\partial z_m}{\partial \chi_b} &= \frac{-A_b \alpha_2 L_2^{'}}{D_a \chi_m} < 0 \\
\frac{\partial s_b}{\partial \chi_m} &= \frac{-\chi_b \alpha_2 L_2'(\alpha_m L_m + \alpha_2 L_2)}{D_a \chi_m} < 0 \\
\frac{\partial s_b}{\partial \chi_b} &= \frac{s_b}{\chi_b} + A_b \chi_b \alpha_b L_b' + \frac{A_b \chi_2 \alpha_2 \alpha_m L_2' L_m'}{D_a} \gtrless 0 \\
\frac{\partial \iota_b}{\partial \chi_m} &= \frac{(1+\iota_b)\chi_b \alpha_2 L_2'(\alpha_m L_m + \alpha_2 L_2)}{(1+s_b) D_a \chi_m} > 0 \\
\frac{\partial \iota_b}{\partial \chi_b} &= -\left(\frac{1+\iota_b}{1+s_b}\right)\left[s_b/\chi_b + A_b \chi_b \alpha_b L_b' + A_b \chi_b \alpha_2 L_2' \alpha_m L_m'/D_a\right] \gtrless 0.
\end{aligned}$$

The effects on ϕ_b are similar to ι_b with opposite sign. ■

B: Long Bonds

Now consider long-term bonds, where main difference from short-term bonds is that $z_b = (\phi_b + \delta) A_b$ is endogenous,. When the constraints bind in all meetings, the effects of ι_0 are

$$\begin{aligned}
\frac{\partial z_m}{\partial \iota_0} &= \frac{\chi_b^2(\alpha_b L_b' + \alpha_2 L_2') - \delta(1+r)A_b/z_b^2}{D_l \chi_m^2} < 0 \\
\frac{\partial z_b}{\partial \iota_0} &= \frac{-\chi_b \alpha_2 L_2'}{D_l \chi_m} > 0 \\
\frac{\partial q_b}{\partial \iota_0} &= \frac{-\chi_b \alpha_2 L_2'}{D_l \chi_m v_b'} > 0 \\
\frac{\partial q_2}{\partial \iota_0} &= \frac{\chi_b^2 \alpha_b L_b' - \delta(1+r)A_b/z_b^2}{D_l \chi_m v_2'} < 0,
\end{aligned}$$

where $D_l > 0$ is given by

$$D_l = \left[\chi_b^2 \left(\alpha_b L_b' + \alpha_2 L_2'\right) - \delta(1+r)A_b/z_b^2\right]\alpha_m L_m' + \left[\chi_b^2 \alpha_b L_b' - \delta(1+r)A_b/z_b^2\right]\alpha_2 L_2'.$$

The effects on q_m are similar to the effects on z_m. For financial variables,

$$\begin{aligned}
\frac{\partial s_b}{\partial \iota_0} &= \frac{-\delta(1+r)A_b\chi_b\alpha_2 L_2'}{D_l\chi_m z_b^2} > 0 \\
\frac{\partial \phi_b}{\partial \iota_0} &= \frac{-\chi_b\alpha_2 L_2'}{D_l\chi_m A_b} > 0 \\
\frac{\partial \iota_b}{\partial \iota_0} &= \frac{1}{1+s_b} + \frac{(1+\iota_b)\,\delta(1+r)A_b\chi_b\alpha_2 L_2'}{(1+s_b)D_l\chi_m z_b^2} \gtrless 0
\end{aligned}$$

The effects of OMO's are

$$\begin{aligned}
\frac{\partial z_m}{\partial A_b} &= \frac{\chi_b\alpha_2 L_2'\gamma(1+r)}{D_l\chi_m z_b} < 0 \\
\frac{\partial z_b}{\partial A_b} &= \frac{-\delta(1+r)(\alpha_m L_m' + \alpha_2 L_2')}{D_l z_b} > 0 \\
\frac{\partial q_b}{\partial A_b} &= \frac{-\delta(1+r)(\alpha_m L_m' + \alpha_2 L_2')}{D_l z_b v_b'} > 0 \\
\frac{\partial q_2}{\partial A_b} &= \frac{-\delta(1+r)\chi_b\alpha_m L_m'}{D_l z_b v_2'} > 0 \\
\frac{\partial s_b}{\partial A_b} &= \frac{-\delta(1+r)\chi_b^2\left(\alpha_m\alpha_b L_m' L_b' + \alpha_m\alpha_2 L_m' L_2' + \alpha_b\alpha_2 L_b' L_2'\right)}{D_l z_b} < 0 \\
\frac{\partial \iota_b}{\partial A_b} &= \frac{\delta(1+r)\,(1+\iota_b)\,\chi_b^2\left(\alpha_m\alpha_b L_m' L_b' + \alpha_m\alpha_2 L_m' L_2' + \alpha_b\alpha_2 L_b' L_2'\right)}{(1+s_b)\,D_l z_b} > 0 \\
\frac{\partial \phi_b}{\partial A_b} &= \frac{-\phi_b\delta(1+r)\,(1+\iota_b)\,\chi_b^2\left(\alpha_m\alpha_b L_m' L_b' + \alpha_m\alpha_2 L_m' L_2' + \alpha_b\alpha_2 L_b' L_2'\right)}{(\iota_b-\pi)\,(1+s_b)\,D_l z_b} < 0.
\end{aligned}$$

Again the effects on q_m are similar to the effects on z_m. In all these results, the one ambiguous effect is $\partial\iota_b/\partial\iota_0$, due to the Fisher and Mundell effects, as in the baseline model. ■

C: More on Directed Search

Consider a directed search model with only one asset with real value $z = \phi A$ and a spread s between the return on it and an illiquid bond, and normalize the measure of buyers to $\mu = 1$. Market makers post (q, z, σ) to solve a version of the problem in the text with s instead of ι_0 and Π instead of Π_m. Generically there is a unique solution, with $U^b(s, \Pi)$ decreasing in both s and Π. The FOC's wrt q and σ are given similar to those in the text with s and Π instead of ι_0 and

Π_m. This generates a correspondence $\sigma(\Pi)$, like a demand correspondence with σ quantity and Π price, and it is decreasing (Rocheteau and Wright 2005, Lemma 5).

One approach in the literature assumes n is fixed, so in equilibrium $\sigma = n$. Then $\sigma(\Pi) = n$ pins down Π, and it is easy to check $\partial q/\partial s = c'/[\alpha u'' - (\alpha+s)c''] < 0$ and $\partial q/\partial n = \alpha'(u'-c')/[\alpha u'' - (\alpha+s)c''] > 0$. Other effects are complicated, in general, so suppose ε is constant, as it is with a Cobb-Douglas matching function, truncated to keep probabilities between 0 and 1. Letting $\varepsilon = \sigma\alpha'(\sigma)/\alpha(\sigma) \in (0,1)$, we have

$$\frac{\partial z}{\partial s} = \frac{\alpha\left\{u'c'\left[\alpha + s(1-\varepsilon)\right] - \varepsilon(1-\varepsilon)(u-c)\left[\alpha u'' - (\alpha+s)c''\right]\right\}}{\left[\alpha + s(1-\varepsilon)\right]^2\left[\alpha u'' - (\alpha+s)c''\right]} < 0$$
$$\frac{\partial z}{\partial n} = \frac{\iota\alpha'\left\{\varepsilon(1-\varepsilon)(u-c)\left[\alpha u'' - (\alpha+s)c''\right] - u'c'\left[\alpha + s(1-\varepsilon)\right]\right\}}{\left[\alpha + s(1-\varepsilon)\right]^2\left[\alpha u'' - (\alpha+s)c''\right]} > 0.$$

Another approach assumes a perfectly elastic supply of homogeneous sellers, with fixed entry cost κ, so that in equilibrium $\Pi = \kappa$ and $\sigma = \sigma(\kappa)$ is endogenous. Then

$$\frac{\partial q}{\partial s} = \frac{c'\alpha''(u-c)}{D} < 0$$
$$\frac{\partial q}{\partial \kappa} = -\frac{\alpha'[1 + s(1-\varepsilon)/\alpha](u'-c')}{D} < 0.$$

with $D = [\alpha u'' - (\alpha+s)c'']\left[\alpha''(u-c) + s\kappa(1-\varepsilon)\alpha'/\alpha^2\right] - \alpha'^2(u'-c')^2 > 0$. While D cannot be signed globally, in equilibrium $D > 0$ by the SOC's. Also, if ε is constant, then

$$\frac{\partial \sigma}{\partial s} = \frac{[\alpha u'' - (\alpha+s)c'']\kappa(1-\varepsilon)/\alpha - \alpha'(u'-c')c'}{D} < 0$$
$$\frac{\partial \sigma}{\partial \kappa} = \frac{[\alpha u'' - (\alpha+s)c''][1 + s(1-\varepsilon)/\alpha]}{D} < 0$$
$$\frac{\partial z}{\partial s} = \frac{\kappa(1-\varepsilon)^2\left[\alpha u'' - (\alpha+s)c''\right] + c'^2\left[\alpha(u-c)\alpha'' - s\kappa(1-\varepsilon)\alpha'\right]}{\alpha^2 D} < 0$$
$$\frac{\partial z}{\partial \kappa} = \frac{-\iota\alpha'\left\{u'\left[\alpha + s(1-\varepsilon)\right] + \varepsilon(1-\varepsilon)c\left[\alpha u'' - (\alpha+s)c''\right]\right\}}{\alpha\left[\alpha + s(1-\varepsilon)\right]D} \gtrless 0.$$

In these results, the only ambiguous effect is $\partial\hat{z}/\partial\kappa$. ■

D: Interest on Reserves

Consider the model with currency plus reserves. To conserve notation, we allow $\iota_r > 0$ but set $\iota_c = 0$, which is without loss in generality since all that matters is $s_c = (\iota_0 - \iota_c)/(1+\iota_c)$.

Given policy (ι_0, ι_r, A_b), equilibrium reduces to two Euler equations in (z_c, z_r), then bargaining determines $\mathbf{q}$, and s_b is pinned down by the equation for z_b. From the Euler equations we derive

$$\mathbf{J}\begin{bmatrix} dz_c \\ dz_r \end{bmatrix} = \begin{bmatrix} -\chi_c\chi_b(\alpha_{cb}L'_{cb} + \alpha_3 L'_3)dA_b \\ \frac{-(1+\iota_0)}{(1+\iota_r)^2}d\iota_r - \chi_r\chi_b(\alpha_{rb}L'_{rb} + \alpha_3 L'_3)dA_b \end{bmatrix},$$

where

$$\mathbf{J} = \begin{bmatrix} \chi_c^2(\alpha_c L'_c + \alpha_{cr}L'_{cr} + \alpha_{cb}L'_{cb} + \alpha_3 L'_3) & \chi_c\chi_r(\alpha_{cr}L'_{cr} + \alpha_3 L'_3) \\ \chi_c\chi_r(\alpha_{cr}L'_{cr} + \alpha_3 L'_3) & \chi_r^2(\alpha_r L'_r + \alpha_{cr}L'_{cr} + \alpha_{rb}L'_{rb} + \alpha_3 L'_3) \end{bmatrix}.$$

Therefore, the effects of changing A_b are

$$\frac{\partial z_c}{\partial A_b} = \frac{\chi_b}{\chi_c D_g}[\alpha_{cr}L'_{cr}(\alpha_{rb}L'_{rb} - \alpha_{cb}L'_{cb}) - \alpha_{cb}L'_{cb}(\alpha_r L'_r + \alpha_3 L'_3 + \alpha_{rb}L'_{rb}) - \alpha_r\alpha_3 L'_r L'_3]$$
$$\frac{\partial z_r}{\partial A_b} = \frac{\chi_b}{\chi_r D_g}[\alpha_{cr}L'_{cr}(\alpha_{cb}L'_{cb} - \alpha_{rb}L'_{rb}) - \alpha_{rb}L'_{rb}(\alpha_c L'_c + \alpha_3 L'_3 + \alpha_{cb}L'_{cb}) - \alpha_c\alpha_3 L'_c L'_3],$$

where $D_g > 0$ is given by

$$D_g = (\alpha_c L'_c + \alpha_{cb}L'_{cb})(\alpha_r L'_r + \alpha_{rb}L'_{rb}) + (\alpha_{cr}L'_{cr} + \alpha_3 L'_3)(\alpha_c L'_c + \alpha_r L'_r + \alpha_{cb}L'_{cb} + \alpha_{rb}L'_{rb}).$$

These effects ambiguous without some restriction. If $\alpha_{rb}L'_{rb} = 0$, e.g., which says no one accepts reserves and bonds but not currency, then $\partial z_c/\partial A_b < 0$. Similarly, if $\alpha_{cb}L'_{cb} = 0$, e.g., then $\partial z_r/\partial A_b < 0$. So under reasonable restrictions OMO's that increases A_b decrease both currency and reserve liquidity. Also, given $\chi_c \geq \chi_r$, e.g., in the natural specification $\chi_c = 1$, increasing A_b must lower at least one of them, since $\partial z_c/\partial A_b + \partial z_r/\partial A_b < 0$.

As regards the effects on $\mathbf{q}$, we have $\partial q_c/\partial A_b \simeq \partial z_c/\partial A_b$ and $\partial q_r/\partial A_b \simeq \partial z_r/\partial A_b$, where $\simeq$ indicates the two sides have the same sign. For the rest, obviously $\partial q_b/\partial A_b > 0$, and

$$\frac{\partial q_{cr}}{\partial A_b} = -\frac{\chi_b}{D_g v'_{cr}}\Phi_{cr} < 0$$
$$\frac{\partial q_{cb}}{\partial A_b} = \frac{\chi_b}{D_g v'_{cb}}\Phi_{cb} > 0$$
$$\frac{\partial q_{rb}}{\partial A_b} = \frac{\chi_b}{D_g v'_{rb}}\Phi_{rb} > 0$$
$$\frac{\partial q_3}{\partial A_b} = \frac{\chi_b}{D_g v'_3}\Phi_3 \gtrless 0$$

where

$$\Phi_{cr} = \alpha_{rb} L'_{rb}(\alpha_c L'_c + \alpha_3 L'_3 + \alpha_{cb} L'_{cb}) + \alpha_{cb} L'_{cb}(\alpha_r L'_r + \alpha_3 L'_3 + \alpha_{rb} L'_{rb}) + \alpha_3 L'_3(\alpha_r L'_r + \alpha_c L'_c) > 0$$

$$\Phi_{cb} = \alpha_{cr} L'_{cr}(\alpha_c L'_c + \alpha_r L'_r + 2\alpha_{rb} L'_{rb}) + \alpha_c L'_c(\alpha_r L'_r + \alpha_{rb} L'_{rb}) + \alpha_3 L'_3(\alpha_c L'_c + \alpha_{rb} L'_{rb}) > 0$$

$$\Phi_{rb} = \alpha_{cr} L'_{cr}(\alpha_c L'_c + \alpha_r L'_r + 2\alpha_{cb} L'_{cb}) + \alpha_r L'_r(\alpha_c L'_c + \alpha_{cb} L'_{cb}) + \alpha_3 L'_3(\alpha_r L'_r + \alpha_{cb} L'_{cb}) > 0$$

$$\Phi_3 = \alpha_{cr} L'_{cr}(\alpha_c L'_c + \alpha_r L'_r + \alpha_{cb} L'_{cb}) + \alpha_c \alpha_r L'_c L'_r + \alpha_{rb} L'_{rb}(\alpha_{cr} L'_{cr} - \alpha_{cb} L'_{cb}) \gtreqless 0.$$

The only ambiguous result is $\partial q_3/\partial A_b$, but the above restrictions that deliver $\partial z_c/\partial A_b < 0$ and $\partial z_r/\partial A_b < 0$ (i.e., $\alpha_{rb} L'_{rb} = 0$ or $\alpha_{cb} L'_{cb} = 0$) also deliver $\partial q_3/\partial A_b > 0$.

As for interest on reserves, the effects on real currency and reserve balances are

$$\begin{aligned}
\frac{\partial z_c}{\partial \iota_r} &= \frac{(1+\iota_0)(\alpha_{cr} L'_{cr} + \alpha_3 L'_3)}{\chi_c \chi_r D_g (1+\iota_r)^2} < 0 \\
\frac{\partial z_r}{\partial \iota_r} &= \frac{-(1+\iota_0)(\alpha_{cr} L'_{cr} + \alpha_{cb} L'_{cb} + \alpha_c L'_c + \alpha_3 L'_3)}{\chi_r^2 D_g (1+\iota_r)^2} > 0.
\end{aligned}$$

The effects on $\mathbf{q}$ are

$$\begin{aligned}
\frac{\partial q_c}{\partial \iota_r} &= \frac{\chi_c}{v'_c} \frac{\partial z_c}{\partial \iota_r} < 0 \\
\frac{\partial q_r}{\partial \iota_r} &= \frac{\chi_r}{v'_r} \frac{\partial z_r}{\partial \iota_r} > 0 \\
\frac{\partial q_{cr}}{\partial \iota_r} &= \frac{-(1+\iota_0)(\alpha_c L'_c + \alpha_{cb} L'_{cb})}{\chi_r v'_{cr} (1+\iota_r)^2 D_g} > 0 \\
\frac{\partial q_{cb}}{\partial \iota_r} &= \frac{\chi_c}{v'_{cb}} \frac{\partial z_c}{\partial \iota_r} < 0 \\
\frac{\partial q_{rb}}{\partial \iota_r} &= \frac{\chi_r}{v'_{rb}} \frac{\partial z_r}{\partial \iota_r} > 0 \\
\frac{\partial q_3}{\partial \iota_r} &= \frac{-(1+\iota_0)(\alpha_c L'_c + \alpha_{cb} L'_{cb})}{\chi_r v'_3 (1+\iota_r)^2 D_g} > 0
\end{aligned}$$

plus $\partial q_b/\partial \iota_r = 0$. Remarkably, these are all unambiguous.

As always, these are generic results; some effects can be 0 for special parameter values. To consider one such case, suppose bonds are accepted in a meeting if and only if reserves are accepted; and sometimes only cash is accepted; and sometimes all assets are accepted. This implies $\alpha_{rb}, \alpha_c, \alpha_3 > 0$ and $\alpha_r = \alpha_b = \alpha_{cb} = \alpha_{cr} = 0$. Inserting these into the above general formulae, we get $\partial z_b/\partial A_b = 1$, of course, plus

$$\frac{\partial z_r}{\partial A_b} = \frac{-\chi_b \left(\alpha_{rb} L'_{rb} \alpha_c L'_c + \alpha_{rb} L'_{rb} \alpha_3 L'_3 + \alpha_c \alpha_3 L'_c L'_3\right)}{\chi_r D_g} \text{ and } \frac{\partial z_c}{\partial A_b} = \frac{\partial(\chi_b z_b + \chi_r z_r)}{\partial A_b} = 0.$$

What is interesting is not so much that $\partial z_r/\partial A_b < 0$ is now unambiguous, but that $\chi_b z_b + \chi_r z_r$ and z_c are independent of A_b. This is because in this case bonds and reserves are perfect substitutes, and note that the result does not require $\chi_b = \chi_r$. So OMO's are neutral in this case. However, ι_r still matters; in particular,

$$\frac{\partial z_c}{\partial \iota_r} = \frac{(1+\iota_0)\alpha_3 L'_3}{\chi_c \chi_r D_g (1+\iota_r)^2} < 0 \text{ and } \frac{\partial z_r}{\partial \iota_r} = \frac{-(1+\iota_0)(\alpha_c L'_c + \alpha_3 L'_3)}{\chi_r^2 D_g (1+\iota_r)^2} > 0.$$

Thus, higher ι_r reallocates the monetary base to less currency and more reserves, in real terms, but notice $\chi_r z_r$ rises by more than $\chi_c z_c$ falls, so in a sense money becomes more liquid.

Here is another special case, where there are only type-c, type-r, type-b and type-3 meetings. Then

$$\frac{\partial z_c}{\partial A_b} = \frac{-\chi_b \alpha_r \alpha_3 L'_r L'_3}{\chi_c D_g} < 0 \text{ and } \frac{\partial z_r}{\partial A_b} = \frac{-\chi_b \alpha_c \alpha_3 L'_c L'_3}{\chi_r D_g} < 0.$$

Of course we have $\partial q_c/\partial A_b < 0$, $\partial q_r/\partial A_b < 0$, $\partial q_b/\partial A_b > 0$ and, one can easily check, $\partial q_3/\partial A_b > 0$. Similarly, for ι_r, we have,

$$\begin{aligned}
\frac{\partial z_c}{\partial \iota_r} &= \frac{(1+\iota_0)\alpha_3 L'_3}{\chi_c \chi_r D_g (1+\iota_r)^2} < 0 \\
\frac{\partial z_r}{\partial \iota_r} &= \frac{-(1+\iota_0)(\alpha_c L'_c + \alpha_3 L'_3)}{\chi_r^2 D_g (1+\iota_r)^2} > 0,
\end{aligned}$$

as minor simplifications of the general case. The effects on $\mathbf{q}$ are

$$\begin{aligned}
\frac{\partial q_c}{\partial \iota_r} &= \frac{(1+\iota_0)\alpha_3 L'_3}{\chi_r D_g (1+\iota_r)^2 v'_c} < 0 \\
\frac{\partial q_r}{\partial \iota_r} &= \frac{-(1+\iota_0)(\alpha_c L'_c + \alpha_3 L'_3)}{\chi_r D_g (1+\iota_r)^2 v'_r} > 0 \\
\frac{\partial q_3}{\partial \iota_r} &= \frac{-(1+\iota_0)\alpha_c L'_c}{\chi_r v'_3 (1+\iota_r)^2 D_g} > 0
\end{aligned}$$

The effects are all unambiguous in this special case. ■