Tax and Welfare in Dynamic Monopoly*

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Abstract

This paper considers the welfare effects of tax policy in a model of dynamic monopoly bargaining. By conditioning the tax rate on past trading volumes, the social planner can provide incentives for the seller to abandon sequential screening and clear the market immediately. The first-best tax policy is budget-neutral in (unique) equilibrium and derived independently of the discount factor. Moreover, efficient policy can be designed when the planner does not know the market distribution of individual demands, and to guarantee the seller's participation in absence of rational expectations. We also explore the role of small, constant transaction tax.

Keywords: Bargaining, monopoly, durable good, regulation, optimal taxation, transaction tax, inventory tax, rationalizability

JEL codes: C78, D42, D61, H21, H25, L12

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1 Introduction

Delayed transactions often pose a frustrating case of allocative inefficiency as evidenced by many recurring experiences of unsold durable goods. For instance, it is common to observe rapidly growing urban areas with severe housing shortages that harbor rising inventories at the same time.[1] Governments have also recognized the damages inflicted by such paradoxical market outcomes and introduced policies, such as taxing inventories, to discourage businesses from stockpiling or delaying production.[2]

Identifying the source of delay and other manifestations of inefficient allocation has been a centerpiece of the bargaining theory literature. In durable good models (e.g. Sobel and Takahashi, 1983; Fudenberg, Levine, and Tirole, 1985; Gul, Sonnenschein, and Wilson, 1986), a profit-maximizing monopolist lacking commitment power screens privately informed buyers sequentially, although, as Coase (1972) conjectured, the resulting inefficiency disappears with vanishing transaction costs. The literature is now stocked with a host of channels that generate bargaining inefficiency and the failure of Coase conjecture.[3]

The broad purpose of this paper is to explore the role of tax policy in controlling welfare loss from bargaining frictions. In particular, we are concerned with situations in which price-based regulations, such as the Ramsey-Boiteux pricing, are not feasible due to lack of authority, information, or other reasons. In dealing with monopoly and imperfect competition, the policy question dates back at least to Wicksell (1896), and also Suits and Musgrave (1955), who considered the relative efficacy of different forms of taxation. The issue of optimal monopoly tax was taken up by Guesnerie and Laffont (1978), Katz (1983), and Laffont (1987). These studies are however based on static contexts in which the seller can commit to uniform or non-linear pricing. The bargaining literature is yet to address the welfare consequences of tax policy in a model of dynamic incentives.

While there are multiple sources and outcomes of bargaining frictions, our focus is on mitigating *strategic delay* that arises due to informational asymmetry. To this end, we introduce dynamic tax policy into the durable good monopoly model of Gul, Sonnenschein, and Wilson (1986), henceforth GSW, who consider privately informed buyers. The social planner in our

[1] For a recent experience of India, see http://www.huffingtonpost.in/2015/03/19/india-property-premium-ap_n_6900400.html.

[2] Inventory tax has a long history in the US at state and local government levels. For the specific purpose of reducing unsold housing stock, India lately began taxing developers on estimated annual rentals on unsold properties (e.g. http://www.dnaindia.com/money/report-fresh-tax-on-unsold-stock-effective-from-current-fiscal-2132622).

[3] Ausubel, Cramton, and Deneckere (2002) provide a survey of the early literature on bargaining models with incomplete information. For more recent contributions, see Deneckere and Liang (2006), Hörner and Vieile (2009), Fuchs and Skrzypacz (2010), and Board and Pycia (2014), among others.

model can commit to a policy that adjusts the tax rate according to the history of trading volumes.[4]

Our main result demonstrates the existence of budget-neutral tax policy which (uniquely) induces the *first-best* outcome: the monopolist clears the market immediately with the lowest incentive-feasible price. This result, which is obtained independently of the discount factor, or the degree of transaction costs, is based on a tax policy which features the structure of multi-part tariff with decreasing marginal tax rate. The final step of the scheme provides a subsidy designed to exactly offset the total tax levied on previous transactions.[5] We further show that a first-best and budget-neutral tax policy can be derived even when the social planner is subject to limited information on the distribution of privately informed buyers.[6]

The optimal budget-neutral policy outlined above generates zero net tax expenditure in equilibrium. However, this is not true *off* the equilibrium, and depending on the levels of tax and discounting, an incorrect conjecture about buyers' behavior could potentially leave the seller with significant losses during the market clearing process. To avoid any possibility of market breakdown under strategic uncertainty, we invoke an additional constraint that the profit must be non-negative throughout any *subgame-rationalizable* path (Bernheim, 1984; Pearce, 1984). Under budget-neutral policy, the tax rates can indeed be set sufficiently low to achieve this participation constraint as well as (equilibrium) efficiency.

Finally, we explore the possibility of welfare-improving constant tax, in the spirit of throwing "sand in the wheels" of a market (Tobin, 1978). Though not optimal, nudging the market with small transaction tax carries practical appeal, and the effectiveness of such policies have been extensively discussed in a variety of financial market contexts. We show that a small tax can serve to reduce bargaining delay in our bargaining model but the improvement in social welfare does not result from direct increase in consumer surplus; rather, it mainly appears in the form of government surplus and the producer may also benefit.

In a recent study, Fuchs and Skrzypacz (2015) examine the effects of government intervention in a dynamic market model with privately informed sellers.[7] In their model with competitive buyers, inefficient delay occurs because higher value sellers hold out for higher prices. By linking

[4]A typical financial statement reports the firm's revenue and inventory information but not the details of individual transactions. Although our analysis is conducted in terms of transaction taxes, our main messages can also be obtained via taxing inventories instead. See Section 6 for a discussion of alternative forms of tax policy.

[5]The role of subsidy to improve welfare in monopoly markets is well-known. Such incentives induce the firm to produce more than its optimal quantity (e.g. Guesnerie and Laffont, 1978). In our dynamic model, subsidies generate benefits from speeding up trade.

[6]Katz (1983) addresses the issue of informationally robust policy design in another monopoly context.

[7]Philippon and Skreta (2012) and Tirole (2012) consider government intervention in static lemons market models.

government policy to trading time, this paper shows that optimal intervention involves initial subsidy followed by post-opening market shutdown. Trades take place without delay but some sellers are excluded. While we also address the issue of inefficient delay, our bargaining setup allows for fully strategic players on both sides of price formation, and we consider tax policy that depends on past sales instead of bargaining interval. In terms of results, our first-best policy features decreasing marginal tax schedule that achieves budget neutrality and involves low rates of tax.[8]

Our paper is also related to the long-standing literature on financial transaction taxes (FTTs), which have seen a revival of policy interests after the latest financial crisis.[9] FTTs can serve to curtail excessive volatility but the overall welfare implications are typically inconclusive in competitive models of financial markets. One channel via which such taxes negatively impact economic activity is that they pose an additional obstacle for efficient information revelation. This latter observation has been pursued in various models of market microstructure (e.g. Lee, 1998; Dupont and Lee, 2007). Our model, in contrast, demonstrates a case of strategic price formation in which transaction tax has a positive welfare effect.

The rest of the paper is organized as follows. The next section describes the model of dynamic monopoly bargaining with tax policy. Section 3 presents our main results on tax policies that achieve the first-best outcome. Section 4 addresses the seller's participation issue under strategic uncertainty by considering rationalizability. We discuss welfare implications of simple (small) tax policy in Section 5. Some concluding remarks are offered in Section 6. Formal proofs, unless otherwise stated, are relegated to the Appendix, which also contains some additional analyses left out from the main text for expositional reasons.

2 The Model

2.1 Dynamic monopoly bargaining

We introduce taxation into GSW's durable goods monopoly model with discrete consumer types. A seller, whose commonly known constant unit cost is $c > 0$, serves a market consisting of a continuum of buyers, each of whom demands one unit. Buyers are indexed by a type $i \in N := \{1, \ldots, n\}$, and each type i buyer's valuation for the good is $v_i > 0$ such that $v_1 > v_2 \cdots > v_n > c$.[10] For each type i, there is a measure q_i of buyers. Let $Q_k := \sum_{i=1}^{k} q_i$ and

[8] The implications of time-contingent tax policy in our model are discussed in Section 6.

[9] See Burman et al. (2016) and Hemmelgarn et al. (2016) for recent reviews of the theory and practice of FTTs.

[10] We assume the "gap" case to simplify exposition of our main analysis in Sections 3 and 4. The material in Section 5 is based on equilibrium uniqueness under the assumption.

$Q := Q_n$. Each buyer's valuation is private information but the market demand schedule, as well as the seller's cost, are common knowledge.[11]

A (dynamic or non-linear) tax policy, f, is a (Riemann) integrable function

$$f : \mathbb{R}_+ \to \mathbb{R}$$

such that, for all $q \in \mathbb{R}$, $f(q)$ specifies the rate of *ad valorem* tax levied at the transaction after accumulated volume of trade q.[12] Note that subsidies (negative tax rates) are feasible. For any $q, q' \in [0, Q]$ such that $q < q'$, we define the average interval tax rate as

$$\bar{f}(q, q') = \frac{1}{q' - q} \int_q^{q'} f(s) ds.$$

For a given tax policy f, the extensive form is as follows. In each discrete period $t = 1, 2, \ldots$, the seller posts a post-tax price, p_t, and the buyers who remain in the game simultaneously accept or reject the price. If a buyer of type i accepts the price, he leaves the game with payoff $\delta^{t-1}(v_i - p_t)$, where $\delta \in (0, 1)$ is the common discount factor. If a buyer rejects the price, he continues as an active player; his reservation utility is zero.

Fix any t and any t-period sequences of price offers $(p_1, \ldots, p_t)$ and cumulative trading volumes $(Q_1, Q_2, \ldots, Q_t)$. Let $Q_0 = 0$. Then, the seller's payoff is given by

$$\begin{aligned} & \sum_{\tau=1}^{t} \delta^{\tau-1} \int_{Q_{\tau-1}}^{Q_\tau} \big[(1 - f(s))p_\tau - c\big] ds \\ = & \sum_{\tau=1}^{t} \delta^{\tau-1} (Q_\tau - Q_{\tau-1}) \left[\left(1 - \bar{f}(Q_{\tau-1}, Q_\tau)\right) p_\tau - c\right]. \end{aligned}$$

The (discounted) tax revenue amounts to

$$\begin{aligned} & \sum_{\tau=1}^{t} \delta^{\tau-1} \int_{Q_{\tau-1}}^{Q_\tau} f(s) p_\tau ds \\ = & \sum_{\tau=1}^{t} \delta^{\tau-1} (Q_\tau - Q_{\tau-1}) \bar{f}(Q_{\tau-1}, Q_\tau) p_\tau. \end{aligned}$$

All our results below are independent of δ, which is therefore fixed throughout.

[11]We later relax the common knowledge assumption to consider a social planner who faces uncertain market demand.

[12]Laffont (1987) considers the use of non-linear taxes against price-discriminating (static) monopoly. Wicksell's original argument, later formalized in a variety of settings by Suits and Musgrave (1955), Skeath and Trandel (1994), and others, is that ad valorem tax is a superior form of tax than unit tax in terms of welfare.

2.2 Strategies and equilibrium

A history is any finite sequence of price offers and the sets of buyers who accept the price offer in each period. Formally, for any $\tau \geq 1$, let p_τ and B_τ denote the price offer and the set of buyers accepting p_τ in period τ, respectively. Then, a typical t-period history, h, is a sequence $\{p_\tau, B_\tau\}_{\tau=1}^t$, where $(B_1, \cdots, B_t)$ are mutually disjoint. The set of all t-period histories is denoted by H^t. Let $H = \bigcup_{t=0}^{\infty} H^t$, where $H^0 = \emptyset$.

The seller's strategy is given by

$$\sigma_S : H \rightarrow \triangle \mathbb{R}_+$$

such that, for all $h \in H$, $\sigma_S(h)$ specifies a distribution of non-negative price offers. Without loss of generality, we assume that each type i buyer adopts a symmetric strategy, which is written as

$$\sigma_i : H \times \mathbb{R}_+ \rightarrow [0, 1]$$

such that, for any t and $h \in H^t$ satisfying $i \notin \bigcup_{\tau=1}^t B_\tau$, and any $p \in \mathbb{R}_+$, $\sigma_i(h, p)$ specifies the buyer's acceptance probability. Let Σ_S and Σ_i denote the set of all strategies of the seller and type i buyer, respectively. For any $i \in N$, any $\sigma_i \in \Sigma_i$ and any $h \in H$, $\sigma_i|h$ denotes the continuation strategy induced by σ_i at history h. We similarly define $\sigma_S|h$ as the seller's continuation strategy.

Given tax policy f, we say that a strategy profile $(\sigma_S, \{\sigma_i\}_{i=1,\ldots,n})$ is a *bargaining equilibrium* if (i) it is a subgame perfect equilibrium (SPE) and (ii) for every buyer type i and any price offer p, $\sigma_i(h, p) = \sigma_i(h', p)$ if h and h' are identical except that the sets of buyers accepting at each period differ at most by measure zero sets.

Condition (ii) above is the regularity condition imposed by GSW, and this implies that a buyer's unilateral deviation does not affect any continuation play.[13] In verifying a bargaining equilibrium, therefore, we only need to consider off-the-equilibrium paths induced by the seller's unilateral deviations. Moreover, this implies that, in all such off-the-equilibrium paths, all buyers expect the same future price sequence and hence higher valuation types transact earlier. For any date $t \geq 1$, any history that matters for equilibrium behavior can then be referred to simply as a sequence of price-quantity pairs $(p_1, Q_1, \ldots, p_t, Q_t)$.

[13] The regularity condition makes the analysis of GSW equivalent to the Bayesian analysis of Fudenberg, Levine, and Tirole (1985).

3 First-best Tax Policy

We begin by investigating the possibility of tax policy that induces the first-best outcome while satisfying budget neutrality.

Definition 1 *A tax policy f is budget-neutral first-best (BNFB) if the corresponding game admits a bargaining equilibrium in which the market clears immediately and the tax revenue is zero.*

Theorem 1 *Consider any tax policy f^* that satisfies the following properties:*

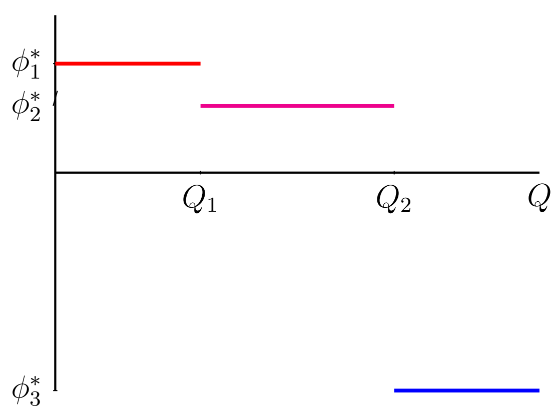
- *For any $i \in N \setminus \{n\}$ and any $q \in (Q_{i-1}, Q_i]$, $f^*(q) = \phi_i^* \geq 1 - \frac{v_n}{v_i}$.*
- *For any $q \in (Q_{n-1}, Q_n]$, $f^*(q) = \phi_n^* := \frac{-\sum_{i=1}^{n-1} \phi_i^* q_i}{q_n}$.*

Then, f is BNFB, and moreover, the bargaining equilibrium outcome is unique.

Proof. See Appendix A. ■

The lowest tax policy in Theorem 1 is a decreasing step function with the following properties: (i) fixed tax rates are levied over n intervals, where the size of each interval $i = 1, \ldots, n$ matches the measure of type i consumers q_i, and (ii) a subsidy is provided for the final segment of transactions (of measure q_n) in a way that offsets the tax expenditure accumulated over all previous transactions. Note, however, that the tax revenue would still be positive if the game generates equilibrium delay, due to discounting and decreasing equilibrium prices. Figure 1 depicts the lowest tax policy for a case of $n = 3$.

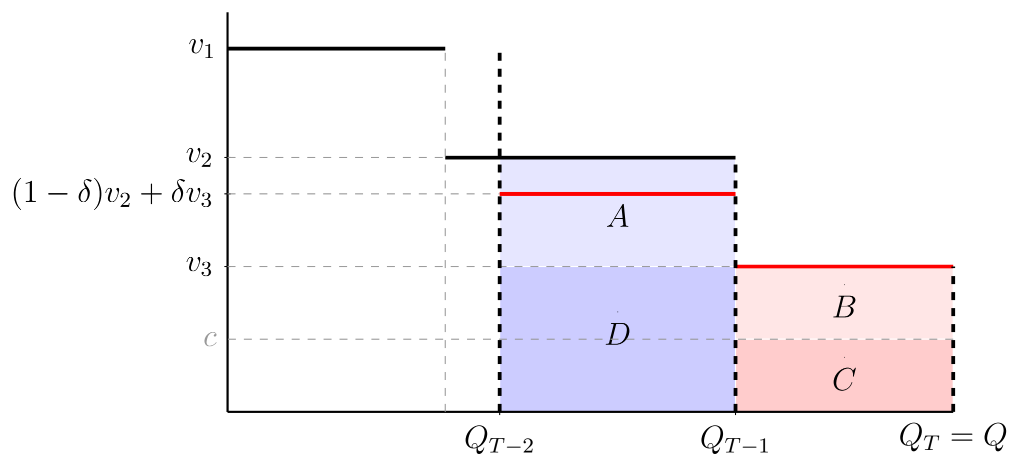
Figure 1: First-best tax policy



A sketch of the proof of Theorem 1 is as follows. The first step of the proof shows that the "skimming property" remains true in the game with tax policy. Since the seller posts post-tax prices, and since the seller would never offer a price below v_n, each type i buyer follows a stationary cutoff strategy such that he would accept (reject) any offer below (above) $(1-\delta)v_i + \delta v_n$. Second, as in GSW, any bargaining equilibrium of our game ends up clearing the market in finite time. That is, the lowest valuation buyers are eventually be served with price v_n.

The critical step of our arguments derives that, at any on- or off-the-equilibrium history of price offers and (cumulative) trading volumes, the seller has incentives to clear the remaining segments of the market immediately with the lowest incentive-feasible price v_n. Figure 2 illustrates an example of such incentives (where A to D measure rectangular areas).

Figure 2: Costs and benefits of delay



Suppose to the contrary that, after some history, the seller practices sequential screening such that the game ends in period T. Consider the pan-ultimate period $T-1$ of the corresponding continuation path with history Q_{T-2}. In the illustrated example, $p_{T-1} = (1-\delta)v_2 + \delta v_3$. By offering v_3 instead, the seller can complete the sales without delay.

What are the costs and benefits of speeding up sales? On the one hand, a lower transaction price implies a forgone (taxed) profit which amounts to $(1-\delta)(1-\phi_2^*)A$. On the other hand, the benefit consists of not only the non-discounted profit from the lowest valuation group, i.e. $(1-\delta)B$, but also the subsidy that gets brought forward, i.e. $-\phi_3^*(B+C)(1-\delta)$. Therefore, a sufficient condition for inducing quicker market clearing is

$$-\phi_3^*(B+C) \geq (1-\phi_2^*)A. \tag{1}$$

Now, notice that, since $-\phi_3^* q_3 = \phi_1^* q_1 + \phi_2^* q_2$, we have $-\phi_3^*(B+C) \geq \phi_2^* D$. Thus, (1) is ensured by setting

$$\phi_2^* D \geq (1-\phi_2^*)A, \tag{2}$$

or indeed,

$$\phi_2^* \geq \frac{A}{A+D} = 1 - \frac{v_3}{v_2}, \tag{3}$$

as in Theorem 1. It thus follows that any bargaining equilibrium of the model under f^* results in immediate market clearing with price offer equal to v_n. Clearly, the total tax revenue is zero and hence f^* is budget-neutral.

It is important to emphasize the role played by the subsidy and the budget neutrality constraint in our optimal taxation problem. First, note that additional costs of delay can be imposed simply by setting high tax rates on low volumes of trade. However, relying on tax alone to induce faster trades may involve tax rates that are too large, and the firm may prefer to withdraw from the market altogether.[14] By subsidizing late transactions, one generates extra benefits from speeding up the bargaining; this not only secures budget neutrality but also serves to reduce the required tax rates. As a result, the subsidy provides a key ingredient in ensuring the seller's participation, even in the absence of rational expectations about consumers' behavior. This issue will be addressed formally in the next section.

Second, f^* does not depend on the precise value of trading history q, and this is another artifact of the requirement of budget neutrality. Note that, if the game were to move beyond the first period, total subsidy to be spent in the final period of trade must negate not just the previous period's tax revenue but the total tax revenue accumulated from the very beginning of the game. It is precisely this logic which, for any given history $Q_{T-2} = q$, transforms (1) to (2) in deriving (3), the lower bound on ϕ_2^*, which ends up being independent of the quantity $Q_{T-1} - q$. To compute a delay-proof (average) tax rate over the interval $(q, Q_{T-1}]$, we only need information on the valuations of those who remain in the market.

This latter implication of budget neutrality enables us to introduce informational robustness to the design of optimal policy. In particular, there is no need for pinpointing when to adjust the tax rate, as our next result demonstrates.

Theorem 2 *Suppose that the social planner only knows that, for each i, q_i belongs to some interval $[\underline{q}_i, \overline{q}_i]$. Let $\overline{Q}_k := \sum_{i=1}^k \overline{q}_i$ and $\underline{Q}_k := \sum_{i=1}^k \underline{q}_i$. Then, if $\overline{Q}_{n-1} < \underline{Q}_n$, a tax policy f^{**} satisfying the following properties is BNFB (with unique equilibrium outcome) for any $\{q_i\}_{i=1}^n$:*

- *For any $i \in N \setminus \{n\}$ and any $q \in (\overline{Q}_{i-1}, \overline{Q}_i]$, $f^{**}(q) = \phi_i^{**} \geq 1 - \frac{v_n}{v_i}$.*

[14] For an extreme case in point, consider f such that $f(q) = 1$ for $q \in (0, Q_{n-1}]$ and $f(q) = 0$ otherwise.

- *For any* $q \in (\overline{Q}_{n-1}, \underline{Q}_n]$, $f^{**}(q) = \phi_n^{**} := \frac{-\sum_{i=1}^{n-1} \phi_i^{**} \overline{q}_i}{\underline{Q}_n - \overline{Q}_{n-1}}$.
- *Otherwise,* $f^{**}(q) = 0$.

Proof. See Appendix B. ■

This implies that the first-best tax policy can also accommodate further informational asymmetry on the part of social planner vis-à-vis market participants. We discussed above why f^* depends only on the valuations of remaining buyers; in fact, the planner does not need exact information on this, since it suffices to set the tax rate in terms of the largest valuation that he believes possible in the continuation game. Therefore, a welfare-maximizing policy can be derived without full knowledge of the measure of each buyer type. That is, the planner can operate even with an unknown *distribution* of privately informed players.

Theorem 2 provides a sufficient condition on the extent of third party informational imperfection that can be reconciled with our policy. It requires that the planner's information on q_n, the measure of lowest valuation buyers, and Q_{n-1}, the aggregate measure of all other types, are not too diffuse. This ensures correct timing of introducing subsidy that would satisfy the budget neutrality constraint under all possible market demand functions.

Note also that the optimal policy makes no use of any knowledge of the seller's cost. Even if the planner was unaware of the precise value of c, Theorem 2 would remain valid as long as its largest possible value is below v_n. Our focus is on situations in which there is no regulatory uncertainty about the socially optimal allocation itself (which in this case corresponds to trade of Q units). Such uncertainty is the subject of the extensive literature on incentive regulation (e.g. Baron and Myerson, 1982; Laffont and Tirole, 1993; Armstrong and Sappington, 2007), which typically assumes the availability of price-contingent controls. We may similarly consider *menu* of tax policies.

4 Rationalizable Participation

A potential pitfall of the first-best tax policy proposed above is that it may involve tax rates that are too high under strategic uncertainty. To see this, fix tax policy f^* that is BNFB, and consider the equilibrium in which the seller offers v_n and clears the market in the first period. Suppose that a fraction $q \leq q_n$ of the lowest valuation buyers, who are indifferent between accepting and rejecting the price, do not follow the equilibrium strategy and reject the offer. Then, the seller's period 1 profit is equal to

$$\left[(1 - \bar{f}^*(0, Q - q))v_n - c\right](Q - q),$$

which would be negative if

$$\bar{f}^*(0, Q - q) \gg 1 - \frac{c}{v_n}.$$

This raises the concern that, unless the players' behavioral conjectures are correct, a liquidity-constrained seller may be reluctant to participate in the market under fiscal intervention.

Our next result is an attempt to address this issue. We construct a tax policy that is not only BNFB, but also satisfies an additional constraint that the seller's profit must be non-negative throughout any *rationalizable* path of the corresponding game. For our dynamic game setting, the following notion of subgame-rationalizability, due to Bernhaim (1984) and Pearce (1984), is adopted.

Definition 2 *For S and any $i \in N$, σ_S and σ_i, respectively, are subgame-rationalizable if, for any $h \in H$, $\sigma_S|h$ and $\sigma_i|h$ are rationalizable in the subgame that follows h. A path $(p_1, Q_1, p_2, Q_2, \ldots)$ is subgame-rationalizable if it is induced by some subgame-rationalizable strategies.*

We formally introduce the participation constraint below.

Definition 3 *A tax policy f satisfies the rationalizable participation constraint (RPC) if, in any subgame-rationalizable path $\{p_t, Q_t\}_{t=1,2,\ldots}$ of the corresponding game, we have*

$$\int_{Q_{t-1}}^{Q_t} \big[(1 - f(s))p_t - c\big] ds \geq 0 \quad \forall t.$$

The next result establishes that, if the seller's cost is sufficiently small, the additional restriction does not limit the scope of tax policy in our model.

Theorem 3 *Suppose that $c < \min \left\{ \frac{v_n[(1-\delta)v_{i+1} + \delta v_n]}{v_i} \right\}_{i \in N}$. Then, there exists a tax policy that satisfies RPC and is BNFB with unique equilibrium outcome.*

Proof. See Appendix C. ■

Thus, the tax rates can be set sufficiently low to ensure the seller's participation even when the players respond to incorrect conjectures about others' strategies in any continuation game. This feature is indeed due to the structure of our tax policy, which employs a subsidy to offset tax revenue.

To understand the result, we first invoke the procedure of iteratively deleting strategies that induce continuation strategies that are "never best responses". In the game with any given tax policy f, the iteration leaves the seller's (subgame-)rationalizable price offer bounded below by v_n. This means that, although the buyers' expectations of future prices may differ under rationalizable strategies, they at least share the expectation that the price would never fall below v_n. Therefore, each type i buyer accepts any offer below $(1-\delta)v_i + \delta v_n$.

Next, fix any sequence $(p_1, Q_1, \ldots, p_t, Q_t)$, and suppose that some rationalizable strategies in the continuation game induce (p_{t+1}, Q_{t+1}). From the previous step of arguments, we can then derive the seller's worst possible profit in period $t+1$ and impose an upper bound on the average interval tax rate $\bar{f}(Q_t, Q_{t+1})$ which would be consistent with the non-negative profit condition. The last step is to show that the arguments behind Theorem 1 can also deliver a BNFB tax policy if c is not too large.

Theorem 3 can be interpreted as considering a policy maker who compares different tax regimes in terms of their equilibrium implications, but at the same time, is aware of the issue of strategic uncertainty that the players face. The policy maker's objective with regard to the latter issue then is to eliminate the possibility of market breakdown along any rationalizable outcome path (and not just at the beginning of the game).

A yet stronger design approach would be to directly compute the welfare properties of rationalizable outcomes under different tax regimes. However, rationalizablity in our dynamic, and potentially infinite-horizon, game does not allow any further selection of players' strategies beyond what has been mentioned above. In particular, without rational expectations, the skimming property fails hold since the players' expectations about the future price sequence may diverge and hence it is possible for low valuation buyers to accept a higher price than high valuation buyers. Thus, for any given tax policy, the bargaining game admits a large set of rationalizable paths.[15]

5 Simple Tax: Sand in the Wheels

In this section, we explore welfare consequences of *constant* tax policy. There are several reasons for asking this question. A constant tax policy is simple and hence less costly to implement than a policy that relies on tracking market transactions. If the tax rate is sufficiently small, the intervention would not significantly affect the seller's profitability and participation decision. A small tax would also be robust to incorrect specification of market demand conditions. Indeed, the idea of nudging the market with a small amount of tax, or throwing "sand in the wheels", has been an important concept elsewhere in economics, especially in financial market regulation (e.g. Tobin, 1978; Stiglitz, 1989; Summers and Summers, 1989).

Note first that, in the model with zero tax, i.e. $f \equiv 0$, since $v_2 > c$, we know from GSW that the bargaining equilibrium outcome path is unique with market clearing in finite time.

[15] For a mechanism design approach with rationalizability and other related solution concepts, see Bergemann, Morris, and Tercieux (2011) and Bergemann and Morris (2012), for example.

Introducing a constant ad valorem tax (subsidy) is then equivalent to raising (reducing) the seller's unit cost by the corresponding proportion.

Lemma 1 *Suppose that a strategy profile* $(\sigma_S, \{\sigma_i\}_{i=1,\ldots,n})$ *is a bargaining equilibrium of the game with constant tax policy* $f \equiv \phi$ *and unit production cost* c*. Then, the profile is a bargaining equilibrium of the game with no tax and cost* $\frac{c}{1-\phi}$*.*

Proof. Consider $(\sigma_S, \{\sigma_i\}_{i=1,\ldots,n})$ in the game with no tax and cost $\frac{c}{1-\phi}$. Fix any date t and any history $h \in H^t$. Due to the regularity condition, the strategy profile induce an identical future path $(p_{t+1}, Q_{t+1}, \ldots)$ in the new game.

Since, for each $i \in N$, $\sigma_i|h$ is optimal in the original game (with $f \equiv \phi$ and cost c), $\sigma_i|h$ must also be optimal in the new game. To see that $\sigma_S|h$ is optimal, note that

$$\sum_{\tau \geq t+1} \delta^{\tau-1} (Q_\tau - Q_{\tau-1}) [(1-\phi) p_\tau - c] = (1-\phi) \sum_{\tau \geq t+1} \delta^{\tau-1} (Q_\tau - Q_{\tau-1}) \left(p_\tau - \frac{c}{1-\phi}\right),$$

which implies that the seller's preference over $(p_{t+1}, Q_{t+1}, \ldots)$ in both games is the same. ■

By GSW, the previous lemma implies the following.

Corollary 1 *For* $\phi < 1 - \frac{c}{v_n}$*, there exists a unique bargaining equilibrium path in the game with tax policy* $f \equiv \phi$*.*

We obtain an unambiguous welfare improvement result for the case of binary consumer types, i.e. $N = \{1, 2\}$. In this case, the unique equilibrium path with $f \equiv 0$, which can be represented as a sequence $(p_1, Q_1, \ldots, p_T, Q_T)$, can be characterized by the following (backward) recursive structure:

- $T = \max\left\{t = 1, 2, \ldots \mid \sum_{\tau=2}^{t} L_\tau \leq q_1\right\}$;
- $Q_t - Q_{t-1} = \begin{cases} q_1 - \sum_{\tau=2}^{T-1} L_\tau & \text{if } t = 1 \\ L_{T+1-t} & \text{if } t = 2, \ldots, T; \end{cases}$
- $p_t = \pi_{T+1-t}$ for $t = 1, \ldots, T$,

where

$$\begin{aligned} \pi_1 &= v_2 \\ v_1 - \pi_{t+1} &= \delta(v_1 - \pi_t) \text{ for } t \geq 1 \end{aligned} \tag{4}$$

and

$$\begin{aligned} L_1 &= q_2 \\ \sum_{\tau=1}^{t+1} \delta^{t+1-\tau}(\pi_\tau - c)L_\tau &= (\pi_t - c)(L_t + L_{t+1}) + \sum_{\tau=1}^{t-1} \delta^{t-\tau}(\pi_\tau - c)L_\tau \text{ for } t \geq 1 \end{aligned} \tag{5}$$

Now, consider any constant tax policy $f \equiv \phi < 1 - \frac{c}{v_2}$. Let the unique equilibrium path be denoted by $\left(p_1^\phi, Q_1^\phi, p_2^\phi, Q_2^\phi, \ldots, p_{T(\phi)}^\phi, Q_{T(\phi)}^\phi\right)$, where $T(\phi) < \infty$ is the finite period in which the game ends.[16] Then, from the recursive structure of the equilibrium, we can define the corresponding *social welfare* by

$$W(\phi) := \delta^{T(\phi)-1} q_2(v_2 - c) + \sum_{t=1}^{T(\phi)-1} \delta^{t-1}(Q_t^\phi - Q_{t-1}^\phi)(v_1 - c).$$

Theorem 4 *Suppose that $n = 2$. There exists $\bar{\phi} \in \left(0, 1 - \frac{c}{v_2}\right)$ such that, for all $\phi \in (0, \bar{\phi})$, $W(\phi) \geq W(0)$, with the equality being strict when $T(0) > 2$.*

Proof. See Appendix D. ■

This result follows from a comparative static analysis on the unique equilibrium path characterized recursively above. We first show that, for a sufficiently small tax rate ϕ, the amount of equilibrium delay is unchanged, i.e. $T(\phi) = T(0)$.[17] The main step of the proof then establishes, for the case of binary consumer types, that a small positive tax has the effect of reducing the volume of trade at each round of screening after the first period. As a consequence, more sales occur without delay and at a higher price. It is worth noting that a subsidy here generates opposite effects, and hence, reduces welfare.

Note from (4) that the equilibrium price path is in fact independent of ϕ. Consumer surplus is not affected, since each buyer is indifferent between accepting a higher price now and a lower price with delay. Therefore, welfare improvement occurs in the (discounted) sum of producer surplus and tax revenue. The simple tax policy is not budget-neutral and may result in a higher seller profit.

Theorem 4 does not extend beyond the case of binary consumer types. In the general case, the equilibrium trading volume in each period no longer responds monotonically to a slight change in c. As a consequence, the sign of welfare effects depends on parameters that determine the demand and supply conditions of the market. In Appendix E, we present some

[16] Even with two types, sequential screening may be a lengthy process when there is a large profit to be extracted from high valuation buyers, e.g. when $v_1 \gg v_2$.

[17] Note that one can find a small enough value of ϕ such that $T(\phi) = T(0)$ even when the social planner is imperfectly informed about the market demand.

numerical examples to illustrate these comparative static possibilities that can occur with more than two consumer types. Note again that constant tax is not optimal; indeed, our earlier results demonstrate the general welfare gain that can be achieved from using more flexible tax policy that depends on past history of transactions.

6 Concluding Remarks

This paper has demonstrated the possibility of engineering a dynamic tax policy to induce the socially efficient outcome in a bargaining model that otherwise generates inefficient delay. With marginal tax rates that decrease with trading volume, the social planner can provide incentives for the seller to abandon sequential screening and induce quicker trades after any history. This first-best policy is budget-neutral in unique equilibrium and derived independently of the discount factor. Moreover, a welfare-maximizing policy can be designed with limited information on the market demand conditions, as well as to ensure the seller's participation in absence of rational expectations. In addition, we explored the role of small, constant tax.

As previously mentioned, the idea of using tax policy to reduce delayed transactions is motivated by real world applications of taxing inventories. Our analysis considers an alternative form of taxation, i.e. transaction taxes. Nonetheless, the same intuition applies to a policy that taxes unsold goods instead. In Appendix F, we derive one such tax scheme that achieves the first-best outcome.

Another class of dynamic tax policy, as considered by Fuchs and Skrzypacz (2015) in a competitive setup, conditions the tax rate on time. One practical issue with this kind of policy is that a "period" in a bargaining game is meant to model the interval between one offer and the next, and as such, it may not correspond with policy maker's observations of calendar time. Trading volumes, in contrast, are typically revealed in business practices. This paper proposes tax policies that depend only on the sum of past trading volumes.

Furthermore, even if it were possible to align policy with actual bargaining intervals, we would not always obtain the first-best outcome under time-contingent tax policy. To see this, suppose instead that there exists such a policy inducing immediate market clearing. But, if there were many consumers with high valuations, the seller would find it profitable to deviate by offering a higher price in the first period and exclude low valuation consumers. Note that such a deviation works here because the tax rate is fixed in each period; under our transaction-contingent scheme, the deviation would lead to higher average tax and lower payoff.

Bargaining is one of the fundamental mechanisms via which resources are allocated, and the literature identifies numerous sources of frictions and welfare loss that can arise in a variety of situations. For instance, in dynamic monopoly bargaining, Board and Pycia (2014) observe that the seller would always charge the monopoly price if the buyers were endowed with outside options. Such an outcome would however fail to be incentive-compatible under a tax policy with decreasing marginal tax rate that this paper considers. Understanding the full implications of tax policy in other strategic settings of price formation offers many outstanding questions for future research.

Appendix

A Proof of Theorem 1

Note first that, for any $q \in [0, Q)$, we have

$$\int_q^Q f^*(s)ds \leq 0, \tag{6}$$

that is, the seller's total tax expenditure in any continuation game cannot be positive as long as he serves the entire market Q.

Consider any bargaining equilibrium $(\sigma_S^*, \{\sigma_i^*\}_{i \in N})$. Let $T^* \leq \infty$ denote the period in which the game ends with no remaining buyer in this equilibrium. We proceed in the following steps.

Step 1: Fix any $h \in H$. For any i remaining in the game, $\sigma_i^*(h, p) = 1$ if $p < (1-\delta)v_i + \delta v_n$.

Proof. It suffices to prove that, for all $i \in N$, $\sigma_i^*(h, p) = 1$ if $p < v_n$. Suppose not. So, there exists some i and some $\epsilon > 0$ such that $\sigma_i^*(h, p) < 1$ for $p > v_n - \epsilon$. Let $\tilde{p} \in [0, v_n)$ represent the supremum of prices which all the remaining buyers would accept for sure.

Clearly, by (6), the seller never offers a price below $\tilde{p}$. Thus, for any $i = 1, \ldots, n$, $\sigma_i(h, p) = 1$ if $p < (1-\delta)v_i + \delta\tilde{p}$. But then, $(1-\delta)v_i + \delta\tilde{p} > \tilde{p}$ for all i, which contradicts that $\tilde{p} < v_n$.

Step 2: Fix any $h \in H$. For any $p < v_n$, $\sigma_S^*(h)\{p\} = 0$.

Proof. Given (6), this follows immediately from the previous step.

Step 3: $T^* < \infty$.

Proof. Suppose not; so, suppose that $T^* = \infty$. There are two cases to consider.

Case 1: There exists some finite $\widetilde{T} > t$ from which no trade takes place.

Let $\widetilde{Q} < Q$ denote the final trading volume. But then, consider the seller offering a price arbitrarily smaller than v_n in period $\widetilde{T}$. By Step 1, all remaining buyers would accept it. Since $v_n > c$, and by (6), the deviation clearly improves the seller's continuation payoff.

Case 2: There does not exist any finite $\widetilde{T} > t$ after which no trade takes place.

For each $t' > t$, let $Q_{t'} < Q$ denote the corresponding accumulated trading volume. First, suppose that $\lim_{t' \to \infty} Q_{t'} \leq Q_{n-1}$. Then, for any $\epsilon > 0$, there must exist some $\widetilde{T} > t$ such that

$\lim_{t' \to \infty} Q_{t'} - Q_{\widetilde{T}} < \epsilon$. Consider the seller offering a price arbitrarily smaller than v_n in period $\widetilde{T}+1$. By (6), the deviation continuation payoff is at least

$$(v_n - c)(Q - Q_{\widetilde{T}}), \tag{7}$$

while the corresponding equilibrium continuation payoff is at most

$$(v_1 - c)\epsilon. \tag{8}$$

Clearly, for sufficiently small ϵ, (7) exceeds (8). This is a contradiction.

Next, suppose that $\lim_{t' \to \infty} Q_{t'} \in (Q_{n-1}, Q]$. Then, there exists $\widetilde{T} > t$ such that $Q_{\widetilde{T}} > Q_{n-1}$. Consider the seller offering a price arbitrarily smaller than v_n in period $\widetilde{T}+1$. The deviation payoff is approximately

$$((1 - \phi_n^*)v_n - c)(Q - Q_{\widetilde{T}}), \tag{9}$$

while the equilibrium payoff is at most

$$((1 - \phi_n^*)v_n - c)(Q_{\widetilde{T}+1} - Q_{\widetilde{T}}) + \delta((1 - \phi_n^*)v_n - c)(Q - Q_{\widetilde{T}+1}) \tag{10}$$

Since $\delta < 1$, it is clear that (9) is greater than (10), a contradiction.

Step 4: Fix any $t \geq 0$. Also, fix $h = \emptyset$ or any $h = (p_1, Q_1, \ldots, p_t, Q_t) \in H^t$ such that $t \geq 1$ and $Q_t < Q$. Then, the game ends in period $t+1$.

Proof. Suppose not. So, given Step 3, let $T < \infty$ denote the period in which the market clears in the given continuation game, and suppose that $T \geq t+2$.

Consider period $T-1$. Suppose that $Q_{T-1} \in (Q_{k-1}, Q_k]$ for some $k = 1, \ldots, n-1$.[18] By Step 1 and backward induction, it must then be that $p_T = v_n$ and $p_{T-1} = (1-\delta)v_k + \delta v_n$.

Consider the seller deviating to offering a price arbitrarily close to v_n from below in period $T-1$. By Step 1, such a deviation would induce all remaining buyers to accept immediately, and the corresponding continuation payoff is approximately

$$\int_{Q_{T-2}}^{Q} \big[\,(1 - f^*(s))\, v_n - c\big]\, ds. \tag{11}$$

The equilibrium continuation payoff is

$$\int_{Q_{T-2}}^{Q_{T-1}} \big[(1 - f^*(s))p_{T-1} - c\big]\, ds + \delta \int_{Q_{T-1}}^{Q} \big[(1 - f^*(s))v_n - c\big]\, ds. \tag{12}$$

[18]Note here that we cannot have $Q_{T-1} \in (Q_{n-1}, Q_n]$ because then $p_{T-1} = v_n$ and those who trade in period T would have an incentive to buy in period $T-1$ instead.

To see that (11) exceeds (12), the inequality can be re-written as

$$\begin{aligned}
& \int_{Q_{T-2}}^{Q_{T-1}} (1-f^*(s))(p_{T-1}-v_n)ds < (1-\delta)\int_{Q_{T-1}}^{Q} \big[(1-f^*(s))v_n - c\big]ds \\
\Leftrightarrow\quad & \int_{Q_{T-2}}^{Q_{T-1}} (1-f^*(s))(v_k-v_n)ds + \int_{Q_{T-1}}^{Q} f^*(s)v_n ds < \int_{Q_{T-1}}^{Q} (v_n-c)ds \\
\Leftrightarrow\quad & \int_{Q_{T-2}}^{Q_{T-1}} (1-f^*(s))(v_k-v_n)ds - \int_{0}^{Q_{T-1}} f^*(s)v_n ds < \int_{Q_{T-1}}^{Q} (v_n-c)ds. \qquad (13)
\end{aligned}$$

Since, for all $s \in (Q_{T-2}, Q_{T-1})$, $f^*(s) \geq 1 - \frac{v_n}{v_k}$, and hence, we have

$$(1-f^*(s))(v_k-v_n) \leq f^*(s)v_n,$$

the LHS of (13) is non-positive. Thus, the inequality in (13) holds, implying a profitable deviation. We have a contradiction.

Step 5: Fix any $h \in H$. For any i remaining in the game, $\sigma_i^*(h,p) = 0$ if $p > (1-\delta)v_i + \delta v_n$.

Proof. Suppose not, So, for some i and some $p > (1-\delta)v_i + \delta v_n$, we have $\sigma_i^*(h,p) > 0$.

By accepting p, the corresponding payoff to buyer i is equal to $v_i - p$ which is less than $\delta(v_i - v_n)$. If i rejects p, by the regularity condition, and by applying Steps 2 and 4 to the continuation game, the seller offers price v_n next period, and hence, the corresponding payoff is $\delta(v_i - v_n) > v_i - p$. This contradicts that $\sigma_i^*(h,p) > 0$.

Step 6: $T^* = 1$ and the seller offers price equal to v_n in period 1.

Proof. The first part is immediate from Steps 3 and 4. Given this, the second part is immediate from Steps 1, 2 and 5.

Now, consider the following strategy profile: for all $h \in H$,

$$\sigma_S(h)\{v_n\} = 1$$

and, for all $i \in N$ and for all $h \in H$,

$$\sigma_i(h,p) = \begin{cases} 1 & \text{if } p \leq (1-\delta)v_i + \delta v_n \\ 0 & \text{otherwise.} \end{cases}$$

The buyers' strategies are optimal at any $h \in H$ given Steps 1 and 5. To see that the seller has no profitable deviation after any history, we can similarly invoke the arguments for Step 4. By the definition of f^*, the corresponding total tax revenue is zero. Step 6 establishes the uniqueness of bargaining equilibrium outcome.

B Proof of Theorem 2

We proceed as follows.

Step 1: Fix any $\{q_i\}_{i=1}^n$. For any $i = 1, \ldots, n$ and $q \in (Q_{i-1}, Q_i]$, $f^{**}(q) \geq 1 - \frac{v_n}{v_i}$.

Proof. If $q \in (\overline{Q}_{i-1}, \overline{Q}_i]$ then $q > Q_{i-1}$, and hence, $q \in (Q_{j-1}, Q_j]$ for some $j \geq i$. Since $v_j < v_i$, it follows that $f^{**}(q) \geq 1 - \frac{v_n}{v_i} \geq 1 - \frac{v_n}{v_j}$, which establishes the claim.

Step 2: f^{**} is BNFB with unique equilibrium outcome.

Proof. While f^{**} does not satisfy the second property of f^*, we have that $\bar{f}^{**}(0, Q) = 0$ and $\bar{f}^{**}(q, Q) < 0$ for all $q \in [0, Q]$. Thus, by Step 1, it is straightforward to show that Step 4 of the proof of Theorem 1 continues to hold. This suffices to verify the existence and outcome uniqueness of bargaining equilibrium in which the market clears immediately.

C Proof of Theorem 3

Consider a tax policy f^R that satisfies the following properties:

- For any $i = 1, ..., n-1$ and any $q \in (Q_{i-1}, Q_i]$,

$$f^R(q) = \phi_i^R := 1 - \frac{c}{(1-\delta)v_{i+1} + \delta v_n} - \frac{Q_{i-1}}{q_i}\left(\frac{c}{(1-\delta)v_{i+1} + \delta v_n} - \frac{c}{(1-\delta)v_i + \delta v_n}\right).$$

- For any $q \in (Q_{n-1}, Q_n]$, $f^R(q) = \frac{-\sum_{i=1}^{n-1} \phi_i^R q_i}{q_n}$.

Claim 1: The average interval tax rate $\bar{f}^R$ satisfies the following:

(i) $\bar{f}^R(0, Q_i) = \begin{cases} 1 - \frac{c}{(1-\delta)v_{i+1} + \delta v_n} & \text{if } i = 1, \ldots, n-1 \\ 0 & \text{if } i = n. \end{cases}$

(ii) $\bar{f}^R(0, q)$ is decreasing in q.

(iii) For any $q, q' \in [0, Q]$ such that $q < q'$, $\bar{f}^R(q, q') \leq \bar{f}^R(0, q')$.

Proof of Claim 1. Parts (i) is derived via simple calculation. For part (ii), note that, for $q \in (Q_{i-1}, Q_i]$,

$$\begin{aligned}
\bar{f}^R(0, q) &= \frac{1}{q}\left\{\bar{f}^R(0, Q_{i-1})Q_{i-1} + \bar{f}^R(Q_{i-1}, q)(q - Q_{i-1})\right\} \\
&= \frac{1}{q}\left\{\left(1 - \frac{c}{(1-\delta)v_i + \delta v_n}\right)Q_{i-1} + \phi_i^R(q - Q_{i-1})\right\}.
\end{aligned}$$

Then, part (ii) follows since

$$\phi_i^R < 1 - \frac{c}{(1-\delta)v_{i+1} + \delta v_n} < 1 - \frac{c}{(1-\delta)v_i + \delta v_n}.$$

Finally, for part (iii), note that

$$\bar{f}^R(q, q') = \frac{1}{q'-q}\left(\bar{f}^R(0,q')q' - \bar{f}^R(0,q)q\right) < \bar{f}^R(0,q'),$$

where the last inequality is obtained since $\bar{f}^R(0,q) > \bar{f}^R(0,q')$ by part (ii) above.

Claim 2: f^R satisfies RPC.

Proof of Claim 2. Fix any subgame-rationalizable strategy profile $\left(\sigma_S^R, \{\sigma_i^R\}_{i\in N}\right)$. For any $k = 1, 2, \ldots$, let $\widetilde{\Sigma}_S^k \subset \Sigma_S$ and $\widetilde{\Sigma}_i^k \subset \Sigma_i$, respectively, denote the set of strategies of the seller and type i buyer that survive k rounds of iterated deletion of strategies that induce a "never-best-response (NBR)" continuation strategy at some $h \in H$. Also, for any $\sigma_S \in S$ and $h \in H$, define $supp(\sigma_S(h)) = \{p \in \mathbb{R}_+ | \sigma_S(h)\{p\} > 0\}$. The claim is proved via the following steps.

Step 1: Suppose that, for some $k = 1, 2, \ldots$ and some $\tilde{p} \in [0, v_n)$,

$$(-\infty, \tilde{p}) \cap \bigcup_{(\sigma_S, h)\in\widetilde{\Sigma}_S^k\times H} supp(\sigma_S(h)) = \emptyset.$$

Then, we have

$$(-\infty, (1-\delta)v_n + \delta\tilde{p}) \cap \bigcup_{(\sigma_S, h)\in\widetilde{\Sigma}_S^{k+1}\times H} supp(\sigma_S(h)) = \emptyset.$$

Proof. Suppose not. So, for some $\sigma_S \in \widetilde{\Sigma}_S^{k+1}$ and some $h \in H$, there exists some $\epsilon > 0$ such that

$$(-\infty, (1-\delta)v_n + \delta\tilde{p} - \epsilon) \cap supp(\sigma_S(h)) \neq \emptyset.$$

By assumption, the seller never offers a price below $\tilde{p}$. Therefore, for any $i \in N$, any $\sigma_i \in \widetilde{\Sigma}_i^k$ and any $\tilde{h} \in H$, $\sigma_i(\tilde{h}, p) = 1$ if $p < (1-\delta)v_i + \delta\tilde{p}$.

Now, consider another continuation strategy $\sigma'_S|h$ which is identical to $\sigma_S|h$ except that

$$\sigma'_S|h(\emptyset)\{(1-\delta)v_n + \delta\tilde{p} - \epsilon\} = \sigma_S|h(\emptyset)(-\infty, (1-\delta)v_n + \delta\tilde{p} - \epsilon].$$

Since all the survived strategies of each type i buyer accepts for sure any price below $(1-\delta)v_n + \delta\tilde{p}$, the deviaion is profitable, which is a contradiction.

Step 2: For any $h \in H$, we have the following:

(i) The support of $\sigma_S^R(h)$ is disjoint with $[0, v_n)$.

(ii) $\sigma_i^R(h,p) = 1$ if $p < (1-\delta)v_i + \delta v_n$.

Proof. By Step 1, the claim follows from applying induction from $\tilde{p} = 0$.

Step 3: Fix any $t \geq 0$. Fix $h = \emptyset$ or any $h = (p_1, Q_1, \ldots, p_t, Q_t) \in H^t$ such that $t \geq 1$. Also, fix any possible outcome (p_{t+1}, Q_{t+1}) that can be induced by $\left(\sigma_S^R|h, \{\sigma_i^R|h\}_{i\in N}\right)$ in period $t+1$. Then, we have

$$\int_{Q_t}^{Q_{t+1}} \left[(1 - f^R(s))p_{t+1} - c\right] ds \geq 0.$$

Proof. The inequality is equivalent to

$$(1 - \bar{f}^R(Q_t, Q_{t+1}))p_{t+1} - c \geq 0 \quad \Leftrightarrow \quad \bar{f}^R(Q_t, Q_{t+1}) \leq 1 - \frac{c}{p_{t+1}}. \tag{14}$$

Since $\left(\sigma_S^R|h, \{\sigma_i^R|h\}_{i\in N}\right)$ are subgame-rationalizable, $p_{t+1} \geq v_n$. Thus, there exists $i = 1, \ldots, n-1$ such that

$$(1-\delta)v_i + \delta v_n > p_{t+1} \geq (1-\delta)v_{i+1} + \delta v_n.$$

By Step 2, all the remaining buyers with valuation up to, and including, v_i accept p_{t+1} for sure. Thus, $Q_{t+1} \geq Q_i$.

Then, we have

$$\bar{f}^R(Q_t, Q_{t+1}) \leq \bar{f}^R(0, Q_{t+1}) \leq \bar{f}^R(0, Q_i) = 1 - \frac{c}{(1-\delta)v_{i+1} + \delta v_n} \leq 1 - \frac{c}{p_{t+1}},$$

where the first and second inequalities follow from part (iii) and part (ii) of Claim 1, respectively. This establishes (14).

Claim 3: f^R is BNFB, and the bargaining equilibrium is unique.

Proof of Claim 3. It suffices to establish that, in any bargaining equilibrium, the market clears immediately in any continuation game. The rest of the proof is identical to that of Theorem 1.

Fix any $t \geq 0$. Also, fix $h = \emptyset$ or any $h = (p_1, Q_1, \ldots, p_t, Q_t) \in H^t$ such that $t \geq 1$ and $Q_t < Q$. Assume that the market clears in period $T > t+1$. By the corresponding arguments in the proof of Theorem 1 (see Step 3), it must be that $T < \infty$. Suppose that $T \geq t+2$.

Consider period $T-1$ such that $Q_{T-1} \in (Q_{k-1}, Q_k]$ for some $k = 1, \ldots, n-1$. We know that that $p_T = v_n$ and $p_{T-1} = (1-\delta)v_k + \delta v_n$. If the seller deviates in period $T-1$ by offering a price slightly below v_n, all remaining buyers accept immediately and the continuation payoff is approximately

$$\int_{Q_{T-2}}^{Q} \left[\left(1 - f^R(s)\right) v_n - c\right] ds. \tag{15}$$

The equilibrium continuation payoff is

$$\int_{Q_{T-2}}^{Q_{T-1}} \big[(1-f^R(s))p_{T-1}-c\big]ds + \delta \int_{Q_{T-1}}^{Q} \big[(1-f^R(s))v_n - c\big]ds. \tag{16}$$

To see that (15) exceeds (16), the inequality can be re-written as

$$\int_{Q_{T-2}}^{Q_{T-1}} (1-f^R(s))(p_{T-1}-v_n)ds < (1-\delta)\int_{Q_{T-1}}^{Q} \big[(1-f^R(s))v_n - c\big]ds$$

Because $f^R(s) < 1$ for all $s \in [0, Q]$, this inequality holds if it does when $Q_{T-2} = 0$, or

$$\begin{aligned}
&\int_{0}^{Q_{T-1}} (1-f^R(s))(p_{T-1}-v_n)ds < (1-\delta)\int_{Q_{T-1}}^{Q} \big[(1-f^R(s))v_n - c\big]ds \\
\Leftrightarrow\ &\int_{0}^{Q_{T-1}} (1-f^R(s))(v_k-v_n)ds + \int_{Q_{T-1}}^{Q} f^R(s)v_n ds < \int_{Q_{T-1}}^{Q} (v_n - c)ds \\
\Leftrightarrow\ &\int_{0}^{Q_{T-1}} (1-f^R(s))(v_k-v_n)ds - \int_{0}^{Q_{T-1}} f^R(s)v_n ds < \int_{Q_{T-1}}^{Q} (v_n - c)ds.
\end{aligned} \tag{17}$$

Note that, for all $s \in (Q_{k-1}, Q_k]$,

$$\bar{f}^R(0,s) \geq \bar{f}^R(0,Q_k) = 1 - \frac{c}{(1-\delta)v_{k+1}+\delta v_n} > 1 - \frac{v_n}{v_k},$$

where the first inequality follows from part (ii) of Claim 1 and the last inequality from the assumption that

$$c < \min\left\{\frac{v_n\left[(1-\delta)v_{i+1}+\delta v_n\right]}{v_i}\right\}_{i\in N}.$$

It then follows that

$$(1-\bar{f}^R(0,Q_{T-1}))(v_k - v_n) < \bar{f}^R(0,Q_{T-1})v_n,$$

and hence, the LHS of (17) is negative. We therefore have a profitable deviation.

D Proof of Theorem 4

We proceed in the following steps.

Step 1: For every $t \geq 1$, L_t is decreasing and continuous in c.

Proof. We use induction to establish this step. First, note that $L_1 = q_2$ and, using (5), we have

$$L_2(\pi_2 - \pi_1) = (1-\delta)L_1(\pi_1 - c),$$

which, since $\pi_1 = v_2$, implies

$$L_2 = \frac{(1-\delta)q_2}{\pi_2 - v_2}(v_2 - c).$$

Clearly, L_1 and L_2 are decreasing and continuous in c.

Next, fix any $t \geq 2$, and suppose that, for all $\tau \leq t$, L_τ is decreasing and continuous in c. Consider $\tau = t+1$.

By (5), we obtain

$$(\pi_{t+1} - \pi_t)L_{t+1} = (1-\delta)\sum_{\tau=1}^{t} \delta^{\tau-1}(\pi_{t+1-\tau} - c)L_{t+1-\tau},$$

which is equivalent to

$$L_{t+1} = \frac{(1-\delta)}{\pi_{t+1} - \pi_t}\sum_{\tau=1}^{t} \delta^{\tau-1}(\pi_{t+1-\tau} - c)L_{t+1-\tau}. \tag{18}$$

By assumption, $L_{t+1-\tau}$ is decreasing and continuous in c for all $\tau = 1, 2, \ldots, t$. Then, the RHS of (18), and hence, L_{t+1}, are also decreasing and continuous in c.

Step 2: There exists $\bar{\phi} \in \left(0, 1 - \frac{c}{v_2}\right)$ such that $T(\phi) = T(0)$ for all $\phi \in \left(0, \bar{\phi}\right)$.

Proof. We write $L_t = L_t(c)$ for ease of exposition. Note that, by the recursive structure of the unique equilibrium path above, we have

$$T(0) = \max\left\{ t = 1, 2, \ldots \mid \sum_{\tau=2}^{t} L_\tau(c) \leq q_1 \right\},$$

which means that

$$\sum_{\tau=2}^{T(0)} L_\tau(c) \leq q_1 < \sum_{\tau=2}^{T(0)+1} L_\tau(c).$$

Since, by Step 1, $L_t(\cdot)$ is decreasing and continuous for all $t \geq 1$, it is clear that, for small enough $\phi > 0$, we have

$$\sum_{\tau=2}^{T(0)} L_\tau\left(\frac{c}{1-\phi}\right) \leq q_1 < \sum_{\tau=2}^{T(0)+1} L_\tau\left(\frac{c}{1-\phi}\right).$$

By Lemma 1, this implies that $T(\phi) = T(0)$.

Step 3: Fix any $\phi \in [0, \bar{\phi})$. Then, Q_1^ϕ is increasing in ϕ, but $Q_t^\phi - Q_{t-1}^\phi$ is decreasing in ϕ for all $t = 2, 3, \ldots, T$.

Proof. By Lemma 1 and its corollary, the game with $f \equiv \phi$ admits a unique equilibrium path $\left(p_1^\phi, Q_1^\phi, p_2^\phi, Q_2^\phi, \ldots, p_{T(\phi)}^\phi, Q_{T(\phi)}^\phi\right)$ and, by Step 2, $T(\phi) = T(0)$. Moreover, from the recursive structure, we have

$$Q_t^\phi - Q_{t-1}^\phi = \begin{cases} q_1 - \sum_{\tau=2}^{T(0)-1} L_\tau\left(\frac{c}{1-\phi}\right) & \text{if } t = 1 \\ L_{T(0)+1-t}\left(\frac{c}{1-\phi}\right) & \text{if } 2 \le t \le T(0) \end{cases}$$

Thus, the claim follows immediately from Step 1.

Step 4: $W(\phi)$ is increasing in $\phi \in [0, \bar{\phi})$.

Proof. Since $Q_{T-1}^\phi = q_1$, we can write

$$\begin{aligned} W(\phi) &= \delta^{T-1} q_2 (v_2 - c) + \sum_{t=1}^{T-1} \delta^{t-1}(Q_t^\phi - Q_{t-1}^\phi)(v_1 - c) \\ &= \delta^{T-1} q_2 (v_2 - c) + q_1(v_1 - c) - \sum_{t=2}^{T-1}(1 - \delta^{t-1})(Q_t^\phi - Q_{t-1}^\phi)(v_1 - c). \end{aligned} \tag{19}$$

By Step 3, the RHS of (19) is increasing in $\phi \in [0, \bar{\phi})$.

E Simple Tax: Numerical Examples

Define $\widehat{Q} := \{q_1, \ldots, q_n\}$ and $\widehat{V} := \{v_1, \ldots, v_n\}$. Suppose that $n = 3$, $\widehat{Q} = \{0.6, 0.3, 0.1\}$, $\widehat{V} = \{4, 2, 1\}$, $c = 0.8$, and $\delta = 0.6$. Figure 3 below illustrates equilibrium social welfare $W(\phi)$ (left panel) and seller profit (right panel) as a function of constant tax rate $\phi \ge 0$.

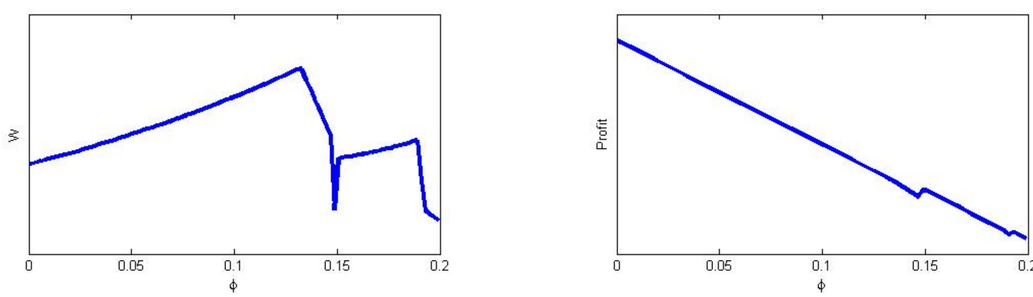


Figure 3: Social welfare and producer surplus

Social welfare increases monotonically for low values of ϕ. In this region of tax rates, pricing remains unchanged and the quantity sold in period 1 rises. These are shown in Figure 4.

We next plot the welfare function under varying demand and supply conditions. First, the welfare improvement from a small tax is greater when the demand schedule is more elastic.

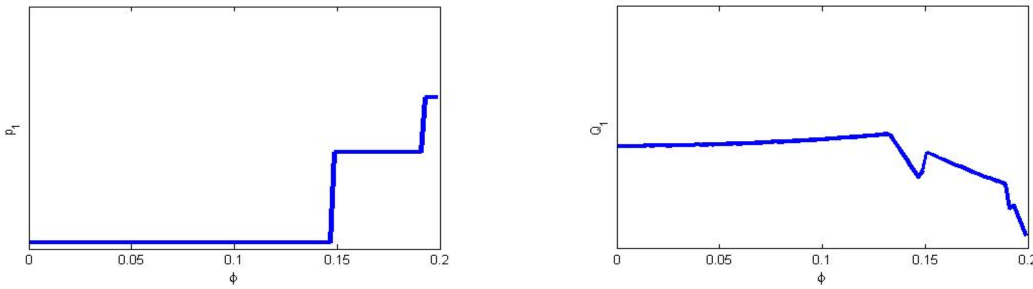


Figure 4: Price and quantity

The left panel of Figure 5 illustrates $W(\phi)$ under $V^0 = \{4, 2, 1\}$, $V' = \{2.5, 1.5, 1\}$ and $V'' = \{10, 4, 1\}$, respectively. In the last case, $W(\phi)$ is decreasing for ϕ near zero. Second, the sign of welfare effect is sensitive to the cost. In the right panel of Figure 5, we consider $c^0 = 0.8$, $c' = 0.75$ and $c'' = 0.93$ (under V^0). Here, negative welfare effects are observed in the latter two cases.

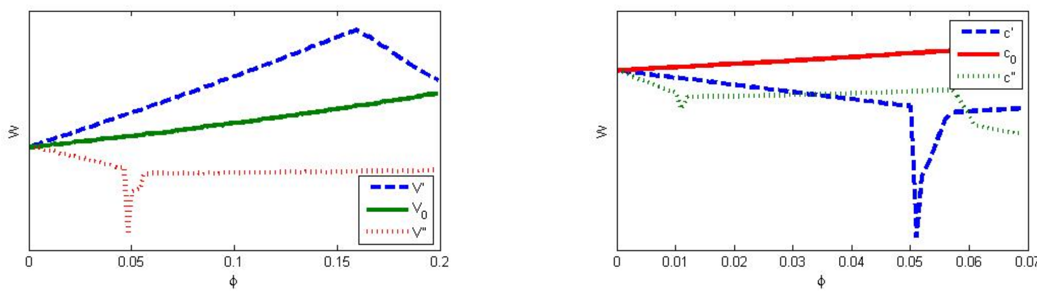


Figure 5: Welfare effects by varying demand and supply

F Inventory Tax

An *inventory* tax policy, I, is a continuous function

$$I : [0, Q] \to \mathbb{R}$$

such that, for all $q \in [0, Q]$, $I(q)$ specifies the rate of *unit* tax levied at the remaining inventory $Q - q$. For any t and any $(p_1, Q_1, \ldots, p_t, Q_t)$, the seller's payoff is given by

$$\sum_{\tau=1}^{t} \delta^{\tau-1} \big[(p_\tau - c)(Q_\tau - Q_{\tau-1}) - I(Q_\tau)(Q - Q_\tau) \big],$$

and the tax revenue is

$$\sum_{\tau=1}^{t} \delta^{\tau-1} I(Q_\tau)(Q - Q_\tau).$$

Theorem 5 *Consider any tax policy I^* such that, for any $i = 1, ..., n-1$ and any $q \in (Q_{i-1}, Q_i]$,*

$$I^*(q) \geq (1-\delta) \left[\frac{(v_i - c)q - (v_n - c)Q}{Q - q} \right].$$

Then, I^* is BNFB.

Proof. The proof is analogous to that of Theorem 1. It suffices to check that Step 4 continues to hold. Fix any tax policy I^* satisfying the stated property and any bargaining equilibrium.

Claim: Fix any $t \geq 0$. Also, fix $h = \emptyset$ or any $h = (p_1, Q_1, \ldots, p_t, Q_t) \in H^t$ such that $t \geq 1$ and $Q_t < Q$. Then, the game ends in period $t+1$.

Proof. Suppose not. So, let $T < \infty$ denote the period in which the market clears, and suppose that $T \geq t+2$.

Consider period $T-1$. Suppose that $Q_{T-1} \in (Q_{k-1}, Q_k]$ for some $k = 1, \ldots, n-1$. Then, it must be that $p_T = v_n$ and $p_{T-1} = (1-\delta)v_k + \delta v_n$.

Consider the seller deviating to offering a price arbitrarily close to v_n from below in period $T-1$. Such a deviation would induce all remaining buyers to accept immediately, and the corresponding continuation payoff is approximately

$$(v_n - c)(Q - Q_{T-2}). \tag{20}$$

The equilibrium continuation payoff is

$$\left[(p_{T-1} - c)(Q_{T-1} - Q_{T-2}) - I^*(Q_{T-1})(Q - Q_{T-1})\right] + \delta(v_n - c)(Q - Q_{T-1}). \tag{21}$$

To see that (20) exceeds (21), the inequality can be re-written as

$$\begin{aligned} & (p_{T-1} - v_n)(Q_{T-1} - Q_{T-2}) - I^*(Q_{T-1})(Q - Q_{T-1}) < (1-\delta)(v_n - c)(Q - Q_{T-1}) \\ \Leftrightarrow \quad & (1-\delta)\left[(v_k - v_n)(Q_{T-1} - Q_{T-2}) - (v_n - c)(Q - Q_{T-1})\right] < I^*(Q_{T-1})(Q - Q_{T-1}), \end{aligned}$$

which holds if it holds when $Q_{T-2} = 0$, that is,

$$(1-\delta)\left[(v_k - c)Q_{T-1} - (v_n - c)Q\right] \quad < \quad I^*(Q_{T-1})(Q - Q_{T-1}) \tag{22}$$

Clearly, by the definition of I^*, (22) holds, implying a profitable deviation. We have a contradiction. ■

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