Aggregate Fluctuations and the Role of Trade Credit

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Abstract

In an economy where production takes place in multiple stages and is subject to financial frictions, how firms finance intermediate inputs matters for aggregate outcomes. This paper focuses on trade credit—the lending and borrowing of input goods between firms—and quantifies its aggregate impacts during the Great Recession. Motivated by empirical evidence, our model shows how trade credit alleviates financial frictions through a process of credit redistribution and creation, thus leading to a higher output level in the steady state. However, in the face of financial market distress, suppliers cut back trade credit lending, further tightening their customers' borrowing constraint. The decline in economic activities following financial shocks is in turn amplified by disruptions in trade credit. Our model simulation suggests that the drop in trade credit during the Great Recession can account for almost one-fourth of the observed decline in output.

JEL classification: E32, E44, L23, L14

Keywords: Aggregate fluctuations, financial crisis, financial frictions, produc-

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1 Introduction

Financial shocks are associated with severe contractions in real economic activities. One prominent example is the 2007–09 financial crisis, followed by what is regarded as the most severe recession since the Great Depression. Following the seminal contribution of Jermann and Quadrini (2012), many papers have studied the macroeconomic effects of financial shocks, in particular in the context of the 2007–09 financial crisis.

So far, almost all of these papers focus on studying how the disruption of credit flows from the financial sector to the real sector affects real economic activities. These existing theories, however, do not take into account the fact that U.S. firms rely heavily on trade credit—their suppliers' lending of inputs—to meet their working capital needs in production, and that the collapse of trade credit played a key role in creating the liquidity shortage faced by U.S. firms during the 2007–09 crisis.¹

In this paper, we explore quantitatively the role played by trade credit in the financial crisis. To this end, we incorporate trade credit into a dynamic general equilibrium model with heterogeneous entrepreneurs, which allows us to study jointly the dynamics of trade credit, bank credit, aggregate productivity, and output.

In the model, the production of final goods takes place in two stages: the intermediate goods stage and the final goods stage. In each stage, there is a continuum of heterogeneous entrepreneurs operating a decreasing return to scale production technology. Homogeneous workers provide labor and enjoy leisure, but do not have access to the asset markets; i.e., they are "hand-to-mouth."

The model has one key new ingredient, that is, the coexistence of trade credit and bank credit as a means of financing working capital. Due to banks' limited enforcement over the repayment of loans, the size of bank loans entrepreneurs can take out is limited by a collateral constraint. Compared with banks, suppliers of inputs—in this case the intermediate goods entrepreneurs—have a comparative advantage in lending to their customers. However, unlike banks, intermediate goods entrepreneurs do not have access to unlimited funds at the equilibrium interest rate.

¹In 2006, the year before the crisis, the aggregate size of the trade credit liability of the nonfinancial corporate sector was approximately one-third the size of its quarterly GDP. From 2007Q4 to 2009Q2, total short-term liability of the nonfinancial corporate firms dropped by more than US \$400 billion, of which approximately 70 percent can be explained by the drop in trade credit.

Lending inputs can be very costly for the intermediate goods entrepreneurs if they themselves are financially constrained. The marginal willingness to lend inputs, therefore, is positively correlated with the intermediate goods entrepreneurs' access to bank credit.

We use a calibrated version of our model to study the role played by trade credit during the 2007–09 financial crisis. A bank credit crunch in our model leads to a larger aggregate output loss compared with a counterfactual model in which bank credit is the only source of financing. The tightening of bank credit makes the entrepreneurs more constrained; as a result, intermediate goods entrepreneurs cut back their lending of trade credit. This results in a larger drop in trade credit relative to output in the equilibrium. Because the final goods entrepreneurs that rely on trade credit are on average more productive, the drop in trade credit essentially leads to a shift of resources that exacerbates the aggregate loss of productivity. This indirect effect through the contraction of trade credit, vis-à-vis the direct effect through the tightening of bank credit, is the driving force behind the larger aggregate output loss in our model economy with the coexistence of trade credit and bank credit.

Related literature There exists a long strand of literature on the theoretical foundations and the empirical properties of trade credit.² Theoretically, our paper builds on the insight that the existence of trade credit reflects a certain comparative advantage of the suppliers in lending inputs to their customers compared with the financial intermediaries (Biais and Gollier, 1997; Burkart and Ellingsen, 2004; Cuñat, 2007). Empirically, our results confirm the "redistributive view" of trade credit in the literature (Meltzer, 1960; Love, Preve and Sarria-Allende, 2007). That is, trade credit helps channel financial resources to flow from financially advantaged firms to disadvantaged ones. We find that the drop in trade credit during the 2007–09 financial crisis can be attributed to the tightening of firms' access to bank credit. A similar conclusion is found by Love, Preve and Sarria-Allende (2007) for the emerging market financial crises. In general, our paper contributes to the empirical literature by providing new firm-level evidence with new identification strategies.

This paper is also related to the literature on the propagation of shocks through

²See Petersen and Rajan (1997) and Cuñat and Garcia-Appendini (2012) for excellent surveys of the literature.

trade credit. Kiyotaki and Moore (1997) build a theory illustrating how shocks to one firm propagate in a network through a chain of trade credit default. This theory is tested by Raddatz (2010) using cross-country sectoral-level data, and by Jacobson and von Schedvin (2015) using Swedish matched firm-to-firm data. The framework of Kiyotaki and Moore (1997) is also used to study the interbank lending market (see Boissay and Cooper, 2016; Lee, 2015; Zhang, 2014). The theoretical framework employed in our paper differs from the papers mentioned above in two ways. First, it models jointly the production and the lending of inputs, whereas all of these papers abstract from production. Second, the propagation of shocks in our paper does not depend on trade credit default, but works through the changes in trade credit supply and demand on the intensive margin.³

More broadly speaking, this paper contributes to two recent developments in the literature that studies the real impacts of financial shocks.

One recent development is to take into account explicitly the input-output linkages in production. Among these papers, Zetlin-Jones and Shourideh (2017) emphasize the real linkages and show that financial shocks can be amplified if there is strong enough complementarity between the intermediate input goods in the production function. Kalemli-Ozcan et al. (2014) build on Kim and Shin (2012), in which trade credit helps sustain long production chains that are more productive than short ones. Financial shocks are amplified in this environment because longer production chains are less viable in financial crises. In this literature, perhaps the paper by Bigio and La'O (2014) is the closest to ours. By assuming that only a fixed fraction of the inputs is purchased using trade credit, Bigio and La'O (2014) show that the input-output structure itself can amplify financial shocks because the multiple financing of inputs increases the aggregate liquidity needs to sustain production. Our paper is complementary to Bigio and La'O (2014). Instead of studying how different input-output structures affect the propagation and amplification of financial shocks, we take as given a simple two-stage production chain, and we focus on exploring the causes and aggregate implications of trade credit dynamics as a result of firm heterogeneity.

Another new development in this literature is to incorporate the producer het-

³The only exception is perhaps Boissay and Cooper (2016), who find that in the process of lending to firms, banks create "inside collateral," which can be used to borrow in the interbank lending market. The creation of collateral gives rise to multiple equilibria in the interbank lending market and makes it more fragile.

erogeneity into the quantitative dynamic general equilibrium framework. Contributions to this development include Buera and Moll (2015), Buera, Fattal-Jaef and Shin (2015), Jermann and Quadrini (2012), and Khan and Thomas (2013). Our paper makes a contribution to this strand of literature by looking beyond the disruptions of credit flows from the financial sector to the nonfinancial sector. We explore instead the aggregate implications of credit flows between heterogeneous firms within the nonfinancial sector.

The rest of the paper is organized as follows: section 2 presents the empirical motivation for the model, section 3 presents the model, section 4 defines and analyzes the recursive competitive equilibrium, section 5 provides a quantitative analysis of the model, and section 6 concludes.

2 Empirical motivation

This section presents empirical evidence that motivates our model. Section 2.1 examines the financial determinants of the distribution of trade credit across different firms in normal times; section 2.2 examines the financial determinants of the dynamics of trade credit during the 2007–09 financial crisis.

Before we present the empirical evidence, the measure of trade credit deserves some discussion. Since trade credit is essentially the lending and borrowing between firms, ideally, we want to have a measure of trade credit *flows* between firms. However, the construction of such a measure requires information about the trade credit contracts—the value of goods sold, trade credit as a share of sales, and the trade credit interest rate (see for example the data used in Klapper, Laeven and Rajan, 2012). To our knowledge, such data are not available on a large scale. In this paper, following the existing literature, we measure trade credit using its *stock*. More formally, we use accounts receivable (AR) to measure firms' lending of trade credit to other firms, accounts payable (AP) to measure firms' borrowing of trade credit from other firms, and net accounts receivable (Net AR=AR-AP) to measure firms' net lending of trade credit.

2.1 The financial determinants of trade credit in normal times

In this section, we test whether financially constrained firms rely more on trade credit than do unconstrained firms. This empirical test is motivated by the observation that small firms rely much more heavily on trade credit than do large firms, and that the smaller firms are on average more financially constrained. As shown in Figure A1, the ratio of net accounts receivable to sales, a measure of net lending of trade credit, is slightly more than 50 percent for firms whose total asset value is higher than US \$500 million. In contrast, for the firms whose total asset value is less than US \$0.5 million, the net lending of trade credit is essentially 0.4

Data To construct our sample of firms, we combine the Compustat North America annual database with the Survey of Small Business Finances (SSBF) database for the years when the SSBF data are available (1987, 1993, 1998 and 2003). Firms in the financial sector (SIC 60-69) and wholesale and retail sector (SIC 50-59) are dropped.⁵

We first consider the sample consisting of only Compustat firms. Following Almeida and Campello (2007), we create three different dummy variables indicating whether a firm is constrained (I_constrained $_{it} = 1$). The first one is based on payout ratio—a firm with a zero payout ratio in year t is identifies as being financially constrained in that year. In the second definition, a firm is identified as financially constrained if it has neither a long-term nor a short-term bond rating from Standard & Poor's (S&P). The third one is based on asset size of firms. A firm is financially constrained if it is among the bottom 30 percentile in asset size distribution.

Second, we augment the above sample of Compustat firms with the SSBF data, which contain relatively small and private firms. This combined Compustat-SSBF sample offers a more comprehensive coverage of the whole population of U.S. firms. For this sample, we define a firm as financially constrained if it belongs to the bottom

⁴The fact that small firms rely more on trade credit than do large firms is first documented by Meltzer (1960).

⁵The financial sector is excluded because we focus on nonfinancial firms in this paper. The decision to exclude the wholesale and retail sectors is based on two facts. First, previous research shows that the choice of trade credit between retailers and their suppliers is affected by the monopolistic power of large retail stores such as Walmart. Second, accounts receivable of retailers and wholesale firms might contain consumer credit. The result does not change by much if we include the retail and wholesale sectors.

30 percentile in asset size distribution.⁶

Empirical specification We apply the following specification to estimate the effect of financial constraint on firms' choice of trade credit,

$$y_{ist} = \alpha I_{constrained_{it}} + \chi_i + \phi_{st} + \epsilon_{ist},$$
 (1)

where y_{ist} is one of the three measures of trade credit—AR/sales, AP/sales, and net AR/sales—of firm i in sector s of year t, ϕ_{st} is the sector-year fixed effect, and χ_i is a set of other time-invariant firm characteristics such as whether it is a corporation.⁷ The estimated coefficient α of the dummy variable I_constrained_{it} is the object of interest. We expect α to be significant and positive if the dependent variable is the borrowing of trade credit; we expect it to be significant and negative if the dependent variable is the lending (or net lending) of trade credit.

Results In Panel (A) of Table 1, we run specification 1 with net AR/sales as the dependent variable. Compared with the unconstrained firms, the financially constrained firms—in net terms—lend out significantly less trade credit. It is 6.2 percentage points lower for the firms with a zero payout ratio (column 1), 5.8 percentage points lower for the firms that do not have an S&P rating (column 2), 11.5 percentage points (column 3) and 17.1 percentage points (column 4) lower for firms that belong to the bottom 30 percentile of the asset distribution in the Compustat sample and the Compustat-SSBF sample, respectively.

In Panels (B) and (C), we run specification 1 using AP/sales and AR/sales as dependent variables, respectively. As shown in Panel (B), financially constrained firms maintain a significantly larger accounts payable; i.e., a larger fraction of their inputs are borrowed. However, perhaps more interestingly, as shown in Panel (C), the impact of being financially constrained on the lending of trade credit is much smaller and more ambiguous than that on the borrowing of trade credit.

One possible explanation for the weaker correlation between being financially

⁶Compustat has a decent number of small firms. In this Compustat–SSBF sample, approximately 22 percent of the financially constrained firms are Compustat firms.

⁷Unfortunately, since Compustat only has information on the year of firms' initial public offering (IPO), but not the year of incorporation, we cannot control for firm age in these regressions, which admittedly is an important factor affecting the choice of trade credit.

constrained and the lending of trade credit is the existence of *accounts receivable financing*, which is the issuance of accounts-receivable-collateralized loans by financial intermediaries.⁸ Consider the case in which accounts receivable cannot be used as collateral to take out bank loans: lending one dollar of trade credit means one dollar of liquidity loss for the firm. With the help of accounts receivable financing, the liquidity loss associated with lending trade credit is reduced. In an extreme case, if the advance rate of accounts receivable is 100 percent, the cost of lending trade credit, even for liquidity constrained firms, is essentially 0.⁹

For the purpose of motivating our model and the quantitative analysis, it is important to note that the existence of accounts receivable financing changes the nature of trade credit. Without it, trade credit serves merely as a redistribution channel, directing credit from unconstrained to constrained firms. With it, a collateralizable asset (accounts receivable) is created whenever firms lend trade credit to other firms. Through the process of collateral creation, accounts receivable financing increases the collective access to bank credit for both trade credit lenders and borrowers.

2.2 The financial determinants of trade credit in a financial crisis

In this section, we explore the reasons behind the huge drop in trade credit relative to output during the 2007–09 financial crisis. The goal is to test whether the drop in trade credit during the crisis can be attributed to the disruptions in firms' access to the financial market that makes them cut back their trade credit lending. Since the drop in trade credit is an equilibrium outcome, the key to this exercise is to identify the supply side forces driving the drop in trade credit.

To this end, we adopt a similar strategy as in Chodorow-Reich (2014), which uses

⁸Accounts receivable financing in the United States was a financial innovation first appeared in the early 1900s (see Murphy, 1992). It has always been an important part of the trade credit practice in reality, but is often neglected in the existing literature.

⁹Because of the lack of data, we do not know the aggregate size of accounts receivable financing in the United States. However, the Thomson Reuters DealScan data on loans issued in the syndicated loan market indicate that accounts receivable financing is rather important. Take the secured credit-line facilities that were opened during 2004–06 as an example: 46.3 percent of them require accounts receivable as collateral, while the rest require other types of assets such as equipment and property. Accounts receivable also has a much higher collateral value than other assets: the average advance rate of accounts receivable is 87 percent, much higher than the 59 percent advance rate for "inventory of all kinds" and the 29 percent advance rate for "property, plant, and equipment."

the performance of firms' relationship banks as an exogenous variation in their access to the bank credit. As argued by Chodorow-Reich (2014) and Sufi (2007), a certain degree of information friction is associated with bank lending. Over time, firms establish a borrowing and lending relationship with a certain bank. The relationship bank accumulates superior information about this firm; therefore, it is costly for the firm to switch to a new lender because the accumulated information would be lost during the switch. A firm's access to bank credit is hindered if its relationship bank goes into financial distress. Therefore, by using the performance of firms' relationship banks as an exogenous source of variation in bank credit availability, we are able to estimate the supply side forces behind the drop in trade credit; i.e., firms cut back their trade credit supply in response to a tightening access to bank credit.

Data Due to data limitations, we focus on a group of Compustat firms that borrow from the syndicated loan market.¹⁰ With the help of the loan-level information of the syndicated loan market taken from the Thomson Reuters DealScan database and the link table provided by Chava and Roberts (2008), we can match the Compustat firms with their lenders in the syndicated loan market.¹¹

The syndicated loan is a type of loan whereby two or more lenders jointly issue funds to a firm. By the nature of the loans, firms have multiple lenders in the syndicated loan market. These lenders can be categorized into two types: lead lender and participants. The lead lender differs from the participants by accumulating superior information regarding the borrower (see Sufi, 2007 and Chodorow-Reich, 2014). We therefore treat the lead lender as the firm's relationship bank in our exercise.

To construct the DealScan-Compustat sample with firms and their relationship banks, we first drop the observation (a loan facility) in the DealScan database if it falls into one of the following categories: 1) the borrower is in the financial, insurance, retail, and wholesale sector, 2) the facility has multiple lead lenders, 3) the fa-

¹⁰Over the past several decades, the syndicated loan market has become one of the most important channels for firms, especially large firms in the U.S., to obtain funds. According to Ivashina and Scharfstein (2010), the syndicated loan market also played an important role for firms to obtain liquidity during the 2007–09 financial crisis.

¹¹The DealScan database contains records of the syndicated loans issued globally and in the United States. Its coverage of the U.S. syndicated loan market is very comprehensive, especially in the post-1995 era. Each observation in the data is a facility (loan). Detailed information about the loan, such as loan type, size, and maturity, is gathered from SEC filings, including 13-Ds, 14-Ds, 13-Es, 10-Ks, 10-Qs, 8-Ks, and S-series.

cility is not open during the period from January 1, 2004 to December 31, 2006, and 4) the lead lender is not among the top 43 lenders as defined in Chodorow-Reich (2014). We then use the link table provided by Chava and Roberts (2008) to match the lead lender of each loan facility in the DealScan database with the borrower from the Compustat database. If a firm has only one open facility during the period from January 1, 2004 to December 31, 2006, we define the lead lender of that facility to be its pre-crisis relationship bank. If a firm has multiple open facilities during that period, we define the lead lender of the newest facility as its relationship bank.

The above process yields a panel of 1,219 firm-bank pairs over the period 2007Q1 to 2009Q4 at a quarterly frequency. The sample is a good representation of the whole universe of Compustat firms in terms of sectoral composition. However, compared with the average Compustat firm, firms in this DealScan-Compustat sample are much larger. The average DealScan-Compustat firm is eight times as large as the rest of the Compustat firms. Among the DealScan-Compustat firms, 393 have a third-party credit rating. In short, the DealScan-Compustat sample consists of very large and financially advantaged firms.

Empirical specification We define a crisis indicator Crisis_t, which takes value 1 during the period of crisis (2007Q4 to 2009Q4). For each firm-bank pair in the DealScan-Compustat sample, we define a dummy variable Unhealthy_i, which takes value 1 if the bank belongs to the bottom 50 percentile in terms of the percentage drop in the issuance of new loans during the crisis period.¹²

The dependent variable is $AR_{it}/Sales_{it}$ of firm i and time t. Our baseline specification is a fixed-effect regression of the following form,

$$\begin{split} AR_{it}/Sales_{it} &= \beta_1 AP_{it}/Sales_{it} + \beta_2 Crisis_t + \beta_3 Crisis_t \times Unhealthy_i \\ &+ \beta_4 Crisis_t \times Rating_i + Crisis_t \times \gamma_s + Crisis_t \times \psi_i \\ &+ \chi_i + \varepsilon_{it}, \end{split} \tag{2}$$

where χ_i is a set of firm-level fixed effects, which absorbs time-invariant differences

¹²The information about banks' new loan issuance is taken from Chodorow-Reich (2014). Unhealthy banks in the DealScan-Compustat sample include BMO Capital Markets, Banco Santander, Bank of New York Mellon, Bear Stearns, CIT Group, CIBC, Citi, Credit Suisse, Deutsche Bank, GE Capital, Goldman Sachs, JP Morgan, KeyBank, Lehman Brothers, M&T Bank, Merrill Lynch, Morgan Stanley, National City, Scotiabank, UBS, and Wachovia.

in terms of trade credit lending. We include the ratio of accounts payable to sales $(AP_{it}/Sales_{it})$ to control for firms' borrowing of trade credit. The crisis indicator $Crisis_t$ captures the average changes in the accounts receivable to sales ratio during the crisis. The interaction term of $Crisis_t \times Unhealthy_i$ thus captures the additional change of the accounts receivable to sales ratio of the firms with an unhealthy relationship bank. Other control variables include the interaction of the crisis indicator with the sectoral fixed effects (γ_s) , the third-party bond rating indicator (Rating_i), and the firm size fixed effects (ψ_i) , capturing, respectively, the sectoral-level trend, and the different responses of large firms and firms with access to the bond market during the crisis.

The coefficient of the interaction term $Crisis_t \times Unhealthy_i$, β_3 , is the object of interest. We expect $\hat{\beta}_3$ to be negative and significant, indicating that having an unhealthy relationship bank during the crisis reduces firms' lending of trade credit more than firms with healthy banks.

Results First, it is important to note that the contraction in the syndicated loan market was rather severe during the 2007–09 financial crisis. Figure A2 plots the changes in several key characteristics of newly opened credit line facilities from 2006 to 2010. There are significant drops in the number, size, and maturity of all three types of credit line facilities. Take the accounts-receivable-collateralized credit line facility as an example. Compared with the pre-crisis level in 2006, total number of newly opened facilities dropped by approximately 60 percent, the total size of new facilities dropped by almost 60 percent, and average maturity dropped by approximately 20 percent. Property of the contraction in the syndicated loan market was rather severe during the 2007–09 financial crisis. Figure A2 plots the changes in several key characteristics of newly opened credit line facilities from 2006 to 2010. There are significant drops in the number, size, and maturity of all three types of credit line facilities. As a property of the characteristics of newly opened facilities dropped by approximately 20 percent. The contraction of the characteristics of newly opened credit line facilities from 2006 to 2010.

The results of specification 2 are displayed in Table 2. Since firms in the DealScan-Compustat sample are very large and financially integrated, not surprisingly, the crisis per se does not seem to have a significant impact on trade credit lending. The coefficient on $Crisis_t$ is insignificant and slightly positive. The estimated coefficients on the interaction term $Crisis_t \times Unhealthy_t$, however, show that having

¹³See Ivashina and Scharfstein (2010) for detailed discussions about the syndicated loan market during the 2007–09 financial crisis.

¹⁴The three types of credit line facilities are: 1) unsecured, 2) secured, with accounts receivable as collateral, and 3) secured, with other types of assets as collateral.

¹⁵Interestingly, the advance rate (borrowing base percentage) of the secured credit line facilities does not change much during the same period.

an unhealthy bank during a crisis significantly reduces the firms' lending of trade credit. Firms whose relationship bank turned unhealthy during the crisis cut back their lending of trade credit, measured by the ratio of accounts receivable to sales, by 1.3 to 1.8 percentage points more than firms with a healthy relationship bank. The estimated results hold true when we include different sets of control variables.

3 Model

In this section, we introduce trade credit into a rather standard macroeconomic model with financial frictions and heterogeneous entrepreneurs. We start by describing the economic environment and production technology (section 3.1 and 3.2). We then show the coexistence of bank credit and trade credit as a means of working capital financing, which is where our model diverges from the standard model (section 3.3).

3.1 Economic environment

The time is discrete with an infinite horizon. There are two types of goods in the economy. Final goods are used for consumption and investment. Intermediate goods are used as inputs to produce final goods.

The production of final goods takes place in two stages. Each stage is populated by a measure 1 of heterogeneous entrepreneurs. Entrepreneurs in the same stage differ from each other by wealth (a) and productivity (z). The productivity process z is stochastic and exogenous. It is parameterized by a Poisson process with death rate π and new draws of productivity from the distribution G(z). The wealth process a is endogenously chosen by the entrepreneurs.

There is a measure N of homogeneous workers. Workers provide labor and consume. They do not have access to the asset markets; i.e., they are "hand-to-mouth."

The banking sector is perfectly competitive. There is a representative bank operating in the sector and making zero profit.

3.2 Preferences, endowments, and production technology

The preferences of workers are time separable, with instantaneous utility function $\mathfrak{u}(c_t^h,h_t)$, such that,

$$U^h(c^h,h) = \sum_t \beta^t u(c^h_t,h_t), \ u(c_t,h_t) = c^h_t - \psi \frac{h_t^{1+\theta}}{1+\theta},$$

where β is the discounting factor, ψ represents disutility from working, and θ is the inverse of Frisch elasticity.¹⁶

The preferences of entrepreneurs are time separable with instantaneous utility function of $log(c_t)$. The expected utility of the entrepreneur can be written as

$$U^{e}(c) = \mathbb{E} \sum_{t} \beta^{t} \log(c_{t}),$$

where the expectation is taken over the stochastic processes of productivity z and wealth a.

Intermediate goods entrepreneurs operate a decreasing return to scale production technology ($\mu_1 < 1$) that transforms capital and labor into intermediate goods, such that

$$y_1 = A_1 z F_1(k, l) = A_1 z (k^{\alpha} l^{1-\alpha})^{\mu_1}.$$

Final goods entrepreneurs operate a decreasing return to scale production technology ($\mu_2 < 1$) that transforms capital, labor, and intermediate goods into final goods, such that

$$y_2 = A_2 z F_2(k, l, x_1) = A_2 z ((k^{\alpha} l^{1-\alpha})^{1-\chi} x_1^{\chi})^{\mu_2}.$$

Since the production technologies in the economy are decreasing return to scale, there exists an optimal production scale for the entrepreneurs given their productivity z.

¹⁶It will become clearer later that workers do not face idiosyncratic or aggregate shocks; hence, there are no expectation terms in their utility.

3.3 Financing production

At the beginning of each period, entrepreneurs carry over from the previous period their wealth α . After the idiosyncratic productivity shock z is realized, entrepreneurs make decisions about their current period production k, l, x_1 , the borrowing and lending of trade credit AR, AP, consumption c, and saving i. To finance these activities, the entrepreneurs take out an inter-temporal bank loan d, with interest rate r, to cover capital expenditure, and an intra-temporal bank loan d, with d interest rate, to cover working capital. Then the production takes place. Entrepreneurs and workers consume and save. After that, entrepreneurs decide whether or not to default on bank loans, then settle their trade credit payments. A renegotiation process starts if the entrepreneurs decide to default on their bank loans. After the renegotiation, the entrepreneurs carry their wealth d into the next period. d

Following Jermann and Quadrini (2012), we assume that the intra-temporal loan m needs to cover 1) savings into the next period i = a' - a, 2) consumption c, 3) interests payment r(k - a), and 4) production costs: $\delta k + wl$ for the intermediate goods entrepreneurs and $\delta k + wl + p_1x_1$ for the final goods entrepreneurs.¹⁸ The inter-temporal loan has to cover the capital expenditure of this period k - a.

The fundamental financial friction of the economy lies in the bank's limited enforcement over the repayment of bank loans. As mentioned in the discussion of the timing, at the end of each period, entrepreneurs can default on their bank loans. Upon default, the bank has the option to liquidate entrepreneurs' collateral. With some probability, the liquidation is successful and the bank recovers the full value of the collateral. The bank and the entrepreneurs can also renegotiate the debt contract before the liquidation option is exercised. More specifically, entrepreneurs could make a take-it-or-leave-it offer to the bank. In this case, entrepreneurs would only offer to pay the expected liquidation value of the collateral to the bank. The resulting incentive-compatible bank loan contract gives rise to a bank loan limit as a function of the value of entrepreneurs' collateral.

Trade credit exists because we assume that the intermediate goods entrepreneurs have a perfect enforcement over the repayment of trade credit. This gives them a

¹⁷The timing is summarized in Figure A4.

¹⁸It can be shown, using the budget constraint of the entrepreneurs, that the sum of these costs is equal to the current period output. The details can be found in Appendix A.1.

comparative advantage in lending to the final goods entrepreneurs. We model the trade credit contracts by assuming that there is a Walrasian market for the intermediate goods and trade credit. An intermediate goods entrepreneur enters the market with a contract consisting of intermediate goods of value p_1y_1 and a loan of size $AR \in [0, p_1y_1]$. Once the contract is accepted by the market, the intermediate goods entrepreneur proceeds to the production stage and expects to collect a payment of size $p_1y_1 + (1 + r^{tc})AR$ from the market by the end of this period, where p_1 is the price of the intermediate goods and r^{tc} is the trade credit interest rate. A final goods entrepreneur enters the market to purchase a contract with intermediate goods of value p_1x_1 and a loan of size $AP \in [0, p_1x_1]$. By signing the contract, the final goods entrepreneur receives a loan of size AP and commits to purchase intermediate goods of value p_1x_1 . They are expected to make a payment of size $p_1x_1 + (1 + r^{tc})AP$ at the end of this period.

There exist equilibrium prices p_1 and r^{tc} that equate the aggregate demand and supply of both the intermediate goods and trade credit. Since the intermediate goods are identical and infinitely divisible, both the supply and the demand of the contracts can be divided infinitely. Therefore, there exists an algorithm—a contract division and allocation rule—that clears the market under the equilibrium prices.

With the help of the entrepreneurs' budget constraints, we can write their working capital constraints as the following:

intermediate:
$$p_1A_1F_1(k,l) + (1+r^{tc})AR \leq \gamma_1\alpha' + \gamma_2AR$$
, (3)

$$\mbox{final}: \quad A_2 F_2(k,l,x_1) - (1+r^{\rm tc}) A P \leqslant \gamma_1 \alpha', \eqno(4)$$

where γ_1 and γ_2 are the probability for the bank to successfully liquidate wealth α' and accounts receivable AR upon entrepreneurs' default.¹⁹ The existence of the trade credit increases the intermediate goods entrepreneurs' need for intra-temporal loans and decreases the final goods entrepreneurs' need for inter-temporal loans by the same amount. At the same time, accounts receivable AR are created and can be used as collateral.²⁰

¹⁹The derivation of the working capital constraints can be found in Appendix A.1.

²⁰Figure A3 summarizes the flow of goods and credit in the economy.

Discussions In the paper, we model trade credit in a rather abstract fashion. First, we make the assumption that suppliers have a comparative advantage in lending to their customers (perfect enforcement of trade credit repayment) without providing a micro foundation. This assumption is motivated by the previous literature, including Biais and Gollier (1997), Burkart and Ellingsen (2004), and Cuñat (2007). They all postulate that trade credit exists because of suppliers' comparative advantage, but the form and the source of the comparative advantage differ in these papers. Since it is not our goal to understand the theoretical foundation of trade credit, we chose a particular type of comparative advantage that yields a simple quantitative framework without providing a deep theory for it.

Second, trade credit in reality is an implicit loan—a delay of payments in the presence of a mismatch of timing between the outflow of cost and the inflow of revenue. However, trade credit in our paper is modeled in an abstract way, only trying to capture the impacts of trade credit on firms' liquidity positions. We introduce an alternative setting in Appendix A.2, in which trade credit is modeled explicitly as a delay of payment. The working capital constraints derived under the alternative setting shows a similar impact of trade credit on firms' liquidity positions. However, the alternative setting introduces another state variable into the entrepreneurs' recursive problem, which greatly increases the computation burden and is the main reason why it was not adopted for our quantitative analysis.

Third, trade credit in reality is also a contract between two firms. However, it is technically challenging to introduce the firm-to-firm linkages into a dynamic model with financial frictions.²¹ Instead, to simplify our quantitative analysis, we introduce a Walrasian market for trade credit contracts to capture the supply and demand of trade credit.

4 Recursive competitive equilibrium

In this section, we present the problem of the workers and the entrepreneurs, define recursive competitive equilibrium, and analyze entrepreneurs' optimal choice

²¹Papers with firm-to-firm linkages include Eaton, Kortum and Kramarz (2015), Lim (2017), and Oberfield (2017), all of which abstract from capital and financial frictions.

of trade credit (section 4.1).

The problem of workers is stationary. It can be written simply as follows:

$$\max_{c^{h},h} c^{h} - \psi \frac{h^{1+\theta}}{1+\theta}, \text{ s.t. } c^{h} = wh.$$
 (5)

Given the current state variables (a, z), intermediate goods entrepreneurs choose input goods k, l, the lending of trade credit AR, consumption c, and next period wealth a'. The choices are subject to an inter-temporal budget constraint 7 and a working capital constraint 8. The two additional constraints on accounts receivable require that it is non-negative and does not exceed the value of output. We also require that entrepreneurs' wealth be always non-negative. The problem of intermediate goods entrepreneurs can be written recursively as follows:

$$V_1(\alpha, z) = \max_{c, k, l, AR, \alpha'} \log(c) + \beta \mathbb{E}_{z'} V_1(\alpha', z'), \tag{6}$$

s.t.
$$c + \alpha' = (1 + r)\alpha + p_1 A_1 z F_1(k, l) - (r + \delta)k - w l + r^{tc} A R,$$
 (7)
 $p_1 A_1 z F_1(k, l) + (1 + r^{tc}) A R \leq \gamma_1 \alpha' + \gamma_2 A R,$ (8)
 $0 \leq A R \leq p_1 A_1 z F_1(k, l), \alpha' \geq 0.$

Similarly, we can write the problem of final goods entrepreneurs as follows:

$$\begin{split} V_{2}(\alpha,z) &= \max_{c,k,l,x_{1},AP,\alpha'} \log(c) + \beta \mathbb{E}_{z'} V_{2}(\alpha',z'), \\ \text{s.t.} & c + \alpha' = (1+r)\alpha + A_{2}zF_{2}(k,l,x_{1}) \\ &- (r+\delta)k - wl - p_{1}x_{1} - r^{tc}AP, \\ &A_{2}zF_{2}(k,l,x_{1}) - (1+r^{tc})AP \leqslant \gamma_{1}\alpha', \\ &0 \leqslant AP \leqslant p_{1}x_{1},\alpha' \geqslant 0, \end{split} \tag{9}$$

where equation 10 is the inter-temporal budget constraint and inequality 11 is the working capital constraint.

We are now ready to define the recursive competitive equilibrium.

Definition 1 The recursive competitive equilibrium consists of the interest rate of bank credit r, wage rate w, intermediate goods price p_1 , and the interest rate of trade credit r^{tc} , value functions of entrepreneurs $V_1(\alpha, z)$ and $V_2(\alpha, z)$, policy functions of entrepreneurs $c_1(\alpha, z)$, $c_2(\alpha, z)$, $k_1(\alpha, z)$, $k_2(\alpha, z)$, $a_1'(\alpha, z)$, $a_2'(\alpha, z)$, $l_1(\alpha, z)$, $l_2(\alpha, z)$, $x_1(\alpha, z)$, $AR(\alpha, z)$,

AP(α , z), consumption and labor supply of workers { c^h , h}, and distributions of entrepreneurs $\Phi_1(\alpha, z)$ and $\Phi_2(\alpha, z)$, such that,

- 1. Given prices, value functions and policy functions solve the optimization problems of entrepreneurs 6 and 9.
- 2. Given prices, consumption and labor supply solve the workers optimization problem 5.
- 3. Labor market clears,

$$\int l_1(a,z)d\Phi_1(a,z) + \int l_2(a,z)d\Phi_2(a,z) = N \cdot h.$$

4. Inter-temporal debt market clears,

$$\int (k_1(a,z)-a)\cdot d\Phi_1(a,z) + \int (k_2(a,z)-a)\cdot d\Phi_2(a,z) = 0.$$

5. Intermediate goods market and trade credit market clear,

$$\begin{split} \int A_1 z F_1(k(\alpha,z),l(\alpha,z)) d\Phi_1(\alpha,z) &= \int x_1(\alpha,z) d\Phi_2(\alpha,z), \\ \int A R(\alpha,z) d\Phi_1(\alpha,z) &= \int A P(\alpha,z) d\Phi_2(\alpha,z). \end{split}$$

6. The stationary distributions evolve according to the following law of motion;

$$egin{array}{lll} \Phi_1(\mathfrak{a}',z') &=& \int \mathbb{I}_{\mathfrak{a}'=\mathfrak{a}'_1(\mathfrak{a},z)}\pi(z'|z)d\Phi_1(\mathfrak{a},z), \ \Phi_2(\mathfrak{a}',z') &=& \int \mathbb{I}_{\mathfrak{a}'=\mathfrak{a}'_2(\mathfrak{a},z)}\pi(z'|z)d\Phi_2(\mathfrak{a},z). \end{array}$$

4.1 Trade credit choices

In this section, we describe entrepreneurs' choices of trade credit with the following three propositions. In Figure 1, we provide a graphic illustration of these propositions.

The first proposition characterizes the state of being financially constrained.

Proposition 1 *There exist functions* $g_1(z)$ *and* $g_2(z)$ *such that*

- 1. For intermediate goods entrepreneurs with wealth α and productivity z, the working capital constraint 8 is not binding if $\alpha > g_1(z)$; it is binding if $\alpha \leqslant g_1(z)$.
- 2. For final goods entrepreneurs with wealth α and productivity z, the working capital constraint 11 is not binding if $\alpha > g_2(z)$; it is binding if $\alpha \leqslant g_2(z)$.

The proof of this proposition can be found in Appendix B.2. It says that the state of being constrained follows a cut-off rule. An increase in wealth a leads to a larger bank loan limit and relaxes the working capital constraint. The entrepreneurs are financially unconstrained if their wealth is large enough to finance the optimal scale of production.

In the second proposition, we analyze firms' borrowing and lending of trade credit.

Proposition 2 *There exist functions* $h_1(z)$ *and* $h_2(z)$ *such that,*

- 1. For intermediate entrepreneurs with wealth α and productivity z, AR>0 if $\alpha\geqslant h_1(z)$, and AR=0 if $\alpha< h_1(z)$.
- 2. For final goods entrepreneurs with wealth α and productivity z, AP = 0 if $\alpha > h_2(z)$, and AP > 0 if $\alpha \leqslant h_2(z)$.

The proof of this proposition can be found in Appendix B.3. It says that the entrepreneurs' decisions regarding the borrowing and lending of trade credit also follow a cut-off rule. This is a very intuitive result. Take an intermediate goods entrepreneur with productivity z as an example. The marginal cost of lending trade credit is $\mu(1-\gamma_2)$, in which μ , the shadow value of liquidity, declines with wealth α , while the marginal benefit, the trade credit interest rate r^{tc} , does not change with α . It follows that there exists a threshold value for α , such that the marginal benefit of lending trade credit exceeds the marginal cost at the threshold. A similar argument can be applied for the final goods entrepreneurs.

Proposition 3 describes the relationship between choices of trade credit and being financial constrained.

 $^{^{22}}$ This can be seen from the first-order condition (FOC) with respect to AR, $r^{tc}=\mu(1-\gamma_2)+\tau_1-\tau_2$, in which μ is the Lagrangian multiplier on the working capital constraint, and τ_1 and τ_2 are the Lagrangian multipliers of two accounts receivable constraints (AR $\geqslant 0$ and AR $\leqslant p_1A_1zF_1(k,l)$, respectively).

Proposition 3 *The following properties hold if* $r^{tc} > 0$:

- 1. If $\gamma_2 \in [0, 1]$, for any z, $h_1(z) \leq g_1(z)$.
- 2. For any z, $h_2(z) \leq g_2(z)$.

The proofs of the proposition can be found in Appendix B.4. The two claims in this proposition say that all unconstrained intermediate goods entrepreneurs lend trade credit, and only constrained final goods entrepreneurs borrow trade credit. It is important to note that this proposition does not rule out the possibility that some constrained intermediate goods entrepreneurs lend trade credit. It also does not rule out the possibility that some constrained final goods entrepreneurs do not borrow trade credit.

5 Quantitative analysis

In this section, we provide quantitative analysis of the model. In section 5.1, we discuss the calibration strategy and some quantitative properties of the calibrated model. Using the calibrated version of the model, we provide a quantitative analysis of the role of trade credit in normal times (section 5.2), during the 2007–09 financial crisis (sections 5.3 and 5.4), and more generally over the U.S. business cycle (section 5.5).

5.1 Calibration strategy and results

One period of the model corresponds to one quarter in the data. The workers' utility function follows the form set out by Greenwood, Hercowitz and Huffman (1988). We pick $\theta = 0.5$, which gives a Frisch elasticity of $2.^{23}$ Another parameter in the utility function ψ , representing the disutility from providing labor, is calibrated such that 30 percent of workers' time is spent on working, i.e., h = 0.3. Entrepreneurs' instantaneous utility function is in log form. We calibrate the discount factor β of

²³This value is well within standard macro estimations (see for example Chetty et al., 2011, and Keane and Rogerson, 2012).

entrepreneurs to match an annual interest rate of 4 percent. Since the share of entrepreneurs in the U.S. data is around 10 percent, we pick the measure of workers N=18 so that the share of entrepreneurs in the model matches the data.

There are two sectoral production functions in the model. In both sectors, we fix the capital share α to be 1/3. Consequently, the labor share is 2/3. Following Yi (2003), the intermediate goods share χ is fixed to be 2/3. The capital depreciation rate δ is chosen to be 0.025 so that the annual depreciation rate of capital is equal to 10 percent. The Poisson death rate π , which governs the persistence of the idiosyncratic productivity shock, is fixed at 10 percent, following Buera, Kaboski and Shin (2011).

We assume that scale parameters in the two sectors are the same, i.e., $\mu_1 = \mu_2$. The productivity distribution G(z) is assumed to be Pareto with scale parameter 1 and tail parameter ν . Following Buera, Kaboski and Shin (2011), we calibrate the scale parameter in the production function, μ_1 , μ_2 and the Pareto tail ν to match the top 5 percentile of the individual earnings share and top 10 percentile of the employment share of the firms, respectively. Lastly, we pick γ_1 and γ_2 , the collateral constraint on wealth α' and accounts receivable AR, such that the model delivers the ratio of credit market liability to nonfinancial assets and the ratio of accounts receivable to gross value added in the data.²⁴ Table 3 presents a summary of the calibrated parameters, targets, and calibration results.²⁵

In the following paragraphs, we present and discuss some quantitative properties of the calibrated model in the steady state.

Trade credit and heterogeneous entrepreneurs In Table 4, we present the trade credit choices of entrepreneurs by their wealth and productivity. As shown in the table, conditional on their productivity level, entrepreneurs with a lower wealth borrow more trade credit from other firms (a higher AP/sales) and lend less trade credit to other firms (a lower AR/sales). Perhaps more interestingly, conditional

²⁴The model moment of credit market liability is the sum of the inter-temporal and intratemporal loans. The aggregate inter-temporal loan can be written as $\int \max(k_1(\alpha,z)-\alpha,0)d\Phi_1(\alpha,z)+\int \max(k_2(\alpha,z)-\alpha,0)d\Phi_2(\alpha,z)$. The size of intra-temporal loan of all intermediate goods entrepreneurs is $\int [p_1y_1(\alpha,z)+AR(\alpha,z)]d\Phi_1(\alpha,z)$. The size of intra-temporal loan of all final goods entrepreneurs is $\int [y_2(\alpha,z)-AP(\alpha,z)]d\Phi_2(\alpha,z)$. We can then write the aggregate intra-temporal loan as $\int p_1y_1(\alpha,z)d\Phi_1(\alpha,z)+\int y_2(\alpha,z)d\Phi_2(\alpha,z)$, given that in equilibrium $\int AR(\alpha,z)d\Phi_1(\alpha,z)=\int AP(\alpha,z)d\Phi_2(\alpha,z)$.

²⁵ The algorithm to solve the stationary equilibrium can be found in Appendix C.1.

on having the same level of productivity, entrepreneurs' wealth level has a much larger impact on the borrowing of trade credit (AP/sales) than on the lending of trade credit (AR/sales). This can be explained by the rather high collateral value of accounts receivable in our calibration ($\gamma_2 = 0.95$). These patterns are consistent with our analysis of the optimal trade credit choice in section 4.1 and the empirical evidence in section 2.1.

Interest rate of trade credit One prominent empirical characteristics of trade credit is its high interest rate. Petersen and Rajan (1997) documents that the effective annual interest rate is around 43 percent for one of the most commonly used trade credit contracts in retail businesses. Costello (2014) calculates that the annual interest rate of trade credit is between 12 percent and 16 percent by comparing firms' gross profit margin before and after the use of trade credit. In our calibrated model, the quarterly interest rate of trade credit is 2.7 percent, yielding an annual interest rate of 11.8 percent, which is very close to the calculation in Costello (2014).

The high interest rate of trade credit observed in the data indicates that trade credit cannot be merely a tool for firms to park their unused cash. Through the lens of our model, the high interest rate of trade credit also indicates that the marginal productivity of the final goods entrepreneurs who borrow trade credit, and the marginal productivity of the intermediate goods entrepreneurs who lend trade credit, are very high.

A decomposition of trade credit by its nature As discussed before, trade credit serves two roles in the data and in our model. First, it redistributes unused credit from unconstrained to constrained entrepreneurs. Second, it creates credit through accounts receivable financing. Using the calibrated model, we could decompose trade credit by these two roles: *credit redistribution* and *credit creation*.²⁶ The decomposition result shows the importance of the credit creation channel: 87 percent of the aggregate trade credit is used for creating credit, while only 13 percent of the trade credit is pure credit redistribution.

 $^{^{26}}$ The credit creation part of trade credit \bar{AR}^c is the amount of trade credit that is used by intermediate goods entrepreneurs as collateral to obtain bank loans. More specifically, it is calculated as $\bar{AR}^c = \frac{1}{\gamma_2} \int \max\{0, p_1 y_1(a,z) + (1+r^{tc})AR(a,z) - \gamma_1 a'\}d\Phi_1(a,z).$ Consequently, the credit redistribution part of trade credit is calculated by $\bar{AR}^r = \int AR(a,z)d\Phi_1(a,z) - \bar{AR}^c.$

5.2 Reallocation effects of trade credit in normal times

To quantify the role of trade credit in steady state, we consider a counterfactual economy in which trade credit is shut down, meaning that all transactions are forced to be made on the spot.²⁷

Quantitatively, we take the calibrated parameters in the benchmark economy (Table 3) and feed them into the counterfactual economy. In particular, we set $\tilde{\gamma}_1 = \gamma_1$, making the collateral value of entrepreneurs' wealth to be the same across two economies.

In Table 5, we present the differences between the counterfactual economy and the benchmark economy in terms of the aggregate and sectoral level output, hours, capital stock, and TFP. Compared to the benchmark, aggregate output of the counterfactual economy is 23.9 percent lower, which can be decomposed into a 15.3 percent lower capital stock, a 24.4 percent lower labor, and an 8.4 percent lower aggregate TFP.

Why is output higher in the benchmark economy? In short, the existence of trade credit alleviates borrowing constraints of the entrepreneurs. Therefore resources are allocated more efficiently in the benchmark, leading to higher aggregate productivity and output.

A further examination of the sectoral differences between the two economies provides a clearer picture. As shown in the last column of Table 5, aggregate TFP of the counterfactual economy is 8.4 percent lower than that of the benchmark economy, indicating a higher degree of resource misallocation. Furthermore, the difference in aggregate TFP is almost completely explained by the difference in the TFP of the final goods sector (7.5 percent), while the difference in the intermediate goods sector TFP is very small (0.9 percent). This is not surprising since trade credit mainly relaxes the borrowing constraints of the final goods entrepreneurs. Although trade credit has a very small impact on the TFP of the intermediate goods sector, its impact on the output of the intermediate goods sector is rather large because of a general equilibrium effect; i.e., higher demand from the final goods sector (26.4 percent).

²⁷See section A.3 for the definition of the recursive competitive equilibrium of the counterfactual economy.

5.3 Simulation of the 2007–09 financial crisis

In this section, we use the calibrated model to study the 2007–09 financial crisis. To this end, we engineer a financial crisis in the model by reducing the collateral value of assets so that the drop in the ratio of credit market liability to nonfinancial assets and the drop in the ratio of trade credit to nonfinancial assets match the observed data moments of the U.S. nonfinancial corporate sector during the 2007–09 financial crisis.²⁸

To simulate the financial crisis in the model, we hit the collateral value γ_1 and γ_2 with a common shock ρ_t .²⁹ That is, $\gamma_{1,t} = \rho_t \gamma_1$ and $\gamma_{2,t} = \rho_t \gamma_2$, in which γ_1 and γ_2 take their steady state value. In Figure 2, we plot the dynamics of the ratio of credit market liability to nonfinancial assets (left panel) and the dynamics of the ratio of trade credit to nonfinancial assets (right panel). Our simulation captures the magnitude of the drop very well. In particular, in the data (dotted line), following the 2007–09 financial crisis, from peak to trough, the ratio of credit market liability to nonfinancial assets dropped by around 10 percent while credit market liability to nonfinancial assets dropped by around 13 percent. As a comparison, our simulated model delivers a drop of 11 percent and 12.5 percent, respectively.³⁰

In general, our model performs rather well in terms of quantitatively matching the other aggregate dynamics during the crisis. As shown in Figure 3, in our model, output drops by 6 percent, matching the approximately 6 percent deviation from trend observed in the data, but it is slightly smaller than the peak-to-trough drop. The model also generates approximately an 8 percent drop in total hours, a 2 percent drop in aggregate TFP, and a 1 percent drop in total capital stock. Compared with the percentage deviations from trend in the data, the model generates a higher drop in hours and a lower drop in TFP.³¹

²⁸The algorithm to solve the transitional dynamics following the financial crisis can be found in Appendix C.2.

²⁹The assumption of a common shock ρ_t is motivated by Figure A2, which shows that during the crisis, the characteristics of different credit line facilities exhibit a very similar dynamics.

³⁰We calibrate the shock process $ρ_t$ as the following: $\{ρ_1, ρ_2, ρ_3, ρ_4\} = \{0.975, 0.95, 0.925, 0.9\}, ρ_t = ρ_{t-1} + 0.014$ for t = 5, ..., 10, and $ρ_t = 1$ for $t \geqslant 11$.

³¹Compared with the peak-to-trough drop in the data, the model generates a drop of very similar magnitude in hours, and a drop of a smaller magnitude in TFP and capital stock.

5.4 Amplification effects of trade credit during the 2007–09 crisis

In this section, we examine quantitatively the role played by trade credit during the 2007–09 financial crisis. To answer this question, we introduce into the counterfactual economy (the one used in section 5.2) the same financial shock that was used to generate the 2007–09 financial crisis in the benchmark economy, and study the dynamics of the counterfactual economy following the shock.

We first recalibrate the steady state of the counterfactual economy so that it is comparable to the benchmark. More specifically, we increase the collateral value of wealth in the counterfactual economy $\tilde{\gamma}_1 = 1.43\gamma_1 = 0.4$, so that the output of the counterfactual economy and of the benchmark economy are at the same level in steady state.³²

After the recalibration, we hit the steady state of the counterfactual economy with the one-time and unexpected shock ρ_t , as calibrated in section 5.3. That is, $\tilde{\gamma}_{1,t} = \rho_t \tilde{\gamma}_1$, in which $\tilde{\gamma}_1$ is the collateral value of entrepreneurs' wealth in the steady state. As shown in Figure 4, the recession is significantly milder in the counterfactual economy than in the benchmark; that is, the drop in output, hours, TFP, and capital are all smaller. In particular, the drop in output is around 1.4 percentage points smaller in the counterfactual economy, which accounts for approximately 23 percent of the total decline in output in the benchmark. Based on these results, we draw the conclusion that the existence of trade credit amplifies the financial shock.³³

The existence of the amplification effects hinges on the underlying entrepreneur heterogeneity. Intuitively speaking, the reason why the economy with trade credit fares so poorly following a financial crisis is that, compared with a drop in bank credit, the negative impacts of a drop in trade credit are disproportionately borne by the most productive entrepreneurs.

More formally, to see the mechanism that gives rise to the amplification effects, it is useful to look into the model dynamics of trade credit. Following the financial shock, entrepreneurs on average become more constrained. Intermediate goods entrepreneurs are less willing to lend trade credit, while final goods entrepreneurs

³²Under this calibration, the shares of constrained entrepreneurs weighted by output in the benchmark and the counterfactual economy are also very similar.

³³We also performed the quantitative analysis without recalibrating the steady state of the counterfactual model, i.e., fix $\tilde{\gamma}_1 = \gamma_1$. We find the same qualitative effect of trade credit during the financial crisis, only with a smaller magnitude (17 percent versus 23 percent).

would like to borrow more. According to our calibrated model, the supply side force dominates in equilibrium, generating a large drop in the ratio of trade credit to output (right panel of Figure A5). The shift in supply and demand of trade credit also leads to a spike in the trade credit interest rate (left panel of Figure A5), resulting in a widening credit spread. Some entrepreneurs that relied on trade credit before the crisis can no longer do so during the crisis as trade credit becomes more costly. In other words, the aggregate effect of the financial crisis is amplified because the reallocation effect of trade credit, as discussed in section 5.2, is hindered by the crisis.

Discussion One caveat of our quantitative analysis is the missing "chain effect," which is at the heart of Kiyotaki and Moore (1997). This is because in our model, production takes place in two stages; therefore, the entrepreneurs are either a lender of trade credit, or a borrower, but never both. We assume a two-stage production process partly because of computational tractability, but more importantly, it is because of the lack of data to track the trade credit flows between firms or between sectors as discussed at the beginning of section 2.³⁴ In other words, even if we adopt a more complicated production structure that allows for the chain effect, it will be impossible to discipline the model quantitatively. Still, one might wonder how would the quantitative effect of trade credit differ with the chain effect. This is undoubtedly an important question, and we will leave it to future research when better data become available.

5.5 Trade credit and the U.S. business cycle

We have examined so far that the role of trade credit in normal times and during the 2007–09 financial crisis. A natural question to ask, then, is what is the role of trade credit over the U.S. business cycle? Before answering the question, it is important to note that our model does not feature a full-fledged business cycle. The aggregate

³⁴In Raddatz (2010), the author constructs the cross-sectoral trade credit flows by decomposing the stock of trade credit at the sector level into flows across different sectors. This approach, however, is not suitable for the purpose of our paper, because it relies on assumptions regarding the relationship between the flow of goods and the flow of credit, and does not take into account the effects of firm heterogeneity on cross-sectoral trade credit flows.

shocks in our model are one-time and unexpected events. However, as shown in the rest of the section, we could still learn something regarding the role of trade credit with the help of our model.

First, we find that in our model, the role of trade credit differs under financial shocks and TFP shocks. As shown in Figure A6, the aggregate dynamics are amplified by trade credit following financial shocks; in contrast, the aggregate dynamics following TFP shocks are indistinguishable in the benchmark and the counterfactual economy.

To understand why trade credit does not seem to play an important role under TFP shocks, it is useful to look at the detailed model dynamics. Take the negative TFP shock as an example. Following the shock, the intermediate goods and final goods entrepreneurs all become less productive. With the bank lending conditions unchanged, they are less constrained. As a result, the intermediate goods entrepreneurs demand less trade credit, and the final goods entrepreneurs could supply more. However, the shifts in the marginal willingness to lend and borrow do not seem to be quantitatively significant. Our result shows that the trade credit interest rate is almost unchanged following the TFP shock.³⁵

Second, we find that our model under financial shocks is consistent with the U.S. business cycle properties observed in the data. As shown in Figure A7, in the data, trade credit is strongly pro-cyclical and has a standard deviation almost twice as large as the standard deviation of GDP. In our model, as shown in Figure A8, the percentage change of trade credit is almost twice as large as that of output following financial shocks, which is consistent with the data. In contrast, the percentage change of trade credit is of a similar magnitude as that of output following TFP shocks. Although not a definitive proof, the above results seem to suggest that, through the lens of our model, financial shocks are an important driver behind the U.S. business cycle, and consequently, trade credit has contributed to the aggregate volatility of the U.S. economy.

³⁵The result is not displayed in this paper.

6 Conclusion

To a certain degree, trade credit and its impact on the aggregate economy originates in production specialization and the associated intermediate inputs transactions. At the present time, the production of final goods usually takes place in multiple stages, with different firms operating in different stages. Therefore, the transactions of intermediate inputs are carried out across firm boundaries and need to be financed. This leads to potential misallocation of intermediate input goods and loss of aggregate productivity (see Jones, 2011). One way to alleviate the misallocation is through vertical integration. The vertical integration of two firms in the production chain eliminates the multiple financing of inputs (see Bigio and La'O, 2014) and, through pooling the financial resources of the firms, results in a better allocation of resources.

In this paper, we show that trade credit—resulting from inputs suppliers' comparative advantage in lending to their customers in the production chain—is another way to alleviate the misallocation that originates in production specialization and the intermediate goods transactions. Furthermore, we find that the extent to which input goods suppliers can utilize the comparative advantage depends crucially on their own financial conditions. The comparative advantage is more efficiently utilized when credit market conditions are good. The fluctuation of trade credit over credit cycles therefore contributes significantly to the aggregate volatility of the economy.

Table 1: Trade credit and being financially constrained

Panel A: Net AR/Sales

| | | -, | | |
|---|----------------------------|----------------------------|-----------------|-----------------------|
| | (1) | (2) | (3) | (4) |
| Financially constrained based on payout ratio | -6.198*** (-29.86) | | | |
| | (-29.00) | | | |
| Financially constrained based on S&P rating | | -5.766*** | | |
| | | (-18.22) | | |
| Financially constrained based on size | | | -11.49*** | |
| Thanciany constrained based on size | | | (-40.85) | |
| | | | (20100) | |
| Financially constrained based on size | | | | -17.07*** |
| D 1 ('11 | NI AD /C | NI LAD /C | NI LAD /C | (-38.33) |
| Dependent variable | Net AR/S | Net AR/S | Net AR/S | Net AR/S |
| Sample N | Compustat 26036 | Compustat 26036 | Compustat 26036 | Compustat+SSB 34705 |
| AR2 | 0.130 | 0.113 | 0.183 | 0.219 |
| | | | | |
| Panel | l B : AP/S | Sales | | |
| | (1) | (2) | (3) | (4) |
| Financially constrained based on payout ratio | 6.552*** | | | |
| | (34.05) | | | |
| Financially constrained based on S&P rating | | 6.964*** | | |
| Thianciany constrained based on S&T Tating | | (23.65) | | |
| | | (23.03) | | |
| Financially constrained based on size | | | 10.05*** | |
| | | | (38.05) | |
| Financially constrained based on size | | | | 12.30*** |
| Thatcany constrained based on size | | | | (28.85) |
| Dependent variable | AP/S | AP/S | AP/S | AP/S |
| Sample | Compustat | Compustat | Compustat | Compustat+SSB |
| N | 26036 | 26036 | 26036 | 34705 |
| AR2 | 0.137 | 0.120 | 0.173 | 0.161 |
| Pana | 1 C : AR/ | Saloc | | |
| 1 and | (1) | (2) | (3) | (4) |
| Financially constrained based on payout ratio | 0.354*** | (2) | (3) | (1) |
| , | (3.00) | | | |
| Th. 1.11 1.00 T. | | 4 40044 | | |
| Financially constrained based on S&P rating | | 1.198*** | | |
| | | (6.40) | | |
| Financially constrained based on size | | | -1.435*** | |
| | | | | |
| • | | | (-9.92) | |
| T' '11 ' ' 11 ' ' | | | (-9.92) | 4 575444 |
| Financially constrained based on size | | | (-9.92) | -4.765*** (21.26) |
| | AR/S | AR/S | , , | (-21.26) |
| Dependent variable | AR/S Compustat | AR/S Compustat | AR/S | (-21.26) AR/S |
| | AR/S Compustat 26036 | AR/S Compustat 26036 | AR/S | (-21.26) |

Notes: Our sample includes all but wholesale, retail, and financial firms in the Compustat and the SSBF data set for the fiscal years 1987, 1993, 1998, and 2003. All regressions include two-digit SIC industry-year fixed effects. Column (4) of every panel includes two dummy variables indicating whether the firm is a corporation or a Compustat firm, respectively. The dependent variables are winsorized at the top and bottom 5% for each year. Standard errors are clustered at the firm level.

Table 2: Effects of bank health on trade credit lending

| | (1) | (2) | (3) | (4) |
|---------------------------|----------|----------|----------|----------|
| Crisis X Unhealthy | -1.274* | -1.502** | -1.545** | -1.837** |
| | (0.681) | (0.696) | (0.714) | (0.718) |
| | 0.446 | | | • |
| Crisis | 0.446 | 0.0672 | 0.243 | 2.680 |
| | (0.483) | (0.537) | (1.423) | (6.087) |
| | | | | |
| AP to sales ratio | 0.381*** | 0.382*** | 0.382*** | 0.382*** |
| | (0.0294) | (0.0294) | (0.0293) | (0.0292) |
| Dependent variable | AR/S | AR/S | AR/S | AR/S |
| Crisis X Credit rating FE | N | Y | Y | Y |
| Crisis X Firm size bin FE | N | N | Y | Y |
| Crisis X SIC FE | N | N | N | Y |
| N | 15275 | 15275 | 15275 | 15275 |
| AR2 | 0.171 | 0.171 | 0.172 | 0.176 |

Notes: The dependent variables in these regressions are AR/Sales (percent). The sample includes quarterly data of 1,219 firms from 2007Q1 to 2009Q4. All regressions include a set of firm fixed effects. Standard errors are clustered at the firm level.

Table 3: Summary of calibration

| Parameter | | Value | Target/Source | Data | Model |
|----------------|--------------------------------------|-------|--|------|-------|
| θ | inverse of Frisch elasticity | 1/2 | standard | - | - |
| α | capital share in production function | 1/3 | capital share of 1/3 | - | - |
| χ | intermediate goods share | 2/3 | Yi (2003) | - | - |
| π | Poisson death rate | 0.1 | Buera, Kaboski and Shin (2011) | - | - |
| N | measure of workers | 18 | share of entrepreneurs | 10% | 10% |
| ψ | disutility from working | 1.9 | hours | 0.3 | 0.3 |
| δ | depreciation rate | 0.025 | annual 10% depreciation rate | 10% | 10% |
| β | discount rate | 0.95 | annual 4% interest rate | 0.4 | 0.4 |
| μ_1, μ_2 | scale parameter | 0.85 | top 5 percentile earning share | 0.3 | 0.3 |
| ν | Pareto tail | 4.0 | top 10 percentile employment share | 0.69 | 0.69 |
| γ_1 | collateral value of wealth | 0.28 | credit market liability to nonfinancial assets | 0.36 | 0.36 |
| γ_2 | collateral value of AR | 0.95 | trade receivable to gross value added | 0.31 | 0.31 |

Notes: The data moment for credit market liability to nonfinancial asset and accounts receivable to gross value-added ratio is computed for the nonfinancial corporate sector, averaged over 4 quarters in year 2006. Credit market liability is taken from Flow of Funds Table L.103 line 23. Nonfinancial asset size is taken from Flow of Funds Table B.103 line 2. Trade receivable is is taken from Flow of Funds Table L.103 line 15. Gross value added is taken from NIPA Table 1.14 line 17.

Table 4: Heterogeneity in trade credit

| | low wealth | low wealth | high wealth | high wealth | |
|------------------------|------------------|-------------------|------------------|-------------------|--|
| | low productivity | high productivity | low productivity | high productivity | |
| AR to output ratio (%) | 100.0 | 79.9 | 100.0 | 79.7 | |
| AP to output ratio (%) | 29.8 | 47.9 | 0.0 | 53.0 | |

Notes: An entrepreneur is defined to be low wealth (productivity) if she belongs to the bottom 50 percentile in the wealth (productivity) distribution of her own sector. The accounts receivable (payable) to output ratio for each group of entrepreneurs is defined as the sum of accounts receivable (payable) divided by the sum of output of all entrepreneurs in that group.

Table 5: Difference between counterfactual and benchmark economy (%)

| | output | capital | labor | input goods | TFP |
|---------------------|--------|---------|-------|-------------|------|
| Intermediate sector | -26.4 | -23.8 | -32.2 | _ | -0.9 |
| Final sector | -23.9 | -0.2 | -10.6 | -26.4 | -7.5 |
| Aggregate | -23.9 | -15.3 | -24.4 | | -8.4 |

Notes: This table displays the percent difference of the counterfactual economy relative to the benchmark economy. A negative number in the table suggest that aggregate statistics of the counterfactual economy is lower than that of the benchmark economy.

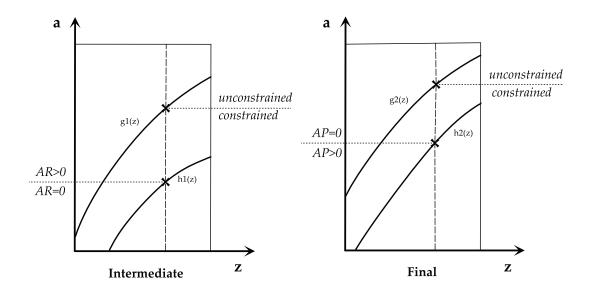


Figure 1: A graphic illustration of Proposition 1-3

Notes: The left panel of this figures illustrates the cut-off properties for intermediate goods entrepreneurs. The two cut-off functions $g_1(z)$ and $h_1(z)$ intersect with the vertical line at two points. These two points represent two cut-off values of wealth that separate constrained entrepreneurs from unconstrained ones, and entrepreneurs who lend trade credit and those who do not. Similarly, the right panel represents the two cut-off functions $g_2(z)$ and $h_2(z)$ for the final goods entrepreneurs.

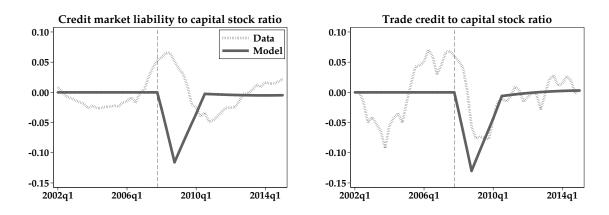


Figure 2: Dynamics of credit market liability and trade credit

Notes: The data used in the above figures are for the U.S. nonfinancial corporate sector. Among them, credit market liability is taken from Flow of Funds Table L.103 line 23. Trade credit is calculated as the average of trade payable (line 30 of Flow of Funds Table L.103) and trade receivable (line 15 of Flow of Funds Table L.103). Capital stock is constructed as the sum of equipment (line 46 of Flow of Funds Table B.103), intellectual property products (IPP) (line 47 of Flow of Funds Table B.103), and nonresidential structural capital (line 51 of Flow of Funds Table B.103), all valued at historical prices. Both credit market liability and trade credit to capital stock ratio are HP-filtered with a smoothing parameter of 1,600, and the percentage derivation from trend is plotted in the figures. The corresponding model moments are normalized to be 0 at t=0.

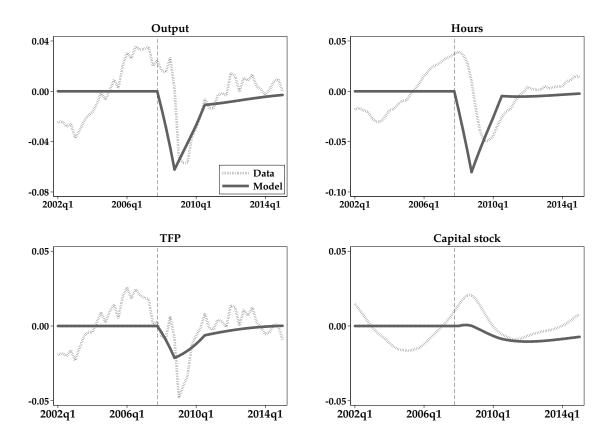


Figure 3: Dynamics of real economic indicators

Notes: The data used in the above figures are for the U.S. nonfinancial corporate sector. Among them, output (gross value added) is taken from NIPA Table 1.14 line 17. Data for hours worked is an index taken from Bureau of Labor Statistics Labor Productivity and Costs database (BLS code PRS88003033). Data for capital stock are constructed in the same way as Figure 2. TFP is then constructed as a Solow-type residual using output, hours, and capital stock.

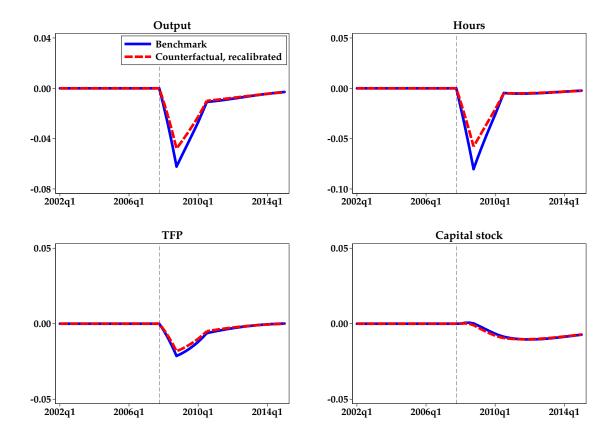


Figure 4: Dynamics after the financial crisis: Benchmark vs. counterfactual

Notes: The figures show the changes in the aggregate economy in terms of output, hours, aggregate TFP, and capital stock after the financial crisis. The solid blue lines represent the benchmark economy (with trade credit), while the dashed red lines represent the counterfactual economy (without trade credit). All lines are normalized to 0 at the beginning of the crisis. The blue solid lines in this figure, which are the dynamics of the benchmark economy following the ρ_t shock, are identical to the solid lines in Figure 3.

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Online Appendix

Not for Publication

A Model

A.1 Deriving the working capital constraint

The amount of intra-temporal bank loan needed for the entrepreneurs is,

intermediate:
$$\hat{m}_1 = \alpha' - \alpha + c + r(k - \alpha) + \delta k + wl + AR$$
, final: $\hat{m}_2 = \alpha' - \alpha + c + r(k - \alpha) + \delta k + wl + p_1x_1 - AP$.

Using budget constraints of the entrepreneurs,

$$\begin{split} \text{intermediate:} & \quad c + \alpha' = (1+r)\alpha + p_1 A_1 F_1(k,l) - (r+\delta)k - w l + r^{tc} AR, \text{ and} \\ & \quad \text{final:} & \quad c + \alpha' = (1+r)\alpha + A_2 F_2(k,l,x_1) - (r+\delta)k - w l - p_1 x_1 - r^{tc} AP, \end{split}$$

we derive the need for intra-temporal bank loans for intermediate goods entrepreneurs as $\hat{\mathfrak{m}}_1 = \mathfrak{p}_1 A_1 F_(k,l) + (1+r^{tc}) AR$, and for final goods entrepreneurs it is $\hat{\mathfrak{m}}_2 = A_2 F_2(k,l,x_1) - (1+r^{tc}) AP$.

Upon default, a renegotiation process begins. Intermediate goods entrepreneurs would propose a take-it-or-leave-it offer to repay only $\gamma_1 \alpha' + \gamma_2 AR$, where $\gamma_2 AR$ is the expected liquidation value of accounts receivable for the bank. The value of default for intermediate goods entrepreneurs is therefore $y + AR - (\gamma_1 \alpha' + \gamma_2 AR)$, and the value of non-default is $y + AR - \hat{m}_1$. The incentive compatibility constraint gives $\hat{m}_1 \leq \gamma_1 \alpha' + \gamma_2 AR$. Similarly, for final goods entrepreneurs, the incentive compatibility constraint leads to a constraint on intra-temporal bank loan $\hat{m}_2 \leq \gamma_1 \alpha'$.

A.2 An alternative way of modeling trade credit

In this section, we show an alternative way of modeling trade credit as a delay of payments.

Timing Consider a different timing for the model, in which the output of the intermediate goods entrepreneur is carried over into the next period. The output is sold at the beginning of next period to generate cash flow.

Suppose that the intermediate goods entrepreneur has two choices regarding selling its goods. First, the goods can be sold on the spot, generating instant cash flow. Second, the goods can be extended as a trade credit loan, which is repaid at the end of the period.

Financial frictions and the existence of trade credit Suppose that at the beginning of each period, the entrepreneurs need to finance working capital. Without loss of generality, assume that working capital includes interest $r_t k_t$, wage bills $w_t l_t$, and for the final goods entrepreneur, also the input goods $p_1 x_1$. The entrepreneurs can finance working capital from two sources: 1) borrow from the bank, and 2) use the cash flow generated by selling goods on the spot market.

First let us consider the case where a bank loan is the only source of financing. Let α_t be the amount of collateral that the entrepreneurs provide to obtain bank loans. Suppose that at the beginning of the period after having been granted the bank loan, the entrepreneur can default. A renegotiation process begins after default. The value of the collateral to the bank is $\chi_t \alpha_t$, where $\chi_t \in (0,1)$. Let λ be the bargaining power of the entrepreneur and $(1-\lambda)$ the bargaining power of the bank. The renegotiation contract would specify that the entrepreneur only needs to repay $\gamma \alpha_t$. Therefore, a renegotiation-proof bank loan contract has a limit of $\gamma \alpha_t$.

Second, consider the scenario in which the suppliers have a certain comparative advantage in lending input goods. Following Burkart and Ellingsen (2004), we assume that unlike bank loans, input goods cannot be diverted. Under the assumptions, 1) supplies will lend trade credit because it is secured, and 2) bank will internalize the comparative advantage by lending against accounts receivable.

Therefore, we can write the working capital constraint of the two entrepreneurs

as the following:

$$\begin{aligned} r_t k_t + w_t l_t &\leqslant & \underbrace{\gamma_1 \alpha_t + \gamma_2 A R_t}_{\text{bank loan}} + \underbrace{(p_{1,t} y_t - A R_t)}_{\text{cash flow}}, \\ r_t k_t + w_t l_t + p_{1,t} x_{1,t} - A P_t &\leqslant & \underbrace{\gamma_1 \alpha_t}_{\text{bank loan}}. \end{aligned}$$

Rearranging the above two equations, we get

$$r_{t}k_{t} + w_{t}l_{t} - p_{1,t}y_{t} + AR_{t} \leq \gamma_{1}a_{t} + \gamma_{2}AR_{t},$$

 $r_{t}k_{t} + w_{t}l_{t} + p_{1,t}x_{1,t} - AP_{t} \leq \gamma_{1}a_{t}.$

Similar to the working capital constraints in our benchmark model (equations 3 and 4), we see that lending trade credit AR_t essentially tightens the borrowing constraint of the intermediate goods entrepreneurs by $(1 - \gamma_2)AR$. They lose an instant cash flow of size AR but gain additional access to bank loan of size γ_1AR . Borrowing trade credit AP_t , on the other hand, relaxes the borrowing constraint of the final goods entrepreneur by AP_t .

Recursive representation of the entrepreneurs' problem Let $V_1(\alpha, y, z)$ be the value function of the intermediate goods entrepreneur and $V_2(\alpha)$ the value function of the final goods entrepreneur.³⁶ We can write their problem recursively as follows.

For the intermediate goods entrepreneurs,

$$\begin{array}{ll} V_{1}(\alpha,y,z) & = & \max_{c,AR,k,l,\alpha'} u(c) + \beta \mathbb{E}_{z'} V_{1}(\alpha',y',z'), \\ \text{s.t.} & c + \alpha' = (1+r)\alpha + p_{1}y_{1} - AR + (1+r^{\text{tc}})AR - (r+\delta)k - wl, \\ & rk + wl - p_{1}y + AR \leqslant \gamma_{1}\alpha + \gamma_{2}AR, \\ & 0 \leqslant AR \leqslant p_{1}y, \\ & \alpha' \geqslant 0, \\ & y' = zA_{1}F_{1}(k,l). \end{array}$$

³⁶We assume that the intermediate goods entrepreneurs carry their output to the beginning of the next period while the final goods entrepreneurs sell their output at the end of the current period. This is why output y enters as a state variable for the intermediate goods entrepreneurs but not for the final goods entrepreneurs.

For the final goods entrepreneurs,

$$\begin{split} V_2(\alpha,z) &= \max_{c,AP,k,l,x_1,\alpha'} u(c) + \beta \mathbb{E}_{z'} V_2(\alpha',z'), \\ s.t. & c + \alpha' = (1+r)\alpha + z A_2 F_2(k,l,x_1) - (r+\delta)k - w l - p_1 x_1 - r^{tc} AP, \\ rk + w l + p_1 x_1 - AP \leqslant \gamma_1 \alpha, \\ 0 \leqslant AP \leqslant p_1 x_1, \\ \alpha' \geqslant 0. \end{split}$$

A.3 Equilibrium definition of the counterfactual economy

The stationary equilibrium of the counterfactual economy is defined as follows:

Definition 2 The recursive competitive equilibrium without trade credit consists of interest rate \tilde{R} , wage rate \tilde{w} , and intermediate goods price \tilde{p}_1 , value functions of entrepreneurs $\tilde{V}_1(\alpha,z)$ and $\tilde{V}_2(\alpha,z)$, policy functions of entrepreneurs $\tilde{c}_1(\alpha,z)$, $\tilde{c}_2(\alpha,z)$, $\tilde{k}_1(\alpha,z)$, $\tilde{k}_2(\alpha,z)$, $\tilde{a}'_1(\alpha,z)$, $\tilde{a}'_2(\alpha,z)$, $\tilde{l}_1(\alpha,z)$, $\tilde{l}_2(\alpha,z)$, $\tilde{x}_1(\alpha,z)$, consumption and labor supply of workers $\{\tilde{c}^h,\tilde{h}\}$ and distributions of entrepreneurs $\tilde{\Phi}_1(\alpha,z)$ and $\tilde{\Phi}_2(\alpha,z)$, such that,

1. Given prices, value functions and policy functions solve the optimization problem of entrepreneurs.

$$\begin{split} \tilde{V}_1(\alpha,z) &= \max_{c,k,l,\alpha'} log(c) + \beta \mathbb{E}_{z'} \tilde{V}_1(\alpha',z'), \\ \text{s.t.} & c+\alpha' = (1+r)\alpha + p_1 A_1 z F_1(k,l) - (r+\delta)k - wl, \\ & p_1 A_1 z F_1(k,l) \leqslant \tilde{\gamma}_1 \alpha', \alpha' \geqslant 0. \end{split}$$

$$\begin{split} \tilde{V}_2(\alpha,z) &= \max_{c,k,l,x_1,\alpha'} log(c) + \beta \mathbb{E}_{z'} \tilde{V}_2(\alpha',z'), \\ \text{s.t.} &\quad c+\alpha' = (1+r)\alpha + A_2 z F_2(k,l,x_1) - (r+\delta)k - w l - p_1 x_1, \\ &\quad A_2 z F_2(k,l,x_1) \leqslant \tilde{\gamma}_1 \alpha', \alpha' \geqslant 0. \end{split}$$

- 2. Given prices, consumption and labor supply solve workers optimization problem 5.
- 3. Labor market clears,

$$\int \tilde{\mathfrak{l}}_1(\mathfrak{a},z)d\tilde{\Phi}_1(\mathfrak{a},z)+\int \tilde{\mathfrak{l}}_2(\mathfrak{a},z)d\tilde{\Phi}_2(\mathfrak{a},z)=\mathsf{N}\cdot\tilde{\mathsf{h}}.$$

4. Capital market clears,

$$\int (\tilde{\mathsf{k}}_1(\mathfrak{a},z)-\mathfrak{a})\cdot \mathrm{d}\tilde{\Phi}_1(\mathfrak{a},z) + \int (\tilde{\mathsf{k}}_2(\mathfrak{a},z)-\mathfrak{a})\cdot \mathrm{d}\tilde{\Phi}_2(\mathfrak{a},z) = 0.$$

5. Intermediate goods market clears,

$$\int A_1 z F_1(\tilde{k}_1(\alpha,z),\tilde{l}_1(\alpha,z)) d\tilde{\Phi}_1(\alpha,z) \ = \ \int \tilde{x}_1(\alpha,z) d\tilde{\Phi}_2(\alpha,z).$$

6. The stationary distributions evolve according to,

$$\tilde{\Phi}_1(\alpha',z') = \int_{\alpha'=\tilde{\alpha}_1'(\alpha,z)} \pi(z'|z) d\tilde{\Phi}_1(\alpha,z),$$

$$ilde{\Phi}_2(lpha',z') \;\; = \;\; \int \mathbb{I}_{lpha'= ilde{lpha}_2'(lpha,z)} \pi(z'|z) d ilde{\Phi}_2(lpha,z).$$

B Proofs

In order to prove the propositions, we first lay out the optimization problem of the entrepreneurs and derive the first-order conditions (FOCs). We prove the first part of each proposition regarding the intermediate goods entrepreneurs. The proof of the second part regarding the final goods entrepreneurs is very similar and hence is omitted.

Intermediate goods entrepreneurs Consider the following problem:

$$\begin{array}{lll} V_{1}(\alpha,z) & = & \displaystyle \max_{c,k,l,AR,\alpha'} \log(c) + \beta \mathbb{E}_{z'} V_{1}(\alpha',z') \\ & \text{s.t.} & c + \alpha' = (1+r)\alpha + p_{1}A_{1}zF_{1}(k,l) - (r+\delta)k - wl + r^{tc}AR, & (12) \\ & & p_{1}A_{1}zF_{1}(k,l) + (1+r^{tc})AR \leqslant \gamma_{1}\alpha' + \gamma_{2}AR, & (13) \\ & 0 \leqslant AR \leqslant p_{1}A_{1}zF_{1}(k,l), & (14) \\ & \alpha' \geqslant 0. & \end{array}$$

The Lagrangian of the problem can be written as,

$$\begin{split} \mathcal{L} &= log((1+r)\alpha + p_1A_1zF_1(k,l) - (r+\delta)k - wl + r^{tc}AR - \alpha') \\ &+ \beta \mathbb{E}_{z'}V_1(\alpha',z') + \mu(\gamma_1\alpha' + \gamma_2AR - p_1A_1zF_1(k,l) - (1+r^{tc})AR) \\ &+ \chi_1(p_1A_1zF_1(k,l) - AR) + \chi_2AR \\ &+ \tau\alpha'. \end{split}$$

The FOCs are:

k:
$$p_1 A_1 z F_{1,k} = \frac{r + \delta}{1 - c\mu + c\chi_1}$$
 (15)

l:
$$p_1 A_1 z F_{1,1} = \frac{w}{1 - c\mu + c\chi_1}$$
 (16)

AR:
$$\frac{1}{c}r^{tc} = \mu(1 + r^{tc} - \gamma_2) + \chi_1 - \chi_2,$$
 (17)

$$\alpha': \quad \frac{1}{c} = \beta \mathbb{E}_{z'} V_{1,\alpha'} + \mu \gamma_1 + \tau. \tag{18}$$

Together with the envelope condition $V_{1,\alpha} = \frac{1}{c}(1+r)$, we derive the Euler equation,

$$\frac{1}{c} = \beta \mathbb{E}_{z'} [\frac{1}{c'} (1+r)] + \mu \gamma_1 + \tau.$$
 (19)

In addition, according to the Kuhn-Tucker condition, the Lagrangian multipliers and the constraints have the following properties:

$$\begin{split} \mu \geqslant 0, \gamma_1 \alpha' - \mathfrak{p}_1 A_1 z F_1(k,l) - (1 + r^{\mathrm{tc}} - \gamma_2) A R \geqslant 0, \\ \chi_1 \geqslant 0, \mathfrak{p}_1 A_1 z F_1(k,l) - A R \geqslant 0 \\ \chi_2 \geqslant 0, A R \geqslant 0 \\ \tau \geqslant 0, \alpha' \geqslant 0, \end{split}$$

with complementary slackness.

Notice that the above problem is a rather standard stochastic optimization problem. According to Theorem 9.7 and 9.8 of Stokey and Lucas (1989), we know that given z, the value function $V(\cdot, z)$ is strictly increasing and strictly concave.³⁷

Before proceeding to the proofs of the propositions, we discuss the monotonicity of the optimal policy function in the following lemma.

Lemma 1 Given z, the policy functions $AR(\alpha, z)$ and $\alpha'(\alpha, z)$ both increase in α .

B.1 Proof of Lemma 1

First, we intend to show that given z, $\alpha'(\alpha, z)$ increases with α . For any $\alpha^h > \alpha^l$ and for any z, with a slight abuse of notation, denote $\{k^h, l^h, AR^h, c^h, \alpha'^h\}$ and $\{k^l, l^l, AR^l, c^l, \alpha'^l\}$ as the optimal choices, respectively. We need to show that $\alpha'^h \geqslant \alpha'^l$.

Suppose not, i.e., $a'^h < a'^l$, it has to be true that $c^h > c^l$ because otherwise $V(\cdot, z)$ cannot be a strictly increasing function.

Since $V(\cdot,z)$ is strictly concave. In order for equation (18) to hold, it has to be true that $\mu^h < \mu^l$, which means that $\mu^l > 0$; i.e., the working capital constraint is binding for (\mathfrak{a}^l,z) . From equation 17 and the complementary slackness conditions, it is easy to see that if $\mu^h < \mu^l$, then $\chi_1^h \geqslant \chi_1^l$ and $\chi_2^h \leqslant \chi_2^l$. Therefore, according to equations

³⁷Stokey, Nancy L. and Robert E. Lucas. 1989. *Recursive Methods in Economic Dynamics*. Harvard University Press.

15 and 16, it holds that $\mathfrak{p}_1A_1z\mathsf{F}_{1,k^h}<\mathfrak{p}_1A_1z\mathsf{F}_{1,k^l}$ and $\mathfrak{p}_1A_1z\mathsf{F}_{2,l^h}<\mathfrak{p}_1A_1z\mathsf{F}_{2,l^l}$. Because the production function F_1 is DRS, the above two conditions give that $\mathsf{k}^h>\mathsf{k}^l$ and $\mathsf{l}^h>\mathsf{l}^l$. Also because $\chi_1^h>\chi_1^l$, we can infer that $\mathsf{AR}^h\geqslant \mathsf{AR}^l$. As a result, the following inequalities hold:

$$a'^{h} \geqslant p_{1}zA_{1}F(k^{h}, l^{h}) + (1 + r^{tc} - \gamma_{2})AR^{h} > p_{1}zA_{1}F(k^{l}, l^{l}) + (1 + r^{tc} - \gamma_{2})AR^{l} = a'^{l}.$$

which contradicts the assumption that $\alpha'^h < \alpha'^l.$

Next we show that given (a, z), the optimal AR increases with a'. Note that given (a, z), the optimization problem of the entrepreneur can be reduced to a combination of static profit maximization problem and a inter-temporal choice of optimal a'. Given any choice of a', the static profit maximization problem can be written as,

$$\begin{aligned} \max_{k,l,AR} & & p_1 z A_1 F_1(k,l) - (r+\delta)k - w l + r^{tc} A R, \\ s.t. & & p_1 z A_1 F_1(k,l) + (1+r^{tc} - \gamma_2) A R \leqslant \gamma_1 \alpha', \\ & & 0 \leqslant A R \leqslant p_1 z A_1 F_1(k,l). \end{aligned}$$

We can show that the optimal choices of the above optimization problem increase with a' using Theorem 2.8.1 from Topkis (1989).³⁸ It is easy to see that the feasibility set increases strictly with a'; therefore, we only need to show that equation 2.8.1 from Topkis (1989) is satisfied.

Denote $W(k, l, AR) = p_1 z A_1 F_1(k, l) - (r + \delta)k - wl + r^{tc}AR$. Consider two sets of choices $\{k_1, l_1, AR_1\}$ and $\{k_2, l_2, AR_2\}$. We need to show that

$$W(k_1, l_1, AR_1) + W(k_2, l_2, AR_2) \le W(k_1 \wedge k_2, l_1 \wedge l_2, AR_1 \wedge AR_2) + W(k_1 \vee k_2, l_1 \vee l_2, AR_1 \vee AR_2),$$

which reduces to

$$\begin{array}{ll} p_1 z A_1 F_1(k_1, l_1) + p_1 z A_1 F_1(k_2, l_2) & \leqslant & p_1 z A_1 F_1(k_1 \wedge k_2, l_1 \wedge l_2) + \\ & p_1 z A_1 F_1(k_1 \vee k_2, l_1 \vee l_2). \end{array}$$

This is straightforward to prove because $\frac{\partial^2 F_1}{\partial k \partial l} > 0$, i.e., F_1 satisfies strictly increasing

³⁸Topkis, Donald M. 1998. Supermodularity and Complementarity. Princeton University Press.

differences in (k, l). As a result of Theorem 2.8.1, the optimal AR increases with a'. Since we already showed that the optimal a' increases with a. It follows that the policy function AR(a, z) is increasing in a. Q.E.D.

B.2 Proof of Proposition 1

Given z, define set $\mathbf{U}^z = \{a | \mu(a, z) = 0\}$. We intend to show that the set \mathbf{U}^z is in the following form (\underline{a}, ∞) .³⁹ To do this, we first show that \mathbf{U}^z has the following property: if $a \in \mathbf{U}^z$ and $\hat{a} > a$, then $\hat{a} \in \mathbf{U}^z$.

Let $a \in U^z$. According to the definition of U^z , we know that $\mu(a, z) = 0$. The complementary slackness condition then implies that for entrepreneur (a, z), the working capital constraint is not binding,

$$p_1A_1zF_1(k,l)+(1+r^{tc})AR<\gamma_1\alpha'+\gamma_2AR.$$

According to equation 17, $\mu = 0$ implies that $\chi_2 = 0$ and $\chi_1 = \frac{1}{c} r^{tc}$. Taking the value of μ , χ_1 , χ_2 back into equations 15 and 16, we get

$$\begin{aligned} k: & & p_1 A_1 z F_{1,k} = \frac{r+\delta}{1+r^{tc}}, \\ l: & & p_1 A_1 z F_{1,l} = \frac{w}{1+r^{tc}}. \end{aligned}$$

Since production function F_1 is decreasing return to scale, there exist optimal k and l that solve the above system of two equations. Denote the solution as k^* and l^* . Since $\chi_1 = 0$, the complementary slackness condition implies that $AR = p_1 A_1 z F_1(k^*, l^*)$.

Now consider the budget constraint 12. Let $m = p_1 A_1 z F_1(k, l) - (r + \delta)k - wl + r^{tc}AR$, and the budget constraint can be re-written as,

$$c + \alpha' = (1 + r)\alpha + m$$
.

It is clear that m is maximized when $k = k^*$, $l = l^*$, and $AR = p_1A_1zF_1(k^*, l^*)$. In other words, entrepreneurs will always choose $k = k^*$, $l = l^*$, and $AR = p_1A_1zF_1(k^*, l^*)$

³⁹Notice that this statement is equivalent to that of Proposition 1.

if they are feasible under the working capital constraint (equation 13).

Consider an entrepreneur with productivity z and wealth $\hat{a} > a$. According to Lemma 1, $a'(\hat{a},z) \geqslant a'(a,z)$. Therefore, since $k=k^*$, $l=l^*$, and $AR=p_1A_1zF_1(k^*,l^*)$ are feasible for entrepreneur (a,z), they must be feasible for entrepreneur (\hat{a},z) as well. Following the above analysis, we know that entrepreneurs will choose $k=k^*$, $l=l^*$, and $AR=p_1A_1zF_1(k^*,l^*)$, and the working capital constraint holds with strict inequality. Using the complementary slackness condition, this implies that $\mu(a',z)=0$.

With the help of this property, we show that \mathbf{U}^z is an interval. Suppose that it is not; then there exists x < w < y, such that $x, y \in \mathbf{U}^z$ but $w \notin \mathbf{U}^z$. This violates the property, since it means $x \in \mathbf{U}^z$, w < x, but $w \notin \mathbf{U}^z$.

Finally, we show that \mathbf{U}^z is unbounded from above. Suppose that it is not; then there exists $w \notin \mathbf{U}^z$ but w > a for all $a \in \mathbf{U}^z$, which is a violation of the property. *Q.E.D.*

B.3 Proof of Proposition 2

Define a set $\mathbf{H}^z = \{a | AR(a, z) > 0\}$. We show that \mathbf{H}^z is in the form of (\underline{a}, ∞) . The proof is very similar to proof of Proposition 1. Essentially, we need to prove that the set \mathbf{H}^z has the following property: if $a \in \mathbf{H}^z$ and $\hat{a} > a$, then $\hat{a} \in \mathbf{H}^z$. It is clear that this property holds since according to Lemma 1, AR(a, z) is an increasing function in a. Therefore, for any $\hat{a} > a$, we have $AR(\hat{a}, z) \geqslant AR(a, z) > 0$. *Q.E.D*.

B.4 Proof of Proposition 3

Proving this proposition is equivalent to showing that $\mathbf{U}^z \subseteq \mathbf{H}^z$. Take any $a \in \mathbf{U}^z$; we know that $\mu(a,z) = 0$ according to the definition of \mathbf{U}^z . Next we show that AR(a,z) > 0.

According to equation 17, if $\mu(\mathfrak{a},z)=0$ then $\frac{1}{c(\mathfrak{a},z)}r^{\mathrm{tc}}=\chi_1(\mathfrak{a},z)-\chi_2(\mathfrak{a},z)$. Since $\frac{1}{c(\mathfrak{a},z)}r^{\mathrm{tc}}>0$, it has to be the case that $\chi_2(\mathfrak{a},z)=0$. Because otherwise if $\chi_2(\mathfrak{a},z)>0$, the complementary slackness condition implies that $AR(\mathfrak{a},z)=0$, which in turn implies that $\chi_1(\mathfrak{a},z)=0$. The equation $\frac{1}{c(\mathfrak{a},z)}r^{\mathrm{tc}}=\chi_1(\mathfrak{a},z)-\chi_2(\mathfrak{a},z)$ therefore cannot hold because the left-hand side is positive but the right-hand side is negative. *Q.E.D.*

C Computation

In this section, we describe the algorithms for computing the benchmark model. Section C.1 contains the algorithms to compute the stationary equilibrium. Section C.2 contains the algorithms to compute the transitional dynamics. The algorithms to compute the counter-factual model are very similar to the benchmark model, only with different sets of FOCs, budget constraint, and working capital constraint. Hence they are omitted here.

C.1 Stationary equilibrium

- Guess equilibrium prices r, w, p₁, r^{tc}.
- Given the prices, solve the household problem.
- Given the prices, solve the entrepreneurs problem as follows:
 - Discretize the state space.
 - Guess policy function c(a, z).
 - For each (a, z), assume that the entrepreneur is unconstrained, i.e., $\mu(a, z) = 0$. Solve for the system of equations that consists of FOCs and budget constraint.
 - Check whether the working capital constraint is satisfied with the solution to the above system of equations.
 - If the working capital constraint is not satisfied, it means that $\mu(\alpha,z)>0$ and working capital constraint holds with equality. Solve the system of equations that consists of FOCs, budget constraint, and working capital constraint (with equality).
 - Use the Euler equation to update the policy function c(a, z) until it converges.
- Given any arbitrary distribution of (a, z), iterate using the policy functions derived above until a stationary distribution is reached.

- Generate the aggregate statistics of the four markets: capital, labor, intermediate goods, and trade credit market.
- Update (r, w, p_1, r^{tc}) until the four markets clear simultaneously.

C.2 Transitional dynamics

To compute the transitional dynamics of the economy, we consider a transition path of $\mathsf{T}=100$ periods. The economy is at the initial stationary equilibrium level in period $\mathsf{t}=1$, and we assume that it converges back to the initial stationary equilibrium at period $\mathsf{t}=\mathsf{T}$.

- Guess a sequence of prices $\{r_t, w_t, p_{1,t}, r_t^{tc}\}_{t=2}^{T-1}$.
- Backward induction. For each t = T 1, T 2, ..., 2,
 - Discretize the state space.
 - Given prices, solve the household problem for period t.
 - Given prices, solve the entrepreneurs policy functions for period t.
 - 1. Guess $c_t(a, z)\mu_t(a, z) = 0$, solve the system of equations that consists of FOCs of period t, budget constraint, and Euler equations (with the next period policy function $c_{t+1}(a, z)$ known).
 - 2. Check whether the working capital constraint is satisfied under the above solution.
 - 3. If the working capital is not satisfied, $c_t(a,z)\mu_t(a,z) > 0$ and the working capital constraint holds with equality. Solve the system of equations that consists of FOCs of period t, budget constraint, Euler equations (with the next period policy function $c_{t+1}(a,z)$ known), and working capital constraint with equality.
- Forward induction. The first period stationary distribution $\Phi_{1,1}(a,z)$ and $\Phi_{2,1}(a,z)$ is set to be the stationary equilibrium distribution. Using the policy functions for period t = 2, ..., T 1, compute the distribution along the transition path $(\Phi_{1,t}(a,z))$ and $\Phi_{2,t}(a,z)$.

- ullet Generate aggregate statistics for the four markets in every period t=2,...,T-1 using the policy functions and the distributions.
- Update $\{r_t, w_t, p_{1,t}, r_t^{tc}\}_{t=2}^{T-1}$ until the four markets clear simultaneously in each period t=2,...,T-1.

D Additional figures

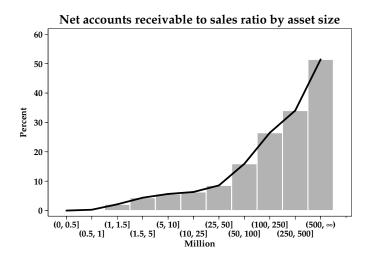


Figure A1: Trade credit and firm size

Notes: The data are taken from the SOI corporate tax return statistical collection. For each firm size class, the ratio of net accounts receivable to sales is calculated by the sum of net accounts receivable of all firms in that class divided by the sum of their business receipts. Financial, retail, and wholesale sectors are excluded.

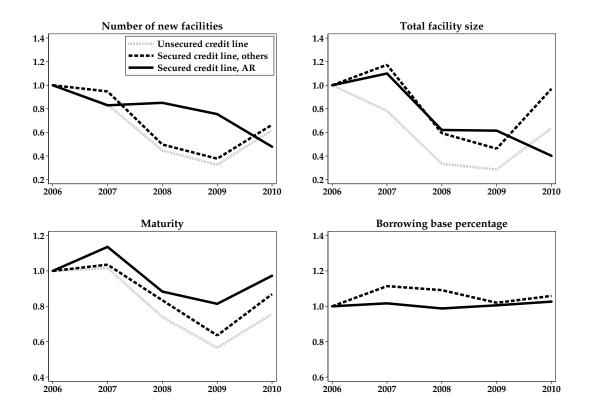


Figure A2: Characteristics of new credit line facilities

Notes: We compute the characteristics of the newly opened credit line facilities of each year as the average of all credit line facilities that are opened in that year. The solid lines in these figures are credit line facilities that require accounts receivable as collateral. The dashed lines are credit line facilities that require other types of assets as collateral. The dotted lines are unsecured credit line facilities. The time series are normalized such that they are 1 in year 2006.

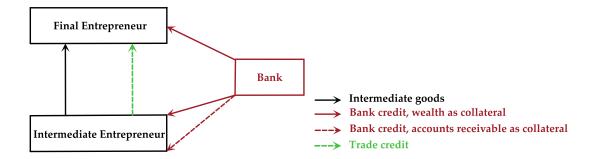


Figure A3: Flow of goods and credit

Notes: This figure shows the flow of goods and credit in the model. Intermediate goods entrepreneurs provide intermediate goods (black solid arrow) and trade credit (green dashed arrow) to final goods entrepreneurs. The bank provides credit to both intermediate goods entrepreneurs and final goods entrepreneurs with either wealth as collateral (red solid arrow) or accounts receivable as collateral (red dashed arrow).

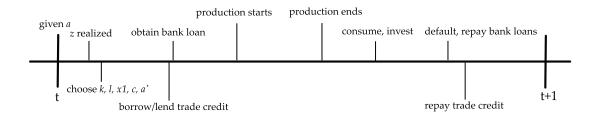


Figure A4: **Timing**

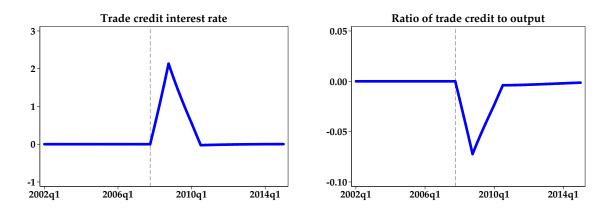


Figure A5: Dynamics of trade credit

Notes: The figures show the changes in trade credit in the benchmark economy after the financial crisis. The left panel shows the trade credit interest rate, and the right panel shows the ratio of trade credit to output. The lines are normalized to 0 at the beginning of the crisis.

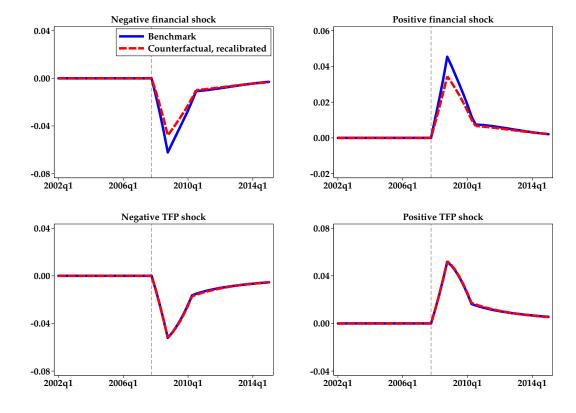


Figure A6: Output dynamics following financial/TFP shocks

Notes: The figures show the changes of output in the benchmark economy and the counterfactual economy after a negative financial shock, a positive financial shock, a negative TFP shock, and a positive TFP shock. The solid blue lines represent the benchmark economy (with trade credit) while the dashed red lines represent the counterfactual economy (without trade credit). All lines are normalized to 0 at the beginning of the crisis.

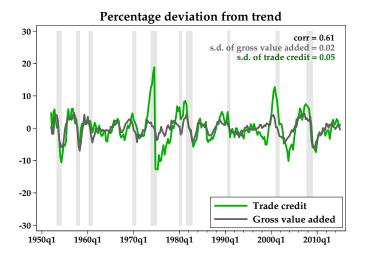


Figure A7: Cyclicality of trade credit

Notes: The data are for the nonfinancial corporate sector. Gross value added is taken from NIPA Table 1.14 line 17. Trade credit is computed as the average of accounts receivable (line 15 of Flow of Funds Table L.103) and accounts payable (line 30 of Flow of Funds Table L.103). Both time series are HP-filtered with a smoothing parameter of 1,600.

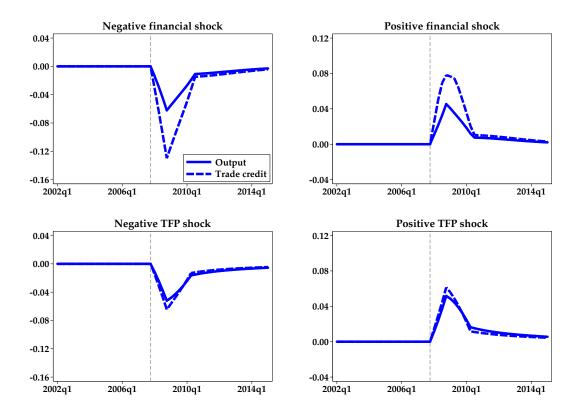


Figure A8: Trade credit and output dynamics following financial/TFP shocks

Notes: The figures show the changes in trade credit and output in the benchmark economy after a negative financial shock, a positive financial shock, a negative TFP shock, and a positive TFP shock. The solid blue lines represent output, while the dashed blue lines represent trade credit. All lines are normalized to 0 at the beginning of the crisis.