Wholesale Funding, Credit Risk and Coordination

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Abstract

This paper presents a model of bank debt rollover to study how credit risk is affected by wholesale funding, short-term debt financing, and capital market liquidity. In the model, a wholesale financer and a continuum of small creditors independently make roll-over decisions based on private information. In equilibrium, wholesale funding is a bouble-edged sword. A higher precision in the wholesale creditor's information on the asset quality of the bank reduces credit risk. However, a larger proportion of wholesale funding does not always reduce credit risk. Moreover, a larger proportion of short-term debt financing, as well as a decrease in market liquidity, reduces the willingness of creditors to roll over, and thereby raises credit risk.

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1 Introduction

Banks such as commercial banks, investment banks and financial institutions alike increasingly rely on rolling over short-term wholesale debt[1] to finance their investment in long-term risky assets (Shin, 2008). Wholesale funds[2] are usually raised on a short-term rollover basis

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[1] As Gorton and Metrick (2010) explain, the main reason for this reliance is the rapid growth of money under management by large creditors such as institutional investors, pension funds, mutual funds, states and municipalities, and nonfinancial firms. These institutions would like to have a demand deposit-like product.

[2] To understand the magnitude of wholesale funding, we show some data on repo and commercial papers. Data on repo are rare and various. For example, Fed shows in March 2008, the total amount of repo in the

with instruments such as repo, commercial papers, interbank deposits, etc. This type of financing model exposes borrowers to the risk that their short-term debts may not be rolled over. Brunnermeier (2009) points out that the deterioration in capital market liquidity coupled with the inability to roll over short-term wholesale debt is one of the direct causes of the failures of Bear Stearns, Lehman Brothers, Washington Mutual, and, eventually, the collapse of a significant part of the U.S. financial system during the 2007-2008 financial crisis.

These features illustrate factors, such as wholesale funding, short-term debt financing, and capital market liquidity, that affect credit risk[3]. In particular, as wholesale financers were critisized during the financial crisis to over rely on information from rating agencies, we are interested in two questions. The first question is whether a better informed wholesale creditor decreases credit risk. The second is whether a larger proportion of wholesale funding lowers credit risk. Theoretical studies on these factors include studies that focus on short-term debt financing (Morris and Shin, 2004), wholesale funding (Calomiris, 1999; Huang and Ratnovski, 2011), market liquidity (Diamond and Rajan, 2005; Brunnermeier and Pedersen, 2009), and market freezes resulting from short-term debt rollover (Plantin, 2009; Acharya et al., 2011). However, to our knowledge, no study exists that addresses how these factors combined affect credit risk.

This paper provides a model to fill in this gap. The key insight that we suggest is that wholesale funding is a double-edged sword. Unlike Calomiris (1999) focusing on the "bright side" and Huang and Ratnovski (2011) stressing the "dark side" of wholesale funding, our model provides general results on the role of wholesale funding. A higher precision in the wholesale creditor's information on the asset quality of the bank reduces credit risk. More interestingly, for a given level of short-term debt financing and market liquidity, a larger proportion of wholesale funding reduces credit risk provided that private information is more precise than public information or the premium of rolling over is sufficiently high. Otherwise, a larger proportion of wholesale funding raises credit risk. In addition, short-term debt financing, as well as deterioration in capital market liquidity, increases credit risk.

Formally, we consider a bank that can be interpreted as an investment bank, a commercial bank, or a financial institution. The bank relies on rolling over short-term debt to finance its investment in long-term risky assets. Its short-term debt is held by a wholesale financer and a continuum of small creditors. When short-term debt matures, holders have to decide independently whether to roll over their loans or not. In a competitive setting, creditors are reluctant to share information about the fundamentals of their debtor. If a creditor believes that, on average, the other creditors are likely to foreclose on their loans, he will foreclose as well. As a result, creditors cannot coordinate perfectly when making their investment

U.S. is 4.5 trillion, while the number from Securities Industry and Financial Markets association in 2005 is 5.21 trillion. Using a different measure, King (2008) estimates that the number is 10 trillion dollars at year-end 2007. Despite of the diverse data sours and measure, it is widely accepted that repo is a very important funding source. The size of commercial papers is relatively smaller than repo, but it is as important as treasure bills. The total short-term asset-backed commercial paper (ABCP) outstanding in the U.S. market grew from US$650 billion in January 2004 to US$1.3 trillion in July 2007. At that time, ABCP was the largest money market instrument in the United States. For comparison, the second largest instrument was Treasury Bills with about $940 billion outstanding.

[3] To measure the risk in financial crisis, Gorton and Metrick (2012) constructed a weighted average of haircut of repo. From September 2007, haircut index kept rising from 5 percent and reached 45 percent at the end of 2008.

decisions, such as whether to roll over their loans or not.

The main structure for our model is a global-games framework. Global games, developed by Calsson and van Damme (1993a, b), have been applied in various contexts in the literature. In focusing on the role of large players, our work is related to Corsetti *et al.* (2004) and Liu and Mello (2011). Corsetti *et al.* (2004) show that the presence of large traders makes small traders more aggressive in currency attacks. In our model, the presence of less informed wholesale financers reduces the willingness of small creditors to roll over. Liu and Mello (2011) show that institutional creditors foreclose if their financial positions deteriorate through the lending channel. In our model, focusing on the borrower's balance sheet, wholesale creditors will foreclose if the borrower is highly leveraged coupled with the deterioration in capital market liquidity.

The main mechanism of the model is as follows. At the refinancing stage, the bank's liquidity depends on how much cash it can raise from the capital markets by pledging its assets as collateral, which, in turn, depends jointly on its asset quality and market liquidity. The bank's risky asset return is not perfectly observable. The inability of observe the risky asset return leads to imperfect coordination between short-term creditors when deciding to rollover or not their loans. The role of wholesale funding is demonstrated in the case when additional foreclosure from the wholesale financer is needed to make the bank fail. In this case, after considering the beliefs of small creditors in the spirit of higher order beliefs of "beauty contest" described by Keynes (1936), if the wholesale financer believes that the bank's financial position is not sustainable whether because there is a deterioration in capital market liquidity or because the asset quality is not good enough, he decides to foreclose. The wholesale financer's foreclosure will make the bank fail. Thus, even abstracting from modeling his financial constraint, the wholesale financer may withdraw upon a hint of negative news[4].

We explicitly model credit risk, which is decomposed, as in Morris and Shin (2010), into insolvency risk and illiquidity risk. Illiquidity risk is defined as the probability that the bank will fail because of a run, when it would not have been insolvent in the absence of a run, and insolvency risk is defined as the probability that the bank will fail if there is no run.

The results show that a higher precision in the wholesale financer's information on the financial capacity of the bank increases the willingness of the small creditors to roll over their loans and thereby reduces credit risk. Intuitively, if the wholesale financer arbitrarily has more precise information on the fundamentals of the debtor, his own switching point is lowered. Because the switching point of the small creditors is positively related to that of the wholesale financer, it is reduced as well. The main reason is that when deciding to roll over or not, each creditor takes into account not only his own belief but also the average opinion of other creditors.

Furthermore, the most interesting result is that analytically the size effect of the wholesale financer is ambiguous. This result suggests that short-term wholesale funding is a double-

[4] As documented by Gorton and Metrick (2012), on August 9, 2007, the French bank BNP Paribas stopped withdrawals from three funds invested in mortgage-backed securities and suspended calculation of net asset values. The interest rate spread of overnight short-term asset-backed commercial paper (ABCP) over the Federal Funds rate increased from 10 basis points to 150 basis points within one day of the BNP Paribas announcement. Subsequently, the market experienced a bank "run" that originated in shadow banking, and ABCP outstanding dropped from \$1.3 trillion in July 2007 to \$833 billion in December 2007.

edged sword such that only under certain conditions, a larger proportion of wholesale funding reduces credit risk. Intuitively, in comparison to the case without wholesale funding, the presence of the wholesale financer reduces the incidence of imperfect coordination resulted from small creditors but add new imperfect coordination between the small creditors and the wholesale financer. Thus, if the decreased part of credit risk due to the presence of the wholesale financer is larger than the increased part, then credit risk will be reduced. Otherwise, credit risk will be increased. After verifying different combinations of parameter values, our numerical solutions reveal two independant conditions. First, an increase in the size of the wholesale financer reduces credit risk provided that private information is more precise than public information. Otherwise, an increase in the size of the wholesale financer raises credit risk. This result explains why when wholesale financers rely on public costly but low quality information from rating agencies, wholesale funding increases credit risk. Second, an increase in the size of wholesale funding reduces credit risk if and the premium of rolling over is sufficiently high. Just for illustration, this result explains why CIT group, the largest financer of small business in the U.S., succeeded in the first time rolling over when its institutional creditors rolled over their loans with very attractive promised return of rolling over and valuable assets as collateral, but failed during the second time rolling over with lower promised return of rolling over and less valuable assets left to serve as collateral.

Last, a larger proportion of short-term debt financing makes the bank more vulnerable to creditor runs, and thereby increases credit risk. In addition, a decrease in market liquidity raises credit risk. If the capital markets are less liquid, the bank's liquidity deteriorates. The deterioration in the bank's liquidity raises both small and large creditors' thresholds to roll over, and, consequently, increases credit risk.

The situation under which a bank can fail because of imperfect coordination among creditors is similar to the situation caused by bank runs. As the seminal paper by Diamond and Dybvig (1983) shows, bank runs occur when numerous depositors withdraw their deposits simultaneously because they believe that the bank is, or might become, insolvent. Diamond and Dybvig's model provides an example of a game with more than one Nash equilibrium. If a depositor expects all of the other depositors to withdraw their funds, then it is rational for the depositor to withdraw his deposit. Hence, bank runs occur in equilibrium. Otherwise, there is equilibrium without a run. The most important policy implication is that deposit insurance has helped to prevent bank runs. However, their model does not provide tools that can predict which equilibrium occurs. Rochet and Vives (2004), Goldstein and Pauzner (2005), and He and Xiong (2011) use global game methods to obtain a unique equilibrium in bank runs. The main difference between our model and these models is that we study bank runs with heterogeneous players.

Studies on credit risk can be traced back to the classic paper by Merton (1974). In that model, employing real-option method, defaulting risk is determined by the debtor's asset quality. The debtor is insolvent only when his asset value is lower than his debt. However, the studies in this framework consider only a single creditor's decision problem and overlook the credit risk resulting from coordination failure between the creditors and thereby underestimate the credit risk. Morris and Shin (2004; 2010) adopt a global-games framework to study how coordination failure between small creditors can increase credit risk. In our model, we study credit risk with both large and small creditors.

This paper is organized as follows. We present the model in Section 1 and solve the equilibrium in Section 2. Then, we present the equilibrium properties in Section 3, and analyze credit risk in Section 4. Finally, Section 5 provides some concluding remarks.

2 The model

This section first describes the players, timing, and payoffs of the game and then displays the information structure of both the small and large creditors. Last, it presents a special case when all of the short-term creditors are small to set a benchmark for the primary results.

2.1 Players, timing, payoffs, and perfect information

The game involves a bank, a continuum of small creditors, and a wholesale creditor. As a typical bank's T-table (cf. Table 1) shows, on the assets side, the bank holds cash and long-term risky assets. On the liabilities side, though holding equity, the bank issues both long-term and short-term debt to finance the holding of long-term risky assets. The bank relies on rolling over short-term debt. Short-term debt includes wholesale debt and retail debt. Short-term wholesale debt is provided by a wholesale financer, while short-term retail debt is borrowed from a continuum of small creditors indexed by the interval $[0, 1]$. The distinguishing feature of the wholesale financer is that he has a sufficiently large amount of funds to finance the bank's short-term debt up to the limit of $p \in (0, 1)$. In contrast, the set of all small creditors together has a proportion of $1 - p$.

There are three event dates, *ex ante* (date 0), interim (date 1), and *ex post* (date 2). There is no discounting, and everyone is risk-neutral. At date 0, the bank, holding equity of E, issues both long-term and short-term debt to acquire A units of risky assets maturing at date 2. The face value of long-term debt is L_2 maturing at date 2, while the face value of short-term debt is S_1 maturing at date 1. Although there is a maturity mismatch between short-term debt financing and long-term asset holding, the expected asset return at date 0 is sufficiently large such that *ex ante* creditors are willing to lend. The loan contract is an incomplete contract.

At date 1, short-term creditors have to decide whether to roll over their loans or not. Whether the bank can remain in operation until date 2 depends on its financial capacity to meet the claims of the short-term creditors that decide not to roll over their loans. When deciding to roll over, a key factor that creditors consider is the bank's financial position, which is determined by its asset return. Each unit of bank assets pays a gross amount of θ_2 in period 2. We denote θ_0 and θ_1 as the expected value of θ_2 in period 0 and 1 respectively such that

$$\begin{aligned}\theta_1 &= \theta_0 + \varepsilon_1 \\ \theta_2 &= \theta_1 + \varepsilon_2,\end{aligned}$$

where ε_1 and ε_2 are independently distributed and follow a normal distribution with mean 0, and precision σ_1 and σ_2, respectively. Their respective cumulative functions are denoted by $F_1(\cdot)$ and $F_2(\cdot)$. Here, θ_1 and θ_2 can be considered to be public signals that are available at date 0 and 1, respectively.

The bank's financial position is illustrated by its balance sheet. On the asset side, the bank holds cash M and A units of risky assets. On the liability side, the bank finances its assets with three sources of funding: long-term debt, short-term debt and equity. The balance sheet at date 2 can be used to deduce the bank's financial position at other periods because of the iterative form of asset return. Let S_2 denote the face value of short-term debt at date 2, which is the amount promised to short-term debt holders at date 2 and E_2 equity at date 2. Thus, the bank's balance sheet at date 2 takes the following form.

Table 1
The bank's balance sheet at date 2.

Assets	Liabilities
Cash, M	Long-term debt, L_2
Risky Assets, $\theta_2 A$	Short-term debt held by the wholesale financer, pS_2
	Short-term debt held by the small creditors, $(1-p)S_2$
	Equity, E_2

The bank is solvent at date 2 if the *ex post* equity, E_2, is positive. That is, if

$$M + \theta_2 A \geq L_2 + S_2,$$

which gives a critical value of insolvency[5] θ_2^* such that

$$\theta_2^* \equiv \frac{L_2 + S_2 - M}{A}.$$

If the bank is insolvent at date 2, it must be liquidated. The recovery rate for both short-term and long-term debt holders is normalized to zero[6].

If some of the short-term creditors choose not to roll over their loans, the bank has limited capacity to raise new funds to repay them by pledging its assets as collateral. As a result, the bank's financial capacity at date 1 depends on how much it can borrow. The amount of cash that can be raised from one unit of a bank asset is $\lambda\theta_1$, where $\lambda \in [0, 1]$ reflecting capital market liquidity. When $\lambda = 0$, no cash can be raised. When $\lambda = 1$, the capital markets are quite liquid. Still, the amount of cash that the bank can raise depends on the expected return of its assets. Thus, the financial capacity of the bank at date 1 is $M + \lambda\theta_1 A$. The liquidity ratio can be defined as

$$\pi \equiv \frac{M + \lambda\theta_1 A}{S_1}.$$

If $\pi \geq 1$, then the bank has sufficient liquidity to repay its short-term creditors and there is no illiquidity risk. Thus, we focus on the case when $\pi < 1$, in which the illiquidity risk

[5] We assume that if the bank remains in operation, then the fundamentals of the risky asset remain unaffected by the extent of the run in the interim stage. In other words, partial liquidation is excluded. Otherwise, as asset returns follow a normal distribution, the analysis in our model would become tremendously complicated. Taking partial liquidation into account would not qualitatively change our results, although it might quantitatively change them.

[6] In general, the recovery rate is positive. The recovery rate is normalized to zero in our model to simplify the algebra. A positive recovery rate will not qualitatively change our results, although it might change them quantitatively.

is positive. If the proportion of creditors not rolling over their loans is larger than π, then the bank fails in a run. If the bank fails in a run, the short-term creditors that rolled over their loans will receive a payoff that is normalized to zero. However, if the bank remains in operation until date 2, the short-term creditors that rolled over will receive a payoff of $r_s = S_2/S_1$. The short-term creditors that decide not to roll over will obtain a payoff of liquidation $r^* > 0$. The matrix of gains for a short-term creditor is given in Table 2.

Table 2
Matrix of gains.

Action/State	Continuation	Liquidation
Roll over	r_s	0
Foreclose	r^*	r^*

If $r_s \leq r^*$, then the dominant strategy is to foreclose. If $r^* < r_s$, there is no dominant dominant strategy. We focus on the case when $0 < r^* < r_s$.

Now, consider the case when, at date 1, the creditors perfectly observe θ_1. For a given λ, the financial capacity of the bank is perfectly known. However, the bank asset return at date 2, θ_2, is still uncertain. The optimal strategy for a creditor is to roll over if the bank is liquid *(i.e.* $M + \lambda\theta_1 A \geq S_1$) and solvent in the next period with a sufficiently high probability (*i.e.* $\Pr(\theta_2 \geq \theta_2^*) \geq r^*/r_s$). Thus, it is optimal to foreclose if the bank is liquid but there is not a high enough probability that it will be solvent (*i.e.,* $\Pr(\theta_2 \geq \theta_2^*) < r^*/r_s$). There is no coordination problem in these two cases. If the bank is illiquid *(i.e.,* $M + \lambda\theta_1 A < S_1$) and $\Pr(\theta_2 \geq \theta_2^*) \geq r^*/r_s$), a creditor's payoff will depend on the other creditors' actions. If the other creditors roll over their loans, a creditor who forecloses will lose the opportunity to obtain r_s. If the mass of creditors that foreclose is large enough to make the bank fail, a creditor who rolls over will receive 0 by losing the opportunity to receive r^*. The mass of creditors who roll over or foreclose is between 0 and 1. Because of the uncertainty regarding θ_2, two types of inefficiencies can occur in the equilibrium. One inefficiency is inefficient liquidation and the other is inefficient rolling over. If coordination failure induces an *ex post* solvent bank to fail at the interim stage, then inefficient liquidation occurs. Following the rolling over of the debt, if the bank is insolvent at date 2, then inefficient roll over occurs.

2.2 Imperfect Information

We consider the most general case when creditors at the interim stage receive imperfect information on θ_1. Both the small creditors and the wholesale financer observe noisy signals x_i and y such that

$$\begin{aligned} x_i &= \theta_1 + e_i \\ y &= \theta_1 + \upsilon \end{aligned}$$

where e_i and υ are normally distributed with mean 0 and precision α and β, respectively. Their respective cumulative functions are denoted by $G(\cdot)$ and $H(\cdot)$. In addition, the creditors are reluctant to share information such that $cov(e_i, \upsilon) = 0$, and $cov(e_i, e_j) = 0$ for $i \neq j$. With imperfect information, the creditors face multiple uncertainties: the financial capacity of the bank to meet its short-term debt claims, the future payoffs in period 2 if rolling over, and the actions of others.

For a creditor, the posterior of his belief in θ_1 is obtained through a simple updating rule. A small creditor's posterior is

$$X_i = \frac{\sigma_2 \theta_2 + \alpha x_i}{\sigma_2 + \alpha}. \tag{1}$$

In a similar way, the wholesale financer's posterior is

$$Y = \frac{\sigma_2 \theta_2 + \beta y}{\sigma_2 + \beta}. \tag{2}$$

Now we consider the strategies of the creditors. A strategy for a creditor is a decision rule that maps each realization of the signal to the action of rolling over his loan or not. The strategy can be naive or sophisticated. A naive strategy is a decision rule that is based only on private information concerning the fundamentals without considering the beliefs of others. A sophisticated strategy is a decision rule that is based not only on private information concerning the fundamentals but also by taking the beliefs of others into account.

For competitive considerations, creditors are reluctant to share information. If a creditor adopts a naive strategy, then he will foreclose if his signals reveal that the fundamentals are not sound; otherwise, he will roll over despite the actions of others. This naive strategy turns the game into a single player's decision problem. Because the payoff of a creditor depends on the actions of others, he is better off adopting a sophisticated strategy.

For a player, it is rational to take higher order beliefs into account. However, when constructing the equilibrium of a game with a continuum of players, it is challenging to keep tracking each layer of each player's anticipation regarding the beliefs of others. Global game methods provide a simple procedure. As shown in Morris and Shin (2004), a simplistic strategy in which each creditor chooses the best action for a uniform belief regarding the proportion of other creditors choosing a certain action generates the same equilibrium outcome as a sophisticated strategy in which each creditor takes the beliefs of others into account. The equilibrium is constructed by assuming that each player adopts a switching strategy, which is a strategy in which a creditor rolls over whenever his estimate of the underlying fundamentals is higher than a given threshold. Otherwise, he forecloses.

To simplify notation, we will set $\sigma_2/\alpha \to 0$, and $\sigma_2/\beta \to 0$. This simplification implies either that the public information $\sigma_2 \to 0$ for α and β finite or that α, $\beta \to \infty$ for a finite σ_2,

$$\lim_{\sigma_2/\alpha \to 0} x = X, \text{ and } \lim_{\sigma_2/\beta \to 0} y = Y.$$

Before solving the game with two types of creditors, we present a brief discussion of a special case when all of the creditors are small to set a benchmark for the main results.

2.3 Small creditors only

The case with the small creditors alone leads to the symmetric game of Morris and Shin (2010) with the difference that, in our case, the financial capacity of the bank is not perfectly observable. Each creditor of the same type possesses, via the same method, the information, and adopts the same switching strategy in which he forecloses if his updated signal falls below a critical value x^*. An equilibrium is a profile of strategies such that the strategy of a creditor maximizes his expected payoff conditional on the information available, when all of the other creditors are following the strategies in the profile. Then the equilibrium is characterized by a critical state θ_1^*, below which the bank will always fail, and a critical value of the individual signal x^*, such that the creditors receiving a signal below this value will always foreclose.

The equilibrium is solved in two steps. The first step is to derive the critical mass condition. If the true state is θ_1, a creditor forecloses whenever his signal is below x^*. The probability that any particular creditor receives a signal below x^* is

$$\Pr(x \leq x^*|\theta_1) = G\left(x^* - \theta_1\right),$$

which is also the proportion of creditors foreclosing. That is, a creditor has a uniform belief regarding the proportion of creditors that foreclose. Then the failure point at which the bank is liquidated θ_1^* is defined by the following critical mass condition

$$\lambda\theta_1^* A = S_1 G\left(x^* - \theta_1^*\right) - M$$

Let D denote the total debt (*i.e.*, $D = S_1 + L_2$), and τ denote the short-term debt ratio (*i.e.*, $\tau = S_1/D$). Then the above mass condition can be rewritten as

$$\lambda\theta_1^* A = \tau D G\left(x^* - \theta_1^*\right) - M. \tag{3}$$

Second, we derive the indifference condition between rolling over and foreclosing. Conditional on the updated signal, the interim probability of insolvency is

$$N_1\left(x_i\right) = \Pr(\theta_2 \leq \theta_2^* \mid x_i) = F_2\left(\theta_2^* - x_i\right).$$

This probability is derived from $\Pr(\theta_2 \leq \theta_2^* \mid x_i) = \Pr(\varepsilon_2 - e_i \leq \theta_2^* - x_i \mid x_i)$. Because $\sigma_2/\alpha \to 0$, it is straightforward that $\varepsilon_2 - e_i$ is normally distributed with precision σ_2, and its cumulative function is $F_2\left(\cdot\right)$. Conditional on the updated signal, given θ_1^*, the creditor has the conditional probability of a successful continuation of

$$\Pr(\theta_2 > \theta_2^*, \theta_1 > \theta_1^*|x) = \left(1 - G\left(\theta_1^* - x\right)\right)\left(1 - F_2\left(\theta_2^* - x\right)\right).$$

The expected payoff of rolling over is $r_s\left(1 - G\left(\theta_1^* - x\right)\right)\left(1 - F_2\left(\theta_2^* - x\right)\right)$, while the payoff to foreclosure is r^*. Hence, the indifference condition between rolling over and foreclosing on the debt is

$$\left(1 - G\left(\theta_1^* - x^*\right)\right)\left(1 - F_2\left(\theta_2^* - x^*\right)\right) r_s = r^*. \tag{4}$$

From the mass condition and the indifference condition, we can solve for θ_1^* and x^*, which characterize the unique equilibrium (See the proof in Morris and Shin, 2004).

The interim illiquidity risk is the probability that the bank will fail in a run but would have been solvent if no run occurs. With small creditors alone, the interim illiquidity risk is

$$L_1\left(\theta_1\right) = \begin{cases} 1 - F_2\left(\theta_2^* - \theta_1\right) & \theta_1 \leq \theta_1^* \\ 0 & \theta_1 > \theta_1^* \end{cases}$$

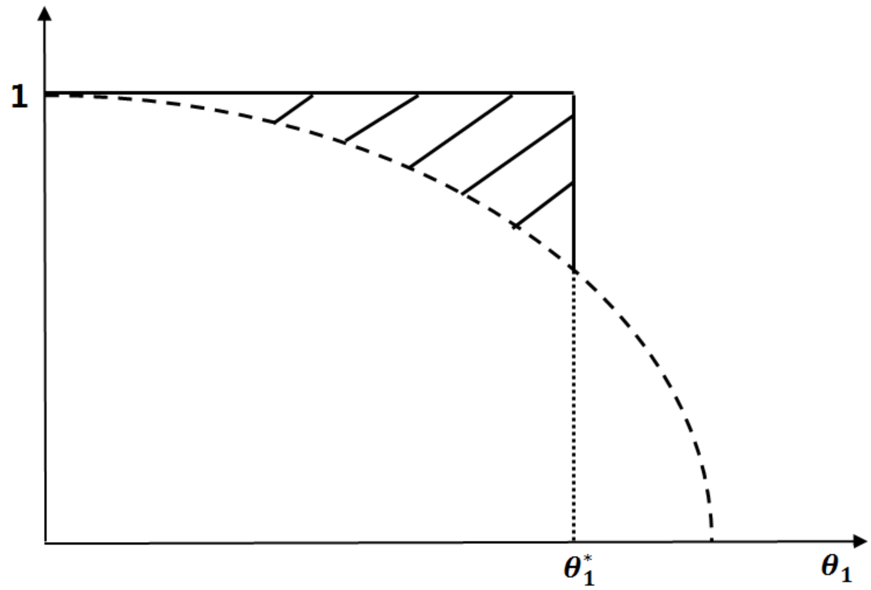


Figure 1: Interim credit risk considering small creditors only. The figure depicts the interim credit risk as a function of the expected asset return θ_1 when all creditors are small. θ_1^* is the critical state below which the bank fails in a run. The broken line represents insolvency risk. In the shaded area, the distance between the horizontal continuous line and the broken line represents the illiquidity risk.

The relationship between the interim insolvency risk and the interim illiquidity risk is shown in Figure 1.

Because θ_1 is the expected value of the risky asset return at date 1, the interim insolvency risk is decreasing in θ_1. The interim illiquidity risk is represented by the distance between the horizontal continuous line and the broken line in the shaded area. The illiquidity risk is the probability that the bank will fail although it would have been solvent without a run. When $\theta_1 > \theta_1^*$, the bank's financial capacity is large enough to meet its short-term claims. The interim illiquidity risk is zero. The insolvency risk on the right side of the critical point θ_1^* represents the probability that the bank will fail even after a successful rollover. Thus, the interim credit risk is decomposed into three parts: the interim illiquidity risk, the interim insolvency risk when the bank's financial capacity is lower than the critical point θ_1^*, and the interim insolvency risk after a successful rollover.

From the point of view of a long-term debt holder, knowing the *ex ante* credit risk is of central importance. The *ex ante* insolvency risk is

$$N_0(\theta_0) = \int_{-\infty}^{+\infty} F_2(\theta_2^* - \theta_1) f(\theta_1 - \theta_0)\, d\theta_1, \tag{5}$$

which is given by the expectation of the area under the broken line indicated in Figure 1.

The *ex ante* illiquidity risk is

$$L_0(\theta_0) = \int_{-\infty}^{\theta_1^*} (1 - F_2(\theta_2^* - \theta_1)) f(\theta_1 - \theta_0) \, d\theta_1, \tag{6}$$

which is given by the expectation of the shaded area indicated in Figure 1.

3 An equilibrium with two types of creditors

We now turn to the case with both small and large creditors, where $p \in (0,1)$. The equilibrium is solved by assuming that both types of creditors follow their trigger strategies around the switching point x^* and y^*, respectively. With two types of creditors, we consider two situations under which the bank fails at the interim stage. The first situation is when the foreclosures by the small creditors alone are sufficient to make the bank fail. The second is when the additional foreclosure by the wholesale financer is needed to make the bank fail.

First, consider the situation under which the foreclosures by the small creditors alone are sufficient to make the bank fail. Conditional on a given θ_1, the mass of small creditors foreclosing is $G(x^* - \theta_1)$. The bank will fail if and only if

$$\tau D(1-p)G(x^* - \theta_1) > M + \lambda\theta_1 A.$$

Let $\underline{\theta}_1$ be the value of θ_1 that makes both sides equal, or equivalently

$$\lambda A \underline{\theta}_1 = \tau D(1-p)G(x^* - \underline{\theta}_1) - M. \tag{7}$$

If the value of θ_1 is lower than this critical value, then the bank will fail due to foreclosures by the small creditors regardless of the wholesale financer's action. When $\theta_1 \geq \underline{\theta}_1$, it does not mean that the bank will remain in operation but that the foreclosures by the small creditors are not sufficient to make the bank fail.

Next, consider the other situation in which the additional foreclosure by the wholesale financer is needed to make the bank fail. An incidence of foreclosure includes two components: foreclosure from the wholesale financer $\tau D p$ and foreclosures from the small creditors $\tau D(1-p)G(x^* - \theta_1)$. Thus, the bank will fail whenever

$$\tau D[p + (1-p)G(x^* - \theta_1)] > M + \lambda\theta_1 A.$$

From the above equation, we can define the other critical value $\overline{\theta_1}$ such that the bank fails if and only if both types of creditors foreclose:

$$\lambda A \overline{\theta}_1 = \tau D\left[p + (1-p)G\left(x^* - \overline{\theta}_1\right)\right] - M. \tag{8}$$

Figure 2 illustrates how $\underline{\theta}_1$ and $\overline{\theta}_1$ are determined. Note that $\underline{\theta}_1 < \overline{\theta}_1$. In the interval $\theta_1 \leq \underline{\theta}_1$, the liquidity of the bank is so low that the foreclosures from the small creditors alone are enough to cause it to fail regardless of the wholesale financer's actions. When $\theta_1 > \overline{\theta}_1$, the bank holds sufficient liquidity to meet the claims of both types of creditors, and

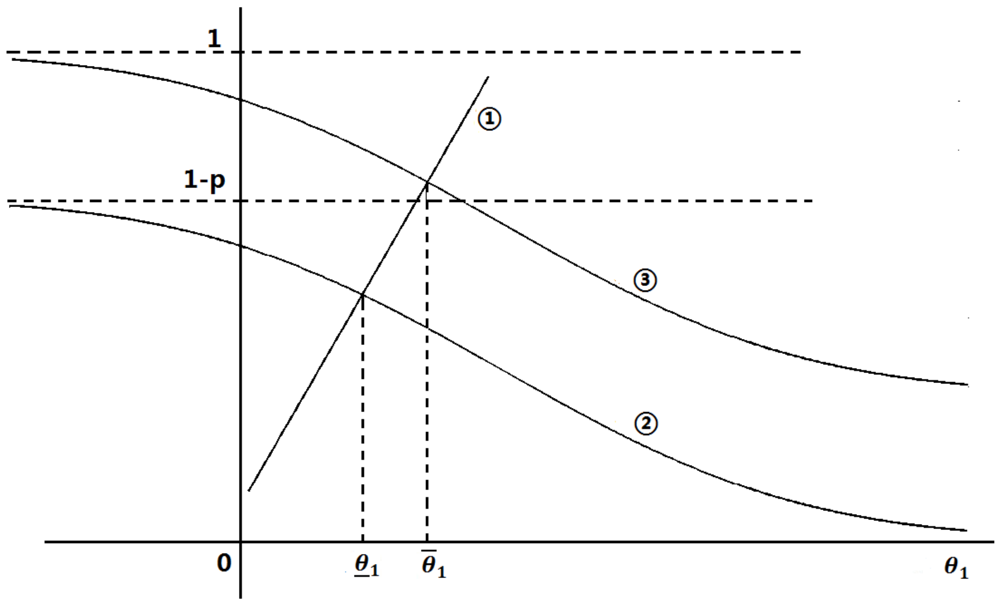


Figure 2: The incidence of foreclsure with two types of creditors. The figure depicts how the two critical states $\underline{\theta}_1$ and $\overline{\theta}_1$ are determined. Line 1 is $\lambda A\theta_1/\tau D + M/\tau D$. Line 2 is $(1-p)G(x^* - \theta_1)$, which represents the incidence of foreclosure when the foreclsoures by the small creditors alone are sufficient to cause the bank to fail. Line 3 is $p + (1-p)\,G(x^* - \theta_1)$, which represents the incidence of foreclosure when the additional foreclosure by the large creditor is needed to cause the bank to fail. Line 1 intersects Line 2 and Line 3, respectively, at $\underline{\theta}_1$ and $\overline{\theta}_1$.

the bank remains in operation. When $\underline{\theta}_1 < \theta_1 \leq \overline{\theta}_1$, the bank fails if the wholesale financer forecloses. Both $\overline{\theta}_1$ and $\underline{\theta}_1$ are functions of the switching point x^*, which, in turn, depends on the wholesale financer's switching point y^* because each creditor's payoff depends on the others' actions. To solve for these two critical points, we need two other equations in terms of $\overline{\theta}_1$, $\underline{\theta}_1$, x^*, and y^*. We appeal to the fact that both types of creditors are indifferent between foreclosing and rolling over at their own switching point, x^* and y^*, respectively.

The wholesale financer, based on the signal he receives, assigns probability $H(\underline{\theta}_1 - y)$ to the event that $\theta_1 \leq \underline{\theta}_1$. Only when $\theta_1 > \underline{\theta}_1$ can the wholesale financer's rollover lead the bank to remain in operation. The insolvency risk that the wholesale financer assigns is $F_2(\theta_2^* - y)$. Thus, the indifference condition for the wholesale financer is

$$(1 - H(\underline{\theta}_1 - y^*))\,(1 - F_2\,(\theta_2^* - y^*))\,r_s = r^*. \tag{9}$$

The wholesale financer will roll over if and only if his signal is larger than his switching point, y^*.

A small creditor's problem is a bit more complicated. In the region $(-\infty, \underline{\theta}_1]$, a small creditor receiving a signal x assigns probability $\int_{-\infty}^{\underline{\theta}_1} g(\theta_1 - x)d\theta_1$ to the event that the bank fails regardless of the actions of the wholesale financer, where $g(\cdot)$ is the density function of $G(\cdot)$. In the region of $(\underline{\theta}_1, \overline{\theta}_1]$, the bank fails if the wholesale financer forecloses. The probability that the wholesale financer forecloses at θ_1, given his trigger strategy around y^*, is $H\,(y^* - \theta_1)$. Hence, the indifference condition is given by

$$\left[1-\left(G\left(\underline{\theta}_1-x^*\right)+\int_{\underline{\theta}_1}^{\overline{\theta}_1} g\left(\theta_1-x^*\right)H\left(y^*-\theta_1\right)d\theta_1\right)\right]\left(1-F_2\left(\theta_2^*-x^*\right)\right)r_s=r^* \qquad (10)$$

where $F_2\left(\theta_2^*-x^*\right)$ is the insolvency risk assigned by the small creditor based on his noisy signal. With these four equations, we prove that there is a unique equilibrium defined by $\{x^*, y^*, \underline{\theta}_1, \overline{\theta}_1\}$. The result regarding to the equilibrium is given by the following proposition.

Proposition 1 *There is a unique dominance solvable equilibrium in the game in which the wholesale financer uses the switching strategy around y^*, while the small creditors use the switching strategy around x^*.*

The proof for this proposition is provided in Appendix A. Basically, we show that there is a unique x^* that solves equation (10). Then, we prove that this unique switching equilibrium is dominance solvable.

4 Equilibrium properties

We can now address the question of how short-term debt financing, capital market liquidity and the presence of the wholesale financer affect the bank's vulnerability to a run. The equilibrium effects of wholesale funding consist of an information effect and a size effect, which leads to two natural questions. Does the involvement of a better informed wholesale financer increase the willingness of the small creditors to roll over? Does an increase in the size of wholesale funding make the small creditors more willing to roll over? In this section, we analyze these equilibrium effects by means of propositions.

What is the effect of having a larger proportion of short-term debt financing in the bank's capital structure? The following proposition summarizes this result.

Proposition 2 *All thresholds $\left(\underline{\theta}_1, \overline{\theta}_1, x^*, y^*\right)$ are increasing in the short-term debt ratio.*

The proof for this proposition is provided in Appendix A. This proposition implies that the more the bank relies on short-term debt financing, the more fragile it will be to creditor runs. A number of studies, such as Bulow and Shoven (1978), White (1980), Morris and Shin (2001) and Detragiache and Garella (1994), find that a larger number of creditors makes debt renegotiation more difficult. However, there is a nuance to this finding. Our result suggests that, given that short-term debt is held by both large and small creditors, a larger proportion of short-term debt financing in the bank's capital structure makes it more vulnerable to creditor runs.

Are creditors more willing to roll over if the capital markets become more liquid? The following proposition provides the answer.

Proposition 3 *All thresholds $\left(\underline{\theta}_1, \overline{\theta}_1, x^*, y^*\right)$ are decreasing with market liquidity.*

The proof for this proposition is provided in Appendix A. Proposition 2 implies that, when the capital markets are more liquid, creditors are more willing to roll over. Conversely, a deterioration in capital market liquidity reduces the bank's liquidity, and thereby raises all thresholds. Here, we focus on the borrower's balance sheet by implicitly assuming that the creditors' balance sheets are not affected. Thus, a creditor does not foreclose because his financial position deteriorates. Intuitively, considering the creditors' balance sheets would amplify this effect.

4.1 Information effect

Does it matter if the wholesale financer has greater precision in its information on the bank's financial capacity[7]? This question raises a central issue in the analysis regarding the equilibrium effect, if any, of improving the quality of the wholesale financer's information. The following proposition synthesizes the result.

Proposition 4 *All thresholds* $\left(\underline{\theta}_1, \overline{\theta}_1, x^*, y^*\right)$ *decrease with the precision of the wholesale financer's information on the financial capacity of the bank.*

The proof for this proposition is provided in Appendix A. *Ceteris paribus*, a higher precision in the information concerning the bank's financial capacity for the wholesale financer increases the willingness of the small creditors to roll over their loans. Intuitively, if the wholesale financer arbitrarily has more precise information on the bank's liquidity, his switching point is reduced. This reduction, in turn, lowers the switching point of the small creditors because their switching point is a function of the wholesale financer's switching point. This result is implied by the behavior of relying on the information of others because each creditor consider not only his own signal but also the average opinion of other creditors.

With a sophisticated strategy, if the value of continuation is *ex post* higher than the value of liquidation, a small creditor relies on precise information from the wholesale financer to minimize the error of foreclosing and losing the opportunity of receiving higher payoffs. The wholesale financer takes into account the risk of an overwhelming foreclosure by small creditors to minimize the error of rolling over when others foreclose so that the bank fails by receiving only 0, which is smaller than r^*. In equilibrium, increasing the accuracy of the wholesale financer's information makes small creditors more willing to roll over.

4.2 Size effect

Does an increase in the size of wholesafe funding make the small creditors more willing to roll over? Interestingly, it is not always possible to analytically provide a definitive answer to the question of whether x^* is decreasing with the size of the wholesale financer. It is decreasing in p if and only if

$$\frac{b_2 b_5 \left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{g\left(x^* - \overline{\theta}_1\right)} < \frac{b_1(b_3 b_6 + b_4) G\left(x^* - \underline{\theta}_1\right)}{g\left(x^* - \underline{\theta}_1\right)},$$

[7] This question refects the feature that institutional creditors can have informational advantage over small creditors. In the model setting, we do not assume that the large creditor has more precise information than the small creditors. The question is what the equilibrium effects are when the large creditor's information becomes more precise.

where parameters b_1, b_2, b_3, b_4, b_5, and b_6 are as defined in Appendix A. When the above condition is satisfied, the thresholds $\underline{\theta}_1$ and y^* are decreasing in p as well. The critical state $\overline{\theta}_1$ is decreasing in p if

$$\frac{dx^*}{dp} > \frac{1 - G\left(x^* - \overline{\theta}_1\right)}{(1-p)g\left(x^* - \overline{\theta}_1\right)}.$$

The ambiguous size effect is interesting, but challenging as well. It is interesting because wholesale funding is a double edged sword rather than simplistically good or bad. It is challenging because the above conditions cannot be straightforwardly interpreted. To further explore the size effect, we proceed in two ways. First, we focus on the limiting case where $\alpha \to \infty$, $\beta \to \infty$, and $\sigma_2 \to 0$. In other words, both types of creditors have precise information, but the variance of the public information tends to be infinite. Second, we numerically solve the model. In the limiting case, (9) can be rewritten as

$$H\left(\sqrt{\beta}(\underline{\theta}_1 - y^*)\right) = 1 - \frac{2r^*}{r_s}, \tag{11}$$

in which as $\sigma_2 \to 0$, $1 - F_2\left(\theta_2^* - y^*\right) = 1/2$ is employed. Because $H\left(\sqrt{\beta}(\underline{\theta}_1 - y^*)\right) \geq 0$, we have $r^*/r_s \leq 1/2$. To make the analysis tractable, we assume $r^*/r_s < 1/2$. As $\beta \to \infty$, we must have $y^* \to \underline{\theta}_1$, or else $H\left(\sqrt{\beta}(\underline{\theta}_1 - y^*)\right)$ will be either zero or one. Hence, the wholesale financer will roll over at states to the right of $\underline{\theta}_1$. When the small creditors have very precise information, they will also roll over at sates to the right of $\underline{\theta}_1$. Thus, in the limit case, we have

$$x^* = y^* = \underline{\theta}_1.$$

The bank fails if and only if $\theta_1 < \underline{\theta}_1$. The question of whether a larger creditor raises the willingness of the small creditors to roll over hinges on the behavior at the critical state $\underline{\theta}_1$.

In solving for the critical state $\underline{\theta}_1$ in the limiting case, we need to distinguish two cases. In the limit, from (7) and (8), we have

$$\begin{aligned}
\underline{\theta}_1 &\in \left[-\frac{M}{\lambda A}, \frac{\tau D(1-p) - M}{\lambda A}\right] \\
\overline{\theta}_1 &\in \left[\frac{\tau Dp - M}{\lambda A}, \frac{\tau D - M}{\lambda A}\right].
\end{aligned}$$

Thus, we can distinguish the case when $\overline{\theta}_1 \leq t$ from the case when $\overline{\theta}_1 > t$, where $t \equiv \left[\tau D(1-p) - M\right]/\lambda A$. In the former case, $\underline{\theta}_1 = \overline{\theta}_1$. However, in the latter case, $\underline{\theta}_1 < \overline{\theta_1}$. The equilibrium value of $\underline{\theta}_1$ in the limit is characterized as follows.

Proposition 5 *In the limit as $\alpha \to \infty$, $\beta \to \infty$, and $\sigma_2 \to 0$, the critical state $\underline{\theta}_1$ tends to $\left[\tau D(1-p)G\left(-\underline{\delta}\right) - M\right]/\lambda A$, where $\underline{\delta} \equiv \sqrt{\alpha}\left(\underline{\theta}_1 - x^*\right)$ and falls under two cases. If $\overline{\theta}_1 > t$, then $\underline{\delta}$ is the unique solution to*

$$1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{+\infty} g\left(k\right) H\left(\sqrt{\frac{\alpha}{\beta}}\left(\underline{\delta} - k\right) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right) dk = \frac{2r^*}{r_s}. \tag{12}$$

If $\overline{\theta}_1 \leq t$, then $\underline{\delta}$ is the unique solution to

$$1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\Gamma} g\left(k\right) H\left(\sqrt{\frac{\alpha}{\beta}}\left(\underline{\delta} - k\right) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right) dk = \frac{2r^*}{r_s}, \tag{13}$$

where $\Gamma = G^{-1}\left(G\left(\underline{\delta}\right) + \frac{p}{1-p}\right)$.

The proof of this result is given in Appendix A. In comparison to the results from the limiting case, we can narrow the range under which $\underline{\theta}_1$ is decreasing in the size of the wholesale financer p. Because $\underline{\theta}_1 = \left[\tau D(1-p)G\left(-\underline{\delta}\right) - M\right]/\lambda A$, the overall effect of p is given by

$$\frac{d\underline{\theta}_1}{dp} = \frac{\tau D}{\lambda A}\left[-G(-\underline{\delta}) - (1-p)g(-\underline{\delta})\right]\frac{d\underline{\delta}}{dp}.$$

When p is large, so that $\overline{\theta}_1 > t$, $d\underline{\theta}_1/dp < 0$. This result occurs because, from (12), $d\underline{\delta}/dp = 0$. However, when $\overline{\theta}_1 < t$, from equation (13), we obtain $d\underline{\theta}_1/dp < 0$. In this case, the sign cannot be determined definitively. We summarize this result in the limiting case as follows.

Proposition 6 *In the limit as $\alpha \to \infty$, $\beta \to \infty$, and $\sigma_2 \to 0$, the critical state $\underline{\theta}_1$ is decreasing in p provided that $\overline{\theta}_1 > t$.*

Hence, a larger institutional creditor raises the willingness of the small creditors to roll over in the limiting case when $\overline{\theta}_1 > t$. For $\overline{\theta}_1 \leq t$, the size effect is ambiguous[8].

We need to emphasize that even when everyone has arbitrarily precise information, the interval of inefficient liquidation or rolling over persists because of strategic uncertainty. Setting α, β, and $\sigma_2 \to \infty$, in the limit, $\underline{\theta}_1 = \overline{\theta}_1 = x^* = y^*$, or $\underline{\theta}_1 = x^* = y^* < \overline{\theta}_1$ but these thresholds are still above 0. The positive thresholds imply that in equilibrium there is always inefficient liquidation or inefficient rollover.

Instead, we numerically solve the model to explore how thresholds change as p increases. We calibrate parameters under two conditions. First, the payoff for foreclosing is quite low relative to the payoff for rolling over so that the insolvency risk is high or/and the capital markets are quite illiquid. We make this choice because in numerically solving the model, we have to avoid functions that are close to step functions so that the variances of distributions are not too small. Because the variance of asset returns represents the risk as well, the liquidation value over the continuation return should be adjusted proportionally with the variances. Second, to be consistent with our model, the liquidity ratio is strictly smaller than 1, or $\pi < 1$. Because θ_1 is endogenous, $\overline{\theta}_1 < (S_1 - M)/\lambda A$. Values for these parameters are given in Table 3. Setting values for the parameters, such as M, S_1, L_2, Y, affects only the size of the bank's balance sheet, and thereby does not affect the primary results. Because insolvency risk is high, the payoff ratio is set to 0.45, and the haircut is 25 percent so that the capital markets are quite illiquid. Both types of creditors' information precision is set to 1, while the public information precision is 1/3.

[8] Corsetti *et al.* (2004) show that when the size effect is locally ambiguous , it holds globally by solving for the critical state in two special cases. One case is when $\alpha/\beta \to 0$, and the other is when $\alpha/\beta \to \infty$. For our model, we can prove that, as $\alpha/\beta \to \infty$, the critical state $\underline{\theta}_1$ is decreasing in p even when $\overline{\theta}_1 \leq t$. However, when $\alpha/\beta \to 0$, the left hand side of (12) and (13) is zero, which means that solvency risk is 1 and $r^*/r_s = 1$. Therefore, we cannot prove that the size effect is globally positive because that it lowers $\underline{\theta}_1$.

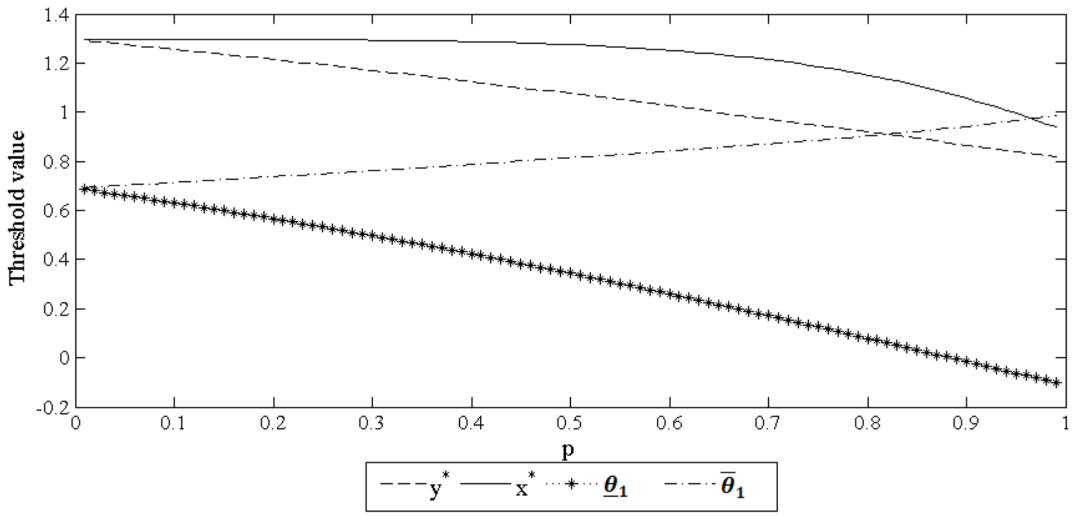


Figure 3: Thresholds as a function of the size of the large creditor p. x^* is the switching point of a small creditor, while y^* is the switching point of the large creditor. $\underline{\theta}_1$ is the critical state when foreclosures by the small creditors alone are sufficient to make the bank fail, while $\overline{\theta}_1$ is the critical stae when the additional foreclosure by the large creditor is needed to make the bank fail.

Table 3
Parameter value for numerical solutions.

Cash, M	10	Assets, A	121
Haircut, $1-\lambda$	0.25	Payoff ratio, r^*/r_s	0.45
Long-term debt, L_2	21	Short-term debt, S_1	100
Private information precision, α, β	1	Public information precision, σ_2	1/3

Then, we numerically solve a system of four nonlinear equations. We plotted all thresholds as a function of p in Figure 3. Figure 3 shows that all thresholds, except $\overline{\theta}_1$, are decreasing in p. Recall that $\overline{\theta}_1$ is defined as the critical state where additional foreclosure by the wholesale financer is needed to make the bank fail. As p increases, this additional foreclosure becomes larger. However, both the large and small creditors' switching points are decreasing in p. Thus, an increase in p not only makes the wholesale financer more willing to roll over, but also raises the willingness of the small creditors to roll over. To verify the robustness of the results, we perform the same computation using different values for $\{\lambda, \alpha, \beta, \sigma_2\}$. The results are proven to be robust.

5 Credit risk

Having established the equilibrium effects, we can now address the primary question of how short-term financing, capital market liquidity and the presence of the wholesale financer affects credit risk. During the interim period, insolvency risk is

$$N_1(\theta_1) = F_2(\theta_2^* - \theta_1),$$

and from the definition of illiquidity riks, it is straightforward that illiquidity risk is

$$L_1(\theta_1) = \begin{cases} 1 - F_2(\theta_2^* - \theta_1) & \theta_1 \leq \underline{\theta}_1 \\ H(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1)) & \underline{\theta}_1 < \theta_1 \leq \overline{\theta}_1 \\ 0 & \theta_1 > \overline{\theta}_1 \end{cases}$$

For θ_1 in the region $(\underline{\theta}_1, \overline{\theta}_1]$, $H(y^* - \theta_1)$ is the probability that the wholesale financer forecloses at θ_1, given his trigger strategy around y^*.

The interim credit risk is $C_1(\theta_1) = N_1(\theta_1) + L_1(\theta_1)$ such that

$$C_1(\theta_1) = \begin{cases} 1 & \theta_1 \leq \underline{\theta}_1 \\ H(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1)) + F_2(\theta_2^* - \theta_1) & \underline{\theta}_1 < \theta_1 \leq \overline{\theta}_1 \\ F_2(\theta_2^* - \theta_1) & \theta_1 > \overline{\theta}_1 \end{cases}$$

The *ex ante* insolvency risk is

$$N_0(\theta_0) = \int_{-\infty}^{+\infty} f_1(\theta_1 - \theta_0) F_2(\theta_2^* - \theta_1) d\theta_1,$$

and the *ex ante* illiquidity risk is

$$\begin{aligned} L_0(\theta_0) &= \int_{-\infty}^{\underline{\theta}_1} (1 - F_2(\theta_2^* - \theta_1)) f_1(\theta_1 - \theta_0) d\theta_1 \\ &\quad + \int_{\underline{\theta}_1}^{\overline{\theta}_1} H(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1)) f_1(\theta_1 - \theta_0) d\theta_1. \end{aligned} \tag{14}$$

The *ex ante* credit risk is

$$C_0(\theta_0) = N_0(\theta_0) + L_0(\theta_0).$$

Note that the changes in thresholds affect the *ex ante* credit risk only through the *ex ante* illiquidity risk. Thus, we focus on how the *ex ante* illiquidity risk is affected. We study first how the *ex ante* illiquidity risk is affected by short-term financing, capital market liquidity and a better informed wholesale financer. The following proposition provides the answer.

Proposition 7 *The ex ante illiquidity risk is increasing in the short term debt ratio. However, it is decreasing in market liquidity, as well as in the precision of the wholesale financer's information on the financial capacity of the bank.*

The proof for this proposition is presented in Appendix A. First, greater short-term debt financing increases the probability of creditor runs and credit risk. Second, an increase in market liquidity reduces credit risk. Conversely, a deterioration in capital market liquidity raises credit risk. Finally, a higher precision in the wholesale financer's information concerning the bank's financial capacity decreases credit risk.

To study the size effect on credit risk, we differentiate (14) with respect to p. Interestingly, it depends on how $\underline{\theta}_1$, y^* and $\overline{\theta}_1$ vary with respect to p (See in Appendix A). Because $\underline{\theta}_1$ and y^* are decreasing in p, while $\overline{\theta}_1$ is increasing in p, the sign of $dL_0(\theta_0)/dp$ cannot be determined definitively. To explain this result, we use the critical state θ_1^*, when all creditors are small, as a benchmark. The interim illiquidity risk with two types of creditors is displayed in Figure 4.

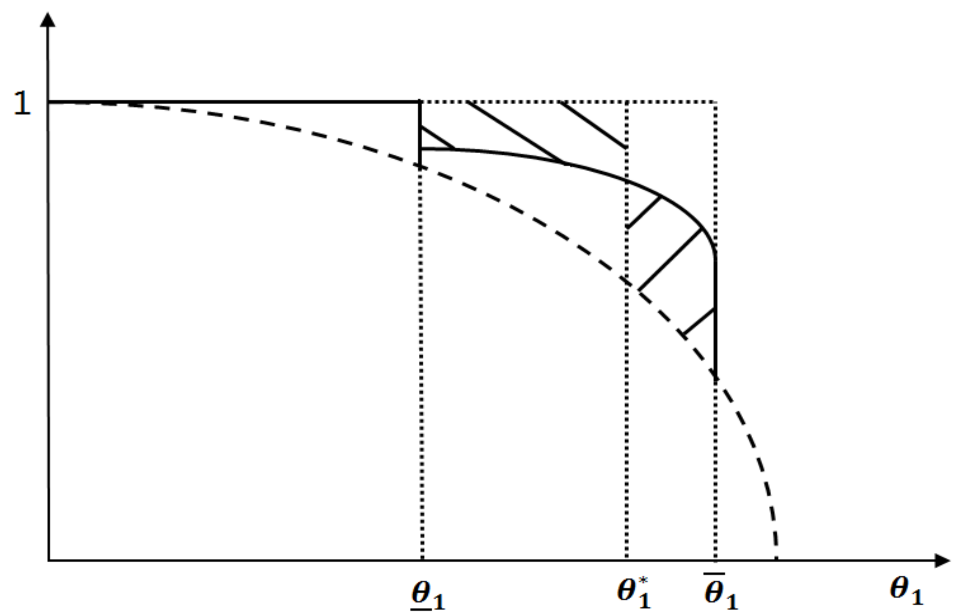


Figure 4: Interim credit risk with two types of creditors. The figure depicts the interim credit risk as a function of the expected asset return θ_1 with two types of creditors. $\underline{\theta}_1$ is the critical state when the foreclosures by the small creditors are sufficient to make the bank fail, while $\overline{\theta}_1$ is the critical state when the additional foreclosure from the wholesale financer is needed to make the bank fail. θ_1^* is the critical state when all creditors are small. The broken line represents the insolvency risk. The distance between the horizontal solide lines and the broken line represents the interim illiquidity risk. The shaded area to the left of θ_1^* represents the portion of the illiquidity risk that is decreased due to the presence of the wholesale financer, while the shaded area to the right of θ_1^* represents the portion of the illiquidity risk that is increased due to the presence of the wholesale financer.

Suppose, initially, that the short-term debt is all held by the small creditors and that the critical sate without wholesale funding is θ_1^*. Now, the short-term debt is held by both the wholesale financer and the small creditors. The presence of the wholesale financer will lower $\underline{\theta}_1$ but will raise $\overline{\theta}_1$. Supposing that $\theta_1^* \in (\underline{\theta}_1, \overline{\theta}_1)$, $\underline{\theta}_1$ moves to the left of θ_1^*, while $\overline{\theta}_1$ moves to the right of θ_1^*. Without the wholesale financer, the interim illiquidity risk is the distance between the horizontal continuous line and the broken line in the area to the left of θ_1^*. With the wholesale financer, because the wholesale financer rolls over with a positive probability $1 - H(y^* - \theta_1)$ for θ_1 in the region $(\underline{\theta}_1, \overline{\theta}_1]$, the shaded area to the left of θ_1^* represents part

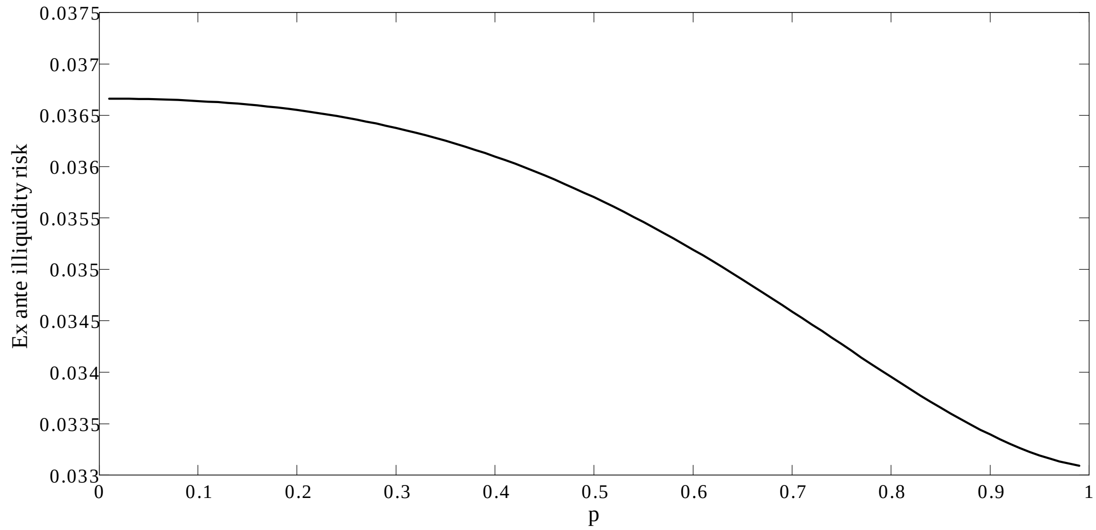


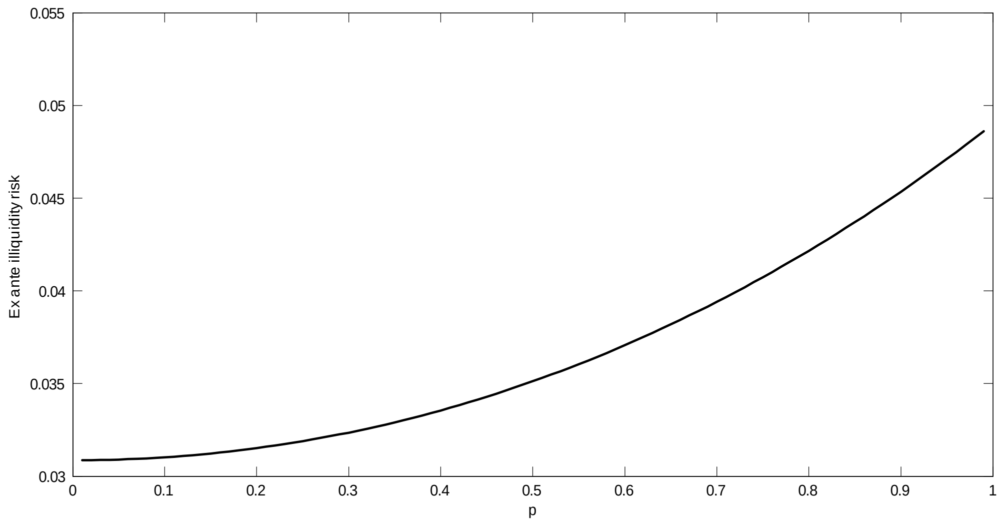
Figure 5: Ex ante illiquidity risk as a function of the size of the large creditor p.

of the illiquidity risk that is decreased due to the presence of the wholesale financer. At the same time, the presence of the wholesale financer pushes $\overline{\theta}_1$ to the right of θ_1^*. The shaded area to the right of θ_1^* represents the part of the illiquidity risk that is increased due to the presence of the wholesale financer. Intuitively, the presence of the wholesale financer reduces the incidence of imperfect coordination resulted from small creditors but add new imperfect coordination between the small creditors and the wholesale financer.

Analytically, considering the expectation of the shaded area indicated in Figure 4, if the decreased part is larger than the increased part, then the *ex ante* illiquidity risk will decrease in p. In this case, an increase in the size of wholesale funding reduces the credit risk. However, if the decreased part is smaller than the increased part in terms of credit risk, then the *ex ante* illiquidity risk will increase in p. Finally, if these opposite effects are equal, then the *ex ante* illiquidity risk is constant in p.

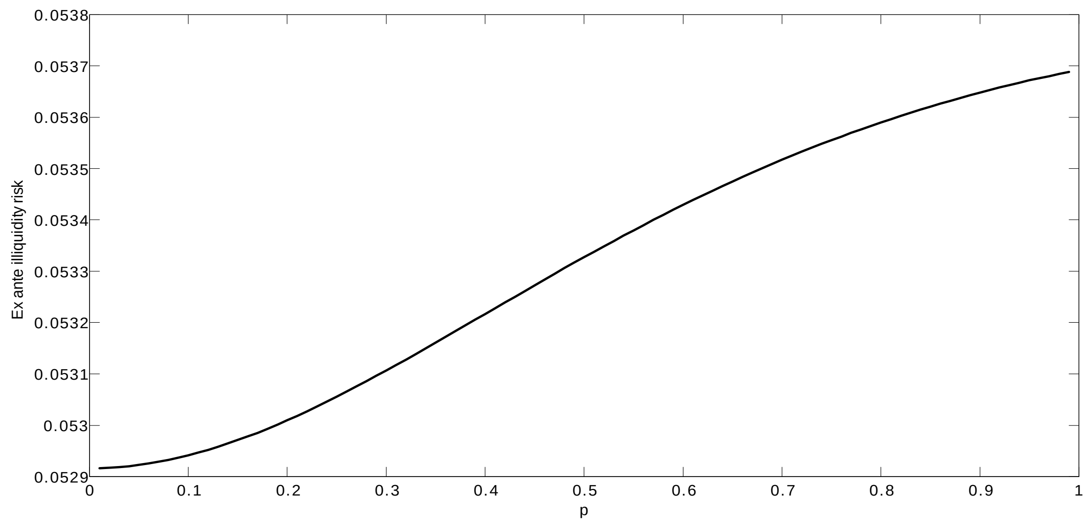
To further explore the size effect on credit risk, we numerically solve the model and compute the *ex ante* illiquidity risk as a function of p using the same parameter setting in Table 3. We plotted the *ex ante* illiquidity risk as a function of p in Figure 5. Figure 5 shows that an increase in the size of wholesale funding lowers credit risk. This result holds provided that public information is less precise than private information and the payoff ratio r^*/r_s is quite low.

We verify the robustness of this result combining different parameter values. However, we identify two independant conditions under which the opposite result can be obtained. We advance these questions in forms of questions. What is the size effect if private information is less precise than public information? The question is relevant, as pointed out by Huang and Ratnovski (2011), that creditors invest less on improving their private information and rely on costly public information provided by rating agencies. We set private and public information precision to 1 and 2 respectively in keeping other parameters unchanged. Interestingly, as Figure 6 shows, an increase in the size of the wholesale financer raises credit risk. When private information is less precise than public information, a larger proportion of wholesale funding raises credit risk. This result explains why when wholesale financers rely on public coarse information provided by rating agencies, wholesale funding played an important role in past runs. For instance, Bear Stearns failed, not because it did not meet regulatory requirement, but because wholesale creditors refused to continue funding. It is an investment bank. Even a commercial bank, Northern Rock failed, not because of the runs of depositors, but because institutional creditors refused to roll over their loans.



Ex ante illiquidity risk as a function of the size of the large creditor p with $\alpha/\sigma_2 = \beta/\sigma_2 = 1/2$.

What is the size effect of the wholesale financer if liquidation value is only slightly lower than continuation value? We set the payoff ratio r^*/r_s to 0.8 keeping other parameters unchanged. As Figure 7 shows, an increase in the size of the wholesale financer raises credit risk if the premium of rolling over is small. Just for illustration, this result explains why CIT group, the largest financer of small business in the U.S., succeeded in the first time rolling over when its institutional creditors rolled over their loans with very attractive promised return of rolling over and valuable assets as collateral, but failed during the second time rolling over with lower promised return of rolling over and less valuable assets left to serve as collateral.



Ex ante illiquidity risk as a function of the size of the large creditor p with $r^*/r_s = 0.8$.

In pulling together our discussion, the overall conclusion that we draw from our analysis is that short-term financing, capital market liquidity and the presence of the wholesale financer are important determinants of credit risk. These conclusions are the most clear cut regarding the effects of short-term financing, capital market liquidity and an increase in the wholesale financer's information. Analytically the size effect of wholesale funding is ambiguous. Our numerical calculations reveal that an increase in the size of wholesale funding lowers credit risk provided that private information is more precise than public information and the payoff ratio r^*/r_s is quite low. However, an increase in the size of wholesale funding raises credit risk if public information is more precise than private information or the payoff ratio r^*/r_s is quite high.

6 Concluding remarks

Economists have documented that factors, such as reliance on short-term debt financing, a lack of liquidity in the capital markets and the unwillingness of wholesale financers to roll over, contributed to the severity of the 2007-2008 financial crisis. In our model, a larger proportion of short-term debt financing, as well as a decrease in market liquidity, increases credit risk. The main channel for these effects is through the deterioration of the borrower's balance sheet. Moreover, wholesale funding has information and size effect on credit risk. The information effect is positive in the sense that a higher precision in the wholesale creditor's information on the asset quality of the bank reduces credit risk. Most interestingly, the size effect is ambiguous. A larger wholesale funding reduces credit risk provided that private information is more precise than public information and the premium of rolling over is high. Otherwise, a larger wholesale funding raises credit risk.

In the model, we focus on the borrower's financial position. When capital market liquidity deteriorates, creditors are more likely to withdraw their loans because of the deterioration in the borrower's financial position. If we take into account the lending channel, then these effects will be amplified.

The main assumption in our model is that all creditors decide to roll over or foreclose simultaneously by abstracting from sequential moves. In practice, creditors can make sequential decisions based on debt seniority if and only if the borrower declares bankruptcy and the court orders reorganization. Otherwise, creditors move simultaneously in deciding to roll over or not. Having said this, the possibility remains that the borrower could negotiate first with the large creditors to ensure that they will roll over. However, if the other creditors cannot observe the large creditors' moves, which is generally the case, then we still have a simultaneous game. Even if institutional creditors can signal their positions, their signals are"cheap talks". Consequently, it is still a simultaneous game.

Appendix A

Proof of Proposition *1.* *First, we will show that there is a unique x^* that solves equation (10). Second, we will show this unique switching equilibrium is dominance solvable.*

Differentiating (7) and (8) with respect to x^*, respectively, provides

$$\begin{aligned}
\frac{d\underline{\theta}_1}{dx^*} &= \frac{\tau D(1-p)g\left(x^*-\underline{\theta}_1\right)}{\lambda A+\tau D(1-p)g\left(x^*-\underline{\theta}_1\right)} \in (0,1), \\
\frac{d\overline{\theta}_1}{dx^*} &= \frac{\tau D(1-p)g\left(x^*-\overline{\theta}_1\right)}{\lambda A+\tau D(1-p)g\left(x^*-\overline{\theta}_1\right)} \in (0,1).
\end{aligned}$$

Let $\underline{\delta}=\underline{\theta}_1-x^*$ $\overline{\delta}=\overline{\theta}_1-x^*$. Both $\underline{\delta}$ and $\overline{\delta}$ are monotonically decreasing in x^* because

$$\begin{aligned}
\frac{d\underline{\delta}}{dx^*} &= \frac{d\underline{\theta}_1}{dx^*}-1<0 \\
\frac{d\overline{\delta}}{dx^*} &= \frac{d\overline{\theta}_1}{dx^*}-1<0.
\end{aligned}$$

Differentiating (9) with respect to x^*, we obtain

$$\frac{dy^*}{dx^*}=b_3\frac{d\underline{\theta}_1}{dx^*} \in (0,1), \tag{A.1}$$

where

$$b_3=\frac{h\left(\underline{\theta}_1-y^*\right)\left(1-F_2\left(\theta_2^*-y^*\right)\right)}{f_2\left(\theta_2^*-y^*\right)\left(1-H\left(\underline{\theta}_1-y^*\right)\right)+h\left(\underline{\theta}_1-y^*\right)\left(1-F_2\left(\theta_2^*-y^*\right)\right)} \in (0,1)$$

We can rewrite (10) as

We can rewrite (10) as

$$\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}} g(k)\,H\left(y^*-x^*-k\right)dk\right]\left[1-F_2\left(\theta_2^*-x^*\right)\right]r_s=r^* \tag{A.2}$$

Differentiating the left hand side of (A.2) with respect to x^*, we obtain

$$-g\left(\underline{\delta}\right)\left(1-H\left(y^*-x^*-\underline{\delta}\right)\right)\frac{d\underline{\delta}}{dx^*}-g\left(\overline{\delta}\right)H\left(y^*-x^*-\overline{\delta}\right)\frac{d\overline{\delta}}{dx^*}$$
$$-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)h\left(y^*-x^*-k\right)\left(\frac{dy^*}{dx^*}-1\right)dk$$
$$+\frac{f_2\left(\theta_2^*-x^*\right)}{1-F_2\left(\theta_2^*-x^*\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)H\left(y^*-x^*-k\right)dk\right]$$

Substituting (A.1) into the above expression shows that the left hand side of (A.2) is strictly increasing in x^*. For sufficiently small x^*, the left hand side of (A.2) is negative, while for sufficiently large x^*, it is positive. The left hand side of (A.2) is continuous in x^*. Thus there is a unique solution to (10). From (9), the wholesale financer's switching point y^* is determined.

We can finish the argument by showing that the unique switching equilibrium is the only equilibrium strategy to survive the iterative elimination of strictly dominated strategies. Consider the expected payoff to rolling over for a small creditor conditional on signal x when all other small creditors follow the switching strategy around $\widehat{x}$, and when the wholesale financer plays his best response against this switching strategy, which is to switch at $y(\widehat{x})$, obtained from (9). Denote this expected payoff by $u(x,\widehat{x})$. It is given by

$$u(x,\widehat{x})=\left[1-\left(G\left(\underline{\theta}_1(\widehat{x})-x\right)+\int_{\underline{\theta}_1(\widehat{x})}^{\overline{\theta}_1(\widehat{x})} g\left(\theta_1-x\right)H\left(y(\widehat{x})-\theta_1\right)d\theta_1\right)\right]\left[1-F_2\left(\theta_2^*-x\right)\right]r_s$$

where $\underline{\theta}(\widehat{x})$ and $\overline{\theta}(\widehat{x})$ indicate the value of $\underline{\theta}$ and $\overline{\theta}$ when small creditors follow the switching strategy around $\widehat{x}$. We allow $\widehat{x}\in\mathbb{R}\cup\{-\infty,\infty\}$ take the values $-\infty$ and ∞, by which the small creditors respectively never and always foreclose. As shown above, $u(.,.)$ is increasing in its first argument and decreasing in its second.

For sufficiently high values of x, rolling over is a dominant action for a small creditor, regardless of the actions of others, small or large. Denote by $\overline{x}^1$ the threshold value of x above which it is a dominant action to roll over for a small creditor. Since all creditors realize this, any strategy to foreclose above $\overline{x}^1$ is dominated by rolling over. Then, it cannot be rational for a small creditor to foreclose whenever his signal is higher than $\overline{x}^2$, where $\overline{x}^2$ solves

$$u(\overline{x}^2,\overline{x}^1)=r^*$$

It is so, since the switching strategy around $\overline{x}^2$ is the best reply to the switching strategy around $\overline{x}^1$ played by other small creditors and to that of the wholesale financer $y(\overline{x}^1)$, and since even the small creditor that assumes the lowest possibility of the continuation of the project believes that the incidence of continuation is higher than that implied by the switching strategy around $\overline{x}^1$and $y(\overline{x}^1)$. Since the payoff to rolling over is increasing in the incidence *of continuation by the other creditors, any strategy that refrains from rolling over for signals higher than $\overline{x}^2$ is strictly dominated. Since*

$$u\left(\overline{x}^1, \infty\right) = u\left(\overline{x}^2, \overline{x}^1\right) = r^*$$

monotonicity of u implies $\overline{x}^1 > \overline{x}^2$. Thus, suppose $\overline{x}^{k-1} > \overline{x}^k$, monotonicity implies that $\overline{x}^k > \overline{x}^{k+1}$. We can generate a decreasing sequence

$$\overline{x}^1 > \overline{x}^2 > \overline{x}^3 ... > \overline{x}^k > ...$$

where any strategy that refrains from rolling over for signal $x > \overline{x}^k$ does not survive k rounds of deletion of dominated strategies. Since the sequence is bounded, assuming $\overline{x}$ is the largest solution to $u(x, x) = r^*$, then monotonicity of u implies that

$$\overline{x} = \lim_{k \to \infty} \overline{x}^k$$

Any strategy that refrains from rolling over for signal higher than $\overline{x}$ does not survive iterated dominance.

Conversely, if $\underline{x}$ is the smallest solution to $u(x, x) = r^*$, any strategy that refrains from foreclosing for a signal below $\underline{x}$ does not survive iterative elimination. If there is a unique solution to $u(x, x) = \overline{L}$, then the smallest solution is the largest solution. Therefore, there is only one strategy that remains after eliminating all iteratively dominated strategies. This strategy is the only equilibrium strategy. This completes the argument.

Proof of Proposition 2. *Differentiating (7) and (8) with respect to τ, respectively, provides*

$$\begin{aligned}
\frac{dx^*}{d\tau} &= \frac{1}{b_1}\frac{d\underline{\theta}_1}{d\tau} - \frac{G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)}, \\
\frac{dx^*}{d\tau} &= \frac{1}{b_2}\frac{d\overline{\theta}_1}{d\tau} - \frac{p + (1-p)\, G\left(x^* - \overline{\theta}_1\right)}{\tau (1-p)\, g\left(x^* - \overline{\theta}_1\right)},
\end{aligned}$$

where

$$\begin{aligned}
b_1 &= \left(1 + \lambda A / \tau D (1-p)\, g\left(x^* - \underline{\theta}_1\right)\right)^{-1} < 1, \\
b_2 &= \left(1 + \lambda A / \tau D (1-p)\, g\left(x^* - \overline{\theta}_1\right)\right)^{-1} < 1.
\end{aligned}$$

Let $\underline{\delta} = \underline{\theta}_1 - x^*$ $\overline{\delta} = \overline{\theta}_1 - x^*$. Then we obtain

$$\begin{aligned}
\frac{d\underline{\delta}}{d\tau} &= (b_1 - 1)\frac{dx^*}{d\tau} + \frac{b_1 G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)} \\
\frac{d\overline{\delta}}{d\tau} &= (b_2 - 1)\frac{dx^*}{d\tau} + \frac{b_2\left[p + (1-p)\, G\left(x^* - \overline{\theta}_1\right)\right]}{\tau (1-p)\, g\left(x^* - \overline{\theta}_1\right)}
\end{aligned}$$

Differentiating (9) with respect to τ, we obtain

$$\frac{dy^*}{d\tau} = b_3 \frac{d\underline{\theta}_1}{d\tau},$$

where

$$b_3 = \frac{h\left(\underline{\theta}_1 - y^*\right)\left(1 - F_2\left(\theta_2^* - y^*\right)\right)}{f_2\left(\theta_2^* - y^*\right)\left(1 - H\left(\underline{\theta}_1 - y^*\right)\right) + h\left(\underline{\theta}_1 - y^*\right)\left(1 - F_2\left(\theta_2^* - y^*\right)\right)} \in (0,1)$$

Then

$$\frac{dy^*}{d\tau} = b_3 b_1 \frac{dx^*}{d\tau} + \frac{b_1 b_3 G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)}.$$

Differentiating (A.2) with respect to τ, we obtain

$$\begin{aligned}
&-g\left(\underline{\delta}\right)\left(1 - H\left(y^* - x^* - \underline{\delta}\right)\right)\frac{d\underline{\delta}}{d\tau} - g\left(\overline{\delta}\right) H\left(y^* - x^* - \overline{\delta}\right)\frac{d\overline{\delta}}{d\tau} \\
&-\int_{\underline{\delta}}^{\overline{\delta}} g(k)\, h\left(y^* - x^* - k\right)\left(\frac{dy^*}{d\tau} - \frac{dx^*}{d\tau}\right) dk \\
&+\frac{f_2\left(\theta_2^* - x^*\right)}{1 - F_2\left(\theta_2^* - x^*\right)}\left[1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\overline{\delta}} g(k)\, H\left(y^* - x^* - k\right) dk\right]\frac{dx^*}{d\tau} = 0
\end{aligned}$$

Let $w = r^* / \left[1 - F_2\left(\theta_2^* - y^*\right)\right] r_s > 0$. Then, $1 - H\left(y^* - x^* - \underline{\delta}\right) = w$. By substitution, we obtain

$$\begin{aligned}
& b_4\left(1 - b_1\right)\frac{dx^*}{d\tau} + b_5\left(1 - b_2\right)\frac{dx^*}{d\tau} + b_6(1 - b_1 b_3)\frac{dx^*}{d\tau} + b_7\frac{dx^*}{d\tau} \\
= \; & \frac{b_1(b_3 b_6 + b_4) G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)} + \frac{b_2 b_5\left[p + (1-p)\, G\left(x^* - \overline{\theta}_1\right)\right]}{\tau(1-p)\, g\left(x^* - \overline{\theta}_1\right)}
\end{aligned}$$

where

$$\begin{aligned}
b_4 &= w g\left(\underline{\delta}\right) > 0 \\
b_5 &= g\left(\overline{\delta}\right) H\left(y^* - x^* - \overline{\delta}\right) > 0, \\
b_6 &= \int_{\underline{\delta}}^{\overline{\delta}} g(k)\, h\left(y^* - x^* - k\right) dk > 0, \\
b_7 &= \frac{f_2\left(\theta_2^* - x^*\right)}{1 - F_2\left(\theta_2^* - x^*\right)}\left[1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\overline{\delta}} g(k)\, H\left(y^* - x^* - k\right) dk\right] > 0.
\end{aligned}$$

Because b_1 and b_2 are smaller than one, all of the coefficients on the left hand side of the above equation are positive. Thus, we have $\frac{dx^*}{d\tau} > 0$, then $\frac{dy^*}{d\tau} > 0$, $\frac{d\underline{\theta}_1}{d\tau} > 0$, and $\frac{d\bar{\theta}_1}{d\tau} > 0$. This completes the proof.

Proof of Proposition 3. *Differentiating (7) and (8) with respect to λ, respectively, yields*

$$\frac{dx^*}{d\lambda} = \frac{1}{b_1}\frac{d\underline{\theta}_1}{d\lambda} + \frac{\underline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \underline{\theta}_1\right)}$$

$$\frac{dx^*}{d\lambda} = \frac{1}{b_2}\frac{d\overline{\theta}_1}{d\lambda} + \frac{\overline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \overline{\theta}_1\right)}.$$

We have

$$\frac{d\underline{\delta}}{d\lambda} = \left(b_1 - 1\right)\frac{dx^*}{d\lambda} - \frac{b_1\underline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \underline{\theta}_1\right)}$$

$$\frac{d\overline{\delta}}{d\lambda} = \left(b_2 - 1\right)\frac{dx^*}{d\lambda} - \frac{b_2\overline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \overline{\theta}_1\right)}$$

Differentiating (9) with respect to λ, we obtain

$$\frac{dy^*}{d\lambda} = b_1 b_3\frac{dx^*}{d\lambda} - \frac{b_1 b_3\underline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \underline{\theta}_1\right)}.$$

Differentiating (A.2) with respect to λ, we obtain

$$\begin{aligned} & b_4\left(1-b_1\right)\frac{dx^*}{d\lambda} + b_5\left(1-b_2\right)\frac{dx^*}{d\lambda} + b_6(1-b_1 b_3)\frac{dx^*}{d\lambda} + b_7\frac{dx^*}{d\lambda} \\ = \; & -\frac{b_1(b_3 b_6 + b_4)\underline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \underline{\theta}_1\right)} - \frac{b_2 b_5\overline{\theta}_1 A}{\tau D\left(1-p\right) g\left(x^* - \overline{\theta}_1\right)} \end{aligned}$$

Thus, we have $\frac{dx^*}{d\lambda} < 0$, $\frac{dy^*}{d\lambda} < 0$, $\frac{d\underline{\theta}_1}{d\lambda} < 0$, and $\frac{d\overline{\theta}_1}{d\lambda} < 0$. This completes the proof.

Proof of Proposition 4. *Differentiating (7) and (8) with respect to the precision of the large lender's information β, we obtain*

$$\begin{aligned} \frac{dx^*}{d\beta} &= \frac{1}{b_1}\frac{d\underline{\theta}_1}{d\beta}, \\ \frac{dx^*}{d\beta} &= \frac{1}{b_2}\frac{d\overline{\theta}_1}{d\beta}, \end{aligned}$$

Then, we have

$$\begin{aligned} \frac{d\underline{\delta}}{d\beta} &= \left(b_1 - 1\right)\frac{dx^*}{d\beta} \\ \frac{d\overline{\delta}}{d\beta} &= \left(b_2 - 1\right)\frac{dx^*}{d\beta} \end{aligned}$$

Moreover, we can write (9) in standard normal and differentiate it with respect to β such that

$$\begin{aligned}&\left(1-F_2\left(\theta_2^*-y^*\right)\right)\left[\phi\left(\sqrt{\beta}\left(\underline{\theta}_1-y^*\right)\right)\left(\sqrt{\beta}\left(\frac{d\underline{\theta}_1}{d\beta}-\frac{dy^*}{d\beta}\right)+\frac{1}{2\sqrt{\beta}}\left(\underline{\theta}_1-y^*\right)\right)\right]\\ =\;&\left[1-H\left(\underline{\theta}_1-y^*\right)\right]f_2\left(\theta_2^*-y^*\right)\frac{dy^*}{d\beta},\end{aligned}$$

which yields

$$\frac{dy^*}{d\beta}=b_1c_1\frac{dx^*}{d\beta}+\frac{c_1}{2\beta}\left(\underline{\theta}_1-y^*\right),$$

where

$$c_1=\frac{\left(1-F_2\left(\theta_2^*-y^*\right)\right)\phi\left(\sqrt{\beta}\left(\underline{\theta}_1-y^*\right)\right)\sqrt{\beta}}{\left(1-F_2\left(\theta_2^*-y^*\right)\right)\phi\left(\sqrt{\beta}\left(\underline{\theta}_1-y^*\right)\right)\sqrt{\beta}+\left(1-H\left(\underline{\theta}_1-y^*\right)\right)f_2\left(\theta_2^*-y^*\right)}\in(0,1).$$

Differentiating (A.2) with respect to β, we obtain

$$\begin{aligned}&-g\left(\underline{\delta}\right)\left(1-H\left(y^*-x^*-\underline{\delta}\right)\right)\frac{d\underline{\delta}}{d\beta}-g\left(\overline{\delta}\right)H\left(y^*-x^*-\overline{\delta}\right)\frac{d\overline{\delta}}{d\beta}\\&-\int_{\underline{\delta}}^{\overline{\delta}}h_x\left(k\right)\phi\left(\sqrt{\beta}\left(y^*-x^*-k\right)\right)\left[\sqrt{\beta}\left(\frac{dy^*}{d\beta}-\frac{dx^*}{d\beta}\right)+\frac{y^*-x^*-k}{2\sqrt{\beta}}\right]dk\\&+\frac{f_2\left(\theta_2^*-x^*\right)}{1-F_2\left(\theta_2^*-x^*\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}}g\left(k\right)H\left(y^*-x^*-k\right)dk\right]\frac{dx^*}{d\beta}=0,\end{aligned}$$

which can be rearranged as

$$\begin{aligned}&-g\left(\underline{\delta}\right)\left(1-H\left(y^*-x^*-\underline{\delta}\right)\right)\left(b_1-1\right)\frac{dx^*}{d\beta}-g\left(\overline{\delta}\right)H\left(y^*-x^*-\overline{\delta}\right)\left(b_2-1\right)\frac{dx^*}{d\beta}\\&-\int_{\underline{\delta}}^{\overline{\delta}}g\left(k\right)\phi\left(\sqrt{\beta}\left(y^*-x^*-k\right)\right)\left[\sqrt{\beta}\left(b_1c-1\right)\frac{dx^*}{d\beta}+\frac{c_1(\underline{\theta}_1-y^*)+y^*-\underline{\theta}_1+\underline{\delta}-k}{2\sqrt{\beta}}\right]dk\\&+\frac{f_2\left(\theta_2^*-x^*\right)}{1-F_2\left(\theta_2^*-x^*\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}}g\left(k\right)H\left(y^*-x^*-k\right)dk\right]\frac{dx^*}{d\beta}=0.\end{aligned}$$

Then, we obtain

$$\begin{aligned}&b_4(1-b_1)\frac{dx^*}{d\beta}+b_5\left(1-b_2\right)\frac{dx^*}{d\beta}+b_6\sqrt{\beta}\left(1-b_1c_1\right)\frac{dx^*}{d\beta}+b_7\frac{dx^*}{d\beta}\\=\;&\int_{\underline{\delta}}^{\overline{\delta}}g\left(k\right)\phi\left(\sqrt{\beta}\left(y^*-x^*-k\right)\right)\frac{c_1(\underline{\theta}_1-y^*)+y^*-\underline{\theta}_1+\underline{\delta}-k}{2\sqrt{\beta}}dk,\end{aligned}$$

Note that $\underline{\theta}_1 - y^* < 0$. *Because* $\underline{\theta}_1 < \overline{\theta}_1$, *the integrand* $y^* - \underline{\theta}_1 + \underline{\delta} - k$ *evaluated between* $\underline{\delta}$ *and* $\overline{\delta}$ *is strictly negative. Hence, that* $\frac{dx^*}{d\beta} < 0$, *then* $\frac{dy^*}{d\beta} < 0$, $\frac{d\underline{\theta}_1}{d\beta} < 0$, *and* $\frac{d\overline{\theta}_1}{d\beta} < 0$ is *straightforward. This completes the proof.*

Proof for the size effect. *Differentiating (7) and (8) with respect to* p provides

$$\begin{aligned} \frac{dx^*}{dp} &= \frac{1}{b_1}\frac{d\underline{\theta}_1}{dp} + \frac{G\left(x^* - \underline{\theta}_1\right)}{(1-p)\,g\left(x^* - \underline{\theta}_1\right)}, \\ \frac{dx^*}{dp} &= \frac{1}{b_2}\frac{d\overline{\theta}_1}{dp} - \frac{1 - G\left(x^* - \overline{\theta}_1\right)}{(1-p)\,g\left(x^* - \overline{\theta}_1\right)}, \end{aligned}$$

and

$$\begin{aligned} \frac{d\underline{\delta}}{dp} &= (b_1 - 1)\frac{dx^*}{dp} - \frac{b_1 G\left(x^* - \underline{\theta}_1\right)}{(1-p)\,g\left(x^* - \underline{\theta}_1\right)} \\ \frac{d\overline{\delta}}{dp} &= (b_2 - 1)\frac{dx^*}{dp} + \frac{b_2\left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{(1-p)\,g\left(x^* - \overline{\theta}_1\right)}. \end{aligned}$$

Differentiating (9) with respect to p provides

$$\frac{dy^*}{dp} = b_1 b_3 \frac{dx^*}{dp} - \frac{b_1 b_3 G\left(x^* - \underline{\theta}_1\right)}{(1-p)\,g\left(x^* - \underline{\theta}_1\right)}.$$

Differentiating (A.2) with respect to p provides

$$\begin{aligned} & b_4(1-b_1)\frac{dx^*}{dp} + b_5(1-b_2)\frac{dx^*}{dp} + b_6(1 - b_1 b_3)\frac{dx^*}{dp} + b_7\frac{dx^*}{dp} \\ = \; & -\frac{b_1(b_3 b_6 + b_4) G\left(x^* - \underline{\theta}_1\right)}{(1-p)\,g\left(x^* - \underline{\theta}_1\right)} + \frac{b_2 b_5\left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{(1-p)\,g\left(x^* - \overline{\theta}_1\right)} \end{aligned}$$

Thus, only when

$$\frac{b_2 b_5\left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{g\left(x^* - \overline{\theta}_1\right)} < \frac{b_1(b_3 b_6 + b_4) G\left(x^* - \underline{\theta}_1\right)}{g\left(x^* - \underline{\theta}_1\right)},$$

we have $\frac{dx^*}{dp} < 0$. *Because, analytically, we cannot prove whether this condition holds or not, we solve the model numerically. We find that* $\frac{dx^*}{dp} < 0$, $\frac{dy^*}{dp} < 0$, *and* $\frac{d\underline{\theta}_1}{dp} < 0$. *However,* $\frac{d\overline{\theta}_1}{dp} > 0$.

Proof of Proposition 5. *First, suppose that* $\lim \underline{\theta}_1 < \lim \overline{\theta}_1$ *so that* $\lim \overline{\theta}_1 \geq \frac{\tau D(1-p) - M}{\lambda A}$. *Because* $x^* \to \underline{\theta}_1$, *we must have* $\overline{\delta} = \sqrt{\alpha}\left(\overline{\theta}_1 - x^*\right) \to +\infty$. *Then (10) in this case is*

$$1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{+\infty} g(k)\, H\left(\sqrt{\frac{\alpha}{\beta}}\left(\underline{\delta} - k\right) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right) dk = \frac{2r^*}{r_s}$$

where $y^* = \underline{\theta}_1 - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)$ *is used.*

Second, consider the case where $\lim \underline{\theta}_1 = \lim \overline{\theta}_1$ *so that* $\overline{\delta}$ *is finite and*

$$(1-p)\left(1-G\left(\underline{\delta}\right)\right) = p + (1-p)\left(1-G\left(\overline{\delta}\right)\right),$$

which yields

$$\overline{\delta} = G^{-1}\left(G\left(\underline{\delta}\right) + \frac{p}{1-p}\right).$$

Hence, in this case, (10) is

$$1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\Gamma} g\left(k\right) H\left(\sqrt{\frac{\alpha}{\beta}}\left(\underline{\delta} - k\right) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right) dk = \frac{2r^*}{r_s},$$

where $\Gamma = G^{-1}\left(G\left(\underline{\delta}\right) + \frac{p}{1-p}\right)$. *This completes the proof.*

Proof of Proposition 7. *Differentiating (14) with respect to the short-term debt ratio provides*

$$\begin{aligned}\frac{dL_0\left(\theta_0\right)}{d\tau} &= \left(1 - H\left(y^* - \underline{\theta}_1\right)\right)\left(1 - F_2\left(\theta_2^* - \underline{\theta}_1\right)\right) f_1\left(\underline{\theta}_1 - \theta_0\right)\frac{d\underline{\theta}_1}{d\tau} \\ &+ H\left(y^* - \overline{\theta}_1\right)\left(1 - F_2\left(\theta_2^* - \overline{\theta}_1\right)\right) f_1\left(\overline{\theta}_1 - \theta_0\right)\frac{d\overline{\theta}_1}{d\tau} \\ &+ \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[h\left(y^* - \theta_1\right)\left(1 - F_2(\theta_2^* - \theta_1)\right)\right] f_1\left(\theta_1 - \theta_0\right) d\theta_1 \frac{dy^*}{d\tau}\end{aligned}$$

Because we have $d\overline{\theta}_1/d\tau > 0$, $d\underline{\theta}_1/d\tau > 0$ *and* $dy^*/d\tau > 0$, *we obtain*

$$\frac{dL_0\left(\theta_0\right)}{d\tau} > 0.$$

Differentiating (14) with respect to market liquidity λ *provides*

$$\begin{aligned}\frac{dL_0\left(\theta_0\right)}{d\lambda} &= \left(1 - H\left(y^* - \underline{\theta}_1\right)\right)\left(1 - F_2\left(\theta_2^* - \underline{\theta}_1\right)\right) f_1\left(\underline{\theta}_1 - \theta_0\right)\frac{d\underline{\theta}_1}{d\lambda} \\ &+ H\left(y^* - \overline{\theta}_1\right)\left(1 - F_2\left(\theta_2^* - \overline{\theta}_1\right)\right) f_1\left(\overline{\theta}_1 - \theta_0\right)\frac{d\overline{\theta}_1}{d\lambda} \\ &+ \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[h\left(y^* - \theta_1\right)\left(1 - F_2(\theta_2^* - \theta_1)\right)\right] f_1\left(\theta_1 - \theta_0\right) d\theta_1 \frac{dy^*}{d\lambda}\end{aligned}$$

Because we have $d\overline{\theta}_1/d\lambda < 0$, $d\underline{\theta}_1/d\lambda < 0$ *and* $dy^*/d\lambda < 0$, *we have*

$$\frac{dL_0\left(\theta_0\right)}{d\lambda} < 0.$$

Differentiating (14) with respect to the precision of the wholesale financer's information gives

$$\begin{aligned}\frac{dL_0(\theta_0)}{d\beta} &= (1 - H(y^* - \underline{\theta}_1))(1 - F_2(\theta_2^* - \underline{\theta}_1)) f_1(\underline{\theta}_1 - \theta_0)\frac{d\underline{\theta}_1}{d\beta} \\ &+ H\left(y^* - \overline{\theta}_1\right)\left(1 - F_2\left(\theta_2^* - \overline{\theta}_1\right)\right) f_1\left(\overline{\theta}_1 - \theta_0\right)\frac{d\overline{\theta}_1}{d\beta} \\ &+ \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[\phi\left(\sqrt{\beta}(y^* - \theta_1)\right)(1 - F_2(\theta_2^* - \theta_1))\right] f_1(\theta_1 - \theta_0)\left[\sqrt{\beta}\frac{dy^*}{d\beta} + \frac{y^* - \theta_1}{2\sqrt{\beta}}\right] d\theta_1\end{aligned}$$

Because $\underline{\theta}_1 < \overline{\theta}_1$*, the integrand* $y^* - \theta_1$ *evaluated between* $\underline{\theta}_1$ *and* $\overline{\theta}_1$*, is strictly negative. Furthermore, we have proven that* $\frac{d\underline{\theta}_1}{d\beta} < 0$*,* $\frac{dy^*}{d\beta} < 0$*, and* $\frac{d\overline{\theta}_1}{d\beta} < 0$*. Hence,*

$$\frac{dL_0(\theta_0)}{d\beta} < 0.$$

Differentiating (14) with respect to the size of the wholesale financer provides

$$\begin{aligned}\frac{dL_0(\theta_0)}{dp} &= (1 - H(y^* - \underline{\theta}_1))(1 - F_2(\theta_2^* - \underline{\theta}_1)) f_1(\underline{\theta}_1 - \theta_0)\frac{d\underline{\theta}_1}{dp} \\ &+ H\left(y^* - \overline{\theta}_1\right)\left(1 - F_2\left(\theta_2^* - \overline{\theta}_1\right)\right) f_1\left(\overline{\theta}_1 - \theta_0\right)\frac{d\overline{\theta}_1}{dp} \\ &+ \int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[h(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1))\right] f_1(\theta_1 - \theta_0)\, d\theta_1 \frac{dy^*}{dp}\end{aligned}$$

Because we have $\frac{d\overline{\theta}_1}{dp} > 0$*, while* $\frac{d\underline{\theta}_1}{dp} < 0$ *and* $\frac{dy^*}{dp} < 0$*, the sign of* $\frac{dL_0(\theta_0)}{dp}$ *is ambiguous. Together, these complete the proof.*

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