Tax Policy and Stability in a General Two-sector Model with Sector-Specific Externalities*

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Abstract

In a recent paper with sector-specific productive externalities and equal factor intensities in both sectors from the private perspective, Guo and Harrison (2001) found that with the empirically realistic value of the investment externality for the US economy, a progressive tax is susceptible to destabilizing the economy. This paper envisages the robustness of the tax policy implications in a general two-sector model of different factor intensities. In this model, the Stolper-Samuelson effect and the Rybczynski effect are stronger yielding larger price, consumption and investment effects. The critical value of the investment externality below which a progressive tax schedule can hold back sunspot expectations, can be increased to a much larger value. The empirically realistic value of the investment externality for the US economy is quantitatively smaller than the critical value. Therefore, a progressive tax schedule in the current US tax code can stabilize the economy.

Keywords: tax policy, stability, two-sector growth model, sector-specific externalities.
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1. Introduction

Since the seminal works put forth by Benhabib and Farmer (1994) and Farmer and Guo (1994), it has been well-known that an otherwise standard one-sector RBC model with sufficiently strong productive externalities can exhibit an indeterminate steady state. One common feature in this class of models is that agents' "sunspot" expectations can be a self-governing impulse to endogenous business cycles. These models thus have policy implications in line with a Keynesian view that policy rules, which operate like automatic stabilizers, are designed to insulate the economy from belief-driven fluctuations. Using the Benhabib-Farmer-Guo model, Guo and Lansing (1998) and Christiano and Harrison (1999) have shown that progressive income taxes can stabilize the economy while, conversely, flat and regressive income taxes are more susceptible to destabilizing the economy.[1]

Recently, Guo and Harrison (2001) have introduced government fiscal policy into a modified two-sector model proposed by Benhabib and Farmer (1996), as in Harrison (2001), where sector-specific productive externalities are present only in the investment sector. These authors considered a progressive (regressive) tax policy wherein the income tax rate rises (falls) with the household's taxable income. By using calibration analysis, they found that when investment externalities are below (above) a certain critical value, a progressive (regressive) tax schedule can stabilize the economy. As the empirically realistic value of the investment externality for the US economy is above the critical value, the result in Guo and Harrison (2001) implies that a progressive tax can destabilize the US economy. These tax implications are opposite to those obtained in a one-sector model.

The results concerning tax policy implications in Guo and Harrison (2001) were obtained under the setup of equal factor intensities in both sectors from the private perspective. Our paper studies the robustness of their results in a general two-sector model.

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[1] The links between tax policies and indeterminacy was studied by Bond et al. (1996) and Schmitt-Grohe and Uribe (1997). In a human-capital based two-sector model without externalities, Bond et al. (1996) showed that asymmetric factor income tax rates can create indeterminacy. Schmitt-Grohe and Uribe (1997) found that if labor income tax rates are determined by a balanced budget rule with a pre-set level of revenue, a one-sector model with a constant returns-to-scale technology can exhibit indeterminacy.

Our study is motivated by the following reasons. Since Uzawa's seminal papers (1962, 1964), many economists have studied a general model with two sectors exhibiting different factor intensities from the private perspective.[2] Moreover, evidence in OECD countries indicates that the consumption sector and the investment sector have different factor intensities.[3] The assumption of equal factor intensities in both sectors is not only restrictive but also inconsistent with conventional wisdom. To highlight the role of different factor intensities, this paper studies a model that is otherwise identical to Guo and Harrison (2001) except for different factor intensities. We find that with a reasonable difference in factor intensities between sectors, the critical value of the investment externality below which a progressive tax schedule can hold back sunspot expectations, can be increased to a much larger value. Quantitatively, the empirically realistic value of the investment externality for the US economy is smaller than the critical value. Thus, a progressive tax schedule can stabilize the economy.

To understand the underlying reasons, when agents anticipate a higher future return to capital, they will switch away from today's consumption (the consumption effect) toward today's investment (the investment effect) which raises tomorrow's capital stock. Higher tomorrow's capital will lower tomorrow's interest rate, but if the investment externality is sufficiently large, the relative price of investment goods will rise (the price effect), thereby raising the rate of return to capital. Then, agents' optimistic expectations are self-fulfilling. The key to successfully holding back sunspot expectations is to reduce the above mechanisms that trigger indeterminacy. In Guo and Harrison, both sectors have equal factor intensities from the private perspective. Only when the investment externality is small, a progressive tax makes the consumption and the price effects to quantitatively outweigh the investment effect and stabilizes the economy.

On the contrary, our two sectors have different factor intensities from the private perspective. There is larger sector reallocation, so the Stolper-Samuelson effect and the

[2] See, e.g., Benhabib and Nishimura (1985), Galor (1992) and Nishimura and Venditti (2002).
[3] See Takahashi et al. (2012).

Rybczynski effects are more substantial. The Stolper-Samuelson effect gives a bigger price effect and the Rybczynski effect yields stronger consumption and investment effects. As all price, consumption and investment effects are larger, a progressive tax schedule can hold back sunspot expectations that would otherwise trigger indeterminacy under a much larger critical value of the investment externality. The empirically realistic value of the investment externality for the US economy is quantitatively smaller than the critical value. Thus, a progressive tax in the current US tax code can stabilize the economy.

Our general two-sector model with an income tax also gives two other new results that add values to existing literature. First, because of a variable income tax schedule, indeterminacy arises in our model not only when the consumption sector is more capital-intensive but also when the consumption sector is less capital-intensive from the private perspective. Second, if income taxes are regressive, indeterminacy emerges in our model even when there is no externality. The first result is different from that of an otherwise identical two-sector model with sector-specific externalities studied by Benhabib et al. (2002). In Benhabib et al., because of no income taxes, indeterminacy arises only when the consumption sector is more capital-intensive than the investment sector from the private perspective. Our second result is different from those of models investigated by Benhabib and Farmer (1994), Guo and Lansing (1998) and Bond et al. (1996). In Benhabib and Farmer (1994), due to zero income taxes, indeterminacy cannot emerge if there are no externalities. In Guo and Lansing, because of no sector reallocation, even with a regressive income tax, indeterminacy arises only if there are positive productive externalities. In Bond et al., because of a sector producing human capital, indeterminacy appears only when capital income and labor income are taxed at very different rates.

A roadmap is as follows. In Section 2, we set up a general two-sector model and analyze the equilibrium conditions and dynamics. In Section 3, we quantify the relationship between the tax schedule and the stability properties. Finally, concluding remarks are offered in Section 4.

2. The Basic Model

Our model economy is otherwise identical to Guo and Harrison (2001) except for the production technology. The economy is populated by a representative household and a representative firm. The representative household lives forever and derives utility from consumption and leisure. There are two production sectors: the consumption sector (Y_c) and the investment sector (Y_I). The representative firm produces goods in both sectors.

2.1 The Firms' Problem

In the consumption sector, output Y_{ct} is produced by competitive firms using the following technology:

$$Y_{ct} = (\bar{K}_{ct}^{a}\bar{L}_{ct}^{1-a})^{\theta_c} K_{ct}^{a} L_{ct}^{1-a}, 0 < a < 1, \theta_c \geq 0, \tag{1}$$

where K_{ct} and L_{ct} are capital and labor in the production of consumption goods. $\bar{K}_{ct}$ and $\bar{L}_{ct}$ are economy-wide average capital and average labor used in the consumption sector and θ_c is the extent to which sector-specific externalities affect the consumption sector. In a symmetric equilibrium, all firms make the same decisions such that $\bar{K}_{ct}$=K_{ct} and $\bar{L}_{ct}$=L_{ct}.

Similarly, output Y_{It} in the investment sector is produced by the following technology:

$$Y_{It} = (\bar{K}_{It}^{b}\bar{L}_{It}^{1-b})^{\theta_I} K_{It}^{b} L_{It}^{1-b}, 0 < b < 1, \theta_I \geq 0, \tag{2}$$

where K_{It} and L_{It} are capital and labor in the production of investment goods. $\bar{K}_{It}$ and $\bar{L}_{It}$ denote economy-wide average capital and average labor employed in the investment sector and θ_I is sector-specific externalities in the investment sector. Symmetric equilibrium implies that $\bar{K}_{It}$=K_{It} and $\bar{L}_{It}$= L_{It}.

Note that if a=b, the capital intensity is the same from the private perspective. The technology in (1) and (2) is reduced to that in Guo and Harrison (2001). On the contrary, if $a \neq b$, the two sectors have different capital intensities from the private perspective.

Profit maximization in both sectors implies:

$$\frac{aY_{ct}}{K_{ct}} = r_t = p_t \frac{bY_{It}}{K_{It}}, \tag{3}$$

$$\frac{(1-a)Y_{ct}}{L_{ct}} = w_t = p_t \frac{(1-b)Y_{It}}{L_{It}}, \tag{4}$$

where r_t is the rental rate of capital, w_t is the wage rate and p_t is the relative price of investment goods in terms of consumption goods.

2.2 The Household's Problem

The household's lifetime utility is represented by:

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{L_t^{1+\chi}}{1+\chi} \right],$$

where C_t denotes consumption goods, and L_t labor supply. Parameter β>0 is the discount factor, $1/\sigma$ is the intertemporal elasticity of substitution (henceforth IES) for consumption and $1/\chi$ is the Frisch labor supply elasticity (henceforth LSE). Following Guo and Harrisson (2001), in the utility we normalize the coefficient of leisure relative to consumption to be unity as it plays no role. The budget constraint faced by the representative household is:

$$C_t + p_t I_t = (1-\tau_t)(r_t K_t + w_t L_t) + TR_t, \tag{5}$$

where K_t is the representative household's capital stock, I_t is investment, τ_t is the income tax rate and TR_t is a transfer from the government. The capital stock evolves according to:

$$K_{t+1} = I_t - (1-\delta)K_t, \quad K_0 \text{ given}, \tag{6}$$

where δ is the depreciation rate.

As in Harrsion and Guo (2001), we use the following form for the income tax rate postulated by Guo and Lansing (1998):

$$\tau_t = 1 - \eta \left(\frac{\bar{Y}}{Y_t} \right)^{\phi}, \ \eta \in (0,1), \tag{7}$$

where Y_t=r_tK_t+w_tL_t is the household's taxable income and $\bar{Y}$ denotes the steady-state income level which is taken as given by the household. The parameters η and ϕ, respectively, govern the level and the slope of the tax schedule. In order to ensure the existence of an interior steady state, a lower bound of ϕ is required in order to ensure that the after-tax interest rate decreases with K_t. Simple calculation indicates that the marginal tax rate is

τ_t^m=τ_t+$\eta\phi(\bar{Y}/Y_t)^\phi$. Then, the marginal tax rate is higher (lower) than the average tax rate when ϕ>(<)0 and thus the tax schedule is progressive (regressive). When ϕ=0, all households face the same tax rate 1-η and there is a flat income tax rate.

The household chooses consumption, labor supply and capital in order to maximize the lifetime utility subject to (5)-(7). The first-order conditions give:

$$C_t^{\sigma} L_t^{\chi} = \eta(1-\phi)\bar{Y}^{\phi} Y_t^{-\phi} w_t, \tag{8a}$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta \frac{\eta(1-\phi)\bar{Y}^{\phi} Y_{t+1}^{-\phi} r_{t+1} + (1-\delta) p_{t+1}}{p_t}, \tag{8b}$$

$$\lim_{t\to\infty} \beta^t \frac{K_{t+1}}{C_t^{\sigma}} = 0. \tag{8c}$$

In these conditions, (8a) equates the marginal rate of substitution between consumption and leisure to the after-tax wage rate, (8b) is the consumption Euler equation, and (8c) is the transversality condition.

2.3 The Government

Following Gou and Harrison (2001, 2011), the government chooses the income tax policy τ_t and balances its budget each period. Moreover, the government returns all its tax revenues to households as a lump-sum transfer so as to focus on the macroeconomic (in)stability effects of the tax policy. The government's flow budget constraint is:

$$TR_t = \tau_t Y_t. \tag{9}$$

2.4 Market Equilibrium

The market clearing conditions for consumption goods and investment goods are C_t=Y_{ct} and I_t=Y_{It}. The market clearing conditions for capital and labor are:

$$K_t = K_{ct} + K_{It} = s_t K_t + (1-s_t) K_t, \tag{10a}$$

$$L_t = L_{ct} + L_{It} = l_t L_t + (1-l_t) L_t, \tag{10b}$$

where $s_t \in (0,1)$ and $l_t \in (0,1)$, respectively, are the fraction of aggregate capital and aggregate labor in the economy allocated to the consumption sector.

An equilibrium is a set of prices and quantities that satisfies (1)-(7), (8a)-(8b), (9), (10a)-(10b) and the two goods market clearing conditions, together with given K_0 and the transversality condition (8c). These conditions are simplified as follows.

First, with production functions (1) and (2), the factor allocation conditions between sectors (3) and (4) and the factor market clearing conditions (10a) and (10b) lead to

$$l_t = l(s_t) \equiv \left[1 + \frac{a(1-b)}{b(1-a)} \left(\frac{1}{s_t} - 1 \right) \right]^{-1}, \tag{11}$$

where $l'(s_t)=[a(1-b)]/[b(1-a)](l_t/s_t)^2>0$. The positive sign is a well-known property that arises due to the complementarity relationship between capital and labor in production. Note that if $a=b$, (11) is reduced to $l_t=s_t$ as is in Guo and Harrison (2001).

Next, combining conditions $C_t=Y_{ct}$, (3), (4), (8a) and (10a)-(10b) yields[4]

$$s_t = s(K_t, C_t), \tag{12}$$

where $\frac{\partial s_t}{\partial K_t} = \frac{-(1+\chi)s_t\Xi}{K_t}$, $\frac{\partial s_t}{\partial C_t} = \frac{[1+\chi+(1-a)(1+\theta_c)(\sigma+\phi-1)]s_t\Xi}{a(1+\theta_c)}$ and $\Xi = \frac{[a+(b-a)s][a(1-b)+(b-a)s]}{[a+(b-a)s]\{[a(1-b)+(b-a)s]+\chi[1-b+(b-a)s]\}+\phi(1-a)[a(1-b)+(b-a)s]}$.

Moreover, using (3)-(4) and (11)-(12), the relative price of investment good can be expressed as

$$\begin{aligned} p_t &\equiv p(s(K_t, C_t), l(s(K_t, C_t)), K_t, C_t) \\ &= \tfrac{a}{b} s_t^{\frac{a(1-b)(1+\theta_I)}{1-a}-1} (1-s_t)^{1-b(1+\theta_I)} [l_t(1-l_t)^{-1}]^{(1-b)(1+\theta_I)} K_t^{\frac{(a-b)(1+\theta_I)}{1-a}} C_t^{1-\frac{(1-b)(1+\theta_I)}{(1-a)(1+\theta_c)}}. \end{aligned} \tag{13}$$

It is easy to show that if $a=b$, $l_t=s_t$ and p_t depends on the stock of capital only indirectly through s_t.[5] However, if $a\neq b$, then p_t is directly affected by the stock of capital. It is worth noting that if $a>(<)b$, the consumption sector is more (less) capital intensive than the investment sector. Then, higher capital in the next period gives a direct effect to increase (reduce) the relative price of investment goods in the next period. Moreover, with different factor intensities from the private perspective, there are additional effects on the relative

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[4] Notice that as Ξ is ambiguous, $\frac{\partial s_t}{\partial K_t}$ and $\frac{\partial s_t}{\partial C_t}$ are ambiguous. However, if $a=b$, $\Xi = \frac{a}{a+\chi+\phi(1-a)} > 0$. Then, $\frac{\partial s_t}{\partial K_t} < 0$ and $\frac{\partial s_t}{\partial C_t} > (<) 0$ if $\sigma+\phi>(<)0$; in the case of $\sigma=1$, $\frac{\partial s_t}{\partial C_t} > (<) 0$ if $\phi>(<)0$.

[5] If $a=b$, $l_t=s_t$ and (13) is reduced to $p_t = s_t^{\theta_I}(1-s_t)^{\theta_I} C_t^{\frac{\theta_c-\theta_I}{1+\theta_c}}$.

price of investment goods via labor shares that are different capital shares.

Finally, with the use of conditions I_t=Y_{It}, (2), and (11)-(13), we rewrite the law of motion for aggregate capital stock (6) and the consumption Euler equation (8b) as:

$$K_{t+1} = Y_{It}(s(K_t, C_t), l(s(K_t, C_t)), K_t, C_t) + (1-\delta)K_t, \tag{14a}$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta\left\{\frac{(1-\phi)(1-\tau_{t+1})r_{t+1} + (1-\delta)p_{t+1}(s_{t+1}, l(s_{t+1}), K_{t+1}, C_{t+1})}{p_t(s_t, l(s_t), K_t, C_t)}\right\}, \tag{14b}$$

where $s_t = s(K_t, C_t)$ is in (12) and $1-\tau_{t+1} = \eta\bar{Y}^{\phi}Y_{t+1}^{-\phi}(s_{t+1}, l(s_{t+1}), K_{t+1}, C_{t+1})$ is in (7). Equations (14a) and (14b) constitute the simplified equilibrium conditions.

To see why a progressive tax schedule can stabilize the economy, notice that (14b) plays an important role in sunspot fluctuations. When agents anticipate higher returns to capital tomorrow, they substitute away from today's consumption C_t toward today's investment I_t thus allocating more resources to the investment sector. Tomorrow's capital stock and thus consumption will increase. Therefore, the left-hand side of (14b) increases. Due to the externality in the investment sector, the relative price of investment goods p_t decreases. If the externality in the investment sector increases p_{t+1}, the right-hand side of (14b) also increases, thereby generating indeterminacy. In Guo and Harrison (2001), the two sectors have equal factor intensities from the private perspective. A progressive tax can suppress investment by reducing after-tax expected returns on capital, but this impact is not strong enough to hold back sunspots equilibrium unless investment externalities are small.

In our model, the two sectors have different factor intensities from the private perspective. Because of larger sectoral reallocation, the Rybczynski effect and the Stolper-Samuelson effect are stronger. Now, the initial fall in p_t brings about factor reallocations between sectors which, through the Stolper-Samuelson effect, in turn induces p_t to increase. Since the Stolper-Samuelson effect is stronger, this offsets the initial price decline and may lower the right-hand side of (14b). Moreover, in the case when the investment sector is more capital-intensive, through the Rybczynski effect, higher K_{t+1}

results in an increase in the output of investment goods and a decrease in the output of consumption goods. This lowers the left-hand side of (14b). Therefore, progressive taxes can hold back the initial self-fulfilling expectations under a much larger critical value of investment externalities. Alternatively, in the case when the consumption sector is more capital intensive, the Rybczynski effect works in an opposite direction. In this case, progressive taxes have a weaker effect and can repress the initial self-fulfilling expectations under a critical value of investment externalities smaller than that in the former case. As we will see, with a reasonable difference in factor intensities between sectors, the empirically realistic value of the investment externality for the US economy is quantitatively smaller than the two critical values of investment externalities. Thus, a progressive tax can stabilize the US economy.

2.5. Steady State and Dynamics

The steady state is determined by $K_{t+1}=K_t=K$ and $C_{t+1}=C_t=C$ for all t. It is straightforward to show that in the steady state, the government transfer as a share in output, the fraction of capital allocated to the consumption sector and the fraction of labor allocated to the consumption sector are, respectively,:

$$\tfrac{TR}{Y}=1-\eta\,,\quad s=1-\frac{b\delta\eta(1-\phi)}{(1/\beta)-1+\delta},\quad \text{and}\quad l=1-\frac{a(1-b)b\delta\eta(1-\phi)}{b(1-a)[(1/\beta)-1+\delta]-(b-a)b\delta\eta(1-\phi)}.$$

With these expressions, the steady-state expressions of all other variables can be easily derived.

Next, we investigate the dynamic properties. We take log-linear approximations to the equilibrium conditions in the neighborhood of the steady state. The log-linear approximations give the following dynamical system:

$$\begin{bmatrix}\hat{K}_{t+1}\\ \hat{C}_{t+1}\end{bmatrix}=J\begin{bmatrix}\hat{K}_t\\ \hat{C}_t\end{bmatrix},\ \ \hat{K}_0 \text{ given,} \tag{15}$$

where a variable with a cap denotes its logarithmic deviation from the steady-state value and J is the Jacobean matrix.[6]

3. Quantitative Analysis

Now, we calibrate our model and quantify the effects of the income tax policy on the stability properties. We look for combinations of parameters whose values are consistent with post-war US data. Except for the factor intensity, we adopt all parameter values used in Guo and Harrison (2001) and are as follows:

$$\{\delta=0.025, \beta=0.99, \eta=0.8, 1/\sigma=1, 1/\chi=4, \theta_C=0, \theta_I=0.108\}$$

First, δ=0.025 and β=0.99 are chosen to match quarterly data. Next, η=0.8 is obtained by targeting the ratio of the government transfer to output in the steady state, TR/Y, at 0.2. Moreover, the IES for consumption is $1/\sigma$=1 so the household has logarithmic utility in consumption and the Frisch LSE is equal to $1/\chi$=4. Finally, the externality in the consumption sector is θ_C=0 and the externality in the investment sector is θ_I=0.108.

3.1 The Stability Properties

Unlike Guo and Harrison (2001), our model allows for different factor intensities. According to the estimations provided by Baxter (1996) and Takahashi et al. (2012), the consumption sector is more capital-intensive than the investment sector in economies like the US and Japan.[7] We set a=0.52 and b=0.32, which are taken from Baxter (1996).

We quantify the effect of income tax policies on the stability properties. Figure 1 illustrates the regions of the size of the investment externality θ_I and the slope of the tax schedule ϕ separating saddles, sinks and sources. It turns out that for this parameterization, the critical value of θ_I at which the model's stability properties change is 0.126. We divide the stability analysis into two cases as follows.

[6] See the appendix for the derivation.

[7] Baxter (1996) estimated the input shares in the nondurable and the durable goods sectors in the U.S.. Takahashi et al. (2012) measured the capital intensities of the consumption sector and the investment sector and they found that the consumption sector was more capital-intensive in more mature economies Japan and the U.S., Canada, France, and West Germany and less capital-intensive in less mature economies like South Korea.

[Insert Figure 1 about here]

Case 1 θ_I>0.126

In this range, the investment externality is very large. Then, like Guo and Harrison (2001), a regressive tax schedule (ϕ<0) can stabilize the economy against sunspot shocks, even when the consumption sector is more capital-intensive than the investment sector.

Case 2 0≤θ_I≤0.126

There are two sub-ranges as follows.

(i) 0≤θ_I≤0.07 In this sub-range, the investment externality is small. A flat tax and a progressive tax schedule can be automatic stabilizers. The stability properties in this sub-range are similar to those in Guo and Harrrison (2001).

(ii) 0.07<θ_I<0.126 In this sub-range, the investment externality is large. As the investment externality increases up to 0.126, a flat or a progressive tax schedule with a smaller degree can stabilize the economy against sunspot shocks. The stability properties in this sub-range are different from those in Guo and Harrison (2001).

In Guo and Harrison (2001) wherein the two sectors have equal factor intensities from the private perspective, a progressive tax schedule can stabilize the economy against sunspot shocks only under a small investment externality in the sub-range (i). Allowing for different factor intensities from the private perspective in our model, a progressive tax schedule can stabilize the economy against sunspot shocks under a much larger value of the investment externality in the sub-range (ii). According to Harrison (1997), θ_{US}=0.108 is the empirically realistic value of the investment externality for the US economy. While the empirically realistic value of the investment externality for the US economy is not in sub-range (ii), it is in sub-range (ii). With θ_I=θ_{US}=0.108, the steady state is a saddle when the degree of the income tax progressivity is $\phi \in (-4.468,-0.513) \cup (-0.461,0.27)$.[8] See the left panel in Table 1. Thus, with θ_I=0.108, a flat and a progressive tax schedule with a progressive degree no more than ϕ=0.27 can stabilize the economy against sunspot shocks.

[8] The bound ϕ >- 4.468 is required in order to ensure the existence of an interior steady state.

[Insert Table 1 about here]

To understand the intuition concerning why a progressive tax schedule can stabilize the economy, we start with how indeterminacy arises in the first place. When agents anticipate a higher future return to capital, they will switch away from consumption (the consumption effect) toward investment (the investment effect) which raises next period's capital stock. Even though higher next period's capital will lower the relative price of investment goods (the price effect), if the external effect in the firms' production is sufficiently large, the rate of return to capital will rise. Thus, agents' optimistic expectations are self-fulfilling. The key to successfully holding back sunspot fluctuations is to reduce the above mechanisms that lead to indeterminacy. In the Guo and Harrison model, the two sectors have the same factor intensity from both the private and the social perspectives. A progressive tax makes the consumption and the price effects to outweigh the investment effect only if the investment externality is below the critical value of θ_I=0.07. As the empirically realistic value of the investment externality for the US economy at θ_{US}=0.108 is above the critical value, it is a regressive tax that makes the consumption and the price effects to outweigh the investment effect and stabilizes the economy.

In our model, the two sectors have different factor intensities from both the private and the social perspectives. Because of large sector reallocation, the Stolper-Samuelson effect strengthens the price effect and the Rybczynski effect augments the consumption and investment effects. Then, flat and progressive taxes can hold back these large effects and suppress the mechanisms that cause sunspot fluctuations under a much larger critical value of investment externalities at θ_I=0.126. The empirically realistic value of the investment externality for the US economy is smaller than the critical value. Thus, a progressive tax schedule can stabilize the US economy.

How do differences in the factor intensity between the two sectors affect such a result? To see this, we fix the degree of the capital intensity in the investment sector and investigate how the result is affected if the degree of the capital intensity in the consumption sector is changed. See the results in Figure 2. It turns out that if the difference in the factor intensity between the two sectors is within a range, a progressive tax can always stabilize the economy. The larger is the difference, the easier the stabilization. Given that the capital intensity is 0.32 in the investment sector by calibration, we find that a flat and a progressive tax can stabilize the economy when the degree of the capital intensity is as small as 0.42 in the consumption sector. The required difference in factor intensities between the two sectors is in a reasonable range.

[Insert Figure 2 about here]

3.2 Sensitivity Analysis

Our results above indicate that under a reasonable difference in the factor intensity between sectors, a flat and a progressive tax schedule can be an automatic stabilizer in the US economy. We wonder whether or not these results are sensitive to the IES for consumption ($1/\sigma$) and the Frisch LSE ($1/\chi$). For sensitivity analysis, we consider the baseline parameterization except for four other parameterizations of IES and LES, which are summarized in Table 2.

[Insert Table 2 about here]

The effects of the tax policy on the stability properties are illustrated in Figure 3. We focus on the empirically realistic value of θ_I=θ_{US}=0.108 for the US economy. Panels A and B are concerning different degrees of the IES of consumption. Panel A indicates that with θ_I=θ_{US}=0.108, if the IES is higher, the investment effect is stronger. Then, the highest degree of the income tax progressivity to remove sunspot fluctuations (ϕ=0.134) is smaller than that in Figure 1 which is reminded to be ϕ=0.27. On the other hand, Panel B suggests that if the IES is lower, the highest degree of the income tax progressivity to repress sunspot equilibria (ϕ=0.387) increases. Intuitively, a higher (lower) IES makes the agent more (less) willing to substitute today's consumption for investment, thus generating a stronger (weaker) investment effect. As a result, the highest degree of the income tax progressivity to control sunspot equilibria decreases (increases).

[Insert Figure 3 about here]

Panels C and D are the sensitivity of determinacy concerning different values of the Frisch LSE. Panel C indicates that with $\theta_I=\theta_{US}=0.108$, if the Frisch LSE is higher, agents are more willing to substitute leisure for labor yielding a stronger investment effect. Thus, the highest degree of the income tax progressivity to restrain sunspot fluctuations decreases ($\phi=0.19$). Alternatively, Panel D reveals that if the Frisch LSE is lower, the investment effect is stronger. Thus, the highest degree of the income tax progressivity to suppress sunspot equilibria increases ($\phi=0.387$).

Overall, Figures 1-3 suggest that a flat and a progressive income tax schedule can be automatic stabilizer for the US economy and the results are robust to different degrees of the IES for consumption and different values of the Frisch LSE.[9]

3.3 The Stability Properties When The Consumption Sector Is Less Capital-intensive

It is interesting to envisage the alternative case when the consumption sector is less capital-intensive than the investment sector. To analyze this case, we maintain our baseline parameterization except for the capital intensity. We set $a=0.32$ and $b=0.52$, so the relative capital intensity in the two sectors are exactly opposite to that in sub-section 3.1. The effect of the tax schedule on the stability properties are illustrated in Figure 4.

[Insert Figure 4 about here]

It turns out that for this parameterization, the critical value of the investment externality θ_I at which the model's stability properties change is 0.2203. For $\theta_I \leq 0.2203$, a flat and a progressive tax schedule can be automatic stabilizers. It is worth noting that the critical value of the investment externality at $\theta_I=0.2203$ is larger than the corresponding value $\theta_I=0.126$ in Figure 1 because the Rybczynski effect works in an opposite direction. As the investment sector is more capital-intensive now, through the Rybczynski effect, higher next period's capital increases the output of investment goods and decreases the output of consumption goods. The effect lowers the growth rate of consumption goods in the next

[9] It is worth noting that when income taxes are very progressive in Guo and Harrison (2001), the steady state is a source and the economy is unstable. Our model rules out such a possibility.

period suppressing the initial self-fulfilling expectations. Thus, a progressive tax schedule can hold back sunspot equilibria that would otherwise emerge under a larger critical value of the investment externality.

The empirically realistic value of θ_{US}=0.108 for the US economy is smaller than the critical value 0.2203. Under θ_I=0.108, the steady state is a saddle when $\phi \in$(-2.365, -1.16)∪(-0.45, 0.505].[10] See the right panel in Table 1. With θ_I=0.108, because the investment sector is more capital-intensive, sunspot equilibria can be held back by a progressive income tax at the highest degree of ϕ=0.505 which is much larger than the corresponding ϕ=0.27 in Figure 1.

To see how differences in factor intensities affect the result, we envisage the effects on determinacy when the investment sector has different capital intensities with fixed capital intensities in the consumption. Figure 5 illustrates the results. Given that the capital intensity is fixed at 0.32 in the consumption sector, Figure 5 indicates that a progressive tax can stabilize the economy when the degree of the capital intensity in the investment sector is as small as 0.34. The difference in factor intensities is very small.

[Insert Figure 5 about here]

3.4 Robustness When the Consumption Sector Is Less Capital-intensive

In this sub-section we envisage the sensitivity of the results in sub-section 3.3 under alternative parameterizations concerning the IES of consumption and the Frisch LSE as listed in Table 2. The results are in Figure 6.

[Insert Figure 6 about here]

As compared to Figure 4, under θ_I=θ_{US}=0.108, Panels A and B indicate, respectively, that a higher IES decreases the highest degree of the income tax progressivity to repress sunspot equilibria (ϕ=0.459) and that a lower IES increases the highest degree of the income tax progressivity to suppress sunspot equilibria (ϕ=0.553). Moreover, under

[10] The bound $\phi \geq$-2.365 is required to assure that there exists an interior steady state.

θ_I=θ_{US}=0.108, Panels C and D suggest, respectively, that a higher Frisch LSE decreases the highest degree of the income tax progressivity to hold back sunspot equilibria (ϕ=0.502) and that a lower Frisch LSE increases the highest degree of the income tax progressivity to restrain sunspot equilibria (ϕ=0.51).

3.5 Other Results

Other than the effects of the income tax schedule on the stability properties, our general model also obtains other results that add values to existing literature.

First, with a variable income tax schedule, our model yields a steady state that is locally indeterminate not only when the consumption sector is more capital-intensive from the private perspective (cf. Figure 1) but also when the consumption sector is less capital-intensive from the private perspective (cf. Figure 4). This result is different from that in an otherwise identical two-sector model with sector-specific productive externalities and without taxes set forth by Benhabib et al. (2002). These authors found that their steady state is locally indeterminate only when the consumption sector is more capital-intensive than the investment sector from the private perspective.

Moreover, with θ_c=θ_I=0, our model collapses to a standard two-sector RBC model without externalities. In such a model economy, we find that if the income tax schedule is regressive,[11] indeterminacy emerges even when there are no externalities (cf. Figures 1 and 4). The result is different from a one-sector model without taxes put forth by Benhabib and Farmer (1994) wherein indeterminacy arises only when there are positive externalities. Moreover, our result is different from that of the one-sector model investigated by Guo and Lansing (1998). These authors found that if the income tax schedule is regressive, indeterminacy arises only when there are positive aggregate externalities. Finally, in a human-capital based two-sector model without externalities studied by Bond et al. (1996), indeterminacy appears only when capital income and labor income are be taxed at different

[11] Under $a>b$, the steady state is a sink if $\phi\in$(-1.022, -0.51] in Figure 1 and under $a<b$, the steady state is a sink if $\phi\in$(-1.27, -0.508] in Figure 4.

rates. If capital income and labor income are taxed at the same rate, as is in our model, indeterminacy cannot develop in the in Bond et al. (1996) model.

5. Concluding Remarks

Guo and Harrison (2001) introduced government tax policy into a two-sector model with sector-specific productive externalities and equal factor intensities in both sectors from the private perspective. They found that when investment externalities are below (above) a critical level, a progressive (regressive) tax schedule can stabilize the US economy. Since the empirically realistic value of the investment externality for the US economy is larger than their critical value, their results imply that a progressive tax in the current US income tax code is susceptible to destabilizing the economy. Our paper envisages the robustness of the tax policy implications in a general two-sector model wherein the sectors have different factor intensities from the private perspective.

To highlight the role of the factor intensity, our paper studies a model that is otherwise identical to the Guo and Harrison model except for different factor intensities between the two sectors from the private perspective. With different factor intensities in the two sectors from the private perspective, because of larger sector reallocation, the Stolper-Samuelson effect and the Rybczynski effect are stronger yielding larger price, consumption and investment effect. Then, a flat and a progressive tax schedule can hold back the initial self-fulfilling expectations under a much larger critical value of investment externalities. By using calibration analysis, we find that under a reasonable difference in the factor intensity between two sectors, the empirically realistic value of the investment externality for the US economy is smaller than the critical value of the investment externality. Our results thus suggest that a progressive tax in the current US income tax code can stabilize the economy.

Appendix

The dynamic system consists of equations (14a) and (14b). When we take log-linearization in the neighborhood of the steady state, we obtain the following form:

$$\begin{bmatrix}\hat{K}_{t+1}\\ \hat{C}_{t+1}\end{bmatrix}=J\begin{bmatrix}\hat{K}_{t}\\ \hat{C}_{t}\end{bmatrix},\hat{K}_0 \text{ given,}$$

where the log-deviations from the steady state are identified with a cap above the variable.

The elements in the Jacobean matrix are:

$$J_{11}=\frac{\delta(1+\theta_I)}{1-a}\left[b-a-\frac{[1-b+(b-a)s](1+\chi)\Xi}{1-s}\right]+1-\delta,$$

$$J_{12}=\frac{\delta(1+\theta_I)}{(1-a)(1+\theta_c)}\left[1-b-\frac{[1-b+(b-a)s][1+\chi-(1-a)(1+\theta_c)(1-\phi-\sigma)]\Xi}{a(1-s)}\right],$$

$$J_{21}=\frac{1}{\Omega}\left\{\left[\frac{[a(1-\phi)+(b-a)s](1+\chi)\Xi}{a+(b-a)s}-1\right][1-\beta(1-\delta)]J_{11}-[1-\beta(1-\delta)J_{11}]\Gamma\right\},$$

$$J_{22}=\frac{1}{\Omega}\left\{\sigma+\left[\frac{[a(1-\phi)+(b-a)s](1+\chi)\Xi}{a+(b-a)s}-1\right][1-\beta(1-\delta)]J_{12}+\beta(1-\delta)J_{12}\Gamma-\Phi+\frac{(1-b)(1+\theta_I)}{(1-a)(1+\theta_c)}-1\right\},$$

$$\Gamma=-\frac{(b-a)(1+\theta_I)}{1-a}-\frac{\{[1-b+(b-a)s](1+\theta_I)-(1-a)\}(1+\chi)\Xi}{(1-a)(1-s)},$$

$$\Omega=(\sigma+\phi-1)+\frac{a(1-\phi)+(b-a)s}{a+(b-a)s}\frac{(1-a)(1-s)\Phi[1-\beta(1-\delta)]}{[1-b+(b-a)s](1+\theta_I)-(1-a)}+\beta(1-\delta)\left\{\frac{(1-b)(1+\theta_I)}{(1-a)(1+\theta_c)}-\phi-\Phi\right\}$$

$$\Phi=\frac{[1-b+(b-a)s](1+\theta_I)-(1-a)}{(1-a)(1-s)}\frac{[1+\chi+(1-a)(1+\theta_c)(\sigma+\phi-1)]\Xi}{a(1+\theta_c)}.$$

$$\Xi=\frac{[a+(b-a)s][a(1-b)+(b-a)s]}{[a+(b-a)s]\{[a(1-b)+(b-a)s]+[1-b+(b-a)s]\chi\}+\phi(1-a)[a(1-b)+(b-a)s]}.$$

The determinant of the Jacobean matrix is given by:

$$\begin{aligned}Det(J)=&\frac{(b-a)\delta(1+\theta_I)}{1-a}\frac{(\sigma-1)a(1-s)(1+\theta_c)+[1+\chi+(1-a)(1+\theta_c)(\sigma+\phi-1)]\Xi}{a(1-s)(1+\theta_c)\Omega}\\&-\frac{(1-\delta)}{\Omega}\left\{\frac{[1-b+(b-a)s](1+\theta_I)-(1-a)}{(1-a)(1-s)}\frac{[1+\chi+(1-a)(1+\theta_c)(\sigma+\phi-1)]\Xi}{a(1+\theta_c)}\right\}\\&+\frac{(1-\delta)}{\Omega}\left[\sigma-1+\frac{(1-b)(1+\theta_I)}{(1-a)(1+\theta_c)}\right]+\frac{(\sigma-1)\delta(1+\theta_I)}{1-a}\frac{[1-b+(b-a)s](1+\chi)\Xi}{(1-s)\Omega}.\end{aligned}$$

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Table 1: Benchmark parameterization

a=0.52, b=0.32		a=0.32, b=0.52	
$\phi \in$(-0.513, -0.489]∪[0.27, 1)	sink	$\phi \in$(-1.16, -0.45]∪(0.505, 1)	sink
$\phi \in$(-4.468, -0.513)∪(-0.461, 0.27]	saddle	$\phi \in$(-2.365, -1.16)∪(-0.45, 0.505]	saddle
$\phi \in$(-0.489, -0.461]	source		

Baseline parameterization: {δ=0.025, β=0.99, η=0.8, 1/σ=1, 1/χ=4, θ_C=0, θ_I=0.108}

Table 2: Alternative parameterizations

	$1/\sigma$	$1/\chi$
Benchmark	1	4
High IES	1.2	4
Low IES	0.8	4
High LSE	1	5
Low LSE	1	3

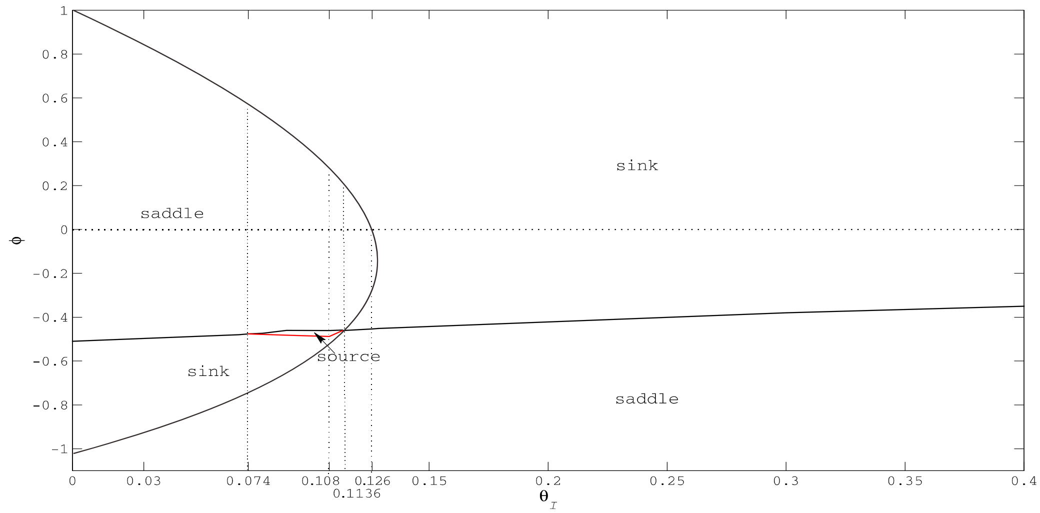

Figure 1. Stability properties under the benchmark parameterization when the consumption sector is more capital-intensive

Parameter values: {δ=0.025, β=0.99, η=0.8, 1/σ=1, 1/χ=4, θ_C=0, a=0.52, b=0.32}

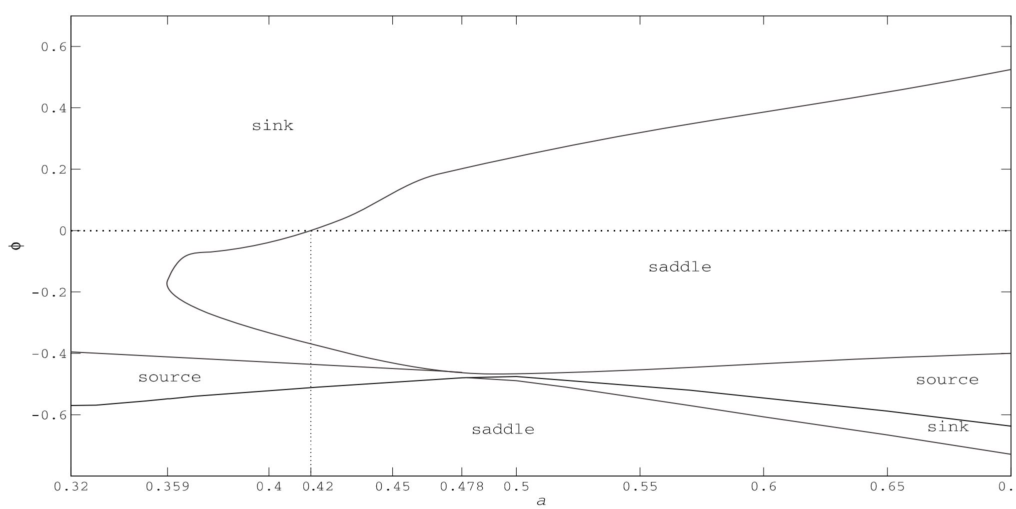

Figure 2. Stability properties under different factor intensities in the consumption sector when the consumption sector is more capital-intensive.

Parameter values: {δ=0.025, β=0.99, η=0.8, 1/σ=1, 1/χ=4, θ_C=0, θ_I=0.108, b=0.32}

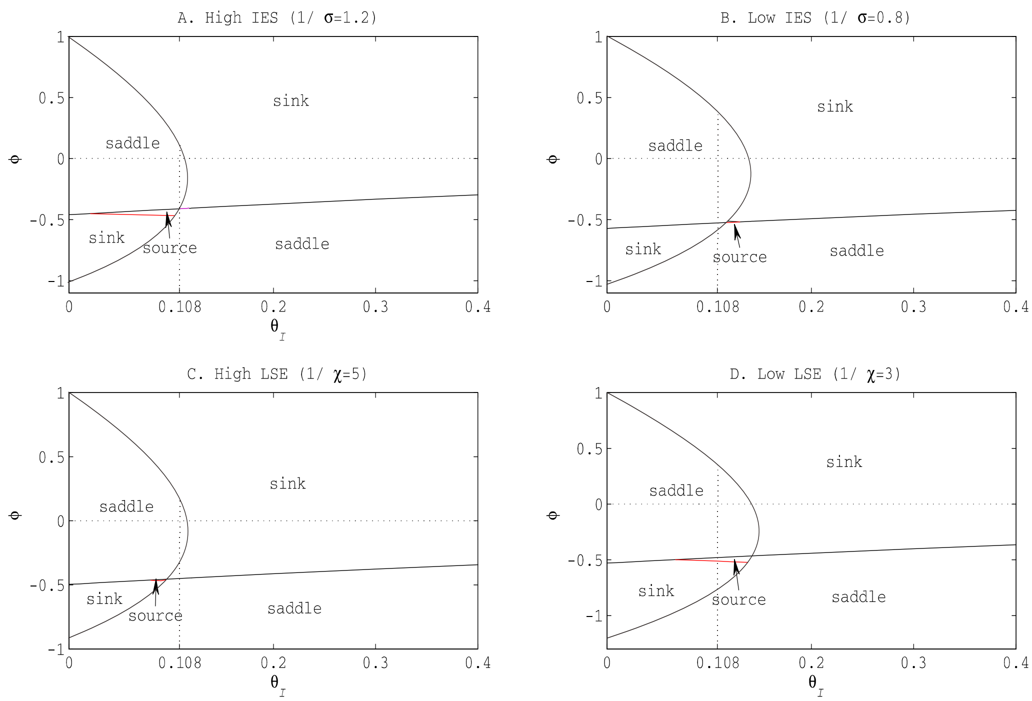

Figure 3. Stability properties under alternative parameterizations when the consumption sector is more capital-intensive.

Parameter values: {δ=0.025, β=0.99, 1/σ=1, 1/χ=4, η=0.8, θ_C=0, a=0.52, b=0.32}

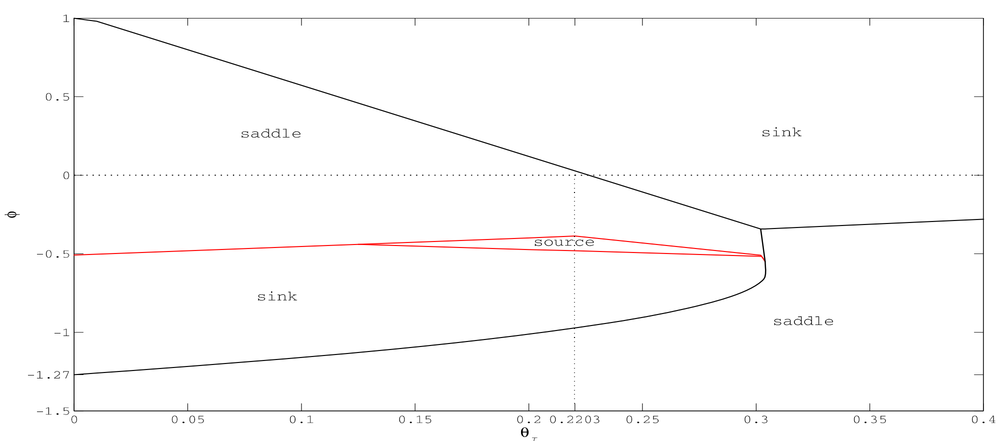

Figure 4. Stability properties under the benchmark parameterization when the consumption sector is less capital-intensive

Parameter values: {δ=0.025, β=0.99, 1/σ=1, 1/χ=4, η=0.8, θ_C=0, a=0.32, b=0.52}

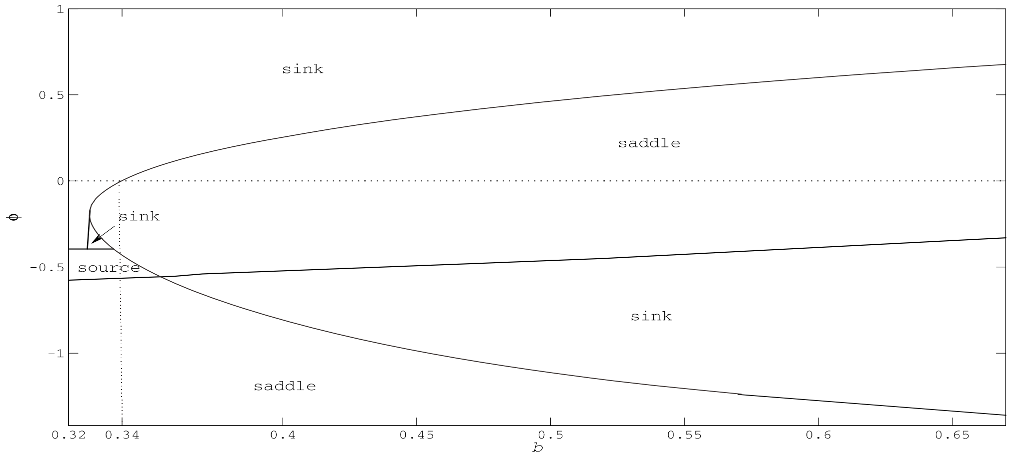

Figure 5. Stability properties under different factor intensities in the investment sector when the investment sector is more capital-intensive.

Parameter values: {δ=0.025, β=0.99, $1/\sigma$=1, $1/\chi$=4, η=0.8, θ_C=0, θ_I=0.108, a=0.32}

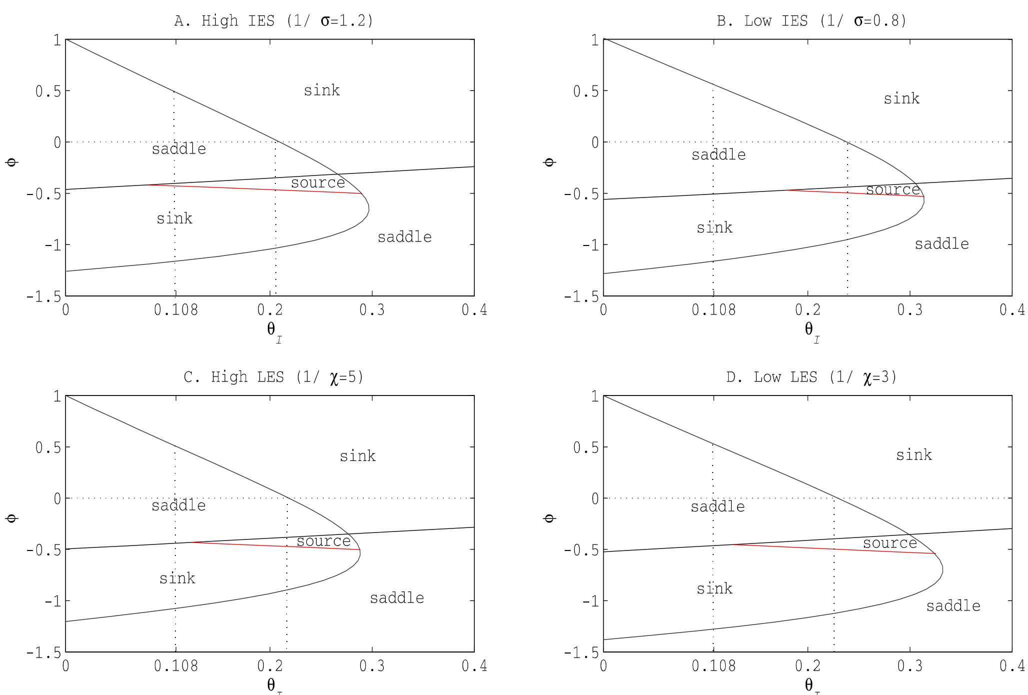

Figure 6. Stability properties under alternative parameterizations when the consumption sector is less capital-intensive

Parameter values: {δ=0.025, β=0.99, $1/\sigma$=1, $1/\chi$=4, η=0.8, θ_C=0, a=0.32, b=0.52}