Price and Advertising Signals of Product Quality with Minimum Demand

Dawen Meng *
Institute for Advanced Research
Shanghai University of Finance and Economics
Shanghai, China. 201600

Guoqiang Tian †
Department of Economics
Texas A&M University
College Station, Texas 77843

November 5, 2011

Abstract

In this paper, we discuss the issue of pricing and advertising strategy of a monopolist who provide an experience good with unknown quality to the consumers. We find that when a minimum consumption is required, the firm's ability of signaling his quality with an upwardly distorted price is limited. We find that the price-quality and advertising-quality relations found in the existing literature does not hold any more.

Keywords: advertising, quality, signaling, minimum demand, intuitive criterion

1 Introduction

In a context where consumers may repeatedly purchase an experience good[1] but don't know exactly its true quality, the firm supplying the high-quality product needs to signal that information to potential consumers in the introductory phase.[2] Examples of such signals are advertising (Nelson 1974; Schmalensee 1978; Wiggins and Lane 1983), and price (Farrell 1980; Gabor and Granger 1966; Leavitt 1954; Scitovsky 1944; Spence 1974,Bagwell and

*Email: devinmeng@yahoo.com.cn

†Email:tian@econmail.tamu.edu

[1]Nelson (1970) differentiated between products on a "search good" versus "experience good" basis. With the former, the relevant characteristics of the product are evident on inspection. With the latter, crucial aspects of the product's quality are impossible to verify except through use of the product.

[2]The product's life cycle is decomposed into two phases: the introductory phase and the mature phase. Signaling occurs during the introductory phase. All consumers know the product quality in the mature phase, say, through word-of-mouth learning or through reading consumer reports. Under these conditions, the firm need not to send a signal in the mature phase.

Riordan (1991)). Three important issues concerning the relationships between quality, price and advertising may therefore arise.

- Do higher prices signal higher quality?(the relationship between product quality and price);
- Are heavily advertised products more likely to be of higher quality than less-advertised products? (the relationship between product quality and advertising);
- Are heavily advertised products more expensive than less-advertised goods? (the relationship between advertising expenditures and price charged).

We focus on the price-quality relationship in this paper. The existing economic and marketing literature has produced various theoretical and empirical explanations on this relationship. For models in which a monopolist signals its quality using only price, Bagwell and Riordan (1991) show that in the introductory phase the high-quality firm will distort its price above the complete-information profit maximizing price, so high price may act as a signal of high quality. Their main argument is that the low quality seller has a lower marginal cost of production (relative to a high quality seller) and therefore finds it more profitable to sell higher quantity at a sufficiently lower price rather than imitate the lower quantityChigher price combination preferred by the high quality seller. But as information about the product diffuses over time, this price distortion lessens or vanishes entirely.

Bagwell (1991) finds that,with a downward sloping demand, the only equilibrium satisfying the Intuitive Criterion (Cho and Kreps 1987) is a separating equilibrium in which the high quality is traded at a higher price but sells less than the low quality. In a model with one seller and one buyer with inelastic demand, Ellingsen (1997) finds that there is a unique equilibrium surviving D1 (Cho and Kreps 1987). The equilibrium is separating: the seller sells with probability one at the low price and with probability less than one (but positive) at the high price. Hence, the general consensus is that, a high quality seller is able to signal quality by distorting his price upwards and reducing the volume of trade relative to the first best.

When price and advertising could be used as joint signals of product quality, the price-quality relationship become more complex. In a multidimensional signalling model, Milgrom and Roberts (1986) identify various conditions under which high quality may be signalled with a high price alone, a low price alone, or a combination of price and advertising expenditure. Their article provides a formal proof of Nelson's "advertising as information" theory that dissipative advertising functions as a signal of quality when the consumers may purchase an experience good repeatedly. [3]In their paper, Milgrom and Roberts also show how to

[3]Adverting which inform potential customers about the existence, characteristics, and prices of the commodities is called informative advertising, most newspaper advertisements (including especially want ads) would seem to be of this sort. advertising conveys no direct information about product is refereed as dissipative advertising.

select (quite generally) an unique separating equilibrium from many through the elimination of dominated strategies. It also introduces the use of the Cho and Kreps (1987) criterion to eliminate pooling equilibria. In their model, with the aid of advertising, a high-quality firm may set an introductory price higher or lower than its perfect information price, and higher or lower than the price charged by the low-quality firm. The distorted prices would reduce current profit, but the present lose will be compensated by the future profit (repeat purchases). So a clever combination of price and advertising enables the monopoly to signal its quality as well as to enlarge its future demand. Milgrom and Roberts' theory appears to be consistent with the stylized facts provided in many empirical studies that price-quality relations are product-specific and weak or even negative in sign [See Oxenfeldt (1950), Morris and Bronson (1969), Sproles (1977), Riesz (1978, 1979), and Geistfeld (1982),etc.]

In this paper, we discuss the pricing behaviors of a monopolist in the context of repeat sales and both price and advertising may be used as signals of quality of an experience good. Our model is distinct to the existing literature in the sense that a minimum level of consumption is considered. An experience good is traditionally defined as good that consumers cannot directly assess its quality before consumption (the only proof of the pudding is in the eating). But in reality, many goods exhibit properties that its quality can only be known after being consumed with an amount no less than certain minimum level (you could not know its proof once take a bite.) We will show in this paper that the introductory pricing strategies of a firm with and without minimum level of consumption are quite different. We also identify the advertising price and minimum consumption as two factors affect the firm's pricing strategy.

The rest of this paper is organized as follows. We describe the benchmark model without minimum consumption in Section 2. In Section 3 the model with minimum consumption is described in detail. Conclusions are presented in Section 4.

2 The Model

A firm provides a new experience good with uncertain quality to the consumers. For simplicity, we assume that quality is either high or low: $\theta \in \{\theta_h, \theta_l\}$. It is of high quality θ_h with probability λ and low quality θ_l with complementary probability $1-\lambda$. A high-quality product is more costly to produce than a low-quality one: $c_h > c_l$. The firm may use both price p and dissipative advertising a as signals for the initially unobservable quality. The binary variable $a \in \{0, 1\}$ denotes whether or not the good was advertised. $k \in (0, \infty)$ is the exogenously determined and publicly observable price of adverting. Let $\rho = \rho(p, a)$ be the

The dissipative advertising has two main characteristics: First it does not directly affect demand (not persuasive nor informative content), and second it is easy to observe that a substantial amount of money has been spent (a celebrity endorsing a coffee brand rather than an anonymous actress/actor).

uninformed consumers' posterior belief that quality is high upon observing p and a. $D(p,\rho)$ denotes the demand function when the firm charge a price p and consumer's posterior belief is ρ. At the beginning of the game, there is a continuum of consumers of unitary mass who are heterogeneous in their willingness to pay for quality. Their preference for quality is described by an index x, which is assumed to be distributed according to cumulative distribution function $G(\cdot)$ over $[0,1]$. For a product perceived to be of quality θ_h with probability ρ, sold at price p, the consumer with an index x will derive net utility

$$u(x,p,\rho) = x[\rho\theta_h + (1-\rho)\theta_l] - p \tag{1}$$

if she purchase it. Each consumer purchases one unit provided she can derive positive utility. Otherwise the consumer does not buy. Given the consumers' utilities shown in (1), the firm's total demand is

$$D(p,\rho) = \Pr\left\{x[\rho\theta_h + (1-\rho)\theta_l] - p > 0\right\} = 1 - G\left(\frac{p}{\rho\theta_h + (1-\rho)\theta_l}\right). \tag{2}$$

Let $\bar{p}(\rho) = \rho\theta_h + (1-\rho)\theta_l$ be the choke price defined by the equation $D(p,\rho) = 0$. It represent the highest possible price a firm may charge when its product is believed to be of high quality with probability ρ.

We need an assumption regarding the cost and quality of firms.

Assumption 1 $\theta_h > c_h > \theta_l > c_l$.

Denote by $\pi(p,\theta_i,\rho) \equiv D(p,\rho)[p-c_i], i \in \{l,h\}$ the one-shot profit of the firm producing quality θ_i, charging a price p and enjoying consumer belief ρ. Let $\pi^*(\theta_h) = \underset{p}{max}\pi(p,\theta_h,1)$ and $\pi^*(\theta_l) = \underset{p}{max}\pi(p,\theta_l,0)$ be, respectively, the optimal profits gained by the high-quality and low-quality firms when the consumers know their types; the corresponding optimal prices are respectively $p^*(\theta_h)$ and $p^*(\theta_l)$. If a high quality firm is perceived as a low quality one, the maximal profit he could gain is

$$\pi^*(\theta_l|\theta_h) = \underset{p}{max}(p-c_h)D(p,0), s.t.: D(p,0) \geqslant 0$$

Under the above assumption, consumers will pay at most θ_l for a product believed to be of low quality, which is strictly lower than its production cost c_h, consequently, the firm will suffer a loss when the market demand is positive, therefore, $\pi^*(\theta_l|\theta_h) = 0$. The profit and optimal price of a low-quality firm being perceived as a high-quality one are denoted respectively by $\pi^*(\theta_h|\theta_l)$ and $p^*(\theta_h|\theta_l)$.

The game proceeds as follows:

- At the beginning of the first (or introductory) phase, nature draws the quality of product according to $\Pr(\theta = \theta_h) = \lambda$;
- The firm sends a binary signal $m = (p,a) \in [0,+\infty] \times \{0,1\}$ to the potential consumers;

- The consumers update their prior belief $\rho_0 = \lambda$ and form a posterior belief $\rho = \Pr(\theta = \theta_h|m)$ and demand $D(p, \rho)$ upon observing the firm's price and advertising decisions;
- The game then enters the second (or mature) phase. The product attracts repeat purchases if and only if it is of high quality and the sales volume in the introductory phase exceeds certain minimum level D_{min}, in that case, customers buy a product repeatedly in every period afterwards knowing its true quality.; the game terminates otherwise.

In economic literature, an experience good, as opposed to search good, is defined as a product or service whose characteristics are difficult to observe in advance, but these characteristics can be ascertained immediately after it is consumed. In this paper, however, we assume that the product's quality could not be fully revealed to the consumers until they consume certain amount of it. (*One can not know the proof of pudding once take a bite.*) Examples are medicines, whose curative effects could not be observed until a lowest possible dose and length of treatment are reached. Another class of examples are durable goods, whose performance levels and failure rates may be revealed only after prolonged use. In our model, each consumer has a potential demand for one unit of product. There exist consumers of unitary mass in the market at the beginning of the first period, everyone of them either makes a purchase or does not, and then leaves the market at the end of the first period. The users the product may share their experiences with the potential consumers outside the market and persuade them to enter or stay out of the market, but they themselves will not enter again. A continuum of new consumers of equal mass will enter the market at the beginning of the second period only if they fully know that the product is of high quality. It is assumed that there exists a critical value of initial purchase of high-quality product (it is also the number of users since each one purchases one unit.) beyond which everyone outside the market may get informed. However, if the number of users is smaller than this critical value, the quality of product is known to only a small coterie of people with negligible small size and almost no one will be persuaded to enter the market. Here we implicitly assume that the information diffuses in the way of word-of-mouth communication rather than via mass media.

Let $\delta \in [0, 1)$ be the firm's discount factor. $\Pi(\theta, \rho, p, a)$ is the expected net profit gained by a $\theta-$type firm enjoying consumer belief ρ and sending an introductory signals (p, a).

$$\Pi(\theta_h, \rho, p, a) = D(p, \rho)[p - c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h)\mathbb{1}[D(p, \rho) \geqslant D_{min}] - ak \tag{3}$$

$$\Pi(\theta_l, \rho, p, a) = D(p, \rho)[p - c_l] - ak \tag{4}$$

$\mathbb{1}[\cdot]$ is an indicator function.

We now consider the separating perfect Bayesian-Nash equilibrium in which high-quality and low-quality firms choose different signals. Then observing price and advertising allows the consumers to be fully informed of the firm's type. Whatever choice $(p(\theta_l), a(\theta_l))$ is made in the separating PBE this choice must yield $\rho(p(\theta_l), a(\theta_l)) = 0$, therefore, the best choice of θ_l-type firm in the introductory phase is $p(\theta_l) = p^*(\theta_l), a(\theta_l) = 0$. That is to say, the low-quality firm has no incentive to signal his quality via distorted introductory price or dissipative advertising. In any separating equilibrium, the high-quality firm's choice (p, a) must satisfies the following two conditions:

$$\begin{aligned} &D(p,1)[p-c_h] + \mathbb{1}[D(p,1) > D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) - ak \\ \geqslant &\max_p \left\{ D(p,0)[p-c_h] + \mathbb{1}[D(p,0) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) \right\} \end{aligned} \tag{5}$$

$$\pi^*(\theta_l) \equiv D(p^*(\theta_l),0)[p^*(\theta_l)-c_l] \geqslant D(p,1)[p-c_l] - ak \tag{6}$$

The inequality (5) asserts that a high-quality firm would rather choose (p, a) and be perceived as high-quality than be perceived as low-quality and optimize accordingly, while (6) assert that the low-quality firm has the reverse preference. Their choices are all supported by the belief $\rho(p, a) = 1$, $\rho(p^*(\theta_l), 0) = 0$ and for all other choice (p', a'), $\rho(p', a')$ is sufficiently small (e.g., zero) that neither player wishes to deviate to (p', a'). Given this belief, the high-quality firm will not advertise if he deviate from equilibrium. His deviating profit $\Pi^d \equiv \max_p \left\{D(p,0)[p-c_h] + \mathbb{1}[D(p,0) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h)\right\}$ and price p^d are determined by his discount factor δ. If $\frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$, $p^d = G^{-1}(1-D_{min})\theta_l$, $\Pi^d = \frac{\delta}{1-\delta}\pi^*(\theta_h) - D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$; if $\frac{\delta}{1-\delta}\pi^*(\theta_h) < D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$, $p^d = \theta_l$, $\Pi^d = 0$. That is to say, when being perceived as low-quality a farsighted high-quality firm will endures the temporary misunderstanding of consumers and charges a low price to attract repeat purchases, because his mature-phase profit is more than enough to cover the loss suffered during the introductory-phase; while a myopic high-quality firm will exit the market when being perceived to be low-quality.

It immediately follows from (5) and (6) that if $a(\theta_h) = 1$, then $p(\theta_h) \in \mathcal{S}(k)$; if $a(\theta_h) = 0$, then $p(\theta_h) \in \mathcal{S}(0)$, where

$$\mathcal{S}(x) = \left\{ p \in \mathbb{R}_+ \,\middle|\, \begin{aligned} &D(p,1)(p-c_h) + \mathbb{1}[D(p,1) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) - \\ &\max_p \left\{ D(p,0)[p-c_h] + \mathbb{1}[D(p,0) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) \right\} \\ &\geqslant x \geqslant D(p,1)(p-c_l) - \pi^*(\theta_l) \end{aligned} \right\}. \tag{7}$$

The following lemma states the existence of the separating PBE.

Theorem 2.1 *There exists a separating PBE if and only if for some $(p, a) \in \mathbb{R}_+ \times \{0, 1\}$, $p \in \mathcal{S}(ak)$; at any separating equilibrium, the high-quality firm chooses (p, a) that satisfies $p \in \mathcal{S}(ak)$, the low-quality firm chooses $(p^*(\theta_l), 0)$. Customers' belief are given by $\rho(p, a) = 1$ for point chosen by the high-quality firm, $\rho(p^*(\theta_l), 0) = 0$, and, for all other out-of-equilibrium points (p', a'), $\rho(p', a')$ is sufficiently small (e.g.,zero) that neither player wishes to deviate to it.*

The situation in which $D_{min} = 0$ and a separating equilibrium exists is depicted in figure 1. From this we find that there are typically many separating equilibria, the Cho-Kreps intuitive criterion can be usefully employed in this environment to select among equilibria. It says that if there exists an out-of-equilibrium pair of price and advertising (p', a') that may be profitable for a high-type (low-quality) firm but which is never profitable for a low-quality (hight-quality) firm no matter what uninformed consumers believe, then uninformed consumers must believe that the deviator is of high-quality (low-quality). In effect, the firm makes an implicit speech as to quality with the selection of the signal (p', a'), and the equilibrium where θ_h chooses (p, a) is overturned..

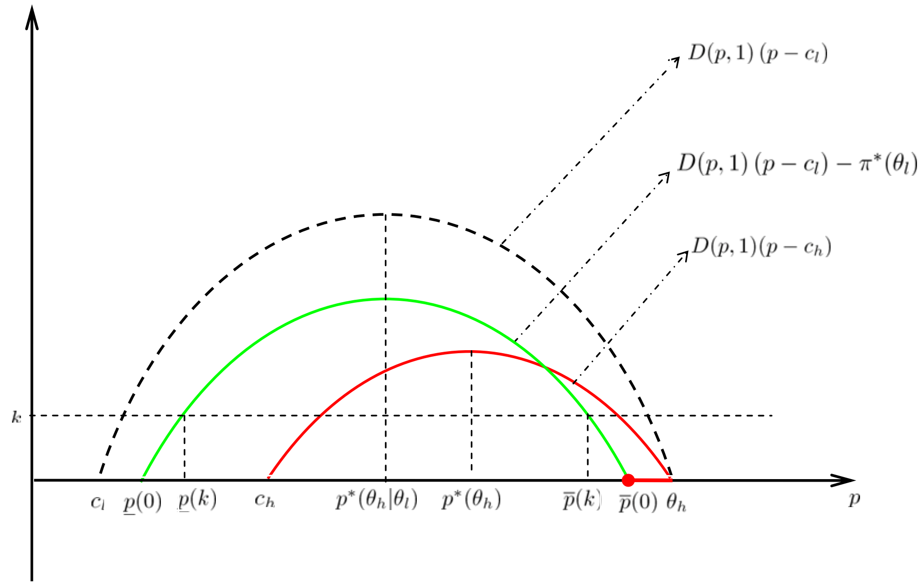


Figure 1: Separating equilibrium without demand constraint.

Theorem 2.2 *Among all separating equilibria,* $(p(\theta_h), a(\theta_h)) = (p^*, a^*)$*,* $(p(\theta_l), a(\theta_l)) = (p^*(\theta_l), 0)$ *are the only pairs of price and advertising surviving a refinement based on the intuitive criterion, where*

$$(p^*, a^*) \equiv \underset{p\in\mathcal{S}(ak), a\in\{0,1\}}{argmax} \left\{ D(p,1)[p-c_h] + \mathbb{1}[D(p,1) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) - ak \right\}.$$

Proof. Suppose that $(p(\theta_h), a(\theta_h)) \neq (p^*, a^*)$, then (p^*, a^*) is an off-equilibrium pair of signals. The equilibrium payoff of high-quality firm is

$$\Pi^*(\theta_h) \equiv D(p(\theta_h),1)(p(\theta_h)-c_h) + \mathbb{1}[D(p(\theta_h),1) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) - a(\theta_h)k.$$

His highest possible profit when deviating to (p^*, a^*) is $\Pi(p^*, a^*, \theta_h) = \underset{\rho\in[0,1]}{max}\,\Pi(p^*, a^*, \rho, \theta_h)$. It is obvious $\Pi(p^*, a^*, \theta_h) \geqslant D(p^*,1)[p^*-c_h] + \mathbb{1}[D(p^*,1) \geqslant D_{min}]\frac{\delta}{1-\delta}\pi^*(\theta_h) - a^*k > \Pi^*(\theta_h)$. The equilibrium payoff of low-quality firm is $\Pi^*(\theta_l) = \pi^*(\theta_l)$, while his highest possible payoff when deviating to (p^*, a^*) is

$$\Pi(p^*, a^*, \theta_l) = \underset{\rho\in[0,1]}{max}\{D(p^*, \rho)(p^*-c_l) - a^*k\} = D(p^*,1)[p^*-c_l] - a^*k \leqslant \pi^*(\theta_l).$$

Then there exists a pair $(p', a^*) \neq (p(\theta_h, a(\theta_h)))$ which satisfies $p' \in \mathcal{S}(a^* k)$ and is sufficiently close to (p^*, a^*) such that the highest possible payoff of the high-quality firm when deviating to (p', a^*) is strictly larger than his equilibrium payoff, the opposite is true for the low-quality firm: $\Pi(p', a^*, \theta_h) > \Pi^*(\theta_h), \Pi(p', a^*, \theta_l) < \Pi^*(\theta_l)$. Therefore, beliefs following observation of (p', a^*) must ascribe probability one to the high-quality firm. This elicits the high-quality firm to deviate to (p', a^*) and thus overturned the equilibrium where $(p(\theta_h), a(\theta_h))$ is chosen.
■

Consider again Figure 1. We express the upper and lower roots of $D(p,1)(p-c_l)-\pi^*(\theta_l) = k$ as $\overline{p}(k)$ and $\underline{p}(k)$. $p(\theta_h)$ satisfying (5) and (6) is not in general unique, there is a continuum of separating equilibria. However, an appeal to the intuitive criterion prunes all candidate signals with the exception of $(p(\theta_h), a(\theta_h)) = (\overline{p}(0), 0)$. This outcome is usually referred to as the efficient equilibrium outcome (or Riley outcome, after Riley,1979), since it is the equilibrium in which high-quality firm spends the least amount of resource in signaling to the consumers their type. We can find that the increasing of advertising price has two opposite effects. On the one hand, advertising can be viewed as a wasteful expenditure. In that case, the equilibrium associated with the lower advertising level is better than that with higher advertising level. On the other hand, advertising allows the high-quality firm to charge a less distorted price and thus increase his profit. With the aid of an expensive advertising, the high-quality firm could distinguish himself with a not-too-high price. Under this interpretation, the equilibrium associated with the higher advertising level is Pareto dominant. The decision to advertise, or not to, depends on the trade-off between these two opposite effects. Since $c_h > c_l$, the high-quality firm prefers to choose a higher price and zero advertising to signal his quality and compensate his production cost as well. If $0 \leqslant D_{min} \leqslant D(\overline{p}(0), 1)$, the highest possible introductory price a high-quality firm might charge without losing the future profits is higher than $\overline{p}(0)$. That is to say, the minimum demand does not impose any restriction on the high-quality firm's decision, and his optimal choice of price and advertising is unaffected.

Theorem 2.3 *If $D_{min} \in [0, D(\overline{p}(0), 1)]$, then in the intuitive equilibrium $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$.*

3 myopic case

We first discuss the myopic case with $\frac{\delta}{1-\delta}\pi^*(\theta_h) < D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$. In this case,

$$\mathcal{S}(x) = \left\{ p \in \mathbb{R}_+ \;\middle|\; \begin{array}{l} D(p,1)(p-c_h) + \mathbb{1}[D(p,1) \geqslant D_{min}]\dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \geqslant x \geqslant D(p,1)(p-c_l) - \pi^*(\theta_l) \end{array} \right\}. \tag{8}$$

The following theorems 3.1 to 3.6 summarizes with the full characterization of the equilibrium advertising and pricing strategies.

Theorem 3.1 *If* $(\delta, D_{min}) \in m_1$,

$$m_1 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \left| \begin{array}{l} [D_{min} - D(\overline{p}(0),1)]\Delta c \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta) \\ \leqslant \max\Big\{ \big[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right),1\right) \\ - D(\overline{p}(0),1)\big]\Delta c, D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]\Big\} \\ D(p^*(\theta_h|\theta_l),1) \geqslant D_{min} \geqslant D(\overline{p}(0),1) \end{array} \right. \right\}$$

then in any intuitive separating equilibrium the high-quality firm's strategies are given as follows:

1. *when* $k^* \leqslant k \leqslant \overline{k}^*$,

$$\begin{array}{rcl} p(\theta_h) & = & \left\{ \begin{array}{lll} max\{\overline{p}(k), p^*(\theta_h)\} & if & G^{-1}(1-D_{min})\theta_h > p^*(\theta_h) \\ G^{-1}(1-D_{min})\theta_h & if & G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h) \end{array} \right. ; \\ a(\theta_h) & = & 1 \end{array}$$

2. *when* $k < k^*$, *or* $k > \overline{k}^*$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$

where

$$k^* \equiv \Upsilon[G^{-1}(1-D_{min})\theta_h] = D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l) \tag{9}$$

$$\overline{k}^* \equiv \frac{\delta}{1-\delta}\pi^*(\theta_h) + D(p(\theta_h),1)[p(\theta_h) - c_h] - D(\overline{p}(0),1)[\overline{p}(0) - c_h] \tag{10}$$

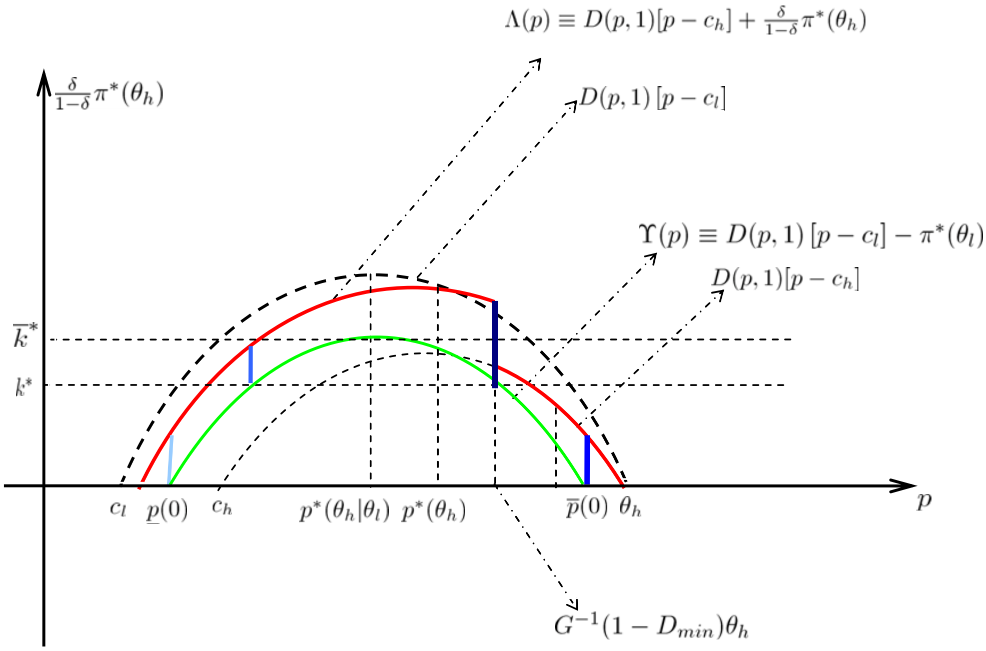
Figure 1 depicts the results of theorem 3.1. If advertising price were low ($k < k^*$), the information content contained in advertising is limited, therefore the high-quality firm has to distinguish himself from his low-quality counterpart using largely distorted price (relative to the complete-information price $p^*(\theta_h)$). Following the same logic as for the case without minimum demand, the pair $(\overline{p}(k), 1)$ is obvious dominated by $(\overline{p}(0), 0)$. Using a downward distorted price and advertising pair $(\underline{p}(k), 1)$, the firm might get extra revenue $\frac{\delta}{1-\delta}\pi^*(\theta_h)$ from future sales. An myopic firm, however, does not pay much attention to it. So it is optimal for a high-quality firm to signal with $(\overline{p}(0), 0)$. If advertising price were extremely high, i.e., $k > \overline{k}^*$, the firm might also choose not to advertise. After all, advertising is a dissipative expenditure. The firm will only advertise for some intermediate k, i.e., $k^* \leqslant k \leqslant \overline{k}^*$.

Theorem 3.2 *If* $(\delta, D_{min}) \in m_2$,

$$m_2 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \left| \begin{array}{l} \max\Big\{\big[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right),1\right) - \\ D(\overline{p}(0),1)\big]\Delta c, [D_{min} - D(\overline{p}(0),1)]\,\Delta c\Big\} \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \\ \min\left\{D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right], \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c\right\} \end{array} \right. \right\}$$

then there exist critical values $\underline{k}^*$,k^*, $\overline{k}^*$ *such that*

1. *when* $k \in (0, \underline{k}^*]$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$;



$$[m_1]: \left\{ \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \geqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant [D_{min} - D(\overline{p}(0),1)]\Delta c \\ \dfrac{\delta}{1-\delta}\pi^*(\theta) \leqslant \left[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right),1\right) - D(\overline{p}(0),1)\right]\Delta c \\ D(p^*(\theta_h|\theta_l),1) \geqslant D_{min} \geqslant D(\overline{p}(0),1) \end{array} \right\}$$

Figure 2: myopic case 1

2. *when* $k \in (\underline{k}^*, k^*]$, $p(\theta_h) = \underline{p}(k), a(\theta_h) = 1$;
3. *when* $k \in (k^*, \overline{k}^*]$,

$$\begin{array}{rcl} p(\theta_h) & = & \left\{ \begin{array}{lll} max\{\overline{p}(k), p^*(\theta_h)\} & if & p^*(\theta_h) < G^{-1}(1-D_{min})\theta_h \leqslant \overline{p}(0) \\ G^{-1}(1-D_{min})\theta_h & if & \underline{p}(0) \leqslant G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h) \end{array} \right. \\ a(\theta_h) & = & 1. \end{array}$$

4. *when* $k \in (\overline{k}^*, +\infty)$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$.

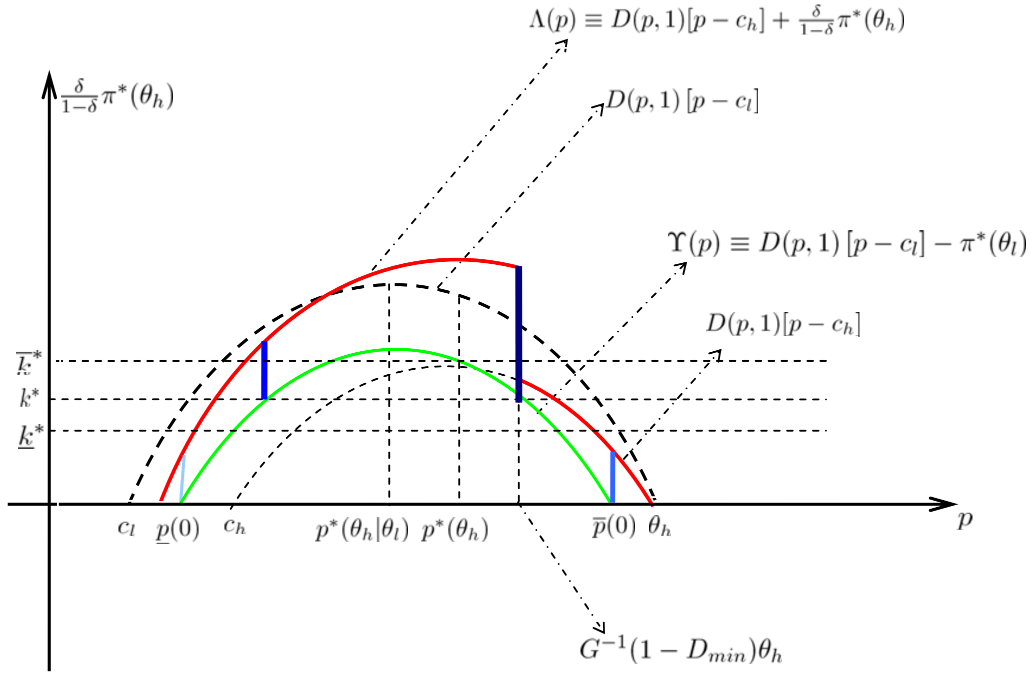
where $\underline{k}^*$ *is given by*

$$k + \left[D(\underline{p}(k),1) - D(\overline{p}(0),1)\right]\Delta c = \frac{\delta}{1-\delta}\pi^*(\theta_h), \tag{11}$$

k^* *and* $\overline{k}^*$ *are given by (9) and (10).*

Theorem 3.3 *If* $(\delta, D_{min}) \in m_3$,

$$m_3 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ D(\overline{p}(0),1) \geqslant D_{min} \geqslant D(\underline{p}(0),1) \end{array} \right\},$$



$$[m_2-1]:\left\{\begin{array}{l}\max\Big\{\big[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right),1\right) - \\ D(\overline{p}(0),1)\big]\Delta c, \left[D_{min} - D(\overline{p}(0),1)\right]\Delta c\Big\} \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \\ \min\left\{D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right], \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c\right. \\ p^*(\theta_h|\theta_l) \leqslant G^{-1}(1-D_{min})\theta_h \leqslant \overline{p}(0)\end{array}\right\}$$

Figure 3: myopic firm: case 2-1

then

1. *when* $k \in (0, k^*)$, $p(\theta_h) = \underline{p}(k), a(\theta_h) = 1$;
2. *when* $k \in (k^*, \overline{k}^{**}]$,

$$\begin{aligned} p(\theta_h) &= \left\{\begin{array}{lll} max\{\overline{p}(k), p^*(\theta_h)\} & if & p^*(\theta_h) < G^{-1}(1-D_{min})\theta_h \leqslant \overline{p}(0) \\ G^{-1}(1-D_{min})\theta_h & if & \underline{p}(0) \leqslant G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h)\end{array}\right. \\ a(\theta_h) &= 1. \end{aligned}$$

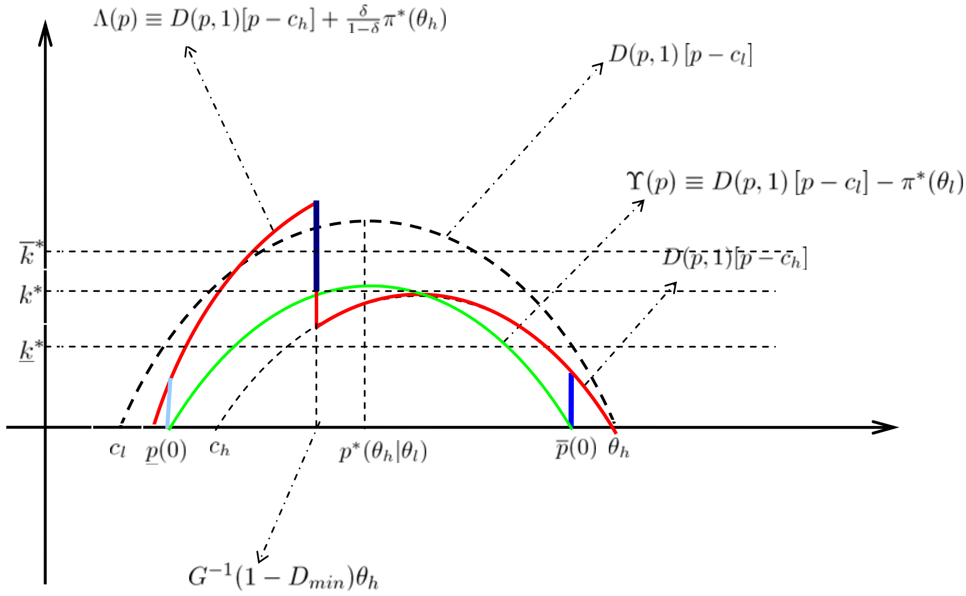
3. *when* $k \in (\overline{k}^{**}, +\infty)$, $p(\theta_h) = \underline{p}(0), a(\theta_h) = 0$.

k^* *is given by (9),*

$$\overline{k}^{**} = D(p(\theta_h),1)[p(\theta_h) - c_h] - D(\underline{p}(0),1)[\underline{p}(0) - c_h]. \tag{12}$$

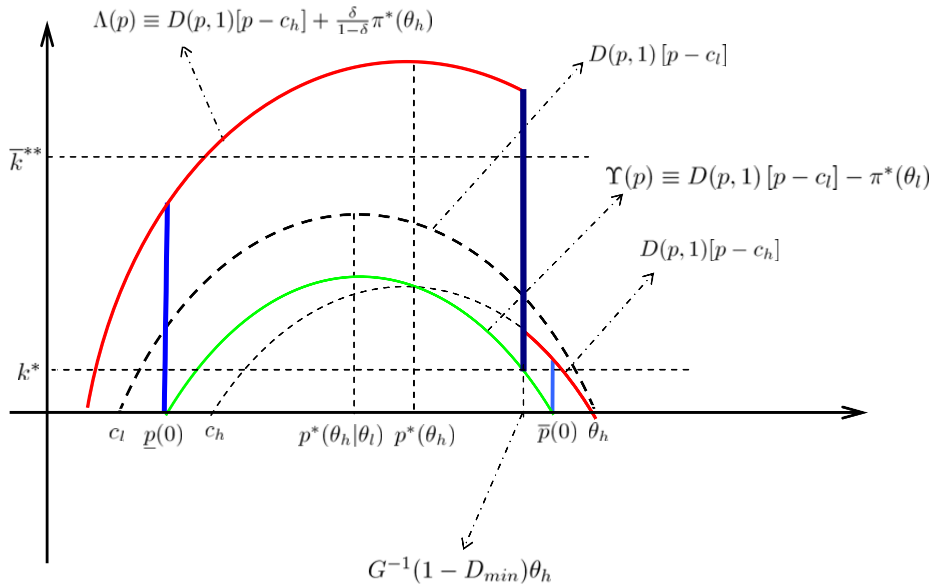
Theorem 3.4 *If* $(\delta, D_{min}) \in m_4$,

$$m_4 \equiv \left\{(\delta, D_{min}) \in [0,1]^2 \,\middle|\, \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \Delta c[D_{min} - D(\overline{p}(0),1)] \\ D(\overline{p}(0),1) \leqslant D_{min} \leqslant D(\underline{p}(0),1) \end{array}\right\},$$



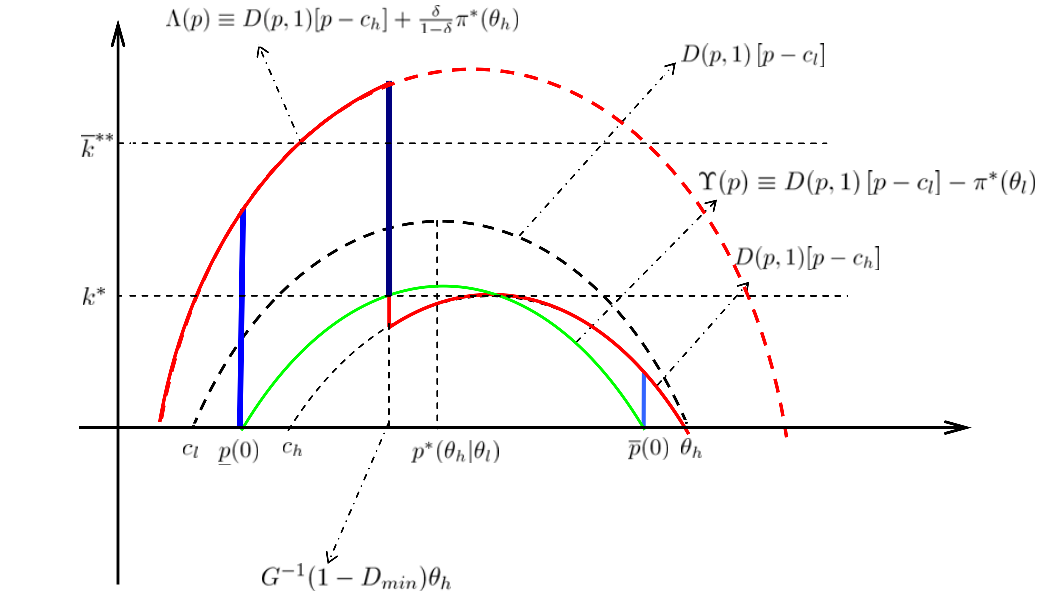
$$[m_2-2]: \left\{\begin{array}{l} \max\Big\{\left[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right),1\right) - \right. \\ \left. D(\overline{p}(0),1)\right]\Delta c, \left[D_{min} - D(\overline{p}(0),1)\right]\Delta c\Big\} \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \\ \min\left\{D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right], \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c \right. \\ \underline{p}(0) \leqslant G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h|\theta_l) \end{array}\right\}$$

Figure 4: myopic firm: case 2-2



$$[m_3-1]: \left\{\begin{array}{l} \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ D(p^*(\theta_h|\theta_l),1) \geqslant D_{min} \geqslant D(\overline{p}(0),1) \end{array}\right\}$$

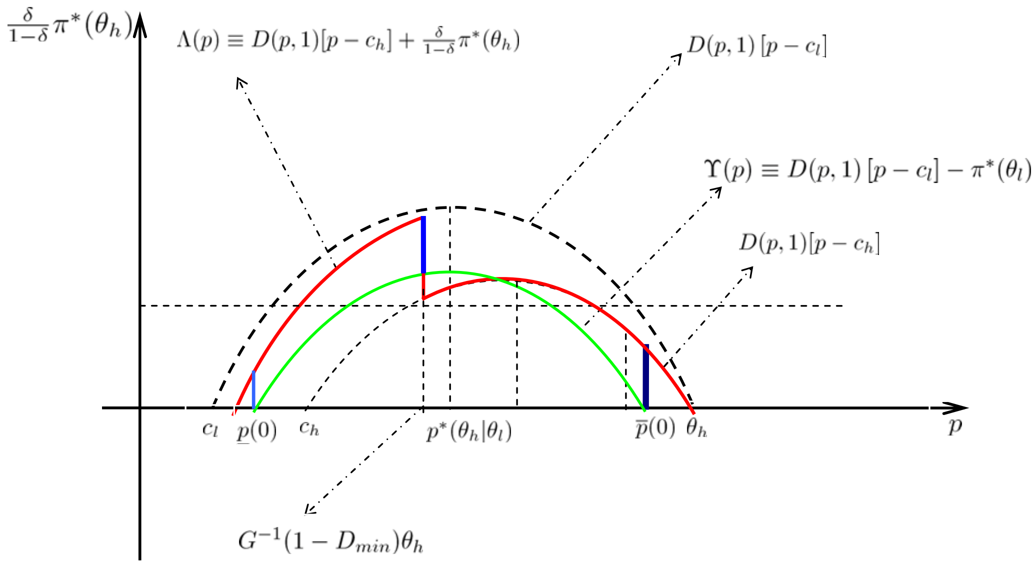
Figure 5: myopic firm: case 3-1



$$[m_3-2]:\left\{\begin{array}{l}\left[D(\underline{p}(0),1)-D(\overline{p}(0),1)\right]\Delta c \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]\\ D(p^*(\theta_h|\theta_l),1) \leqslant D_{min} \leqslant D(\underline{p}(0),1)\end{array}\right\}$$

Figure 6: myopic firm: case 3-2

then $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$ *for all* $k \in (0, +\infty)$.



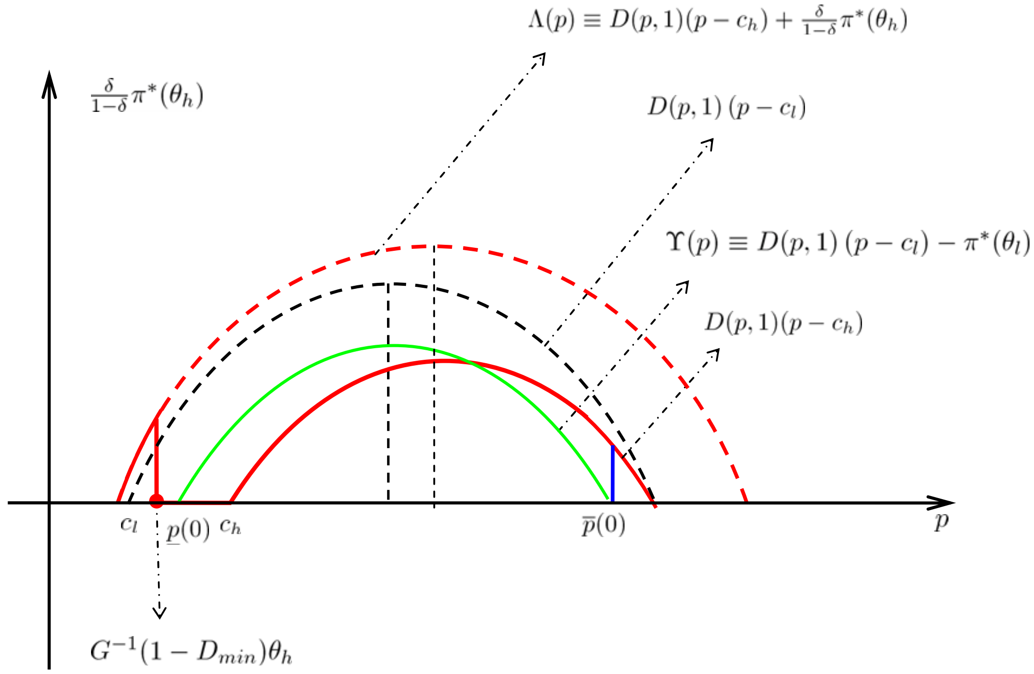
$$[m_4]:\left\{\begin{array}{l}\dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \Delta c[D_{min} - D(\overline{p}(0),1)]\\ D(\overline{p}(0),1) \leqslant D_{min} \leqslant D(\underline{p}(0),1)\end{array}\right\}$$

Figure 7: myopic firm: case 4

Theorem 3.5 *If* $(\delta, D_{min}) \in m_5$,

$$m_5 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \left| \begin{array}{l} \frac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ \frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_h\right] \\ +D(\overline{p}(0),1)[\overline{p}(0)-c_h]1; \geqslant D_{min} \geqslant D(\underline{p}(0),1) \end{array} \right. \right\},$$

then $p(\theta_h) = G^{-1}(1-D_{min})\,\theta_h, a(\theta_h) = 0$, *for all* $k \in (0, +\infty)$.



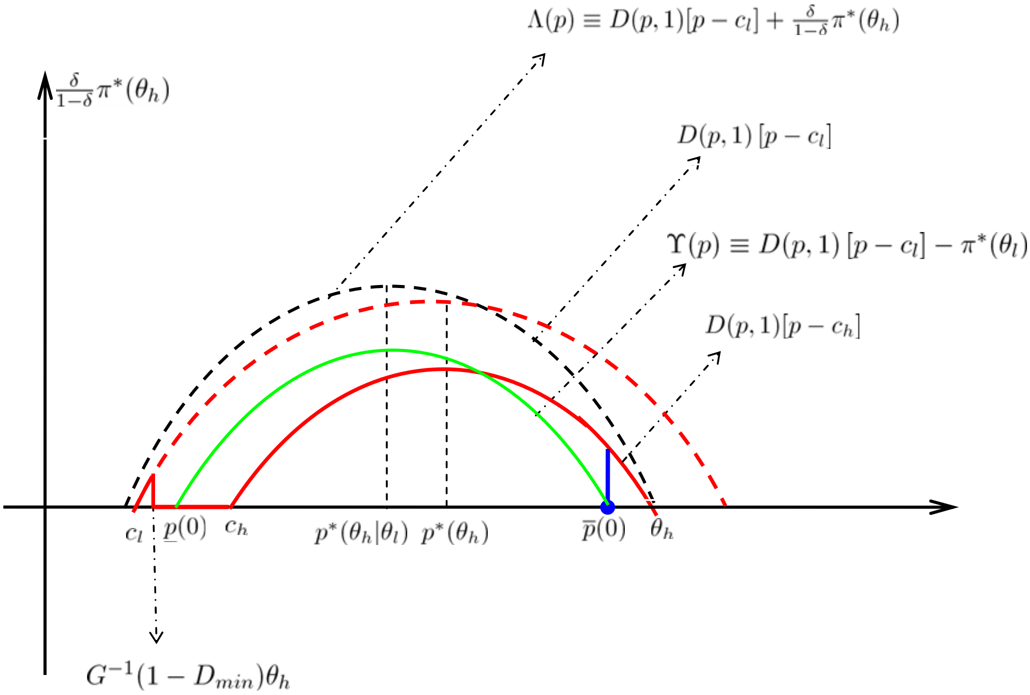
$$[m_5] : \left\{ \begin{array}{l} \frac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ \frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_h\right] + D(\overline{p}(0),1)[\overline{p}(0)-c_h] \\ 1 \geqslant D_{min} \geqslant D(\underline{p}(0),1) \end{array} \right\}$$

Figure 8: myopic firm: case 5

Theorem 3.6 *If* $(\delta, D_{min}) \in m_6$,

$$m_6 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \left| \begin{array}{l} \frac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ \frac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right] \\ 1 \geqslant D_{min} \geqslant D(\underline{p}(0),1) \end{array} \right. \right\}$$

then $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$, *for all* $k \in (0, +\infty)$.



$$[m_6] : \left\{ \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right] \\ \quad 1 \geqslant D_{min} \geqslant D(\underline{p}(0),1) \end{array} \right\}$$

Figure 9: myopic firm: case 6

4 intermediate case

In this section we discuss the intermediate case with $D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \frac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l)$.

Theorem 4.1 *If* $(\delta, D_{min}) \in I_1$,

$$I_1 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \leqslant \max\Big\{ D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l), \\ \left[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right), 1\right)\right. \\ \left. - D(\overline{p}(0),1)\right]\Delta c \Big\} \end{array} \right\}$$

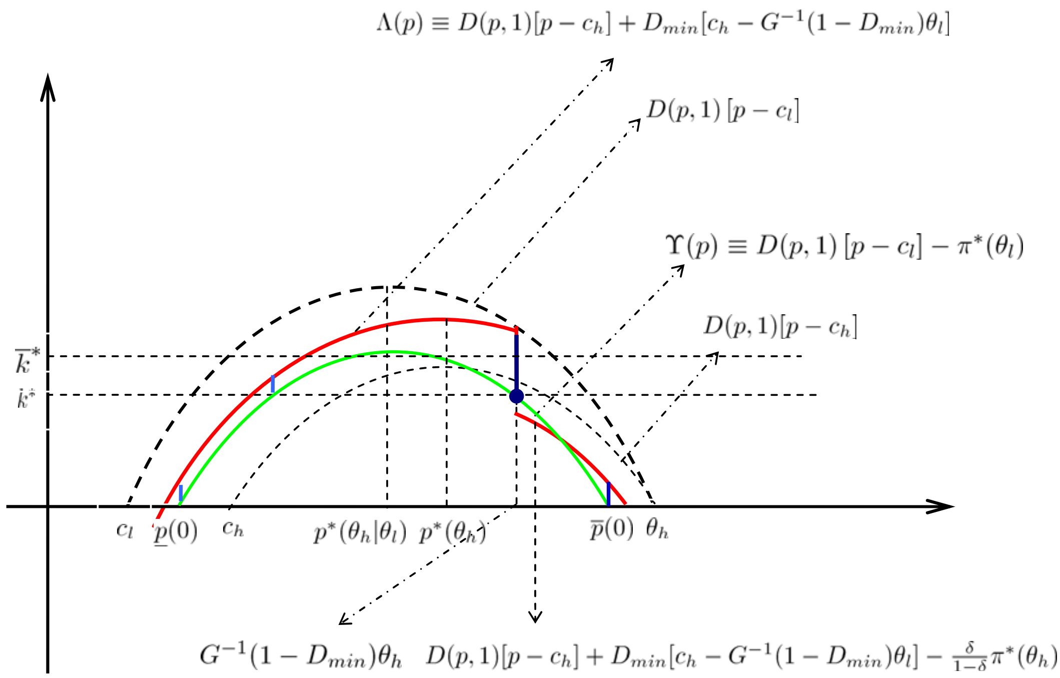
then

1. *when* $k \in [0, k^*)$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$;

2. *when* $k \in [k^*, \overline{k}^*)$,

$$
\begin{aligned}
p(\theta_h) &= \begin{cases} max\{\overline{p}(k), p^*(\theta_h)\} & \text{if} \quad G^{-1}(1-D_{min})\theta_h > p^*(\theta_h) \\ G^{-1}(1-D_{min})\theta_h & \text{if} \quad G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h) \end{cases} \\
a(\theta_h) &= 1;
\end{aligned}
$$

3. *when* $k \in [\overline{k}^*, +\infty)$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$.



$$
[I_1] : \left\{ \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \left[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right), 1\right) - D(\overline{p}(0), 1)\right]\Delta c \end{array} \right\}
$$

Figure 10: intermediate firm: case 1

Theorem 4.2 *If* $(\delta, D_{min}) \in I_2$,

$$
I_2 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l); \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant \\ \left[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right), 1\right)\right. \\ \left. - D(\overline{p}(0), 1)\right]\Delta c \dfrac{\delta}{1-\delta}\pi^*(\theta) \leqslant \left[D(\underline{p}(0), 1) - D(\overline{p}(0), 1)\right]\Delta c \end{array} \right\}
$$

then there exist critical values k^*, $\overline{k}^*$ *and* $\underline{k}^*$ *defined in (9),(10), (11) such that*

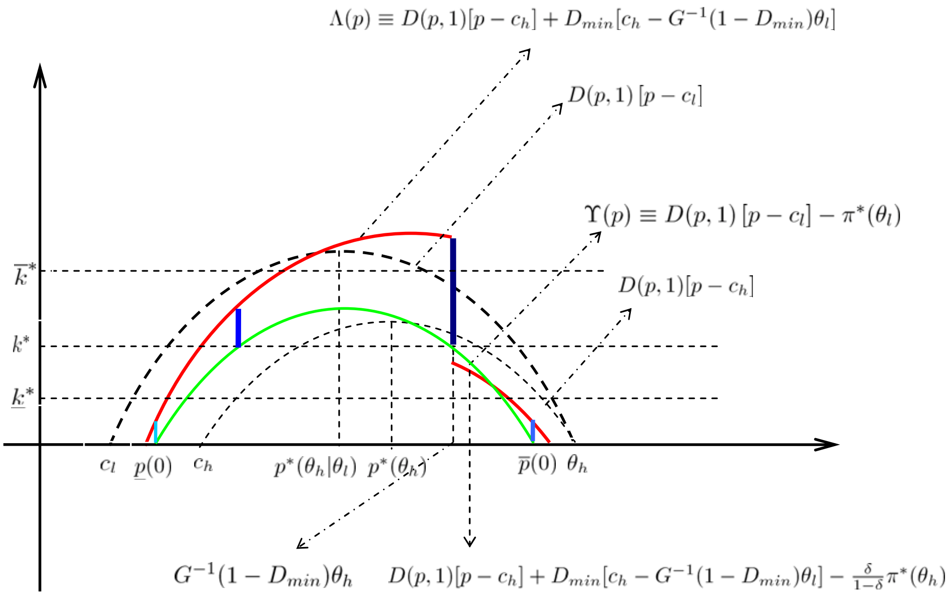
1. *when* $k \in (0, \underline{k}^*)$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$;

2. *when* $k \in [\underline{k}^*, k^*)$, $p(\theta_h) = \underline{p}(k), a(\theta_h) = 1$;

3. *when* $k \in [k^*, \overline{k}^*)$

$$
\begin{aligned}
p(\theta_h) &= \begin{cases} max\{\overline{p}(k), p^*(\theta_h)\} & if \quad G^{-1}(1-D_{min})\theta_h > p^*(\theta_h) \\ G^{-1}(1-D_{min})\theta_h & if \quad G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h) \end{cases} \\
a(\theta_h) &= 1;
\end{aligned}
$$

4. *when* $k \in [\overline{k}^*, +\infty)$, $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$.



$$
[I_2] : \left\{ \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant \left[D\left(\underline{p}\left(D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)\right), 1\right) - D(\overline{p}(0), 1)\right]\Delta c \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant \left[D(\underline{p}(0), 1) - D(\overline{p}(0), 1)\right]\Delta c \end{array} \right\}
$$

Figure 11: intermediate case 2

Theorem 4.3 *If* $(\delta, D_{min}) \in I_3$,

$$
I_3 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l); \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant \left[D(\underline{p}(0), 1) - D(\overline{p}(0), 1)\right]\Delta c \\ D(\overline{p}(0), 1) \leqslant D_{min} \leqslant D(\underline{p}(0), 1) \end{array} \right\}
$$

then

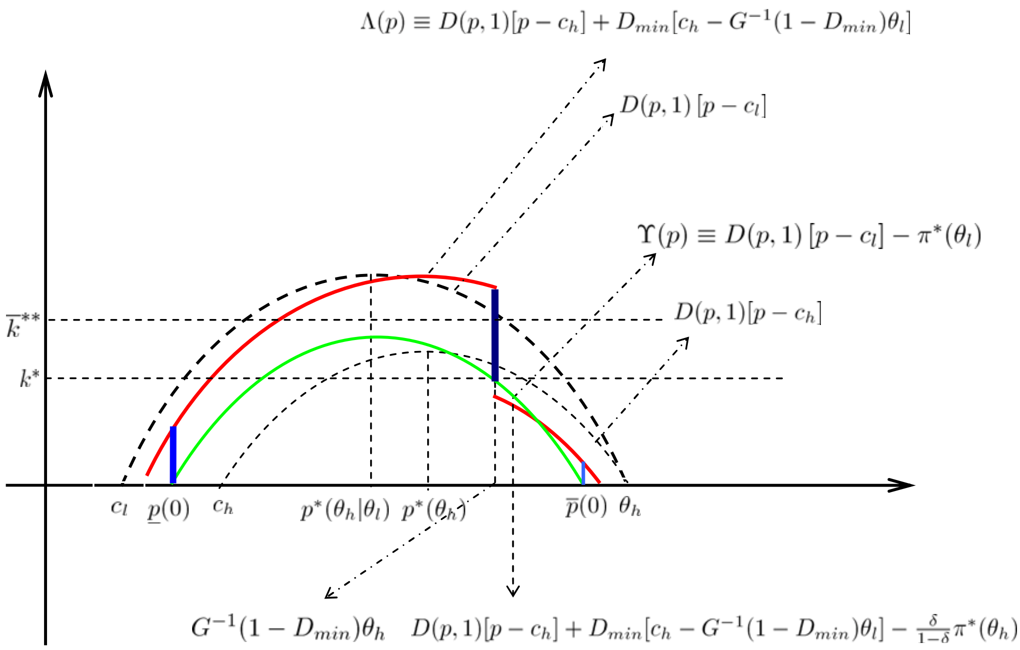
1. *when* $k \in (0, k^*)$, $p(\theta_h) = \underline{p}(k), a(\theta_h) = 1$;

2. *when* $k \in [k^*, \overline{k}^{**})$

$$
\begin{aligned}
p(\theta_h) &= \begin{cases} max\{\overline{p}(k), p^*(\theta_h)\} & if \quad G^{-1}(1-D_{min})\theta_h > p^*(\theta_h) \\ G^{-1}(1-D_{min})\theta_h & if \quad G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h) \end{cases} \\
a(\theta_h) &= 1;
\end{aligned}
$$

3. *when* $k \in [\overline{k}^{**}, +\infty)$, $p(\theta_h) = \underline{p}(0), a(\theta_h) = 0$.

k^* *and* k^{**} *are given by (9) and (12).*



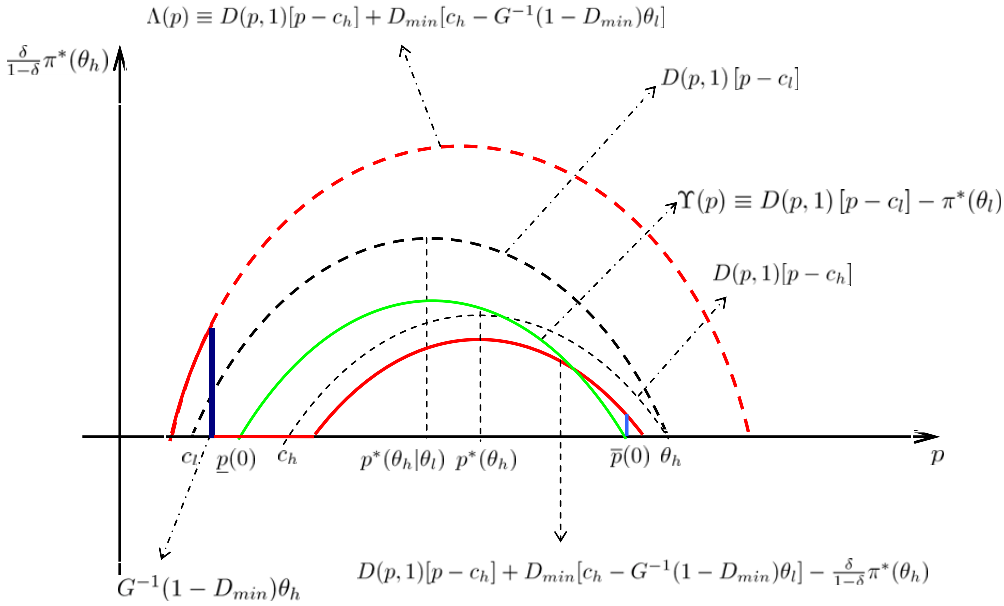
$$
[I3]: \left\{ \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant \left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c \\ D(\overline{p}(0),1) \leqslant D_{min} \leqslant D(\underline{p}(0),1) \end{array} \right\}
$$

Figure 12: intermediate case 3

Theorem 4.4 *If* $(\delta, D_{min}) \in I_4$,

$$
I_4 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \left| \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l); \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_h\right] \\ + D(\overline{p}(0),1)[\overline{p}(0) - c_h]; D(\underline{p}(0),1) \leqslant D_{min} \end{array} \right. \right\}
$$

then $p(\theta_h) = G^{-1}(1-D_{min})\theta_h, a(\theta_h) = 0$ *for all advertising price* $k \in [0, +\infty)$.



$$[I4]: \left\{ \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min}\theta_h)\right] + D(\overline{p}(0),1)[\overline{p}(0) - c_h] \\ D(\underline{p}(0),1) \leqslant D_{min} \end{array} \right\}$$

Figure 13: intermediate case 5

Theorem 4.5 *If*

$$I_5 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l); \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min}\theta_h)\right] \\ + D(\overline{p}(0),1)[\overline{p}(0) - c_h]; D(\underline{p}(0),1) \leqslant D_{min} \end{array} \right\}$$

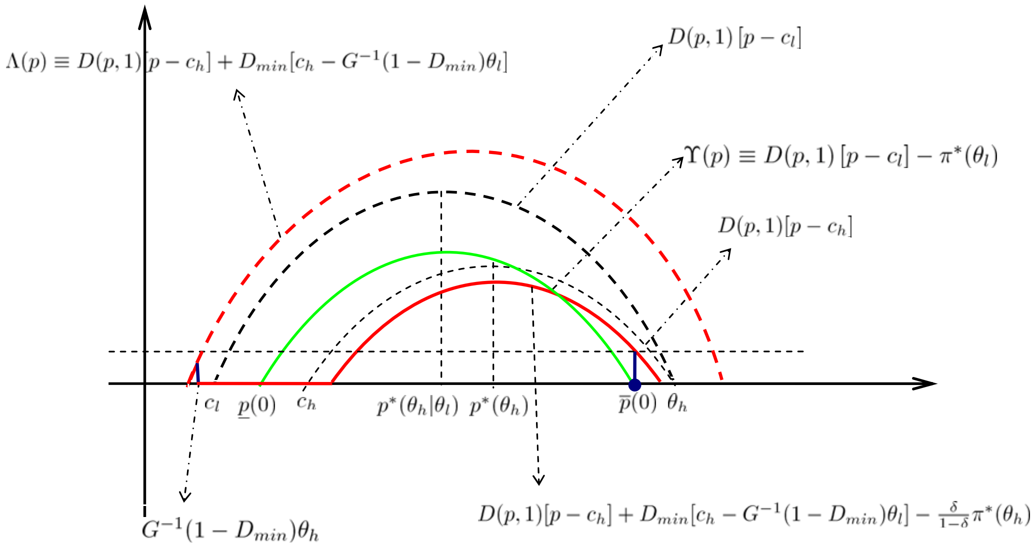
then $p(\theta_h) = \overline{p}(0), a(\theta_h) = 0$ *for all advertising price* $k \in [0, +\infty)$.

5 farsighted case

We now discuss the case with $\frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l)$. In this case,

$$\mathcal{S}(x) = \left\{ p \in \mathbb{R}_+ \;\middle|\; \begin{array}{l} D(p,1)(p-c_h) + \mathbb{1}[D(p,1) \geqslant D_{min}]\dfrac{\delta}{1-\delta}\pi^*(\theta_h) \\ - \dfrac{\delta}{1-\delta}\pi^*(\theta_h) + D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_h\right] \\ \geqslant x \geqslant D(p,1)(p-c_l) - \pi^*(\theta_l) \end{array} \right\}. \tag{13}$$

It is obvious that $\mathcal{S}(x) \cap [G^{-1}(1-D_{min})\theta_h, +\infty) = \emptyset$ for every $x \in \mathbb{R}_+$. So the high-quality firm will by no means charge an introductory price higher than $G^{-1}(1-D_{min})\theta_h$.



$$[I5]: \left\{ \begin{array}{l} D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \leqslant \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \leqslant D_{min}\left[c_h - G^{-1}(1-D_{min}\theta_h)\right] + D(\overline{p}(0),1)[\overline{p}(0)-c_h] \\ D(\underline{p}(0),1) \leqslant D_{min} \end{array} \right\}$$

Figure 14: intermediate case 5

Notice that the curve $\Gamma(p) \equiv D(p,1)[p-c_h] + D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$ will be shifted upwards as the minimum demand D_{min} increases, and thus the high-quality firm's "no-defect" condition (5) are easier to be satisfied. That is because when a high-quality firm deviates from the equilibrium and thus be perceived as his low-quality counterpart, he needs to reduce the price charged and thus suffers a loss at the first period to attract repeat purchases. The higher the minimum demand, the greater will be his price reduction, and thus the greater profit loss he will suffer. In order to guarantee the existence of separating PBE for all advertising price, we assume that when the minimum demand D_{min} are such that the highest possible price he could charge is at most $\overline{p}(0)$, $\Gamma(p) \equiv D(p,1)[p-c_h] + D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$ is above $\Upsilon(p) \equiv D(p,1)[p-c_l] - \pi^*(\theta_l)$ in the first quadrant. When $D(\overline{p}(0),1) = D_{min}$, $\Gamma(p) - \Upsilon(p) = D(\overline{p}(0),1)\left[c_h - \overline{p}(0)\frac{\theta_l}{\theta_h}\right] + \pi^*(\theta_l) - D(p,1)\Delta c$. Because $\Gamma(p) - \Upsilon(p)$ is increasing in p, $\Gamma(p) \geqslant \Upsilon(p)$ for all $p \geqslant \underline{p}(0)$ if $\Gamma(\underline{p}(0)) \geqslant \Upsilon(\underline{p}(0))$. We thus requires the following condition, which we maintain in this section.

Assumption 2

$$D(\overline{p}(0),1)\left[c_h - \overline{p}(0)\frac{\theta_l}{\theta_h}\right] \geqslant D(\underline{p}(0),1)[c_h - \underline{p}(0)]. \tag{14}$$

Figure (5) illustrates the pricing and advertising decisions in a separating equilibrium satisfying the intuitive criterion in the case with $0 \leqslant D_{min} \leqslant D(\overline{p}(0),1)$. For a given k, the segments of horizonal axis and line k lying between $\Gamma(p)$ and $\Upsilon(p)$ represent the set of prices the firm may charge in separating equilibrium, while the price with highest $\Gamma(p)$

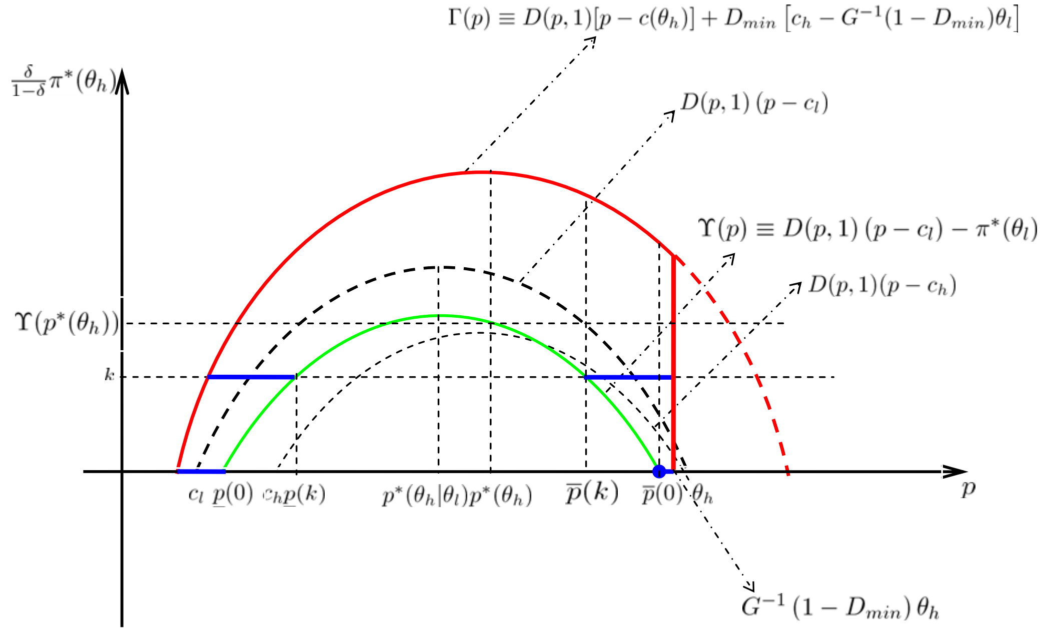


Figure 15: The case with farsighted firm and $0 \leqslant D_{min} \leqslant D(\overline{p}(0), 1)$

may be the price surviving the elimination of intuitive criterion. Advertising functions as a complementary signal of price, higher advertising price enables the high-quality firm to charge a less distorted price. If a high-quality firm choose to advertise, he must separate with $\overline{p}(k)$ instead of a lower price $\underline{p}(k)$ when k is small, i.e., $k < \Upsilon(p^*(\theta_h))$, since the former is closer to the full-information price $p^*(\theta_h)$ than the latter one; while for sufficiently high advertising price, i.e., $k \geqslant \Upsilon(p^*(\theta_h))$, he may charge his full-information price $p^*(\theta_h)$. If he choose not to advertise, however, he may charge a higher price $\overline{p}(0)$. A high-quality firm both gains and loses when he advertises. He gains by charging a less distorted price and thus increasing profit; meanwhile he loses because a dissipative expenditure k has to be paid. Because high-quality production is costly $c_h > c_l$, the high price is the efficient means of separation because the forgone profit is less than the advertising price. Thus, the high-quality firm prefers to separate with the high price and zero advertising choice $(\overline{p}(0), 0)$. If, however, the minimum demand $D_{min} > D(\overline{p}(0), 1)$, high prices may restrict sales and thereby deprive the high-quality firm of the future profits, then the high-quality product is introduced at a low price and rise to a higher price in the mature phase.

Theorem 5.1 *Suppose that* $(\delta, D_{min}) \in F_1$,

$$F_1 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} \frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l), \\ D(\overline{p}(0), 1) \leqslant D_{min} < D(p^*(\theta_h), 1) \end{array} \right\}$$

then the optimal strategies and payoffs of a high-quality firm in an intuitive separating equilibrium dependent on the advertising price k:

1. *if* $0 < k < \Upsilon\left(G^{-1}(1-D_{min})\theta_h\right)$, *then* $p(\theta_h) = \underline{p}(k), a(\theta_h) = 1, \Pi(\theta_h) = D(\underline{p}(k), 1)[\underline{p}(k) -$

$c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - k$;

2. *if* $\Upsilon\left(G^{-1}(1-D_{min})\theta_h\right) \leqslant k < \Upsilon(p^*(\theta_h))$, *then* $p(\theta_h) = \overline{p}(k), a(\theta_h) = 1$, $\Pi(\theta_h) = D(\overline{p}(k),1)[\overline{p}(k) - c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - k$;

3. *if* $\Upsilon(p^*(\theta_h)) \leqslant k \leqslant \hat{k} \equiv \pi^*(\theta_h) - D(\underline{p}(0),1)\left[\underline{p}(0) - c_h\right]$, *then* $p(\theta_h) = p^*(\theta_h), a(\theta_h) = 1$, $\Pi(\theta_h) = \frac{\pi^*(\theta_h)}{1-\delta} - k$;

4. *if* $k > \hat{k}$, *then* $p(\theta_h) = \underline{p}(0), a(\theta_h) = 0$, $\Pi(\theta_h) = D(\underline{p}(0),1)[\underline{p}(0) - c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h)$.

Proof. When $D_{min} \in [D(\overline{p}(0),1), D(p^*(\theta_h),1))$,

$$\begin{aligned}
&\underset{p\in\mathcal{S}(k)}{max}\{D(p,1)[p-c_h]-k\} \\
&= \begin{cases} D(\underline{p}(k),1)[\underline{p}(k)-c_h]-k & \text{if} \quad 0<k<\Upsilon\left(G^{-1}(1-D_{min})\theta_h\right) \\ D(\overline{p}(k),1)[\overline{p}(k)-c_h]-k & \text{if} \quad \Upsilon\left(G^{-1}(1-D_{min})\theta_h\right) \leqslant k < \Upsilon(p^*(\theta_h)) \\ \pi^*(\theta_h)-k & \text{if} \quad k \geqslant \Upsilon(p^*(\theta_h)) \end{cases} \qquad (15) \\
&\underset{p\in\mathcal{S}(0)}{max}\{D(p,1)[p-c_h]\} = D(\underline{p}(0),1)[\underline{p}(0)-c_h]. \qquad (16)
\end{aligned}$$

We get immediately the optimal choices and corresponding profits of high-quality firm. ■

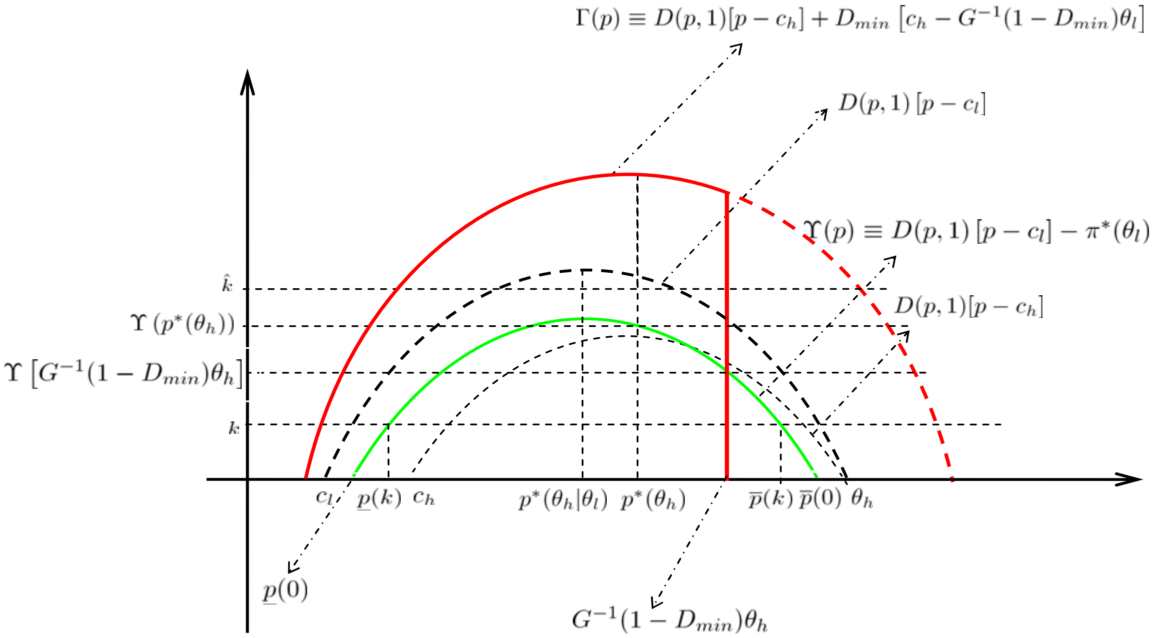
When the advertising price is low, its signaling function might be weak, the high-quality firm has to distinguish himself with an extremely high or low price which are never profitable for the low-quality firm. In the case without minimum demand, and thus the sales during mature phase is independent of sales during introductory phase, the high-quality firm might prefer to choose a high price since it signals its quality and compensates its higher production cost as well. In the present paper, the high-quality product attracts repeated purchases if and only if the volume of introductory sales exceeds certain threshold value. In this case, high prices which discourage sales become a less attractive method of signaling high quality. A farsighted high-quality firm might instead prefer to signal with a low price if the advertising price is low (i.e., $k \in (0, \Upsilon\left(G^{-1}(1-D_{min})\theta_h\right))$, in which case price would tend to rise over time; he may still signal with a high and declining price for an intermediate advertising price (i.e., $k \in [\Upsilon\left(G^{-1}(1-D_{min})\theta_h\right), \Upsilon(p^*(\theta_h)))$; when the advertising price exceeds $\Upsilon(p^*(\theta_h))$, advertising itself may convey enough information about product quality, the firm thus needs not to signal its quality with a distorted price; when its price goes beyond a critical value $\hat{k}$, advertising may become too expensive to be adopted by the high-quality firm.

Theorem 5.2 *Suppose that* $(\delta, D_{min}) \in F_2$,

$$F_2 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l), \\ D(p^*(\theta_h),1) \leqslant D_{min} < D(\underline{p}(0),1) \end{array} \right\}$$

then the optimal strategies and payoffs are given as follows:

1. *if* $k \in \left(0, \Upsilon\left(G^{-1}(1-D_{min})\theta_h\right)\right)$, *then* $p(\theta_h) = \underline{p}(k), a(\theta_h) = 1$, $\Pi(\theta_h) = D(\underline{p}(k),1)[\underline{p}(k) - c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - k$;



$$F_1 : \left\{ \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ p^*(\theta_h) \leqslant G^{-1}(1-D_{min})\theta_h \leqslant \overline{p}(0) \end{array} \right\}$$

Figure 16: The farsighted case with $D(\bar{p}(0),1) < D_{min} \leqslant D(p^*(\theta_h),1)$.

2. *if* $k \in \left[\Upsilon\left(G^{-1}(1-D_{min})\theta_h\right), \tilde{k}\right)$*, then* $p(\theta_h) = G^{-1}(1-D_{min})\theta_h, a(\theta_h) = 1$, $\Pi(\theta_h) = D_{min}[G^{-1}(1-D_{min})\theta_h - c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - k$;
3. *if* $k \in [\tilde{k}, \infty)$*, then* $p(\theta_h) = \underline{p}(0), a(\theta_h) = 0$, $\Pi(\theta_h) = D(\underline{p}(0),1)[\underline{p}(0) - c_h] + \frac{\delta}{1-\delta}\pi^*(\theta_h)$.

where

$$\tilde{k} = D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_h\right] - D(\underline{p}(0),1)[\underline{p}(0) - c_h]. \tag{17}$$

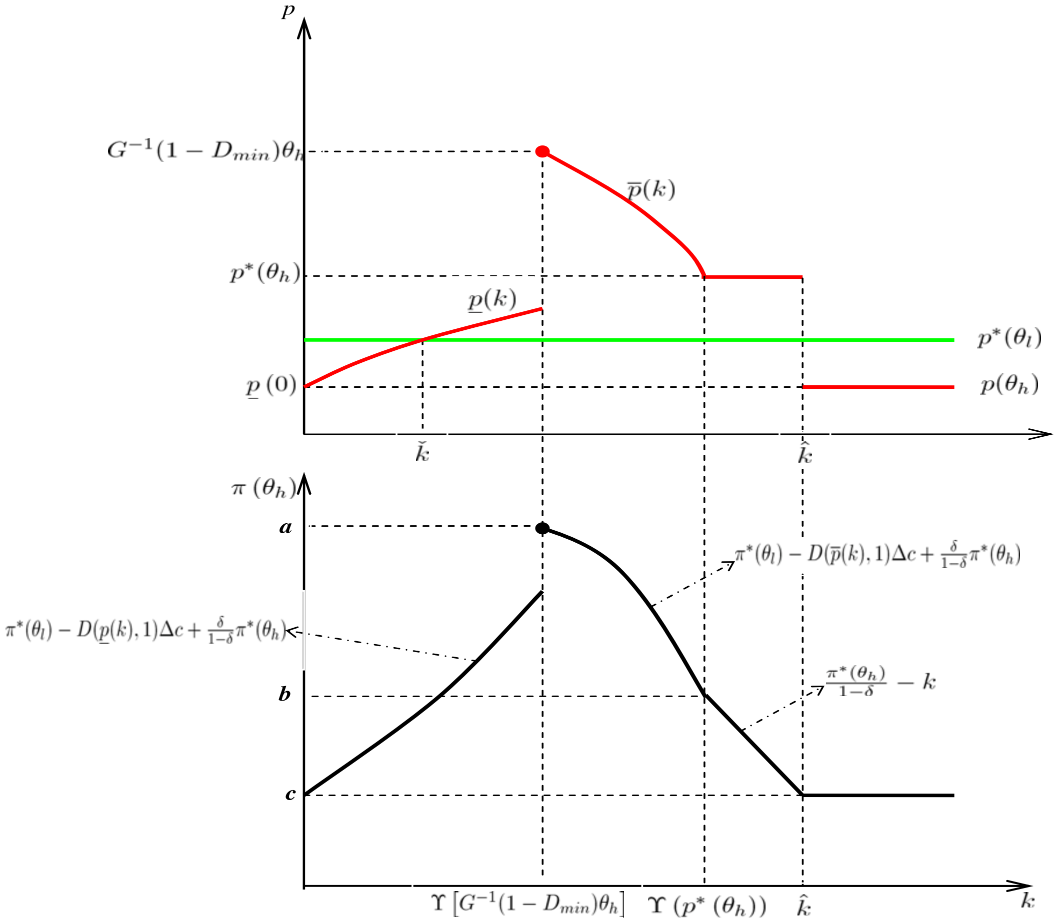
Theorem 5.3 *Suppose that* $(\delta, D_{min}) \in F_3$*,*

$$F_3 \equiv \left\{ (\delta, D_{min}) \in [0,1]^2 \;\middle|\; \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] \\ +\,\pi^*(\theta_l); D(\underline{p}(0),1) < D_{min} \leqslant 1 \end{array} \right\}$$

then $p(\theta_h) = G^{-1}(1-D_{min})\theta_h$*,* $a(\theta_h) = 0$*,* $\Pi(\theta_h) = D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_h\right] + \frac{\delta}{1-\delta}\pi^*(\theta_h)$.

Theorem 5.4 *The price-quality relationships in the introductory phase are:*

1. *if* $D_{min} \in [0, D(\overline{p}(0),1)]$*, then* $p(\theta_h) > p(\theta_l)$*;*
2. *if* $D_{min} \in (D(\overline{p}(0),1), D(p^*(\theta_l),1))$*, then there exist critical values* $\breve{k}$ *and* $\hat{k}$*, such that the high quality firm will charge a price higher than its lower counterpart, e.g.,* $p(\theta_h) > p(\theta_l)$ *if and only if* $k \in (\hat{k}, \breve{k},)$*;*
3. *if* $D_{min} \geqslant D(p^*(\theta_l),1)$*, then* $p(\theta_h) < p(\theta_l)$*.*



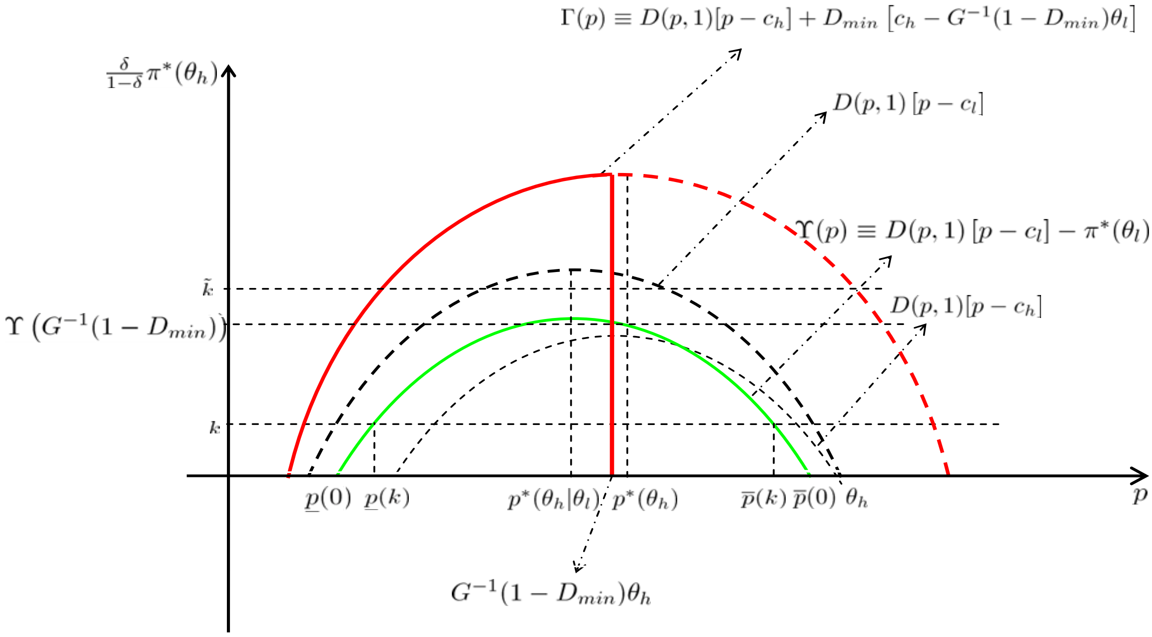
$$
\begin{aligned}
a &= D_{\min}\left[G^{-1}(1-D_{min})\theta_h - c_h\right] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - \Upsilon\left(G^{-1}(1-D_{min})\theta_h\right) \\
b &= \frac{\pi^*(\theta_h)}{1\text{-}\delta} + \pi^*(\theta_l) - D\left(p^*(\theta_h)\,1\right)\Delta c \\
c &= D(\underline{p}(0),1)[\underline{p}(0) - c(\theta_h)] + \frac{\delta}{1-\delta}\pi^*(\theta_h)
\end{aligned}
$$

Figure 17: Price and payoff in the case F_1.

6 Conclusion

We analysis the pricing strategy of an experience good with minimum demand for a monopolist firm in the setting of repeated sales. we find that

1. If there is no minimum demand and the advertising is not free, the high-quality firm will distort upward his introductory price; while if there is no minimum demand and advertising is free, the high-quality firm will distort his introductory price downward.
2. If the minimum demand is not to high, the high-quality firm's introductory price is not a monotonic function of the advertising price k. It increases when k is relative small, then decreases when k is larger than some critical value.
3. If the minimum demand is high enough, the high-quality firm's introductory price is

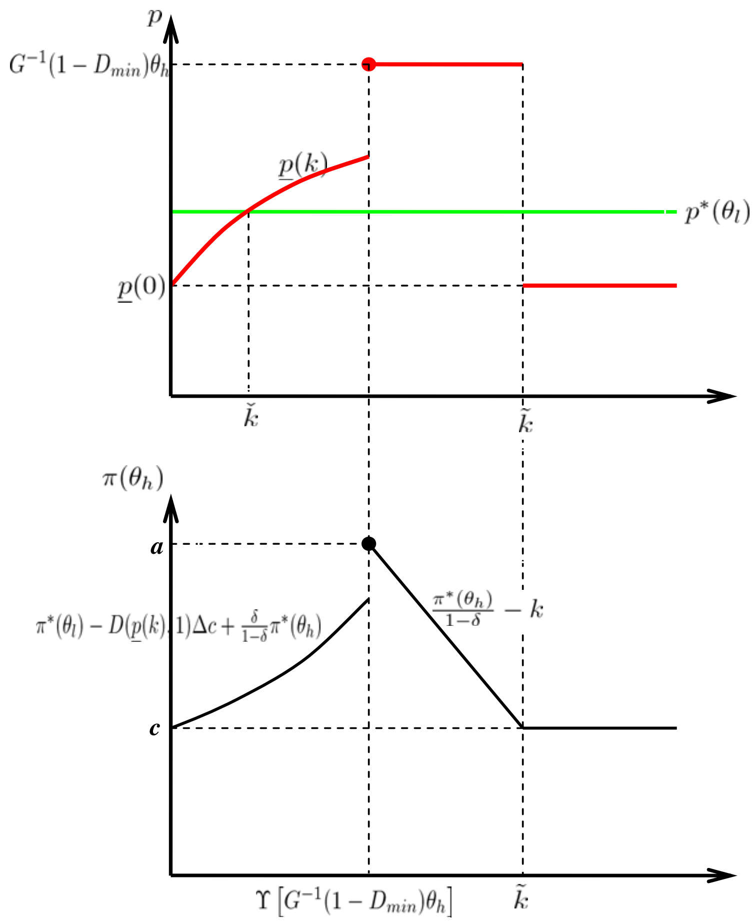


$$F_2 - 1 : \left\{ \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ p^*(\theta_h|\theta_l) \leqslant G^{-1}(1-D_{min})\theta_h \leqslant p^*(\theta_h) \end{array} \right\}$$

Figure 18: The farsighted case with $D(p^*(\theta_h),1) < D_{min} \leqslant D(p^*(\theta_h|\theta_l),1)$

not affected by the advertising price.

4. There exist no strong and positive price-quality correlations and advertising-quality correlations, these relations both depend on the advertising price and minimum demand.



$$
\begin{aligned}
a &= D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_h\right] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - \Upsilon\left[G^{-1}(1-D_{min})\theta_h\right] \\
c &= D(\underline{p}(0),1)[\underline{p}(0) - c(\theta_h)] + \frac{\delta}{1-\delta}\pi^*(\theta_h)
\end{aligned}
$$

Figure 19: Price and profit of high-quality firm in the case $F_2 - 1$: $\frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l); D(p^*(\theta_h),1) < D_{min} \leqslant D(p^*(\theta_h|\theta_l),1)$.

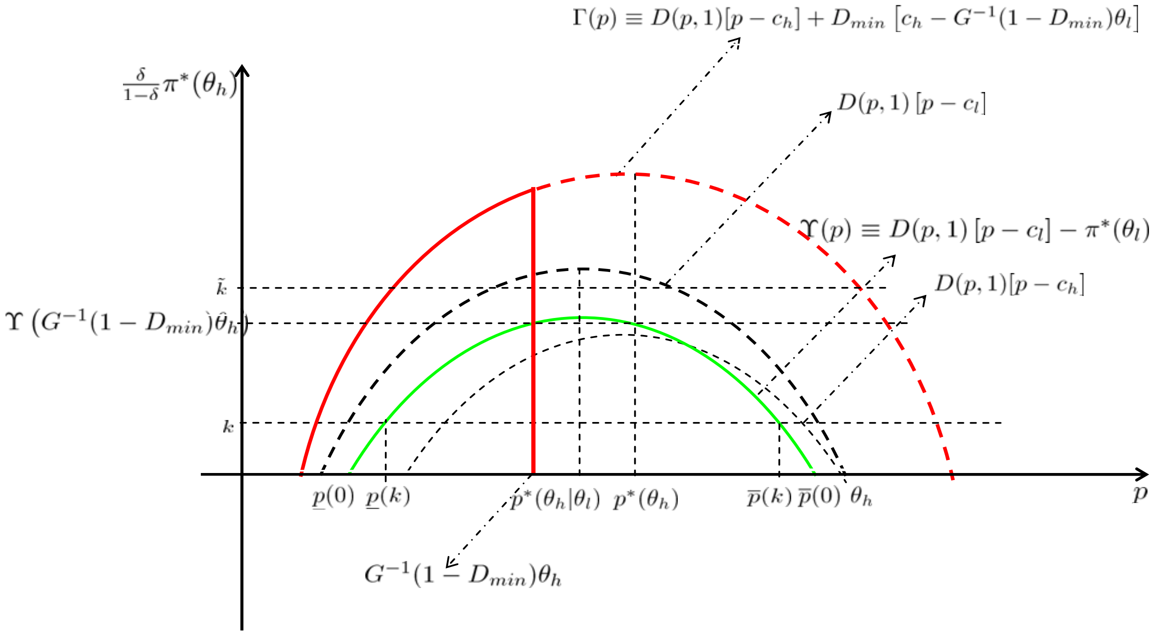
References

[1] Archibald, R., Haulman, C., Moody, C., 1983. Quality, price, advertising, and published quality ratings. Journal of Consumer Research 9 (4), 347-356.

[2] Bagwell, K., 1991. Optimal export policy for a new-product monopoly. American Economic Review. 81, 1156-1169.

[3] Bagwell, K., Riordan, M., 1991. High and declining prices signal product quality. American Economic Review 81 (1), 224-239.

[4] Cho, I.K., Kreps, D.M., 1987. Signaling games and stable equilibria. Quarterly Journal of Economics. 102, 179-221.



$$F_2 - 2: \left\{ \begin{array}{l} \frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l) \\ \underline{p}(0) \leqslant G^{-1}(1-D_{min}) \leqslant p^*(\theta_h|\theta_l) \end{array} \right\}$$

Figure 20: The farsighted case with $D(p^*(\theta_h|\theta_l),1) < D_{min} \leqslant D(\underline{p}(0),1)$

[5] Cho, I.K., Sobel, J., 1990. Strategic stability and uniqueness in signaling games. Journal of Economic Theory. 50, 381-413.

[6] Curry, D., Riesz, P., 1988. Prices and price / quality relationship: a longitudinal analysis. Journal of Marketing 52, 36-51.

[7] Ellingsen, T., 1997. Price signals quality: the case of perfectly inelastic demand. International Journal of Industrial Organization. 16, 43-61.

[8] Farrell, J. (1980), "Prices as Signals of Quality," Ph.D. thesis, University of Oxford.

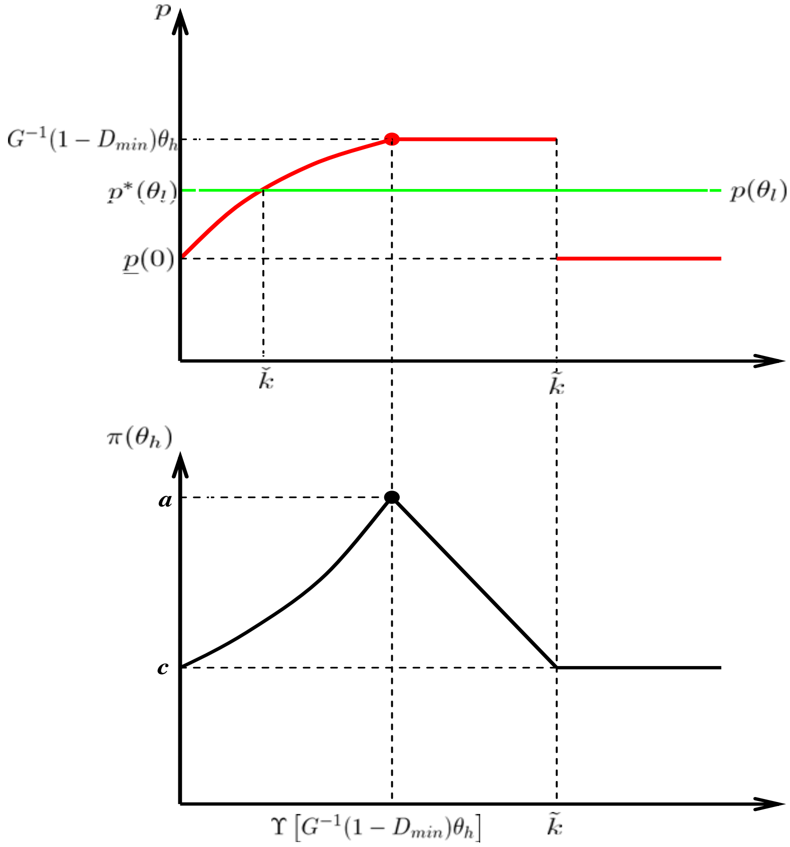
[9] Gabor,A . andC . W. J. Granger(1966), "Pricea s an Indicator of Quality: Report on an Inquiry," Economica, 33, 43-70.

[10] Geistfeld,L orenV . (1982), "The Price-QualityR elationship -Revisited," Journalo f ConsumeAr ffairs,1 6 (Winter)3, 34- 5.

[11] Horstmann, I., MacDonald, G., 1994. When is advertising a signal of product quality? Journal of Economics & Management Strategy 3 (3), 561-584.

[12] Horstmann, I., MacDonald, G., 1995. Is advertising a signal of product quality? Evidence from the compact disc player market. University of Western Ontario, Department of Economics Research Report, 9523, Revised Version 1999.

[13] Kihlstrom, R., Riordan, M., 1984. Advertising as a signal. Journal of Political Economy 92 (3), 427-450.



$$
\begin{aligned}
a &= D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_h\right] + \frac{\delta}{1-\delta}\pi^*(\theta_h) - \Upsilon\left[G^{-1}(1-D_{min})\theta_h\right] \\
c &= D(\underline{p}(0),1)[\underline{p}(0) - c(\theta_h)] + \frac{\delta}{1-\delta}\pi^*(\theta_h)
\end{aligned}
$$

Figure 21: Price and profit of high-quality firm in the case $F_2 - 2$: $\frac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l); D(p^*(\theta_h|\theta_l),1) < D_{min} \leqslant D(\underline{p}(0),1)$.

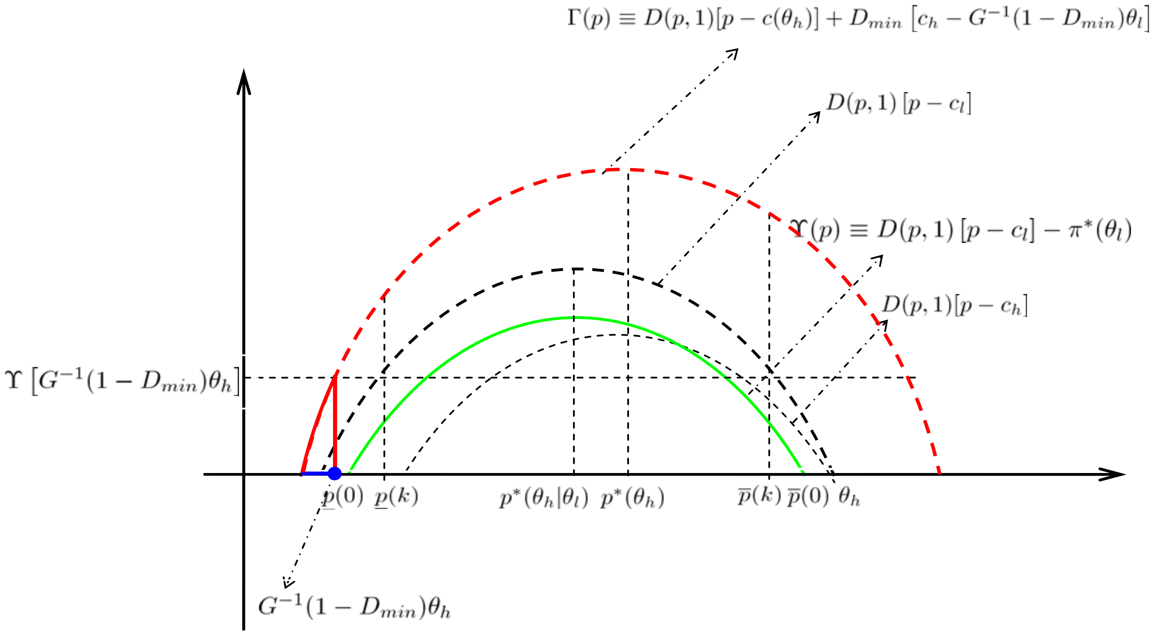
[14] Leavitt, Harold J. (1954), "A Note on Some Experimental Findings About the Meaning of Price," Journal of Business, 27 (July), 205-10.

[15] Linnemer, L., 1998. Entry deterrence, product quality: price and advertising as signals. Journal of Economics & Management Strategy 7 (4), 615-645.

[16] Milgrom, P., Roberts, J., 1986. Price and advertising signals of product quality. Journal of Political Economy 94 (4), 796-821.

[17] Nelson, P., 1974. Advertising as information. Journal of Political Economy 82 (4), 729-754. Schmalensee, R., 1978. A model of advertising and product quality. Journal of Political Economy 86 (3), 485-503.

[18] Schmalensee, R., 1978. A model of advertising and market structure. Journal of Political Economy 86, 485-503.



$$F_3 : \left\{ \begin{array}{l} \dfrac{\delta}{1-\delta}\pi^*(\theta_h) \geqslant D_{min}\left[c_h - G^{-1}(1 - D_{min})\theta_l\right] + \pi^*(\theta_l) \\ G^{-1}(1 - D_{min}) \leqslant \underline{p}(0) \end{array} \right\}$$

Figure 22: The farsighted case with $D_{min} \geqslant D(\underline{p}(0), 1)$.

[19] Scitovsky, T. (1944), "Some Consequences of the Habit of JudgingQ ualityb y Price,"R eviewo f EconomicS tudies, 12, 100-5.

[20] Spence,M . (1974), MarketS ignalling.C ambridgeM, A: Harvard University Press.

[21] Thomas, L., Shane, S., Weigelt, K., 1998. An empirical examination of advertising as a signal of product quality. Journal of Economic Behavior & Organization 37 (4), 415-430.

[22] Wiggins, Steven N. and W. J. Lane (1983), "Quality Uncertainty, Search, and Advertising,"A mericanE conomicR eview (December), 881-9.

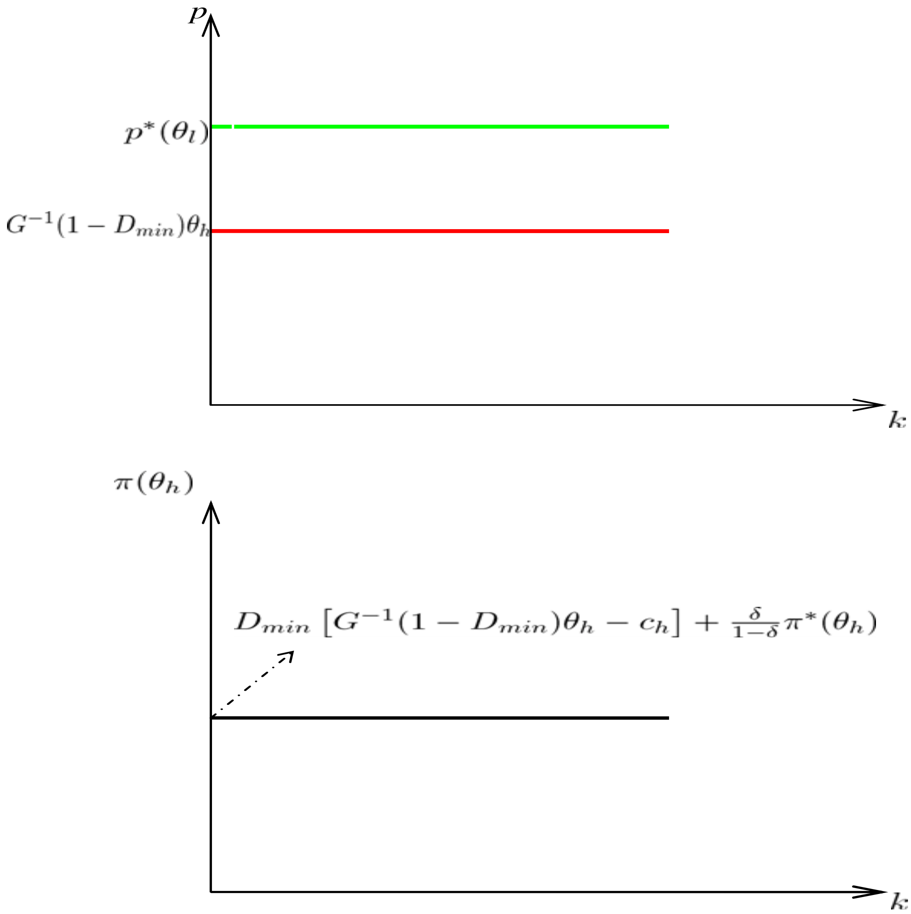


Figure 23: the high-quality firm's price and profit in case F_3.

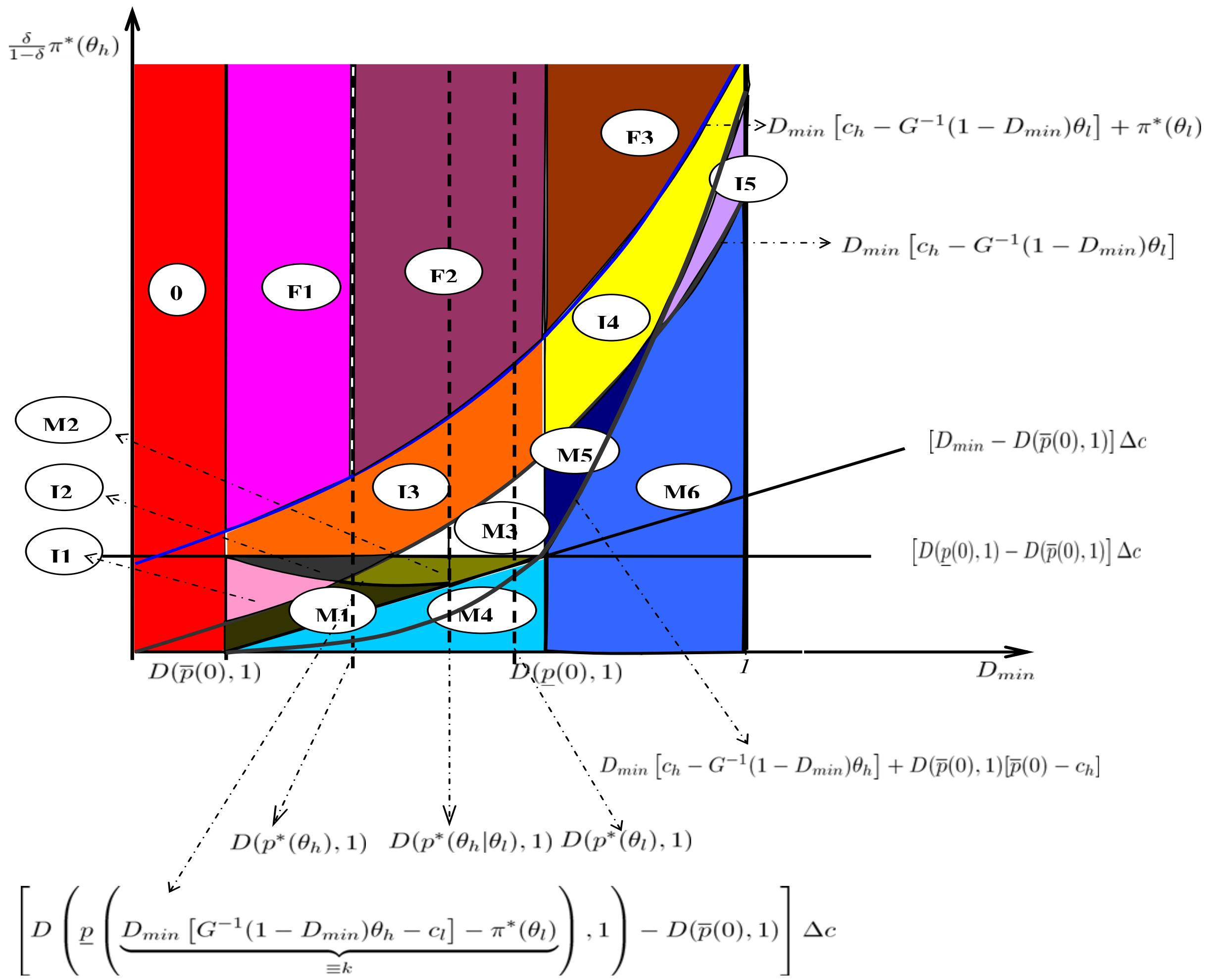

$\frac{\delta}{1-\delta}\pi^*(\theta_h)$
0
F1
F2
F3
I1
I2
I3
I4
I5
M1
M2
M3
M4
M5
M6
$D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right] + \pi^*(\theta_l)$
$D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_l\right]$
$\left[D_{min} - D(\overline{p}(0),1)\right]\Delta c$
$\left[D(\underline{p}(0),1) - D(\overline{p}(0),1)\right]\Delta c$
$D(\overline{p}(0),1)$
$D(\underline{p}(0),1)$
1
D_{min}
$D_{min}\left[c_h - G^{-1}(1-D_{min})\theta_h\right] + D(\overline{p}(0),1)[\overline{p}(0)-c_h]$
$D(p^*(\theta_h),1)$
$D(p^*(\theta_h|\theta_l),1)$
$D(p^*(\theta_l),1)$
$\left[D\left(\underline{p}\left(\underbrace{D_{min}\left[G^{-1}(1-D_{min})\theta_h - c_l\right] - \pi^*(\theta_l)}_{\equiv k}\right),1\right) - D(\overline{p}(0),1)\right]\Delta c$

Figure 24:

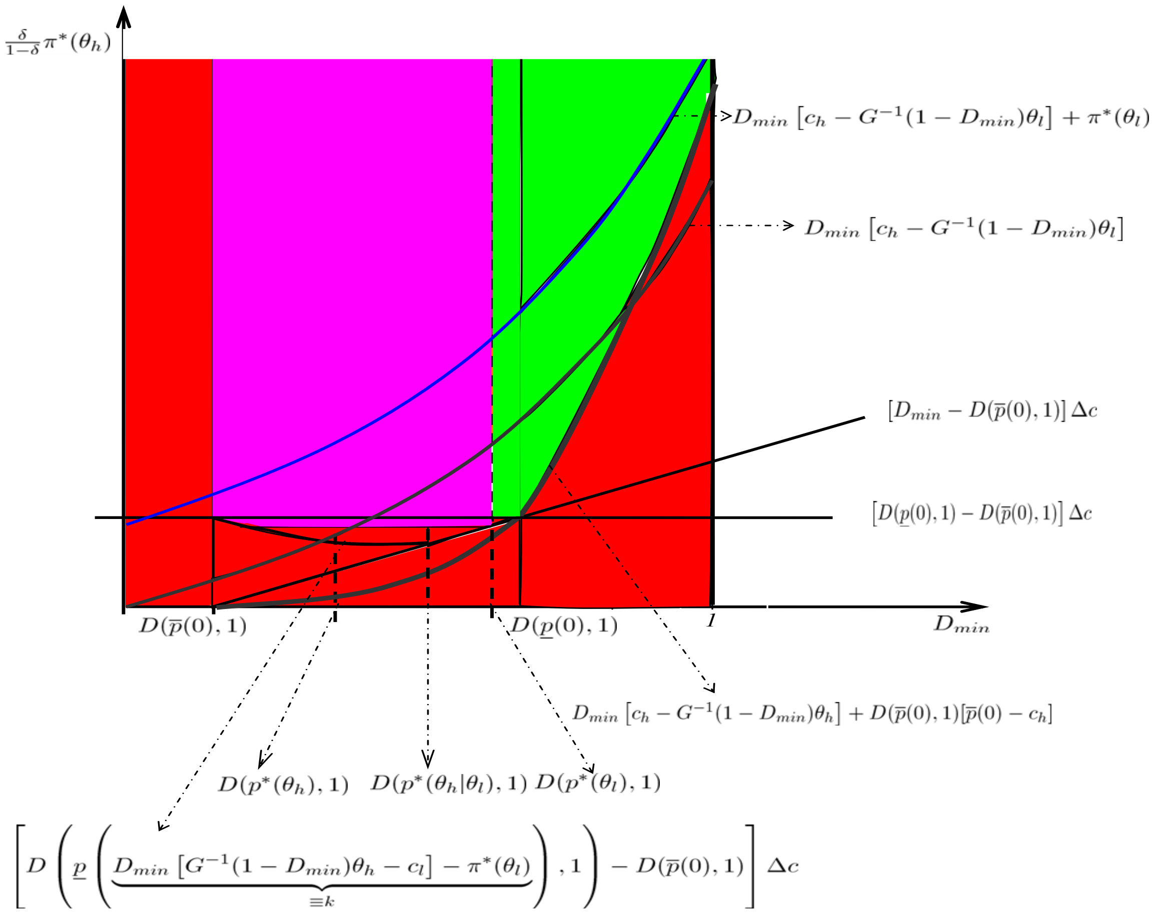


Figure 25: The regions of price-quality relationship

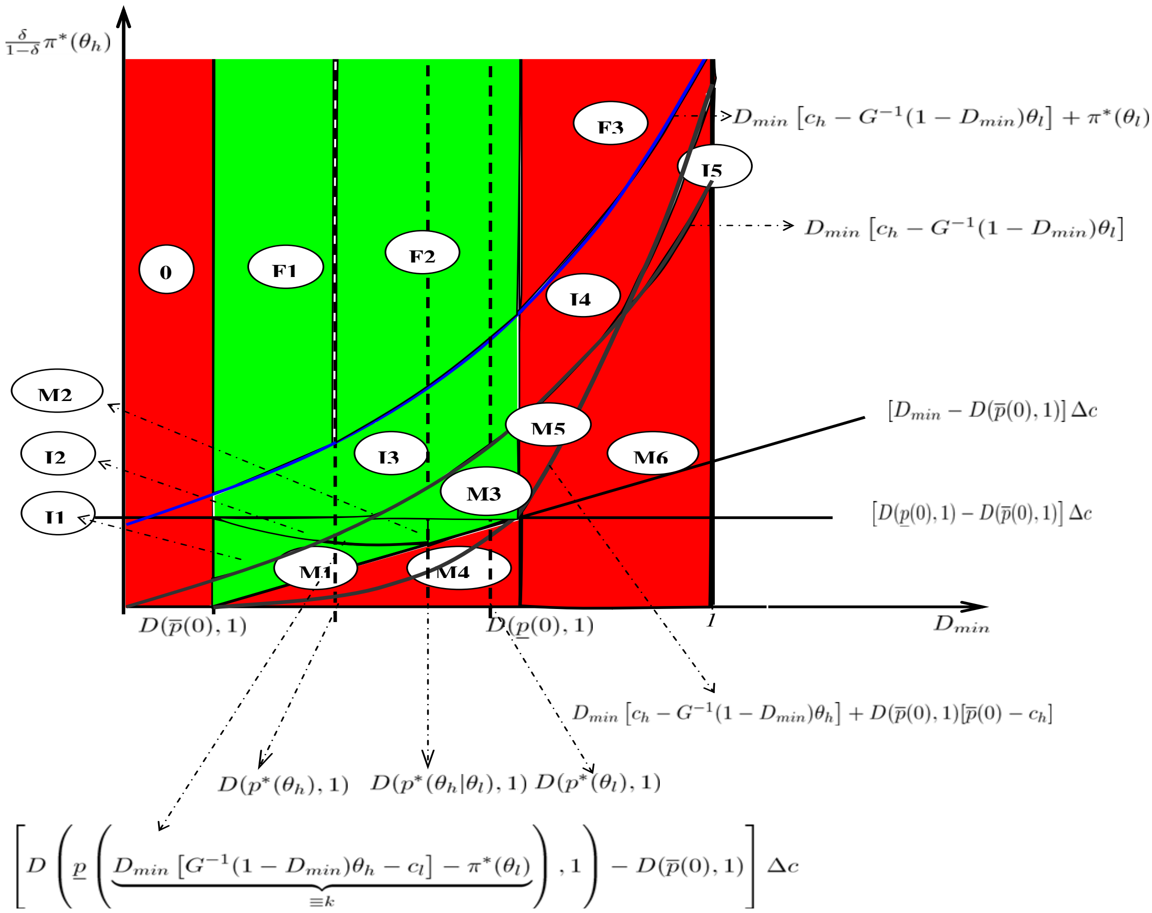


Figure 26: The advertising regions