Auction design with endogenously correlated buyer types

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January 21, 2011

Abstract

This paper studies optimal auction design when the seller can affect the buyers' valuations through an unobservable ex ante investment. The key insight is that the optimal mechanism may have the seller play a mixed investment strategy so as to create correlation between the otherwise (conditionally) independent valuations of buyers. Under the assumption that the seller announces the mechanism before investing, the paper establishes conditions on the investment technology so that a mechanism exists which leaves buyers no information rent and leaves the seller indifferent between his investments. Under these conditions, the seller can, in fact, extract the first best surplus almost fully.

Keywords: Auction, ex ante investment, full surplus extraction, correlation, mechanism design.

JEL Classification No.: C72, D42, D44, D82

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1 Introduction

In many situations a seller can affect buyers' valuations by an unobservable ex ante investment in the object for sale. For example, buyers' valuations for real estate will depend on the effort spent by the construction company. The company's effort, such as work care or the quality of materials, can typically not be directly observed or immediately experienced. In public procurement, the contractor's cost often depend on the cooperation by the government agency, for instance through the number of staff provided to deal with problems that occur during the implementation phase, or on complementary infrastructure investments. Further, in second hand markets, buyers' valuations are affected by the unobservable care with which initial owners have treated the item they sell. A final example is persuasive advertising where the typically unobservable campaign expenditures influence buyers' willingness to pay.

What is the revenue maximizing selling mechanism in such a setting? This is the question I address in this paper. I study a private value environment in which the seller's investment raises buyers' valuations stochastically. Conditional on the seller's investment, valuations are conditionally independent. I assume that the seller's investment is unobservable for buyers and that buyers' valuations are their private information. Consequently, if the seller adopts a pure investment strategy, there are independent private values, and buyers can typically secure information rents.

The purpose of this paper is to show that the seller can reduce information rents by designing a mechanism that induces him to adopt a mixed investment strategy. The key insight is that if the seller randomizes, then, because buyers cannot observe investment, their valuations become correlated in equilibrium. The seller can exploit this correlation to reduce buyers' information rents. To see intuitively why correlation emerges when the seller randomizes, one may think of an urn model where each urn corresponds to a pure investment strategy by the seller. Buyers' valuations are drawn independently from one urn, but if the seller randomizes and buyers do not observe the realized investment, they do not know what the true urn is. Therefore, the realization of a buyer's valuation contains information about the true urn and thus about the valuation of the rival buyer.

The contribution of the paper is to establish conditions so that a mechanism can be constructed which leaves buyers no rent and, at the same time, makes the seller indifferent between his investment options so that randomizing is optimal. The main result is that, under appropriate conditions, the seller can implement any mixed investment strategy and fully extract the resulting surplus. As a consequence, if his mixed investment strategy places almost full probability mass on the efficient investment level, then the seller extracts nearly the first best surplus.

I focus on cases in which the seller announces the mechanism before he chooses his investment. This is plausible for instance in public procurement where an early announcement of the selling procedure is not untypical. Also, in vertical relations downstream firms frequently procure inputs through standardized, pre–announced auction platforms.[1]

To establish my main result, I first construct mechanisms which leave buyers no information rent for a given mixed investment strategy by the seller. Following Cremer and McLean (1988), the existence of such mechanisms is guaranteed as long as a buyer's beliefs are *convexly independent* across his types which means that no type's belief about the rival buyer's type is in the convex hull of the beliefs of the other types.[2] Since in my setup a buyer's beliefs are endogenous, whether they are convexly independent or not is endogenous, too. I show that convex independence holds for *any* mixed strategy by the seller whenever the probability distributions over buyer types, conditional on the various investments, satisfy a certain statistical condition which, intuitively, requires any realization of a buyer's type to be sufficiently informative about the true investment.[3]

The more difficult part of my analysis concerns the question if the seller can be made indifferent between his investments. In fact, at first sight one may wonder how the seller can extract the full surplus and, at the same time, be indifferent when each investment generates a different surplus. The crucial observation here is that, in my setup, the seller's investment is his private information. To illuminate this point, notice that the seller's expected profit is simply the expected payments, where payments depend on the buyers' (reports about their) valuations.

[1]Analytically, assuming that the seller announces the mechanism before investing substantially enhances tractability. If the timing is reversed, the choice of mechanism may signal the true investment, leading to an informed principal problem.

[2]Myerson (1981) was first to point out the possibility of full surplus extraction under correlated valuations. McAffee and Perry (1992) extend this insight to a setup with continuous types.

[3]The condition is for instance (yet not only) violated if there is a buyer valuation who is equally likely under each investment so that observing this type is uninformative about investment.

When each buyer, for each valuation, gets zero utility, then the seller's expected profit equals the full surplus when the expectation is taken with respect to the *unconditional* distribution of buyer valuations. In contrast, in my setup the seller holds private beliefs about the distribution of buyer valuations which, moreover, depends on the actually chosen investment. This has two implications. First, *conditional* on the investment, the seller's expected profit will not equal the full surplus generated by his investment even if buyers get zero utility from the perspective of their own beliefs. (Yet, from an ex ante perspective the seller's expected profit, averaged over the investment distribution, will equal full ex ante surplus.) Second, because different investments induce different seller beliefs about the buyers' valuations, the expected payments, conditional on different investments, respond differently to changes in the payment schedule. It is these differences in beliefs that provide the channel through which a payment schedule can be designed that makes the seller indifferent between his investment opportunities.

When can the conditions for leaving buyers no rent and seller indifference be jointly satisfied? For the simplest case with two buyers, two investment opportunities, and a binary distribution of buyer valuations, I present a geometric argument to show that the seller can always implement any mixed investment strategy and fully extract the resulting surplus. I then extend this result to the case in which there are (weakly) less investments than possible buyer types. For this to work, I require a condition which says that, irrespective of how the seller randomizes, for each pure investment there is one buyer valuation that provides the strongest evidence for this investment to having occurred. This condition is natural in environments in which higher investments induce, on average, higher valuations. It will guarantee that within the class of payment schedules that leave buyers no rent, there are still enough degrees of freedom to make the seller indifferent between his investments. Finally, I consider the case when there are more investments than possible buyer types. Since the number of instruments available to make the seller indifferent is the number of buyer types, there is little hope for a general result in this case. Therefore, I confine myself with considering the model with two buyer types only and demonstrate that there is a mechanism which yields the seller the first best profit almost fully.

In a related paper, Obara (2008) studies an auction model where buyers can take (hidden) actions that influence the joint distribution of their valuations. He demonstrates that this generically prevents the seller from extracting full surplus by a mechanism that implements a pure action profile by buyers. However, almost full surplus extraction can be attained by a mechanism which implements a mixed action profile by buyers and has them report not only their valuation but also the realization of their actions. Similar to my construction, Obara's mechanism thus exploits correlation that is created through mixed strategies. In contrast, in my setup it is the seller who randomizes, and almost full surplus extraction is achieved without having the seller report about the realization of his action.

Full surplus extraction results have come under criticism from a variety of angles. First, full surplus extraction critically relies on risk–neutrality or unlimited liability of buyers (Roberts, 1991, Demougin and Garvie, 1991) or on the absence of collusion by buyers (Laffont and Martimort, 2000). In principle, these concerns apply to a literal interpretation of my construction, too. However, even if the conditions for full surplus extraction are not met, often the correlation among buyer valuations can still be exploited to some extent.[4] The spirit of my argument is likely to carry over to such situations. In this paper, I consider an environment in which zero rent mechanisms exist, because this allows me to focus on the question if there is a mechanism which induces the seller to randomize at all.

Second, a recent literature points out that full surplus extraction depends on strong common knowledge assumptions with respect to the distribution of buyers' valuations and their higher order beliefs. Neeman (2004) has shown that full surplus extraction relies on the property that an agent's beliefs about other agents uniquely determine his payoff.[5] Parreiras (2005) finds that full surplus extraction fails if the precision of agents' information is their private information.[6] While some work studies optimal ("robust") design with weaker common knowledge assumptions (Chung and Ely, 2007, Bergemann and Schlag, 2008) it is an open issue how a seller can exploit correlation in such environments. Note, however, that in my setup the joint distribution

[4]See e.g. Bose and Zhao (2007) who study optimal design when the agents' beliefs violate convex independence, or Dequiedt and Martimort (2009) who consider the case when the designer cannot commit to a grand mechanism but only to bilateral contracts with each agent.

[5]Neeman and Heifetz (2006) and Barelli (2009) demonstrate that this property is generic. To the contrary, Gizatulina and Hellwig (2009) point out that the genericity of the "beliefs determine preferences" property depends on the assumption that beliefs and payoffs are exogenous features of an abstract "type" of the agent. They show that when an agent's beliefs derive from available information, then generically beliefs do uniquely determine payoffs.

[6]For a related observation when agent's can acquire information about each other see Bikhchandani (2010). For a qualification of Parreiras' result see Krähmer and Strausz (2010)

of buyers' valuations emerges endogenously in equilibrium as a result of the seller's investment. Therefore, if there are no significant exogenous information sources that affect buyers' valuations and/or their beliefs, then the common knowledge assumption is simply embodied in the equilibrium concept, it is not an ad hoc assumption on players' exogenous beliefs.

A question related to mine has been raised in industrial economics by Spence (1975) who studies the incentives of a monopolist to invest in product quality. The difference is that in Spence the monopolist cannot price discriminate between consumers. There seems to be relatively little work in the mechanism design literature that considers optimal design with an ex ante action by the designer.[7] Instead, most work focusses on optimal design with ex ante actions by agents, such as investments in their valuation or information acquisition (e.g., Rogerson, 1992, Cremer et al., 1998, Bergemann and Välimäki, 2002, to name only a few).

The paper is organized as follows. The next section presents the model. Section 3 derives the first best benchmark. In section 4, the seller's problem is described, and section 5 contains the main argument. Section 6 concludes. All proofs are in the appendix.

2 Model

There are one risk–neutral seller and two risk–neutral buyers $i = 1, 2$. The seller has one good for sale. Buyer i's valuation for the good (or, his type) is denoted by θ^i. For simplicity, buyers' valuations are assumed to be symmetric and can take on the values $0 < \theta_1 < \ldots < \theta_K$. In what follows, $i, j \in \{1, 2\}$ indicates a buyer's identity, and $k, \ell \in \{1, \ldots, K\}$ indicates a buyer's type. The distribution of buyers' valuations depends on a costly ex ante investment $z \in \{z_1, \ldots, z_M\}$ by the seller. Investing z_m costs $c(z_m) = c_m$. Given z_m, the probability with which a buyer has valuation θ_k is p_{mk}. I assume $p_{mk} > 0$ for all m, k. This rules out deterministic investment technologies and captures buyer heterogeneity. Let

$$p_m = \begin{pmatrix} p_{m1} \\ \vdots \\ p_{mK} \end{pmatrix}$$

[7] An exception is the literature on bilateral trading mechanisms where the seller may invest in the buyer's valuation. See Schmitz (2002), Hori (2006), or Zhao (2008).

be the *type distribution* conditional on investment z_m. Buyers' valuations are assumed to be conditionally independent, conditional on z. Moreover, I assume that buyers cannot directly observe the seller's investment choice.

The seller may randomize between investments. A mixed investment profile is denoted by $\zeta = (\zeta_m)_{m=1}^M$, where ζ_m is the probability with which the seller chooses z_m. In the analysis, an important role will be played by the set of *totally mixed* investment profiles denoted by

$$\mathring{\Delta}_M = \{\zeta \mid \sum_m \zeta_m = 1, \quad \zeta_m > 0 \quad \forall m = 1, \dots, M\}.$$

If the seller adopts ζ, the unconditional probability that a buyer's valuation is θ_k is $\sum_n p_{nk}\zeta_n$. By Bayes' rule, conditional on observing valuation θ_k, a buyer's belief that investment is z_m is $q_{km} = p_{mk}\zeta_m / \sum_n p_{nk}\zeta_n$, and his belief that his rival has valuation θ_ℓ is $\mu_{k\ell}(\zeta) = \sum_m q_{km}p_{m\ell}$. Let $\mu_k(\zeta)$ be the corresponding belief (column) vector. Hence, we can write

$$\mu_k(\zeta) = \sum_m q_{km} p_m. \tag{1}$$

Since $\sum_m q_{km} = 1$, this means that the buyer's belief about his rival is a convex combination of the type distributions. Intuitively, this is because one's own valuation is a noisy signal of the true investment. This observation will be useful below.

The basic point of the paper rests on the insight that if the seller adopts a mixed investment strategy, then valuations are correlated from the point of view of buyers. The reason is that a buyer cannot observe the investment realization. One may think of an urn model where each pure investment corresponds to one urn. Buyers' valuations are drawn independently from one urn, but a buyer does not know from which one. Therefore, the realization of a buyer's own valuation contains information about the true urn and thus about the valuation of the rival buyer.

The objective of the paper is to explore whether the seller can exploit this correlation to extract full surplus. Cremer and McLean (1988, Theorem 2) have shown that full surplus extraction is closely related to a certain form of correlation which requires that beliefs be convexly independent. Formally, a set of vectors $(v_k)_{k=1}^K$ is *convexly independent* if no vector is the convex combination of the other vectors, that is, for no k there are weights $\beta_\ell \geq 0$ with $\sum_{\ell \neq k} \beta_\ell = 1$ so that $v_k = \sum_{\ell \neq k} \beta_\ell v_\ell$.

3 First best

As a benchmark, consider the situation in which the buyers' valuations are public information. In that case, the seller optimally offers the good to the buyer with the maximal valuation at a price equal to that valuation. Therefore, for each realization of valuations, the seller can extract the full ex post surplus $\max\{\theta^1, \theta^2\}$, yielding an ex ante profit of

$$\pi^{FB}(z) = E[\max\{\theta^1, \theta^2\} \mid z] - c(z).$$

Suppose there is a unique first best investment level $z_{\bar{m}}$ given by

$$z_{\bar{m}} = \arg\max_z \pi^{FB}(z).$$

4 Seller's problem

I now turn to the case in which the seller's investment is unobservable and the buyers' valuations are their private information. Therefore, the seller designs a mechanism which makes the assignment of the good and payments conditional on communication by the buyers. I consider the following timing.[8]

1. Seller publicly proposes and commits to a mechanism.
2. Seller privately chooses an investment.[9]
3. Buyers privately observe their valuation.
4. Buyers simultaneously reject or accept the contract.

– If a buyer rejects, he gets his outside option of zero.

5. If buyers accept, the mechanism is implemented.

In general, a mechanism may depend on the (identity of the) participating buyers and specifies for each participating buyer a message set, the probability with which a participating buyer gets the object, and payments from buyers to the seller contingent on messages submitted by the buyers in stage 5. After a mechanism is proposed, a simultaneous move game of incomplete

[8] A similar timing is adopted, e.g., in Cremer et al. (1998). If the stages 1 and 2 are swapped, signaling issues may contaminate the analysis.

[9] My results would hold a forteriori and under weaker assumptions if the seller could ex ante commit to an investment strategy.

information starts at date 2 where the seller chooses an investment and each buyer chooses an acceptance and reporting strategy.[10] I assume that players play a Bayes Nash Equilibrium (in short: equilibrium) of that game. In equilibrium, the seller's investment is a best reply to buyers' acceptance and reporting strategies, and buyer's acceptance and reporting strategies are best replies to the seller's investment and the rival buyer's acceptance and reporting strategy. The objective of the seller is to design a revenue maximizing mechanism subject to the constraint that an equilibrium is played in the game that starts after he proposes the mechanism.[11]

Next, I spell out the seller's problem formally. I begin by considering direct and incentive compatible mechanisms. While the set of possible mechanisms available to the seller is much larger, it will follow by the revelation principle that the search for an optimal mechanism can be restricted to the class of direct and incentive compatible mechanisms. Moreover, I will argue that the seller can restrict attention to mechanisms in which all buyer types participate.

A *direct* mechanism asks each buyer to announce his type after stage 3 and before stage 4, and consists of an assignment rule

$$x_{k\ell} = (x^1_{k\ell}, x^2_{\ell k}), \qquad 0 \leq x^1_{k\ell}, x^2_{\ell k} \leq 1, \qquad x^1_{k\ell} + x^2_{\ell k} \leq 1,$$

which specifies the probabilities $x^1_{k\ell}$, $x^2_{\ell k}$ with which buyer 1 and 2 obtain the good, conditional on the buyers' type announcements $(\theta^1, \theta^2) = (\theta_k, \theta_\ell)$. Moreover, it consists of a transfer rule

$$t_{k\ell} = (t^1_{k\ell}, t^2_{\ell k}),$$

which specifies the transfers $t^1_{k\ell}$, $t^2_{\ell k}$ which buyer 1 and 2 pay to the seller, conditional on the buyers' type announcements $(\theta^1, \theta^2) = (\theta_k, \theta_\ell)$. In vector notation:

$$x^i_k = \begin{pmatrix} x^i_{k1} \\ \vdots \\ x^i_{kK} \end{pmatrix}, \quad t^i_k = \begin{pmatrix} t^i_{k1} \\ \vdots \\ t^i_{kK} \end{pmatrix}.$$

A direct mechanism is *incentive compatible* if each buyer has an incentive to announce his type truthfully, given his beliefs about the rival buyer's type. Note that since a buyer's

[10] This means that a buyer cannot observe the participation decision of the rival buyer, which avoids the complication that a buyer's participation decision reveals information about his type to the other buyer.

[11] Implicit in this formulation of the seller's problem is the (standard) assumption that the seller can select his most preferred equilibrium.

beliefs about the rival buyer's type depend upon (his beliefs about) the seller's investment strategy, incentive compatibility has to be defined for a given investment strategy. The expected probability of winning and the expected transfers of type k of buyer i, given an investment strategy ζ, when he announces type ℓ are respectively given as

$$\sum_{r=1}^{K} x^i_{\ell r}\mu_{kr}(\zeta) = \langle x^i_\ell, \mu_k(\zeta)\rangle, \qquad \sum_{r=1}^{K} t^i_{\ell r}\mu_{kr}(\zeta) = \langle t^i_\ell, \mu_k(\zeta)\rangle,$$

where $\langle \cdot, \cdot \rangle$ denotes the scalar product.[12] The mechanism is incentive compatible, given ζ, if for all i, k, ℓ:

$$\theta_k \langle x^i_k, \mu_k(\zeta)\rangle - \langle t^i_k, \mu_k(\zeta)\rangle \;\geq\; \theta_k \langle x^i_\ell, \mu_k(\zeta)\rangle - \langle t^i_\ell, \mu_k(\zeta)\rangle. \tag{IC$_\zeta$}$$

Finally, a mechanism is *individually rational* if each type of each buyer, given his beliefs, participates in the mechanism at stage 4. Formally, (x, t) is individually rational, given ζ, if for all i, k:

$$\theta_k \langle x^i_k, \mu_k(\zeta)\rangle - \langle t^i_k, \mu_k(\zeta)\rangle \;\geq\; 0. \tag{IR$_\zeta$}$$

A direct mechanism that is incentive compatible and individually rational, given ζ, is called feasible, given ζ.

I shall say that a direct mechanism is *equilibrium feasible w.r.t.* ζ if, following the proposal of the mechanism, the mechanism is feasible, given ζ, and the seller optimally chooses ζ, given all buyer types participate and announce their type truthfully. Let the seller's expected profit from investment z_m be given by

$$\pi_m \quad = \quad \sum_{k,\ell} [t^1_{k\ell} + t^2_{\ell k}] p_{mk} p_{m\ell} - c_m. \tag{2}$$

Thus, a direct mechanism (x, t) is equilibrium feasible w.r.t. ζ if

$$\pi_m = \pi_n \text{ if } \zeta_m, \zeta_n > 0, \quad \text{and} \quad \pi_m \geq \pi_n \text{ if } \zeta_m > 0, \zeta_n = 0, \tag{IND}$$

(IC$_\zeta$) and (IR$_\zeta$) hold.

[12] In what follows, I will adopt both an economic and a geometric interpretation of the scalar product. Interpreting a vector $y \in \mathbb{R}^K$ as a random variable, the scalar product between y and a belief ν is the expected value of y with respect to the belief ν. Geometrically, the scalar product is the length of the orthogonal projection of y onto ν multiplied by the length of ν. In particular, the sign of the scalar product coincides with the sign of the orthogonal projection.

Condition (IND) means that it is optimal for the seller to adopt the investment strategy ζ. The formulation explicitly allows for the optimality of a mixed strategy. The second line means that the mechanism is feasible, given buyers hold correct beliefs about investment.

The revelation principle now implies that the search for an optimal mechanism can be restricted to direct equilibrium feasible mechanisms. Indeed, fix an arbitrary mechanism and consider some equilibrium outcome of the mechanism as a function of buyer types. Now replace the original mechanism by a direct mechanism which asks buyers to report their type (after stage 3) and then implements the outcome of the original mechanism where, if a buyer did not participate in the original mechanism, his probability of winning and payments are zero under the new mechanism. Since the players strategies were an equilibrium under the original mechanism, buyers have an incentive to report their types truthfully under the new mechanism given the seller's investment strategy. Also the seller's investment strategy is optimal given buyers report truthfully. But this means that the new mechanism is equilibrium feasible.

The seller's problem, therefore, simplifies to choosing a mechanism (x, t) and an investment profile ζ which maximizes his profit subject to the constraint that the mechanism be equilibrium feasible w.r.t. ζ:

$$\max_{x,t,\zeta} \sum \pi_m \zeta_m \quad s.t. \quad \text{(IND)}, \ (\text{IC}_\zeta), \ (\text{IR}_\zeta).$$

5 Mechanisms with endogenous correlation

When the seller is restricted to use a pure investment strategy z_m, a buyer's belief is independent of his type: $\mu_k = p_m$ for all k. In that case, a buyer can typically secure an information rent. The basic insight of this paper is that randomizing between investments may allow the seller to concede no information rent to buyers.[13] By this I mean that a buyer's expected utility is zero for each type. For randomizing to occur in equilibrium, the mechanism has to leave the seller indifferent between all investments that he uses with positive probability. My approach to the seller's problem is to first examine if there are mechanisms that permit an equilibrium of the induced game in which the seller randomizes and buyers get no rent. If that is the

[13] The seller would, a forteriori, need to concede no rent if conditional independence is violated. The point of the paper is that this is still true *even though* valuations are conditionally independent.

case, then the seller extracts the full surplus in *ex ante expectation*, i.e. as viewed from the point before investment has realized. Thus, I say that the respective investment strategy is *FSE-implementable.*

I focus on extraction of the full *ex post efficient* surplus. From now on fix x to be the ex post efficient allocation rule which assigns the object to the buyer with the highest valuation. I assume that ties are broken by tossing a fair coin. Thus,

$$x^i_{k\ell} = \begin{cases} 0 & if \quad k < \ell \\ 1/2 & if \quad k = \ell \\ 1 & if \quad k > \ell. \end{cases}$$

I say that an investment strategy ζ is *FSE-implementable* if there is a direct mechanism (x, t) which is equilibrium feasible w.r.t. to ζ so that each buyer makes zero rent. Formally, this means that, next to the seller's optimality condition (IND), we have:

$$(\text{IC}_\zeta) \text{ holds, and } (\text{IR}_\zeta) \text{ is binding for all types } k. \tag{ZR}$$

Condition (ZR) means that, given ζ, the mechanism is incentive compatible, and buyers' expected utility, conditional on their type, is zero. To describe the set of FSE-implementable strategies, I proceed in two steps. I first construct mechanisms which satisfy (ZR) for given ζ. I shall refer to those mechanisms as *zero rent mechanisms.* Then I look among all zero rent mechanisms for one which satisfies the seller's indifference condition (IND).

Zero rent mechanisms

The construction of zero rent mechanisms follows the existing literature. I consider payment rules where buyer i's payment consists of a base payment b^i_k that depends on his own report θ_k only and a contingent payment $\tau^i_{k\ell}$ which depends on the rival's announcement θ_ℓ as well. Intuitively, contingent payments serve to elicit a buyer's belief: when valuations are correlated, there exist contingent payments that entail zero expected payments when telling the truth yet (arbitrarily) large expected payments when lying. Base payments then serve to extract the buyer's gross utility from truth-telling and may be interpreted as entry fees.

I focus on symmetric mechanisms which treat buyers symmetrically: $b^1_k = b^2_k$ and $\tau^1_{k\ell} = \tau^2_{k\ell}$. This allows me to consider only buyer 1 and omit the superindex i. All arguments carry over to buyer 2. The vector of contingent payments is

$$\tau_k = \begin{pmatrix} \tau_{k1} \\ \vdots \\ \tau_{kK} \end{pmatrix}.$$

The next result shows that for constructing zero rent mechanisms, one needs only construct appropriate contingent payments. Base payments are then automatically determined by the allocation rule and the buyers' beliefs.

Lemma 1 *A zero rent mechanism (x, t) exists if and only if there are contingent payments τ with*[14]

$$\begin{aligned} \langle \tau_k, \mu_k \rangle &= 0, && \text{(ZR1)} \\ \langle \tau_\ell, \mu_k \rangle &\geq \theta_k \langle x_\ell, \mu_k \rangle - \theta_\ell \langle x_\ell, \mu_\ell \rangle, \quad \ell \neq k. && \text{(ZR2)} \end{aligned}$$

In this case, base payments are pinned down by $b_k = \theta_k \langle x_k, \mu_k \rangle$.

The condition (ZR1) says that the expected contingent payments from telling the truth are zero. Together with $b_k = \theta_k \langle x_k, \mu_k \rangle$, this makes sure that the buyer's utility from reporting truthfully is zero. The condition (ZR2) guarantees truthtelling: the expected contingent payments from a lie are sufficiently large so that the expected total payments of a lie of type k, $\langle \tau_\ell, \mu_k \rangle + b_\ell$, would give negative utility. Geometrically, (ZR1) means that the contingent payments τ_k are orthogonal to buyer type k's beliefs, and (ZR2) says that the projection of the contingent payments τ_ℓ on the other types' beliefs μ_k is sufficiently large.[15]

Seller indifference

The second step is to ask when there are transfers that leave the seller indifferent between his investment opportunities. It will be usefull to re–arrange (2) as follows: Because of symmetry, expected profits are simply twice the expected payments by buyer 1. To compute this expectation, first condition on buyer 1's type and then average over all types of buyer 1. This yields:

$$\pi_m = 2 \sum_k p_{mk} (\langle \tau_k, p_m \rangle + b_k) - c_m. \tag{3}$$

[14] If it does not create confusion, I shall drop the dependency of μ on ζ.

[15] see footnote 12.

Observe that $\langle \tau_k, p_m \rangle + b_k$ can be interpreted as the expected payment of buyer 1 conditional on buyer 1 being type k and *conditional on the seller having invested* z_m. That is, the expectation over buyer 2's types is taken with respect to p_m. Expected profit π_m is then obtained by averaging over all possible types k of buyer 1, conditional on z_m.

Notice that if each buyer type gets zero rent, then the expected payments are equal to the full surplus provided the expectation is taken with respect to the *unconditional* distribution of buyer types, i.e., before the investment has realized. After having chosen investment, however, the seller has private information about the true investment. Hence, when calculating expected payments, he averages over all possible buyer types, *conditional on* z_m. Since the unconditional and the conditional distribution of buyer types, conditional on investment, do generally not coincide, this implies that the seller's profit from investment z_m is generally not equal to the full surplus generated by that investment.

5.1 Zero rent mechanisms and seller indifference

I now look for FSE-implementable strategies. By Lemma 1 this amounts to looking for contingent payments τ that satisfy (ZR1), (ZR2), and (IND) with π_m given by (3). To build intuition, I begin with the "binary–binary case" in which there are only two types and two investments. I show that any totally mixed investment strategy is FSE-implementable in this case.

5.2 The binary–binary case

Suppose there are two types $k = 1, 2$ and two investments z_1, z_2. I assume that the "low" investment z_1 is more likely than the "high" investment z_2 to bring about the low valuation θ_1: $p_{11} > p_{21}$. This implies that a low valuation buyer assigns a higher probability than a high valuation buyer to the event that he faces a low valuation rival buyer: $\mu_{11} > \mu_{21}$ for all $\zeta \in \mathring{\Delta}_2$. Intuitively, valuations are positively correlated.

Figure 1 illustrates the setup. The horizontal axis displays the first, and the vertical axis the second component of a vector. As probability vectors, p_1, p_2, μ_1, μ_2 are located on a line where the components sum to one. Since μ_k is a convex combination of p_1 and p_2, μ_k is in between p_1 and p_2. Moreover, since observing θ_1 (resp. θ_2) increases the likelihood that investment is low (resp. high), μ_1 is flatter than μ_2.

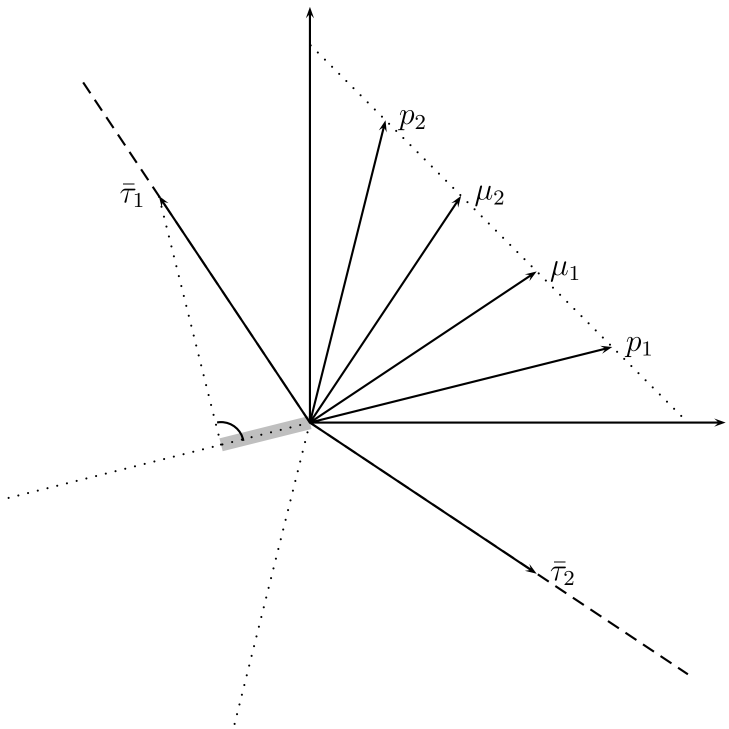

Figure 1: Zero rent mechanisms.

I now illustrate the construction of zero rent mechanisms. By (ZR1), expected contingent payments τ_k are zero for buyer type k. By (ZR2), they have to be such that the expected contingent payment from a lie, $\langle \tau_k, \mu_\ell \rangle$, $\ell \neq k$ is large enough. Geometrically, this means that τ_k is orthogonal to μ_k and directed so that the projection on μ_ℓ is positive and large enough. This is the case if τ_1 points to the north west, and τ_2 points to the south east. Economically, because valuations are positively correlated, this means that buyers have to pay a positive amount $\tau_{k\ell} > 0$ if their reports mismatch, but are rewarded otherwise ($\tau_{kk} < 0$). In the figure, $\bar{\tau}_k$ is meant to indicate the shortest transfer vector which is orthogonal to μ_k and just long enough to meet (ZR2). Hence, all vectors τ_k on the dashed lines that are longer than $\bar{\tau}_k$ leave buyers no rents.

Next, consider seller indifference. By (3):

$$\pi_m = 2(p_{m1}\langle \tau_1, p_m \rangle + p_{m2}\langle \tau_2, p_m \rangle) + 2(p_{m1}b_1 + p_{m2}b_2) - c_m.$$

The key observation is that the sign of the expected contingent payment, conditional on the seller's belief p_m, $\langle \tau_k, p_m \rangle$, is determined by the fact that the seller's beliefs are more dispersed than those of the buyer. Indeed, when the seller invests, say z_1, he assigns a lower probability than buyer type θ_1 to the event that the buyers' types mismatch and, therefore, that the contingent payment is positive. Since buyer type θ_1's expected contingent payment, $\langle \tau_1, \mu_1 \rangle$, is zero, this implies that, from the seller's perspective, the expected contingent payment, $\langle \tau_1, p_1 \rangle$, is negative.[16] With similar arguments it follows that:

$$\langle \tau_1, p_1 \rangle < 0, \quad \langle \tau_2, p_1 \rangle > 0,$$
$$\langle \tau_1, p_2 \rangle > 0, \quad \langle \tau_2, p_2 \rangle < 0.$$

In other words, the seller who invested z_k evaluates the contingent payments τ_k negatively in expectation, while the seller who invested z_ℓ, $\ell \neq k$, evaluates them positively. Crucially, this difference in expectations implies that the seller's profit responds differently to changes in the payment schedule, depending on his investment. Indeed, as τ_1 increases uniformly in the reports of buyer 2, the payments that the seller expects to collect from buyer 1 decrease if he has invested z_1 but increase if he has invested z_2. Thus, increasing the length of τ_1 decreases the expected profit from investing z_1 and increases the expected profit from z_2. Likewise, increasing the length of τ_2 increases π_1 and decreases π_2. Thus, an intermediate value argument implies that a solution to the indifference condition $\pi_1 = \pi_2$ can be found by either increasing τ_1 or τ_2. Hence:

Proposition 1 *In the binary–binary case, any $\zeta \in \mathring{\Delta}_2$ is FSE-implementable.*

The underlying geometric reason why a zero rent mechanism exists that also leaves the seller indifferent is that μ_1 and μ_2 are convex combinations of the type distributions p_1 and p_2. This implies that the line (hyperplane) through μ_k separates the beliefs μ_k, p_k jointly from the beliefs μ_ℓ, p_ℓ, $\ell \neq k$. This means that a payment vector corresponding to the normal vector of the hyperplane is evaluated differently, depending on on which side of the hyperplane one's belief is located: the disagreement about the sign of the expected payments between buyer type k and the buyer type ℓ can be used to construct zero rent mechanisms; at the same time, the disagreement between the seller who invested z_k and the seller who invested z_ℓ can be used to make the seller indifferent.

[16] Geometrically, $\langle \tau_1, p_1 \rangle$ corresponds to (minus a multiple of) the thick grey line segment in Figure 1.

It is illuminating to see how the surplus is divided across buyer types and investments. For the sake of concreteness, suppose z_2 is the efficient investment. Since the seller is indifferent in equilibrium, the seller's ex ante profit $\pi(\zeta)$ is equal to his ex post profit:

$$\pi(\zeta) = \zeta_1\pi_1 + \zeta_2\pi_2 = \pi_1 = \pi_2.$$

At the same time, since buyers get no rent, the seller extracts the full surplus in expectation:

$$\pi(\zeta) = \zeta_1\pi^{FB}(z_1) + \zeta_2\pi^{FB}(z_2).$$

As surplus is higher at the efficient investment level, this implies that the seller receives less than the full surplus at the efficient investment level ($\pi^{FB}(z_2) > \pi_2$) but more than the full surplus at the inefficient investment ($\pi^{FB}(z_1) < \pi_1$). Effectively, when the efficient investment realizes, the payments of the mechanism shift surplus from the seller to the buyer, and vice versa. Holding the buyer type fixed and averaging over investments, this means that the buyer gets a positive utility at the efficient investment but makes a loss at the inefficient one.

Let me emphasize the three main properties used to establish Proposition 1. First, buyer beliefs are convexly independent. Second, the type distributions and buyer beliefs are ordered in a way that μ_k and p_k can jointly be separated from μ_ℓ and p_ℓ, $\ell \neq k$. Third, the seller can be made indifferent by an intermediate value argument. Next, I turn to the case when there are (weakly) fewer investments than types.

5.3 Less investments than types: $M \leq K$

I develop the argument according to the three properties used in the binary–binary case.

Convex independence of beliefs

Cremer and McLean (1988, Theorem 2) have shown that, if beliefs are not convexly independent, ex post efficient zero rent mechanisms may not exist. In my setup, convex independence of beliefs might, in principle, depend on the endogenous investment strategy ζ. However, the next lemma shows that for totally mixed investment strategies this is not the case. Rather, convex independence is a property of the primitives $(p_m)_{m=1}^M$ only. To state the lemma, define by

$$\bar{q}_{km} = \frac{p_{mk}}{\sum_n p_{nk}}$$

the probability with which investment z_m has realized conditional on observing θ_k when the seller adopts the uniform investment strategy which places weight $1/M$ on each investment. Denote by $\bar{q}_k$ the corresponding probability (column) vector.

Lemma 2 *Let $(p_m)_{m=1}^M$ be linearly independent. Then $(\mu_k(\zeta))_{k=1}^K$ is convexly independent for all $\zeta \in \mathring{\Delta}_M$ if and only if $(\bar{q}_k)_{k=1}^K$ is convexly independent.*

The proof uses the separating hyperplane theorem to actually show that $(\bar{q}_k)_{k=1}^K$ is convexly independent if and only if, for an arbitrary mixed strategy ζ, the posteriors about investments $(q_k(\zeta))_{k=1}^K$ are convexly independent. Hence, the convex independence of beliefs $(\mu_k)_{k=1}^K$ is guaranteed if there is only a single investment strategy so that the induced posteriors about investments are convexly independent.

In light of Theorem 2 by Cremer and McLean (1988), Lemma 2 makes clear that if $(\bar{q}_k)_{k=1}^K$ is not convexly independent, then the seller will, in general, not be able to create correlation that allows him to extract full surplus. While the seller might still benefit from creating correlation also in this case, the construction of optimal mechanisms is demanding already in the case with exogenous beliefs (see Bose and Zhao, 2007). To focus on the question when randomizing by the seller can occur at all, I restrict attention to the case in which beliefs are convexly independent, and impose:

Assumption 1 *(a) $(p_m)_{m=1}^M$ is linearly independent,*
(b) $(\bar{q}_k)_{k=1}^K$ is convexly independent.

Assumption 1(b) means that any realization of a buyer's type is sufficiently informative about the true investment. For example, the condition is violated if there is a type k which is equally likely under all investments. In this case, observing k is completely uninformative and so $\bar{q}_k$ is equal to the "prior" ζ which is always in the convex hull of posteriors. For Assumption 1(b) to hold, the probability that an investment brings about a type k has to be sufficiently dispersed across investments. [17]

[17]Notice that if there are only two investments and more than two types, then Assumption 1(b) cannot be satisfied for dimensionality reason: three beliefs over a binary state space cannot be convexly independent. This implies that Assumption 1(b) is not generically satisfied. At the same time it is not generically violated, because if it holds, it is not upset by slightly perturbing the type distribution.

Ordering of type distributions and buyer beliefs

In the binary–binary case, the key idea is to exploit belief differences. More precisely, I have constructed payments so that the seller and a buyer type agree about the sign of the expected payments if and only if the seller has chosen the investment for which the buyer type provides the strongest evidence. I now show how this feature carries over to the general case. Recall that a buyer of type k assigns probability $q_{km}(\zeta) = p_{mk}\zeta_m / \sum_n p_{nk}\zeta_n$ to the event that investment z_m has realized. Consider a type which provides, among all types, the strongest evidence that z_m has occurred:

$$\kappa(\zeta, m) \in \arg\max_k q_{km}(\zeta). \tag{4}$$

Guided by the binary-binary case, I seek to construct payments so that the seller who invests z_m and the buyer type $\kappa(\zeta, m)$ agree about the sign of the expected payments, but disagree with all other buyer types and sellers who invest differently. Geometrically, I seek to separate p_m and $\mu_{\kappa(\zeta,m)}$ jointly from all other type distributions and buyer beliefs. It turns out that this is possible in environments in which $\kappa(\zeta, m)$ is independent of ζ. In what follows, I will focus on such environments.[18] In Lemma 3 below, I will show that $\kappa(\zeta, m)$ is independent of ζ if and only if the following likelihood ratio condition is satisfied.

Assumption 2 *For all m there is a $k = k(m)$ so that for all ℓ, n, $n \neq m$, $\ell \neq k$:*

$$\frac{p_{mk}}{p_{m\ell}} \geq \frac{p_{nk}}{p_{n\ell}}. \tag{5}$$

Assumption 2 says that for each investment z_m there is one valuation θ_k whose likelihood ratio relative to any other valuation θ_ℓ is the highest among *all* investments. This means that any investment brings about some valuation with relatively high likelihood. This is a natural feature when investments are ordered, and higher investments induce, on average, higher valuations, as in the next example.[19]

Example 1 *There are as many investments as types, and investment z_m brings about valuation θ_m with some probability ρ and any other valuation with probability $(1-\rho)/(K-1) < \rho$.*

[18]This is somewhat stronger than needed, but makes the argument more transparent.

[19]Observe that Example 1 also satisfies Assumption 1.

Next, I show that Assumption 2 is necessary and sufficient for the same observation to provide the strongest evidence for a given investment, irrespective of the "prior" ζ with which investments are drawn.

Lemma 3 *$\kappa(\zeta, m)$ is independent of ζ if and only if Assumption 2 holds. In this case, $\kappa(\zeta, m) = k(m)$.*

I now turn to the construction of payments. In the first step, I construct payments for types $k = \kappa(m)$ which provide the strongest evidence for some investment.

Lemma 4 *Suppose Assumptions 1 and 2 hold, and let $\zeta \in \mathring{\Delta}_M$. Then for all m and for all numbers $s_{mn} > 0$, $n \neq m$, there is a vector $\sigma_{\kappa(m)} \in \mathbb{R}^K$ so that*

$$\langle \sigma_{\kappa(m)}, \mu_{\kappa(m)} \rangle = 0, \tag{6}$$

$$\langle \sigma_{\kappa(m)}, \mu_\ell \rangle > 0 \quad \textit{for } \ell \neq \kappa(m). \tag{7}$$

Moreover,

$$\langle \sigma_{\kappa(m)}, p_m \rangle < 0, \tag{8}$$

$$\langle \sigma_{\kappa(m)}, p_n \rangle = s_{mn} \quad \textit{for } n \neq m. \tag{9}$$

In Lemma 4, the vector $\sigma_{\kappa(m)}$ corresponds to a payment vector whose expectation is zero for buyer type $\kappa(m)$ by (6), but positive for buyer types ℓ, $\ell \neq \kappa(m)$ by (7). Thus, when scaled up appropriately, $\sigma_{\kappa(m)}$ satisfies the conditions for the contingent payment for type $\kappa(m)$ of a zero rent mechanism. Moreover, by (8) and (9) the expected value of $\sigma_{\kappa(m)}$ is negative from the view of a seller who invested z_m while for a seller who invested z_n, $n \neq m$, the expectation is positive and equal to s_{mn}. Below, the numbers s_{mn} will be chosen so as to satisfy seller indifference.[20]

The left panel of Figure 2 illustrates Lemma 4 geometrically for the case $M = K = 3$. Each point in the simplex corresponds to a belief vector over $\{1, 2, 3\}$. Any line through a point corresponds to a hyperplane that passes through this point. Recall that two vectors are separated by a hyperplane if the scalar products of these vectors with the hyperplane's normal

[20] The proof actually establishes first the properties (6), (8), and (9) by exploiting the fact that the type distributions are linearly independent and beliefs are convex combinations of the type distributions. Jointly with Assumption 2, this then implies property (7)

vector have opposite signs. Now consider $m = 1$. In Lemma 4, $\sigma_{\kappa(1)}$ is a normal vector of a hyperplane, which by (6) passes through $\mu_{\kappa(1)}$. In the figure, this hyperplane is indicated by the dashed line through $\mu_{\kappa(1)}$. The properties (8) and (9) say that the hyperplane separates the type distribution p_1 from p_2 and p_3. Property (7) says that the belief vectors $\mu_{\kappa(2)}$ and $\mu_{\kappa(3)}$ are both located on the same side of the hyperplane. In fact, (9) implies that they are located on the same side as the type distributions p_2 and p_3. Therefore, the hyperplane separates $\mu_{\kappa(1)}$ and p_1 from all the other vectors $p_2, p_3, \mu_{\kappa(2)}, \mu_{\kappa(3)}$. Lemma 4 says that under Assumptions 1 and 2 such a separation is always possible.

Lemma 4 constitutes the first step in the construction of transfers. As in the binary–binary case, I will define the transfer τ_k as a multiple of an orthogonal vector of type k's belief. Consider first types k for which there is an m with $k = \kappa(m)$. Lemma 4 specifies orthogonal vectors σ_k for these types, yet not uniquely. To guarantee that for each type k there is a unique orthogonal vector, I assume that no type can provide the strongest evidence for two different investments.

Assumption 3 *Let κ be defined as in (4). If $m \neq m'$, then $\kappa(m) \neq \kappa(m')$.*

Together with Lemma 4, Assumption 3 implies that for each type distribution, there is a *different* belief so that the two can be separated from all other type distributions and beliefs. This is again illustrated by the left panel of Figure 2. Observe that for each m a distinct $\mu_{\kappa(m)}$ can be found so that the points p_m and $\mu_{\kappa(m)}$ can jointly be separated from all other points. Observe that Example 1 satisfies Assumption 3 with $\kappa(m) = m$.

The right panel in Figure 2 depicts a constellation which is not covered by Assumption 3. There is no hyperplane that separates μ_3 and exactly one type distribution p_m jointly from all other points. Indeed, consider any hyperplane passing through μ_3 that leaves μ_1 and μ_2 on the same side. Such a hyperplane corresponds to a line located in between the two dashed lines. Therefore, if μ_1 and μ_2 are separated from μ_3, the points p_1 and p_2 are always on the "other" side. It follows that there is no m with $\kappa(m) = 3$. With three investments and three types, this implies that Assumption 3 is violated.

After having specified orthogonal vectors for all μ_k with $k = \kappa(m)$ for some m, I next specify orthogonal vectors for types ℓ for which there is no m with $\ell = \kappa(m)$. For such a type, I define the corresponding orthogonal vector such that the projection on all other belief

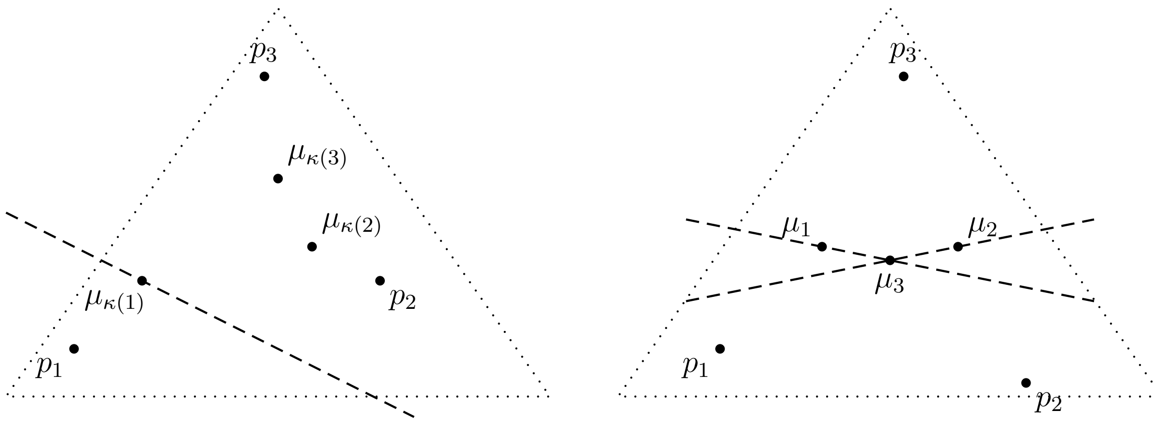

Figure 2: Orderings of beliefs and type distributions.

vectors is positive. By the separating hyperplane theorem, this is always possible since beliefs are convexly independent. Formally, choose σ_ℓ so that

$$\langle \sigma_\ell, \mu_\ell \rangle = 0 \quad \textit{and} \quad \langle \sigma_\ell, \mu_k \rangle > 0 \quad \forall k \neq \ell. \tag{10}$$

Hence, orthogonal vectors σ_k are now defined for all k, and I set transfers equal to a multiple of these:

$$\tau_k = \lambda_k \sigma_k.$$

Then (6) and the left part of (10) imply (ZR1). Moreover, (7) and the right part of (10) imply (ZR2) whenever λ_k is large enough, that is,

$$\lambda_k \geq \frac{\theta_\ell \langle x_k, \mu_\ell \rangle - \theta_\ell \langle x_\ell, \mu_\ell \rangle}{\langle \sigma_k, \mu_\ell \rangle} \equiv \bar{\lambda}_k. \tag{11}$$

Intermediate value argument

In the final step of the construction, I choose the numbers s_{mn} in Lemma 4 so that, similarly to the binary–binary case, an intermediate value argument can be used to make the seller indifferent. I have the following proposition.

Proposition 2 *Under Assumptions 1 to 3, any $\zeta \in \mathring{\Delta}_M$ is FSE-implementable.*

I illustrate the argument for the case with three investments and three valuations: $M = K = 3$. Fix an investment z_m and let n' and n'' be the other two investment indices. Now consider profits (3). By Assumption 3, each type k provides the strongest evidence for a unique investment. Therefore I can express the sum over k in (3) as the sum over $\kappa(m)$, $\kappa(n')$, and $\kappa(n'')$:

$$\begin{aligned}\pi_m &= 2p_{m\kappa(m)}\underbrace{\langle\sigma_{\kappa(m)},p_m\rangle}_{<0}\lambda_{\kappa(m)} + 2p_{m\kappa(n')}\underbrace{\langle\sigma_{\kappa(n')},p_m\rangle}_{=s_{n'm}>0}\lambda_{\kappa(n')} + 2p_{m\kappa(n'')}\underbrace{\langle\sigma_{\kappa(n'')},p_m\rangle}_{=s_{n''m}>0}\lambda_{\kappa(n'')} \\ &\quad +2\sum_k p_{mk}b_k - c_m.\end{aligned}$$

Observe that by Lemma 4, $\langle\sigma_{\kappa(m)},p_m\rangle < 0$ and $\langle\sigma_{\kappa(n)},p_m\rangle = s_{nm} > 0$ for $n = n', n''$: the seller who invested z_m evaluates the payments of the buyer type $\kappa(m)$ in expectation differently from the seller who has not invested z_m. Therefore, π_m decreases in $\lambda_{\kappa(m)}$ and increases in $\lambda_{\kappa(n)}$, $n = n', n''$. I now use this property to construct transfers that leave the seller indifferent. The coefficients s_{nm} can be chosen in such a way that a change of $\lambda_{\kappa(m)}$ changes the profits $\pi_{n'}$ and $\pi_{n''}$ at the *same* rate.[21] Now, consider some zero rent mechanism, that is, $\lambda_k \geq \bar{\lambda}_k$ for all types k. I now argue that the seller can be made indifferent by appropriately increasing some λ_k's (observe that this maintains the zero rent property). Suppose that at the original zero rent mechanism, profits happen to be ranked as $\pi_3 > \pi_1 > \pi_2$. Now increase $\lambda_{\kappa(1)}$. Since this decreases π_1 and raises π_2, there will be a point $\lambda'_{\kappa(1)}$ at which equality holds: $\pi_1 = \pi_2$. Moreover, at this point it is still true that $\pi_3 > \pi_1$, because also π_3 increases in $\lambda_{\kappa(1)}$. Next, increase $\lambda_{\kappa(3)}$. This decreases π_3 and raises π_1 and π_2. In fact, because π_1 and π_2 increase at the same rate, the equality $\pi_1 = \pi_2$ is maintained when $\lambda_{\kappa(3)}$ is increased. Hence, there will be a point $\lambda'_{\kappa(3)}$ at which equality holds between all three profits, and the seller is indifferent.

Proposition 2 provides conditions so that *any* totally mixed strategy is FSE-implementable. Therefore, it directly implies that the seller can attain a profit arbitrarily close to the (ex ante) first best surplus by implementing an investment strategy which places almost full mass on the first best investment:

Proposition 3 *Under Assumptions 1 to 3, the seller can approximately attain the ex ante first best surplus* π^{FB}.

Strictly speaking, therefore, the seller's problem only has an approximate solution: for any mechanism which FSE-implements some investment strategy, there is a better mechanism

[21] This works for $s_{n'm} = p_{n'\kappa(n')}p_{n''\kappa(n')}$ and $s_{n''m} = p_{n'\kappa(n'')}p_{n''\kappa(n'')}$.

FSE-implementing a strategy that places slightly more weight on the first best investment. The important message of Proposition 3 is, however, that by implementing a mixed investment strategy and thus creating correlation between the buyers' valuations, the seller can do better than by offering a mechanism that implements a pure investment strategy and leaves a positive rent to the buyers.

For the seller to attain approximate first best, only investment strategies which are close to the first best investment need to be FSE-implementable. Therefore, Proposition 3 holds under weaker conditions than Assumptions 1 to 3, which guarantee FSE-implementability for *all* totally mixed strategies. In fact, suppose that, instead of globally for all ζ, $\kappa(\zeta, m)$ is independent of ζ and that Assumption 3 holds in a neighborhood around the first best investment strategy. Then the same argument that I presented is applicable to show that any totally mixed strategy in this neighborhood is FSE-implementable, and accordingly, the seller can approximately attain the first best.

A caveat of the previous result is that if the seller places almost all probability mass on the efficient investment, the correlation between the buyers' beliefs gets very small, necessitating large transfer to implement the near first best outcome. Already small amounts of risk aversion or limited liability would suffice to undermine the implementation of the near first best in my construction. Still, this does not necessarily mean that in cases with risk aversion or limited liability, the optimal mechanism has the seller play a pure investment strategy. If risk aversion or limited liability are not too severe, it is likely that the seller would still benefit from reducing information rent by creating correlation.

5.4 More investments than types

The previous section shows that when there are fewer investments than types, any totally mixed strategy is FSE-implementable. When there are more investments than types, this will no longer be true in general. The reason is that the number of transfers available to make the seller indifferent is equal to the number of types. Thus, there are K instruments only to satisfy $M - 1 \geq K$ equations (plus non–negativity constraints on the instruments).[22]

[22] As in the previous section, the seller could still be made indifferent between (less than) K of his investments. The problem is that this does not imply that he prefers those over the remaining investments. If the seller could

For this reason, I now specialize the analysis in two respects. First, I focus exclusively on the question whether the seller can approximately attain the first best profit. Second, for tractability reasons I confine myself with considering the case with two types $k = 1, 2$.

5.4.1 Two types

With two types, investments are ordered according to the probability with which they bring about the low valuation. Assume that, possibly after relabeling indices, investments are ordered as $z_1 < \ldots < z_M$ so that low investments are more likely to bring about the low valuation: $p_{m1} > p_{n1}$ if $m < n$. The next result shows that the seller can almost fully extract the ex ante first best surplus.

Proposition 4 *Suppose there are two types. Then the seller can approximately attain the first best profit* π^{FB}

The idea behind the proof is to let the seller randomize between the first best investment $z_{\bar{m}}$ and either the next smaller or next larger investment. As in the binary–binary case, transfers can be found which make the seller indifferent between these two investments. In fact, any multiple of these transfers leaves the seller indifferent, too. The remaining question is then whether within this set of transfers one can be found so that, in addition, the seller (weakly) prefers the investment $z_{\bar{m}}$ over all other investments z_m. This essentially boils down to the question how the difference in profits $\pi_{\bar{m}} - \pi_m$ changes as transfers are increased. It turns out that this change is always positive, once the probability $\zeta_{\bar{m}}$ with which the seller plays $z_{\bar{m}}$ is sufficiently large. Therefore, if transfers are increased, $\pi_{\bar{m}} - \pi_m$ becomes arbitrarily large, making the seller prefer $z_{\bar{m}}$ over z_m.

What facilitates the analysis in the two type case is that all type distributions and beliefs are ordered on a one–dimensional line. This imposes enough structure to determine the size of the projections of type distribution p_m on transfers τ_k, which, in turn, is needed to figure out the change in the profit difference $\pi_{\bar{m}} - \pi_m$. In higher dimensions, there is no such restriction on the location of type distributions and beliefs.

commit not to use some investments, then he would be back in the case of the previous section. However, given that investments are unobservable, it is questionable that a court could enforce investment commitments.

6 Discussion and Conclusion

Let me discuss some assumptions underlying the analysis. First, while I have considered a private values auction model, all my results will go through for more general mechanism design problems with general allocation spaces and (gross) utility functions of agents, including interdependent values models. The reason is that by Lemma 1, it is essentially enough to construct appropriate contingent transfers which are orthogonal to an agent's *beliefs*. The specific form of agents' *willingness to pay* is irrelevant for the construction of contingent payments, it only pins down base payments. Similarly, the restriction to two symmetric buyers is not substantial and just keeps notation simple.

What is more substantial is the restriction to simple type spaces with the property that any belief goes along with a distinct valuation ("beliefs determine preferences"). A situation in which this need not be true is when buyers possess (imperfect) private information ex ante, and the information they receive after the seller's investment is only *additional* information. In such a case, convex independence of beliefs and thus full surplus extraction may fail (see, e.g. Neeman, 2004, or Parreiras, 2005). As argued earlier, then the construction of optimal mechanisms is demanding already in the case in which the seller cannot affect beliefs. I therefore focus on simple type spaces.

Furthermore, I have confined the analysis to mechanisms which do not condition on a report by the seller. While under the assumptions of Propositions 2 and 4, mechanisms with buyer reports only cannot be improved upon, this may change once these assumptions are violated: because the seller, after having observed the realization of his investment, holds private information, too, allowing for mechanisms with seller reports could extend the set of implementable outcomes. In fact, Obara (2008) shows that when it is buyers who choose ex ante actions, having them report about the realizations of their actions, can be beneficial. However, in Obara's setup, agents' transfers in the extended mechanism can be constructed by standard orthogonality conditions. In my setup, this is not true, because here the seller's transfers are the payments by buyers. Therefore, in my setup, the transfers in the extended mechanism have to respect, in addition, a sort of budget balance condition. Dealing with this requires a rather different line of argument.[23] I leave the full analysis for future research.

[23]See d'Aspremont et al. (2004), or Kosenok and Severinov (2008) who deal with budget balancedness.

I conclude with noting that my analysis raises the more general question about strategies by which a mechanism designer can influence the joint distribution of the agents' valuations. A case in point is disclosure. Standard models of disclosure (e.g. Bergemann and Pesendorfer, 2007, Ganuza and Penalva, 2010) typically fix a selling format such as a first price auction and ask how much information the seller optimally wants to disclose to bidders. If the seller has some discretion over the selling format, then disclosing information in ways such that bidders' information is correlated may be beneficial.

Appendix A: Proofs

Proof of Lemma 1 For a mechanism with base payments b_k and contingent payments τ_k, the feasibility constraints (IC$_\zeta$) and (IR$_\zeta$) are respectively:

$$\begin{aligned} \theta_k\langle x_k, \mu_k\rangle - \langle \tau_k, \mu_k\rangle - b_k &\geq \theta_k\langle x_\ell, \mu_k\rangle - \langle \tau_\ell, \mu_k\rangle - b_\ell \quad \forall k, \ell, && (12)\\ \theta_k\langle x_k, \mu_k\rangle - \langle \tau_k, \mu_k\rangle - b_k &\geq 0 \qquad\qquad\qquad\qquad \forall k, && (13)\end{aligned}$$

where (13) is binding under a zero rent mechanism. With this, the if-part is obvious (simply define $t_{k\ell} = \tau_{k\ell} + b_k$). For the only-if-part, define $b_k = \theta_k\langle x_k, \mu_k\rangle$ and $\tau_{k\ell} = t_{k\ell} - b_k$. It is straightforward to verify (ZR1) and (ZR2). Q.E.D.

Proof of Lemma 2 Let $\zeta \in \mathring{\Delta}_M$. Define by $\alpha_k = \sum_n p_{nk}\zeta_n$ the probability of type k, and recall that $q_{km} = p_{mk}\zeta_m/\alpha_k$ is the posterior over investments conditional on k given ζ. Let q_k be the corresponding probability (column) vector. The lemma follows from the two following claims:

(a) $(\bar{q}_k)_{k=1}^K$ is convexly independent if and only if $(q_k)_{k=1}^K$ is convexly independent.

(b) If $(p_m)_{m=1}^M$ is linearly independent, $(\mu_k(\zeta))_{k=1}^K$ is convexly independent if and only if $(q_k)_{k=1}^K$ is convexly independent.

As for (a). By the separating hyperplane theorem (e.g. Ok, 2007, p. 481), $(\bar{q}_k)_{k=1}^K$ is convexly independent if and only if for all k there is a hyperplane which separates $\bar{q}_\ell, \ell \neq k$ from $\bar{q}_k$. Hence, there is a normal vector $x_k \in \mathbb{R}^M$ of the hyperplane so that:

$$\langle x_k, \bar{q}_\ell\rangle \geq 0 \quad \forall \ell \neq k \quad \textit{and} \quad \langle x_k, \bar{q}_k\rangle < 0. \tag{14}$$

Observe that for all ℓ

$$\bar{q}_{\ell m} = \frac{\alpha_\ell}{\zeta_m \sum_n p_{n\ell}} \frac{p_{m\ell}\zeta_m}{\alpha_\ell} = \frac{\alpha_\ell}{\zeta_m \sum_n p_{n\ell}}\, q_{\ell m}. \tag{15}$$

Let $y_k \in \mathbb{R}^M$ be defined by the components $y_{km} = x_{km}/\zeta_m$. Then (14) is equivalent to

$$\begin{aligned} & \frac{\alpha_\ell}{\sum_n p_{n\ell}}\langle y_k, q_\ell\rangle \geq 0 \quad \forall \ell \neq k \qquad \textit{and} \qquad \frac{\alpha_k}{\sum_n p_{nk}}\langle y_k, q_k\rangle < 0 && (16)\\ \Leftrightarrow \quad & \langle y_k, q_\ell\rangle \geq 0 \quad \forall \ell \neq k \qquad \textit{and} \qquad \langle y_k, q_k\rangle < 0. && (17)\end{aligned}$$

Consequently, the hyperplane with normal vector y_k separates $q_\ell, \ell \neq k$ from q_k, and this proves (a).

As for (b). Consider an arbitrary index k and probability weights β_ℓ, $\ell \neq k$. By (1):

$$\mu_k = \sum_{\ell\neq k} \beta_\ell \mu_\ell \quad \Leftrightarrow \quad \sum_m q_{km} p_m = \sum_{\ell \neq k} \beta_\ell \sum_m q_{\ell m} p_m \quad \Leftrightarrow \quad \sum_m (q_{km} - \sum_{\ell \neq k} \beta_\ell q_{\ell m}) p_m = 0. \quad (18)$$

Now, if $(q_k)_{k=1}^K$ is convexly independent, then there is an m' so that $q_{km'} - \sum_{\ell \neq k} \beta_\ell q_{\ell m'} \neq 0$. Since $(p_m)_{m=1}^M$ is linearly independent, the right equation is, therefore, violated. Hence, also the first equation is violated, but this means that $(\mu_k)_{k=1}^K$ is convexly independent. As for the reverse, suppose $(\mu_k)_{k=1}^K$ is convexly independent so that the first equation is violated. Then also the third equation is violated, and hence $q_{km} - \sum_{\ell \neq k} \beta_\ell q_{\ell m} \neq 0$ for some m. But this means that $(q_k)_{k=1}^K$ is convexly independent. Q.E.D.

Proof of Lemma 3 Note that $\kappa(\zeta, m) = k$ if and only if

$$\frac{p_{mk}\zeta_m}{\sum_n p_{nk}\zeta_n} \geq \frac{p_{m\ell}\zeta_m}{\sum_n p_{n\ell}\zeta_n} \qquad \forall \ell \neq k \quad (19)$$

$$\Leftrightarrow \qquad \sum_n (p_{mk}p_{n\ell} - p_{m\ell}p_{nk})\zeta_n \geq 0 \qquad \forall \ell \neq k. \quad (20)$$

Observe that the inequality is true for all ζ if and only if the term in brackets under the sum are positive for all $n \neq m$. But this is equivalent to Assumption 2. Q.E.D.

Proof of Lemma 4 I begin with establishing properties (6), (8) and (9). Fix m. I write the system of M equations given by (6) and (9) in matrix notation. Define the $K \times M$-matrix $A = (\mu_{\kappa(m)}, p_1, \ldots, p_{m-1}, p_{m+1}, \ldots, p_M)$ and the row vector $v = (0, s_{m1}, \ldots, s_{m,m-1}, s_{m,m+1}, \ldots, s_{mM}) \in \mathbb{R}^M$. Then the equations in (6) and (9) can be stated as

$$\sigma_{\kappa(m)}^T A = v, \quad (21)$$

where the superscript T indicates the transposed. I have to show that for all v there is a solution $\sigma_{\kappa(m)}$ to (21). Indeed, observe that the M vectors $\mu_{\kappa(m)}$, p_n, $n \neq m$, are linearly independent. This is so since the $M - 1$ vectors p_n, $n \neq m$, are linearly independent by Assumption 1, and $\mu_{\kappa(m)}$ is a convex combination of *all* p_n's with a positive weight on p_m. Hence, the matrix A has rank M, and a solution $\sigma_{\kappa(m)}$ to (21) exists, establishing (6) and (9). Moreover, (6) and (9) together with (1) directly imply inequality (8).

It remains to show (7). I begin with the remark that under Assumption 1, for all k, ℓ there is a strict inequality in (5) for some m and n. To the contrary, suppose that there are k, $\ell \neq k$

so that $p_{mk}/p_{nk} = p_{m\ell}/p_{n\ell}$ for all m and n, then

$$(\bar{q}_{km})^{-1} = \frac{\sum_n p_{nk}}{p_{mk}} = \frac{\sum_n p_{n\ell}}{p_{m\ell}} = (\bar{q}_{\ell m})^{-1}. \tag{22}$$

It follows that $\bar{q}_k = \bar{q}_\ell$, a contradiction to the convex independence of $(\bar{q}_k)_{k=1}^K$ posited in Assumption 1.

Finally, I show inequality (7). The fact that $\langle \sigma_{\kappa(m)}, \mu_{\kappa(m)} \rangle = 0$ and (1) imply

$$\langle \sigma_{\kappa(m)}, p_m \rangle = -\frac{\alpha_{\kappa(m)}}{p_{m\kappa(m)}\zeta_m} \sum_{n \neq m} \frac{p_{n\kappa(m)}\zeta_n}{\alpha_{\kappa(m)}} \langle \sigma_{\kappa(m)}, p_n \rangle = \sum_{n \neq m} \frac{p_{n\kappa(m)}\zeta_n}{p_{m\kappa(m)}\zeta_m} \langle \sigma_{\kappa(m)}, p_n \rangle. \tag{23}$$

Using this in (1) for μ_ℓ gives

$$\begin{aligned}
\langle \sigma_{\kappa(m)}, \mu_\ell \rangle &= \frac{p_{m\ell}\zeta_m}{\alpha_\ell} \langle \sigma_{\kappa(m)}, p_m \rangle + \sum_{n \neq m} \frac{p_{n\ell}\zeta_n}{\alpha_\ell} \langle \sigma_{\kappa(m)}, p_n \rangle && (24) \\
&= \sum_{n \neq m} \left[-\frac{p_{n\kappa(m)}\zeta_n}{p_{m\kappa(m)}\zeta_m} \cdot \frac{p_{m\ell}\zeta_m}{\alpha_\ell} + \frac{p_{n\ell}\zeta_n}{\alpha_\ell} \right] \langle \sigma_{\kappa(m)}, p_n \rangle && (25) \\
&= \sum_{n \neq m} \frac{\zeta_n}{\alpha_\ell} \left[-\frac{p_{n\kappa(m)} p_{m\ell}}{p_{m\kappa(m)}} + p_{n\ell} \right] \langle \sigma_{\kappa(m)}, p_n \rangle. && (26)
\end{aligned}$$

By Assumption 2, the term in the square bracket is non–negative for all n, ℓ, and, by the remark at the beginning, is positive for some n, ℓ. Together with the fact that $\langle \sigma_{\kappa(m)}, p_n \rangle > 0$ by (9), this yields the claim. Q.E.D.

Proof of Proposition 2 I construct $\lambda_k \geq \bar{\lambda}_k$ so that $\pi_m = \pi_n$ for all m, n. Let

$$s_{nm} = \prod_{n' \neq m} p_{n'\kappa(n)}, \quad s_n = \prod_{n'} p_{n'\kappa(n)}. \tag{27}$$

Then profits in (3) become

$$\begin{aligned}
\pi_m &= 2p_{m\kappa(m)} \langle \sigma_{\kappa(m)}, p_m \rangle \lambda_{\kappa(m)} + 2 \sum_{n \neq m} s_n \lambda_{\kappa(n)} + && (28) \\
&\quad +2 \sum_{\ell : \nexists n \text{ with } \kappa(n) = \ell} p_{m\ell} \langle \sigma_\ell, p_m \rangle \lambda_\ell + 2 \sum_k p_{mk} b_k - c_m.
\end{aligned}$$

Observe that if $\lambda_{\kappa(m)}$ is increased, then, since $\langle \sigma_{\kappa(m)}, p_m \rangle < 0$ and because $\kappa(m) \neq \kappa(n)$, $n \neq m$, the profit π_m decreases, while all π_n, $n \neq m$, increase. Moreover, all π_n, $n \neq m$ increase at the *same* rate $2s_m$. Therefore, if $\lambda_{\kappa(m)}$ is increased, then the difference $\pi_n - \pi_{n'}$ is unaffected for all $n, n' \neq m$. I now exploit this property to construct the desired λ step by step.

Let $\lambda' \in \mathbb{R}^K$ with $\lambda'_k \geq \bar{\lambda}_k$ for all k. If $\pi_m(\lambda') = \pi_n(\lambda')$ for all m, n we are done. Otherwise, consider the investments that give the lowest payoff:

$$N_{min}(\lambda') = \{n \mid \pi_n(\lambda') = \min_m \pi_m(\lambda')\}. \tag{29}$$

Moreover, denote by $\hat{m}$ the next best investment, which is given by:

$$\pi_{\hat{m}}(\lambda') > \min_m \pi_m(\lambda'), \qquad \pi_{\hat{m}}(\lambda') \leq \pi_m(\lambda') \quad \text{for all } m \notin N_{min}(\lambda'). \tag{30}$$

Now consider an arbitrary $n_0 \in N_{min}(\lambda')$. Because $\pi_{\hat{m}}$ decreases and π_{n_0} increases continuously in $\lambda_{\kappa(\hat{m})}$, there is a $\lambda''_{\kappa(\hat{m})} > \lambda'_{\kappa(\hat{m})}$ so that $\pi_{\hat{m}}(\lambda'') = \pi_{n_0}(\lambda'')$, where λ'' has the same components as λ' except of the $\kappa(\hat{m})$-th component, which is $\lambda''_{\kappa(\hat{m})}$.

Moreover, because all π_n, $n \neq \hat{m}$ increase at the same rate as $\lambda'_{\kappa(m)}$ is increased to $\lambda''_{\kappa(m)}$, we also have that $\pi_{\hat{m}}(\lambda'') = \pi_n(\lambda'')$ for all $n \in N_{min}(\lambda')$, so that at λ'':

$$N_{min}(\lambda'') = N_{min}(\lambda') \cup \{\hat{m}\}. \tag{31}$$

Now proceed repeatedly in the same manner. After at most M steps, this yields a λ with $\lambda_k \geq \bar{\lambda}_k$ for all k, at which all π_m are the same. Q.E.D.

Proof of Proposition 4 Consider the investment strategy ζ which places probability $1-\eta$ on the first best investment $z_{\bar{m}}$ and probability η on $z_{\bar{m}-1}$. (If $\bar{m} = 1$, then randomizing between $z_{\bar{m}}$ and $z_{\bar{m}+1}$ works.) I show that for all η smaller than some $\bar{\eta} > 0$, ζ is FSE-implementable. Consequently, the seller's ex ante profit gets arbitrarily close to the first best profit as η goes to zero.

Because beliefs μ_1 and μ_2 are convexly independent, there are σ_1 and σ_2 in $\mathbb{R}^2$ so that

$$\langle \sigma_\ell, \mu_\ell \rangle = 0 \quad \textit{and} \quad \langle \sigma_\ell, \mu_k \rangle > 0 \quad \forall k \neq \ell. \tag{32}$$

For $k = 1, 2$, define transfers $\tau_k = \lambda_k \sigma_k$ with $\lambda_k \geq \bar{\lambda}_k$, as defined in (11). By (3), the profit difference between investment $\bar{m}$ and any other investment $n \neq \bar{m}$ is

$$\pi_{\bar{m}} - \pi_n \;=\; 2[p_{\bar{m}1}\langle \sigma_1, p_{\bar{m}} \rangle - p_{n1}\langle \sigma_1, p_n \rangle]\lambda_1 2[p_{\bar{m}2}\langle \sigma_2, p_{\bar{m}} \rangle - p_{n2}\langle \sigma_2, p_n \rangle]\lambda_2 + T_1, \tag{33}$$

where T_1 is a constant independent of λ_1, λ_2. Hence, the seller is indifferent between $\bar{m}$ and $\bar{m}-1$ if and only if

$$\lambda_1 = -\frac{p_{\bar{m}2}\langle \sigma_2, p_{\bar{m}} \rangle - p_{\bar{m}-1,2}\langle \sigma_2, p_{\bar{m}-1} \rangle}{p_{\bar{m}1}\langle \sigma_1, p_{\bar{m}} \rangle - p_{\bar{m}-1,1}\langle \sigma_1, p_{\bar{m}} \rangle}\lambda_2 + T_2 \tag{34}$$

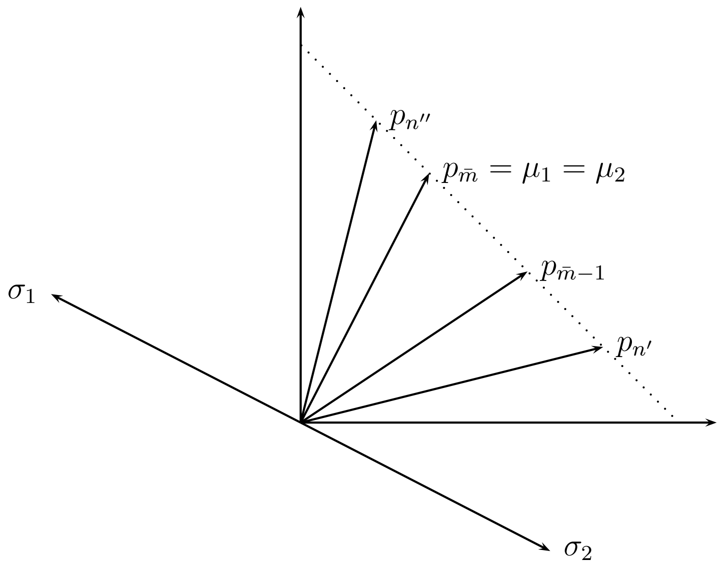

Figure 3: Ordering of beliefs and type distributions for $M > K = 2$ and $\eta = 0$.

for some constant T_2. With this, I can compare the profit of m and any other investment n:

$$\pi_{\bar m} - \pi_n = 2\left\{-\frac{p_{\bar m 2}\langle\sigma_2, p_{\bar m}\rangle - p_{\bar m-1,2}\langle\sigma_2, p_{\bar m-1}\rangle}{p_{\bar m 1}\langle\sigma_1, p_{\bar m}\rangle - p_{\bar m-1,1}\langle\sigma_1, p_{\bar m-1}\rangle}\cdot[p_{\bar m 1}\langle\sigma_1, p_{\bar m}\rangle - p_{n1}\langle\sigma_1, p_n\rangle]\right. \\ \left. +[p_{\bar m 2}\langle\sigma_2, p_{\bar m}\rangle - p_{n2}\langle\sigma_2, p_n\rangle]\right\}\cdot\lambda_2 + T_3, \tag{35}$$

where T_3 is some constant. I now show that when $\eta = 0$, the coefficient in the curly brackets in front of λ_2 is strictly positive for all $n \neq \bar m, \bar m - 1$. By continuity, this coefficient will still be strictly positive for all small $\eta > 0$. This implies that by raising λ_2, the difference $\pi_{\bar m} - \pi_n$ can be made arbitrarily large while keeping $\pi_{\bar m} - \pi_{\bar m-1}$ equal to zero. Therefore, the seller prefers the investments $z_{\bar m}$ and $z_{\bar m-1}$ over all other investments, and randomizing between $z_{\bar m}$ and $z_{\bar m-1}$ is optimal.

Indeed, for $\eta = 0$, it holds that $\mu_1 = \mu_2 = p_{\bar m}$ and, given the orientation of σ_1 and σ_2:

$$\sigma_1 = \begin{pmatrix} -p_{\bar m 2} \\ p_{\bar m 1} \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} p_{\bar m 2} \\ -p_{\bar m 1} \end{pmatrix}. \tag{36}$$

Thus, we have that $\langle\sigma_1, p_{\bar m}\rangle = \langle\sigma_2, p_{\bar m}\rangle = 0$ and $\langle\sigma_1, p_n\rangle = -\langle\sigma_2, p_n\rangle$ for all n. Figure 3 illustrates.

With this, the coefficient in front of λ_2 gets

$$\left\{-\frac{-p_{\bar{m}-1,2}\langle\sigma_2, p_{\bar{m}-1}\rangle}{-p_{\bar{m}-1,1}\langle\sigma_1, p_{\bar{m}-1}\rangle}\cdot[-p_{n1}\langle\sigma_1, p_n\rangle]+[-p_{n2}\langle\sigma_2, p_n\rangle]\right\} \tag{37}$$

$$= \quad \langle\sigma_1, p_n\rangle[-\frac{p_{\bar{m}-1,2}}{p_{\bar{m}-1,1}}\cdot p_{n1}+p_{n2}]. \tag{38}$$

I now argue that this expression is positive. Observe first that the term in the square brackets is positive if and only if

$$-(1-p_{\bar{m}-1,1})p_{n1}+p_{\bar{m}-1,1}(1-p_{n1})>0 \quad \Leftrightarrow \quad p_{\bar{m}-1,1}>p_{n1} \quad \Leftrightarrow \quad n>\bar{m}-1. \tag{39}$$

Observe moreover (see Figure 3) that $\langle\sigma_1, p_n\rangle > 0$ if and only if $n > \bar{m}$. These two observations imply that the term in front of λ_2 in (35) is strictly positive for all $n \neq \bar{m}, \bar{m}-1$. This completes the proof. Q.E.D.

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