Idiosyncratic Risk, Expected Windfall, and the Cross-Section of Stock Returns

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Outline

- A. Literature Review:
  Idiosyncratic Risk and the Idiosyncratic Volatility Puzzle
- B. Data
- C. A Novel Methodology for Robust Estimation of Idiosyncratic Volatility
- D. Cross-Sectional Portfolio Analysis
- E. Cross-Sectional Regression Tests
- F. Concluding Remarks

Contribution

1. Develop A Novel Methodology for Robust Estimation of Conditional Idiosyncratic Volatility
2. Identify Two Risk Factors on Cross-sectional Stock Returns:
   (1) Idiosyncratic Variance,
   (2) Firm-Level Return Skewness (measured by Expected Windfall).

   Solve the Major Piece of the Idiosyncratic Volatility Puzzle!

A. Literature Review

- Systematic Risk (Market Risk) and Idiosyncratic Risk (Firm-Specific Risk)
- Capital Asset Pricing Model (CAMP)
- Idiosyncratic Risk Matters!
  - Coetzmann and Kumar (2004)
  - Falkenstein (1996); Day, Wang, and Xu (2000)
  - Campbell, Lettau, Malkiel and Xu (2001)
- Theoretical Perspective - A **Positive** Idiosyncratic Volatility Effect at the Individual Stock Level
  - Levy(1978), Merton(1987), Malkiel and Xu(2001)
  - Barberis and Huang (2001)
- Empirical Evidence - Still Mixed!
  - A **Negative** Idiosyncratic Volatility Effect: Idiosyncratic Volatility Puzzle
    High (Realized) Idiosyncratic Volatility vs. Low Returns
    (Ang, Hodrick, Xing and Zhang (2006 JoF, JFE forthcoming))
  - A **Positive** Idiosyncratic Volatility Effect
    (Fu (JFE forthcoming); Spiegel and Wang (2007); Eiling (2006); Brockman and Schutte (2007))

A.1 Idiosyncratic Volatility (Literature Review con'd)

- Market Model: Fama-French (1993) Three-Factor Model:

$$r_{i,t} - r_t^f = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \underbrace{u_{i,t}}_{\text{Idiosyncratic Shock}}$$

- Idiosyncratic shocks are measured as OLS residuals $u_{i,t}$, which cannot be priced by market factors.

- Idiosyncratic Volatility $\sigma_{i,t} \equiv Std.Dev\left(u_{i,t}\right)$

A.2 Idiosyncratic Volatility Puzzle (Literature Review con'd)

- OLS Estimation of the Realized Idiosyncratic Volatility with Daily Returns: For the $i-th$ stock within the month t:

$$
\begin{aligned}
r_{i,s} - r_s^f &= \alpha_i + \beta_{i,MKT} MKT_s + \beta_{i,SMB} SMB_s + \beta_{i,HML} HML_s + u_{i,s} \\
\text{where } s &= 1, \ldots, N_{i,t}
\end{aligned}
$$

  Monthly Idiosyncratic Volatility $\widehat{\sigma_{i,t}} = \sqrt{N_{i,t}} \times sd\left(\widehat{u_{i,s}}\right)$

- **Idiosyncratic Volatility Puzzle** (Table VI, Andrew Ang et. al (2006)): *High (Realized) Idiosyncratic Volatility vs. Low Future Return*

Rank	Return	Std.Dev.	Mkt Share
1 (low)	1.04	3.83	53.5
2	1.16	4.74	27.4
3	1.20	5.85	11.9
4	0.87	7.13	5.2
5 (high)	−0.02	8.16	1.9
5 − 1	−**1.06** [−3.01]		

A.3 Positive Idiosyncratic Volatility Effect (Literature Review con'd)

- EGARCH Estimates of Conditional Idiosyncratic Volatility

$$
\begin{aligned}
r_{i,t} - r_t^f &= \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + u_{i,t} \\
u_{i,t} &= \sigma_{i,t} \varepsilon_{i,t},\ \varepsilon_{i,t} \overset{iid}{\sim} N(0,1) \\
\ln \sigma^2_{i,t} &= a_i + b_i \ln \sigma^2_{i,t-1} + c_i \left\{ \theta \varepsilon_{i,t-1} + \gamma \left[|\varepsilon_{i,t-1}| - \sqrt{2/\pi} \right] \right\}
\end{aligned}
$$

- A Positive Relationship Between Conditional Idiosyncratic Volatility and Expected/Average Stock Returns
  (Fu (2008); Spiegel and Wang (2007); Eiling (2006); Brockman and Schutte (2007))

A.3 Positive Idiosyncratic Volatility Effect (Literature Review con'd)

- Portfolios Sorted by EGARCH Estimates of Conditional Idiosyncratic Volatility (Table 6, Spiegel and Wang (2007))

Rank	return	Std.Dev.	Mkt Share
1(low)	0.03	3.72	27.48
2	0.96	7.19	24.36
3	1.19	4.7	16.04
4	1.17	7.57	10.98
5	0.98	6.94	7.34
6	1.00	5.11	5.08
7	0.98	4.25	3.52
8	1.09	5.68	2.42
9	0.96	6.59	1.71
10 (high)	1.36	8.15	1.08
10 − 1	**1.33** [3.21]		

B. Data

- Time Span: July 1964 ~December 2006 (510 months)
- CRSP: Monthly stock returns, stock prices and outstanding share numbers for stocks traded on NYSE, AMEX and NASDAQ
- COMPUSTAT: Book-values of asset/equity
- Kenneth French's Online Database: Risk-free interest rate, Fama-French three factors
- $1,926,356$ return observations for $12,051$ stocks over the span of 510 months

C.1 Revisit: Are We There Yet?

- Time-Persistence of Idiosyncratic Volatility
- Firm-level Gaussian-Innovation Assumption in the EGARCH Model (Fu (2008); Spiegel and Wang (2007); Eiling (2006); Brockman and Schutte (2007))

$$\forall i, \{\varepsilon_{i,t}\}_t \overset{iid}{\sim} N(0,1)$$

Significance Level	1%	5%
$H_0 : Skewness = 0$	74%	81%
$H_0 : ExcessKurtosis = 0$	80%	88%
$H_0 : Normality$	83%	**90%**

- At the 5% significance level, the Gaussian-innovation assumption is rejected by over 90% stocks traded on NYSE, NASDAQ and AMEX!

C.2 Robust Estimation of Conditional Volatility: A Quantile-Regression Based Approach

- Linear TGARCH(1,1)

$$
\begin{aligned}
r_{i,t}-r_t^f &= \alpha_i+\beta_{i,MKT}MKT_t+\beta_{i,SMB}SMB_t+\beta_{i,HML}HML_t+u_{i,t} \\
u_{i,t} &= \sigma_{i,t}\varepsilon_{i,t},\ \underline{\underline{\varepsilon_{i,t}\overset{iid}{\sim}F_i(0,1)}} \\
\sigma_{i,t} &= \theta_{i,0}+\theta_{i,1}\sigma_{i,t-1}+\gamma_{i,1}|u_{i,t-1}|+\gamma_{i,2}|u_{i,t-1}|\times I(u_{i,t-1}<0)
\end{aligned}
$$

- $F_i(\cdot)$: the firm-specific innovation distribution

- Robust Estimation of Conditional Volatility and Quantiles: An Iterative Quantile-Regression Based Algorithm

$$Q_i(\tau)=F_i^{-1}(\tau)$$

$$
\left.\begin{aligned}
(\theta_i,\gamma_i) &= \arg\min_{\theta_i,\gamma_i}\textstyle\sum_t \rho_\tau\left[u_{i,t}-\sigma_{i,t}(\theta_i,\gamma_i)Q_i(\tau)\right] \\
Q_i(\tau) &= \arg\min_{\xi}\textstyle\sum_t \rho_\tau\left[\frac{u_{i,t}}{\sigma_{i,t}(\theta_i,\gamma_i)}-\xi\right]
\end{aligned}\right\}\Rightarrow\begin{cases}\widehat{\sigma_{i,t}} \\ \widehat{Q_i}(\tau)\end{cases}
$$

Quantile Regression: A Very Brief Introduction

$$y = X\beta + \varepsilon$$

- Conditional Mean: Ordinary Least Squares (OLS)

$$\widehat{\beta_{OLS}} = \arg\min_{\beta} \sum_i \left(y_i - x_i'\beta\right)^2, \qquad E[y|X] = X\widehat{\beta_{OLS}}$$

- Conditional Median: Least Absolute Deviations (LAD)

$$\widehat{\beta_{LAD}} = \arg\min_{\beta} \sum_i \left|y_i - x_i'\beta\right|, \qquad Q_y(0.5|X) = X\widehat{\beta_{LAD}}$$

- **Quantile Regression**

$$\begin{aligned} \widehat{\beta_\tau} &= \arg\min_{\beta} \sum_i \rho_\tau(y_i - x_i'\beta) \\ &= \arg\min_{\beta} \sum_{i|y_i - x_i'\beta > 0} \tau \cdot \left|y_i - x_i'\beta\right| + \sum_{i|y_i - x_i'\beta \le 0} (1-\tau) \cdot \left|y_i - x_i'\beta\right| \\ Q_y(\tau|X) &= X\widehat{\beta_\tau} \end{aligned}$$

Check Function: $\rho_\tau(v) \equiv v[\tau - I(v < 0)]$.

C.3 Robustness Comparison

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{T}\sum_{t=1}^{T}\left(\widehat{\sigma_{i,t}} - \sigma_{i,t}\right)^2$$

	Normal	$t(df=5)$	$t(df=3)$	*skewed* $t(df=3, s=5)$
1. MLE of Conditional idiosyncratic Volatility with the Gaussian-Innovation Assumption)				
$T=60$	9.86×10^{-4}	8.42×10^{-3}	6.69×10^{-3}	3.72×10^{-3}
$T=300$	2.61×10^{-4}	4.59	5.26×10^{-3}	0.123
$T=500$	1.45×10^{-4}	4.98×10^{-3}	5.83×10^{-3}	0.235
2. Quantile-Regression Based Estimates of Conditional idiosyncratic Volatility				
$T=60$	1.15×10^{-3}	1.33×10^{-3}	2.27×10^{-3}	1.34×10^{-3}
$T=300$	3.56×10^{-4}	5.77×10^{-4}	9.82×10^{-4}	5.46×10^{-4}
$T=500$	2.85×10^{-4}	4.84×10^{-4}	7.12×10^{-4}	4.28×10^{-4}

- The misspecified Gaussian-innovation assumption causes severe estimation errors in the MLEs of idiosyncratic volatilities!
- The proposed quantile-regression based method is able to robustly estimate conditional idiosyncratic volatilities for different return processes.

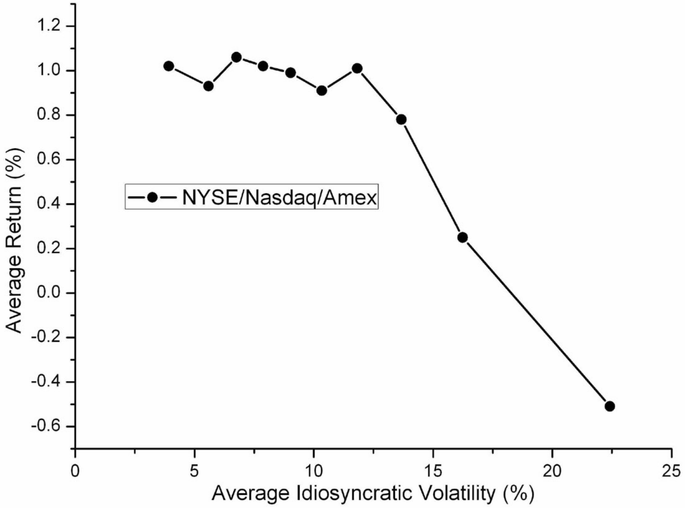
D.1 Reexamine the Idiosyncratic Volatility Puzzle: A Cross-Sectional Portfolio Analysis

Rank	E(IVol)	Return	Std.Dev.	Mkt S	Price
1 (low)	3.92	1.02	3.69	31.04	42.71
2	5.59	0.93	4.16	25.27	67.51
3	6.76	1.06	4.59	15.44	33.44
4	7.88	1.02	5.22	9.87	27.10
5	9.03	0.99	5.82	6.89	23.36
6	10.34	0.91	6.57	4.69	19.83
7	11.82	1.01	7.24	3.03	16.59
8	13.67	0.78	8.31	1.92	13.36
9	16.24	0.25	8.54	1.22	10.40
10 (high)	22.41	−0.51	9.49	0.63	6.86
10 − 1		**−1.53** [−4.19]			

- The Puzzle becomes even more puzzling: a contemporaneous **negative** idiosyncratic volatility effect that is just the opposite of the findings of Fu (2008) and Spiegel and Wang (2007).

D.2 Observation 1

- **Idiosyncratic Volatility Effect — More Than A Linear Effect**
  Average Return (col. Ret) vs. Average Idiosyncratic Volatility (col. E(IVol))



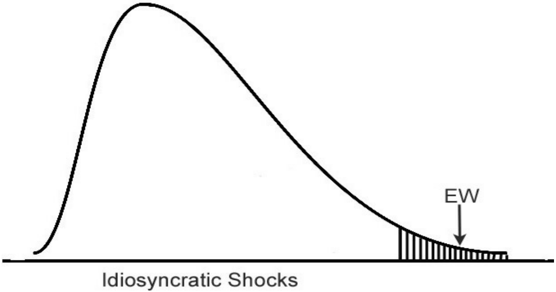
D.3 Observation 2

- The most volatile stocks are **small-cap** and **low-price** stocks!
  Chen, Hong and Stein (2001), Deffee (2002), Zhang (2005) — Small stocks tend to have more positively skewed return distributions than large stocks
- Firm-level expected skewness can be priced.
  "Stocks as Lotteries", Barberis and Huang, 2008, AER.
- The measure of firm-level expected skewness
  - Past skewness
  - Intra-industry skewness (Zhang, WP Yale)

D.3 Observation 2 (Con'd)

- **Expected Windfall** — An alternate measure for firm-level expected skewness

$$EW_x(\tau) \equiv \frac{1}{1-\tau} E[x | x \geq F_x^{-1}(\tau)]$$



- Computional Advantage: Bassett, Koenker and Kordas (2004)

$$\widehat{EW_{i,t}}(\tau) = \frac{1}{1-\tau} \frac{1}{t} \min_{\zeta} \sum_{s=1}^{t} \rho_\tau(u_{i,s} - \zeta) - \frac{\tau}{1-\tau} \frac{1}{t} \sum_{s=1}^{t} u_{i,s}$$

E.1 Fama-MacBeth Cross-Sectional Regression Tests

- Fama-MacBeth (1973) Cross-Sectional Regression Test

$$
\begin{aligned}
R_{i,t} &= \gamma_{0,t} + \sum_{k=1}^{K} \gamma_{k,t} X_{i,k,t} + \nu_{i,t} \\
\widehat{\gamma_k} &= \frac{1}{T} \sum_{t=1}^{T} \widehat{\gamma_{k,t}} \\
var\left(\widehat{\gamma_k}\right) &= \frac{\sum_{t=1}^{T} \left(\widehat{\gamma_{k,t}} - \widehat{\gamma_k}\right)^2}{T(T-1)} \\
t_{FM} &= \frac{\widehat{\gamma_k}}{\sqrt{var\left(\widehat{\gamma_k}\right)}}
\end{aligned}
$$

- $\widehat{\gamma_k}$ indicates the predictive power of the $k-th$ regressor on the expected returns.

E.1 Fama-MacBeth Cross-Sectional Regression Tests (con'd)

- Idiosyncratic Volatility Effect — More Than A Linear Effect

$$\widehat{R_{i,t}} = \widehat{\gamma_{0,t}} + \widehat{\gamma_{1,t}}\sigma_{i,t} + \widehat{\gamma_{2,t}}\sigma^2_{i,t} + \ldots\ldots$$

	const	σ	σ^2	*BETA*	ln (*size*)	*B* / *M*	$ret_{-2:-7}$	$\overline{F}$
(1)	−0.007 [−0.35]	**0.02** [0.845]		0.17 [14.43]	0.012 [18.0]	0.002 [1.26]	0.73 [43.99]	–
(2)	0.043 [2.33]	**0.334** [7.63]	**−1.798** [−10.02]	0.17 [14.29]	0.013 [19.09]	0.002 [1.65]	0.712 [44.01]	**50.3**

- $\frac{\partial \widehat{R_i}}{\partial \sigma_i} = \widehat{\gamma_1} - 2\widehat{\gamma_2}\sigma_i > 0 \Rightarrow \sigma_i < \frac{\widehat{\gamma_1}}{2\times\widehat{\gamma_2}} = 9.28\%$

A positive idiosyncratic volatility effect for stocks comprising about 85% of the total market cap!

E.1 Fama-MacBeth Cross-Sectional Regression Tests (con'd)

- Firm-level Return Skewness — Expected Windfall

	const	σ	σ^2	$EW_{95\%}$	*BETA*	ln (*size*)	*B/M*	$ret_{-2:-}$
(1)	−0.007	0.02			0.172	0.012	0.002	0.73
	[−0.35]	[0.845]			[14.43]	[18.0]	[1.26]	[43.99]
(2)	0.043	0.33	−1.8		0.168	0.0128	0.002	0.712
	[2.33]	[7.63]	[−10.02]		[14.29]	[19.09]	[1.65]	[44.01]
(3)	0.066	**0.69**	−**1.81**	−**0.123**	0.166	0.013	0.0024	0.707
	[3.50]	[9.60]	[−10.0]	[−9.58]	[14.27]	[19.86]	[2.0]	[44.70]

- $\frac{\partial \widehat{R_i}}{\partial \sigma_i} = \widehat{\gamma_1} - 2\widehat{\gamma_2}\sigma_i > 0 \Rightarrow \sigma_i < \frac{\widehat{\gamma_1}}{2 \times \widehat{\gamma_2}} = 19\%$

Solve the major piece of the idiosyncratic volatility puzzle: a positive idiosyncratic volatility effect for stocks comprising about 99% of the total market cap!

- $\sigma_{i,t} \geq 19\% : \overline{price} = \$7.53,\ \overline{MCap} = \$93m$

E.2 Cross-Sectional Quantile-Regression Test

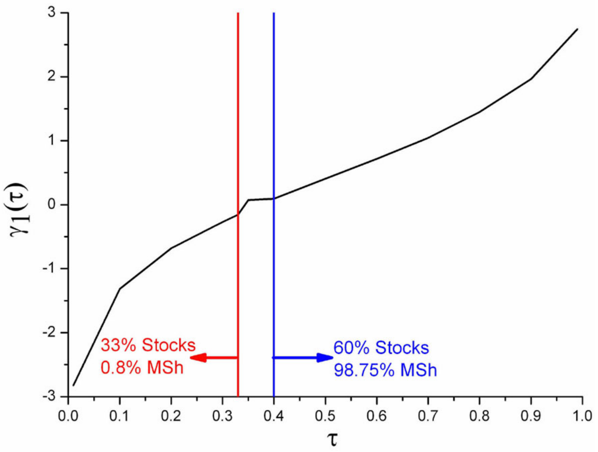
- Quantile Regression Analysis of the predictability of Cross-sectional Returns

$$
\begin{aligned}
R_{i,t}(\tau) &= \gamma_{0,t}(\tau) + \gamma_{1,t}(\tau)\sigma_{i,t} + \sum_{k=2}^{K} \gamma_{k,t}(\tau) X_{i,k,t} + \nu_{i,t}(\tau) \\
\widehat{\gamma_k}(\tau) &= \frac{1}{T} \sum_{t=1}^{T} \widehat{\gamma_{k,t}}(\tau) \\
var\left[\widehat{\gamma_k}(\tau)\right] &= \frac{\sum_{t=1}^{T} \left(\widehat{\gamma_{k,t}}(\tau) - \widehat{\gamma_k}(\tau)\right)^2}{T(T-1)}
\end{aligned}
$$

- $\widehat{\gamma_k}(\tau)$ indicates the predictive power of the $k-th$ regressor on returns at the $\tau - th$ quantile of the cross-sectional stock return distribution.

E.2 Cross-Sectional Quantile-Regression Test (con'd)

- Quantile-dependent Idiosyncratic Volatility Effect



- A positive and *statistically significant* idiosyncratic volatility effect for stocks comprising **98.8**% of the total market cap of the NYSE, NASDAQ and AMEX exchanges!

F. Contribution & Concluding Remarks

- Robustly Estimate Conditional Idiosyncratic Volatility
- Cross-Sectional Portfolio Analysis
  - The idiosyncratic volatility puzzle exists intertemporally!
  - Two Observations
    - Non-linear Idiosyncratic Volatility Effect — Idiosyncratic Variance
    - Firm-level Return Skewness — Expected Windfall
- Cross-Sectional Regression Tests
  - **Solve the major piece of the idiosyncratic volatility puzzle**: A positive idiosyncratic volatility effect for about 99% of the total market capitalization
- Next
  - Distress Risk (Default Risk)
  - China Stock Market