Hot and Cold Seasons in the Housing Market*

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Abstract

Every year during the second and third quarters (the "hot season") housing markets in the UK and the US experience systematic above-trend increases in both prices and transactions. During the fourth and first quarters (the "cold season"), house prices and transactions fall below trend. We propose a search-and-matching framework that sheds new light on the mechanisms governing housing market fluctuations. The model has a "thick-market" effect that can generate substantial differences in the volume of transactions and prices across seasons, with the extent of seasonality in prices depending crucially on the bargaining power of sellers. The model can quantitatively mimic the seasonal fluctuations in transactions and prices observed in the UK and the US.

Key words: housing market, thick-market effects, search-and-matching, seasonality, house price fluctuations.

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1 Introduction

A rich empirical and theoretical literature has been motivated by dramatic boom-to-bust episodes in regional and national housing markets.[1] Booms are typically defined as times when prices rise and there is intense trading activity, whereas busts are times when prices and trading activity fall below trend.

While the boom-to-bust episodes motivating the extant work are relatively infrequent and of unpredictable timing, this paper shows that in several housing markets, booms and busts are just as frequent and predictable as the seasons. In particular, in most regions of the UK and the US, every year a housing boom of considerable magnitude takes place in the second and third quarters of the calendar year (the "hot season"), followed by a bust in the fourth and first quarters (the "cold season"). The predictable nature of house price fluctuations (and transactions) is furthermore confirmed by estate agents, who in conversations with the authors observed that during winter months there is less activity and "owners tend to sell at a discount." Perhaps more compelling, publishers of house price indexes go to great lengths to produce seasonally adjusted versions of their indexes, usually the index that is published in the media. As stated by publishers:

"House prices are higher at certain times of the year irrespective of the overall trend. This tends to be in spring and summer, when more buyers are in the market and hence sellers do not need to discount prices so heavily in order to achieve a sale," and *"...we seasonally adjust our prices because the time of year has some influence. Winter months tend to see weaker price rises and spring/summer see higher increases all other things being equal."* (From Nationwide House Price Index Methodology.)

"Houses prices are seasonal with prices varying during the course of the year irrespective of the underlying trend in price movements. For example, prices tend to be higher in the spring and summer months when more people are looking to buy." (From Halifax Price Index Methodology.)

The first contribution of this paper is to systematically document the existence and quantitative importance of these seasonal booms and busts.[2] For the UK as a whole, we find that the difference

[1] See for example Stein (1995), Muellbauer and Murphy (1997), Genesove and Mayer (2001), Krainer (2001), Brunnermeier and Julliard (2008), and the contributions cited therein.

[2] Studies on housing markets have typically glossed over the issue of seasonality. There are a few exceptions, albeit they have been confined to only one aspect of seasonality (e.g., either quantities or prices) or to a relatively small geographical area. In particular, Goodman (1993) documents pronounced seasonality in *moving patterns* in the US, Case and Shiller (1989) find seasonality in prices in Chicago and—to a lesser extent—in Dallas, and Hosios and Pesando (1991) find seasonality in prices in the City of Toronto; the latter conclude "that individuals who are willing

in annualized growth rates between hot and cold seasons is above 8 percent for nominal prices (6 percent for real prices) and 108 percent for transactions. For the US as a whole, the corresponding differences are above 3 percent for nominal (and real) prices and 148 percent for transactions, though there is considerable variation within the country (particularly for prices).

The predictability and size of seasonal fluctuations in house prices poses a challenge to standard models of durable-good markets. In those models, anticipated changes in prices cannot be large: If prices are expected to be much higher in May than in December, then buyers will shift their purchases to the end of the year, narrowing down the seasonal price differential. More concretely, a typical no-arbitrage condition states that seasonality in prices must be accompanied by seasonality in rental flows or in the cost of housing services. Rents, however, display no seasonality, implying a substantial and, as we shall argue, unrealistic degree of seasonality in service costs.[3,4] A possible explanation for why standard no-arbitrage conditions fail is of course that transaction costs are very high and hence investors do not benefit from arbitrage. Still, the question remains as to why presumably informed buyers do not try to buy in the low-price season. Furthermore, it is not clear why we observe a systematic seasonal pattern. (The lack of scope for seasonal arbitrage does not necessarily imply that most transactions should be carried out in one season, nor does it imply that prices and transactions should be correlated.) To offer answers to these questions, we develop a search-and-matching model for the housing market. The model more realistically captures the process of buying and selling houses and it can more generally shed new light on the mechanisms governing housing market fluctuations.

The model starts from the premise that the utility potential buyers derive from a house is match-specific; so, for example, two individuals visiting the same house may attach a different value to it and hence have different willingness to pay. In that context, buyers are more likely to find a higher-quality match (and thus their willingness to pay is more likely to increase) when there are

to purchase against the seasonal will, on average, do considerably better."

[3]For example, the degree of price seasonality observed in the UK implies that service costs should be at least 200 percent higher in the "cold" season than in the "hot" season—see Appendix 7.2. This seems unlikely, particularly because interest rates and tax rates, two major components of service costs, display no seasonality.

[4]Seasonality in housing markets does not seem to be driven by seasonal differences in liquidity related to overall income. Income is typically high in the last quarter, a period in which house prices and the volume of transactions tend to fall below trend. Beaulieu and Miron (1992) and Beaulieu, Miron, and MacKie-Mason (1992) show that in most countries, including the UK and the US, income peaks in the fourth quarter of the calendar year. There is also a seasonal peak in output in the second quarter, and seasonal recessions in the first and third quarters. House price seasonality thus is not in line with income seasonality: Prices are above trend in the second and third quarters.

more houses for sale. Hence, in a thick market (or hot season), sellers can charge higher prices.[5] Because prices are higher, potential sellers are more willing to sell their houses, better matches are formed, willingness to pay increases, and so on. This mechanism thus leads to a higher number of transactions and prices in the hot season.

In the baseline model, we distinguish seasons by differences in the ex-ante propensity to move. These differences may arise, for example, from the school calendar: Families may prefer to move in the summer, before sending their children to new schools, or from other factors, such as weather. These differences alone, however, are too small to explain the full extent of seasonality we document.[6] Most of the explanatory power of the model is due to the match-quality effect. We show that a slightly higher *ex-ante* probability of moving in a given season can trigger thick-market effects that make it appealing to a large number of agents to buy and sell during that season. This amplification mechanism can create substantial seasonality in transactions; the extent of seasonality in prices, in turn, increases with the bargaining power of sellers. Intuitively better matches in the hot season imply higher surpluses to be shared between buyers and sellers; to the extent that sellers have some bargaining power, this leads to higher prices in the hot season. The calibrated model can quantitatively account for most of the seasonal fluctuations in transactions and prices in the UK and the US.[7]

The contribution of the paper can be summarized as follows. First, it systematically documents seasonal booms and busts in housing markets; it argues that the predictability and high extent of seasonality in prices cannot be quantitatively reconciled with the standard asset-pricing equilibrium condition embedded in most models of housing markets (or consumer durables, more generally). Second, it develops a search-and-matching model that can quantitatively account for the seasonal patterns of prices and transactions observed in the UK and the US. The model is more general than its current application and can be adapted to study lower-frequency movements in house prices and transactions.

[5] The labor literature distinguishes the thick-market effects due to faster arrival of offers and those due to the quality of the match. Our focus is entirely on the quality effect. See for example Diamond (1981) and Petrongolo and Pissarides (2006) and Gautier and Teulings (2008).

[6] For example, parents of school-age children account for only a small fraction of total movers. (See Goodman, 1993.) And although weather may make house search more convenient in the summer, it is unlikely that this convenience is worth so much money to the typical buyer.

[7] Our focus on these two countries is largely driven by the reliability and quality of the data.

The paper is organized as follows. Section 2 presents the empirical evidence. Section 3 introduces the model. Section 4 presents the qualitative results and a quantitative analysis of the model, confronting it with the empirical evidence. Section 5 discusses the efficiency properties of the model and studies the robustness of the results to alternative modelling assumptions. Section 6 presents concluding remarks. Analytical derivations and proofs are collected in the Appendix.

2 Hot and Cold Seasons

In this Section we study seasonality in housing markets in the US and the UK at different levels of geographic disaggregation.[8] As said, publishers of house price indexes produce both seasonally adjusted (SA) and non-seasonally adjusted (NSA) series.[9] In Appendix 7.1 we report the seasonal component implied by their adjustment. In our analysis, we use exclusively the (raw) NSA series to compute the extent of seasonality.

2.1 Data

UK

In the UK two main sources provide quality-adjusted NSA house price indexes: One is the Department of Communities and Local Government (DCLG) and the other is Halifax, one of the country's largest mortgage lenders.[10] Both sources report regional price indexes on a quarterly basis for the 12 standard planning regions of the UK, as well as for the UK as a whole. The indexes calculated

[8] We focus on the US and the UK because of the availability of *quality-adjusted* price data in both countries. This mitigates any concern with compositional changes in the types of houses transacted across seasons. Results for other countries are available from the authors. (Though we find qualitatively similar seasonal patterns in other countries, we are less confident about the comparability of the data, as typically they are not quality adjusted.)

[9] The US National Association of Realtors also produces NSA and SA data for transactions.

[10] Other price publishers, like Nationwide Building Society, report quality adjusted data but they are already SA (the NSA data are not publicly available). Nationwide Building Society, however, reports in its methodology description that June is generally the strongest month for house prices and January is the weakest; this justifies the SA they perform in the published series. In a somewhat puzzling paper, Rosenthal (2006) argues that seasonality in Nationwide Building Society data is elusive; we could not, however, gain access to the NSA data to assess which of the two conflicting assessments (Nationwide Building Society's or Rosenthal's) was correct. We should perhaps also mention that Rosenthal (2006) also reaches very different conclusions from Muellbauer and Murphy (1997) with regards to lower-frequency movements. Finally, the Land Registry data reports average prices, without adjusting for quality.

are 'standardized' and represent the price of a typically transacted house. The standardization is based on hedonic regressions that control for a number of characteristics, including location, type of property (house, sub-classified according to whether it is detached, semi-detached or terraced, bungalow, flat), age of the property, tenure (freehold, leasehold, feudal), number of rooms (habitable rooms, bedrooms, living-rooms, bathrooms), number of separate toilets, central heating (none, full, partial), number of garages and garage spaces, garden, land area, road charge liability, etc. These controls adjust for the possibility of seasonal changes in the composition of the set of properties (for example, shifts in the location or sizes of properties).

The two sources differ in three respects. First, DCLG collects information from a sample of all mortgage lenders in the country, while the Halifax index uses all the data from Halifax mortgages only, which account for an average of 25 percent of the market (re-mortgages and further advances are excluded in both cases). Second, DCLG reports the price at the time of completion of the transaction, while Halifax reports the price at the time of approval of the mortgage. Completion takes on average three to four weeks following the initial agreement, but some agreed transactions do not reach completion. Finally, the DCLG index goes back to 1963 for certain regions, while Halifax starts in 1983.

To compute real price indexes, we later deflate the house price indexes using the NSA retail price index (RPI) provided by the UK Office for National Statistics.

As an indicator of the number of transactions, we use the number of mortgages advanced for home purchases; the data are collected by the Council of Mortgage Lenders (CML) and are also disaggregated by region.

US

The main source of NSA house price indexes for the US is OFHEO; we focus on the purchase-only index, which starts in 1991:01. This is a repeat-sale index calculated for the whole of the US and also disaggregated by Census regions and states. The repeat-sale index, introduced by Case and Shiller (1987), measures average price changes in repeat sales of the same properties; as such, the index is designed to control for the quality of the homes sold. We also study the Case-Shiller index carried out by Standard & Poor's for 20 big cities and a composite of major cities; this index is also a repeat-sale, purchase-only and starts in 1987:01.

To compute real price indexes, we use the NSA consumer price index (CPI) provided by the US

Bureau of Labor Statistics.[11]

Data on the number of transactions come from the National Association of Realtors (NAR), and correspond to the number of sales of existing single-family homes. The data are disaggregated into the four major Census regions.

2.2 Extent of Seasonality

We focus our study on deterministic seasonality, which is easier to understand (and to predict) for buyers and sellers (unlikely to be all econometricians), and hence most puzzling from a theoretical point of view. In the UK and the US, the average quarterly growth rates in prices in both the second and third quarters are above the average growth rates for the periods we analyze, while the quarterly growth rates in both fourth and first quarters are below average. For ease of exposition, we group data into two broadly defined seasons—second and third quarter, or "hot season", and fourth and first quarter, or "cold season". (We use interchangeably the terms hot season and summer term to refer to the second and third quarters and cold season and winter term to refer to the first and fourth quarters.)

In the next set of Figures, we depict in dark (red) bars the average (annualized) price increase from winter to summer, $\ln\left(\frac{P_S}{P_W}\right)^2$, where P_S is the price index at the end of the hot season and P_W is the price at the end of the cold season. Correspondingly, we depict in light (blue) bars the average (annualized) price increase from summer to winter $\ln\left(\frac{P_{W'}}{P_S}\right)^2$, where $P_{W'}$ is the price index at the end of the cold season of the following year. We plot similar Figures for transactions.

The extent of seasonality for each geographical unit can then be measured as the difference between the two bars. This measure nets out lower-frequency fluctuations affecting both seasons. (In the model we later present, we use a similar metric to gauge the extent of seasonality.)

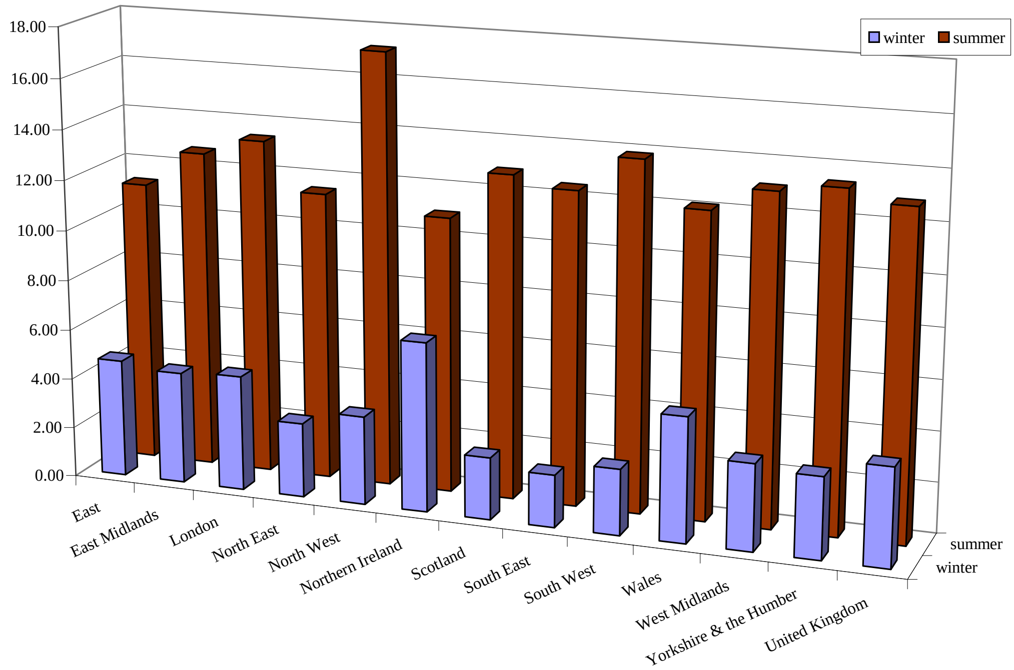
2.2.1 Housing Market Seasonality in the UK

Nominal and Real House Prices Figure 1 reports the average annualized percent price increases in the summer term and the winter term from 1983 through to 2007 using the regional price indexes provided by DCLG. During the period analyzed, the average nominal price increases in the winter term were below 5 percent in all regions except for Northern Ireland. In the summer

[11] As it turns out, there is little seasonality in the US CPI index, a finding first documented by Barsky and Miron (1989), and hence the seasonal patterns in nominal and real housing prices coincide. The CPI is reported at monthly frequency. We take the last month of the quarter to deflate nominal prices.

term, the average growth rates were above 12 percent in all regions, except for Northern Ireland, East Anglia, and the North East. As shown in the graph, the differences in growth rates across the two broad seasons are generally very large and economically significant, with an average of 9 percent for all regions. (For some regions, the DCLG index goes back to 1968, and though the average growth rates are lower in the longer period, the average difference across seasons is still very high at above 8 percent.[12])

Figure 1: Average annualized house price increases in summers and winters. DCLG 1983-2007.



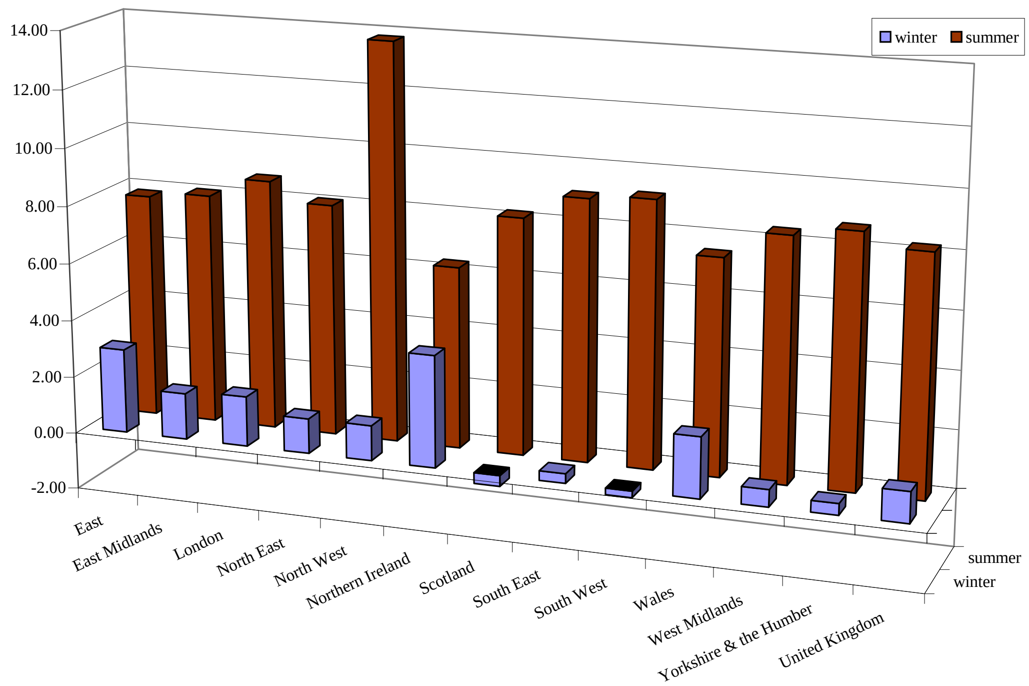
Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DCLG, 1983-2007.

The patterns are qualitatively similar when we use the Halifax index, not reported here in the interest of space (results are available from the authors). The annualized average price growth during the summer term is above 11 percent in all regions, with the exception of the North East and West Midlands, whereas the increase during the winter term is systematically below 5 percent, except for the North East region and London, where the increase is just above 5 percent. The average difference in growth rates across seasons is 7.4 percent. There are some non-negligible quantitative differences between the two sources, which might be partly explained by differences in coverage and by the lag between approval and completion, which, as mentioned, is one important difference between the two indexes. The two sources, however, point to a similar pattern of prices surging in the summer and stagnating in the winter.

[12] Results are available from the authors. We start in 1983 for comparability with the Halifax series.

The previous discussion was based on the seasonal pattern of nominal house prices. The seasonal pattern of *real* house prices (that is, house prices relative to the overall NSA price index) depends also on the seasonality of overall inflation. In the UK, overall price inflation displays some seasonality. The difference in overall inflation rates across the two seasons, however, can hardly "undo" the differences in nominal house price inflation, implying a significant seasonal also in real house prices. (See Figure 2.) Netting out the effect of overall inflation reduces the differences in growth rates between winters and summers to a country-wide average of 7.3 percent using the DCLG series and 5.6 using the Halifax series. We also looked at more disaggregated data, distinguishing between first-time buyers and former-owner occupiers, as well as purchases of new houses versus existing houses. Seasonal patterns were similar across the various groups; in the interest of space, we do not report the results here, but they are available upon request.

Figure 2: Average annualized real house price increases in summers and winters. DCLG 1983-2007

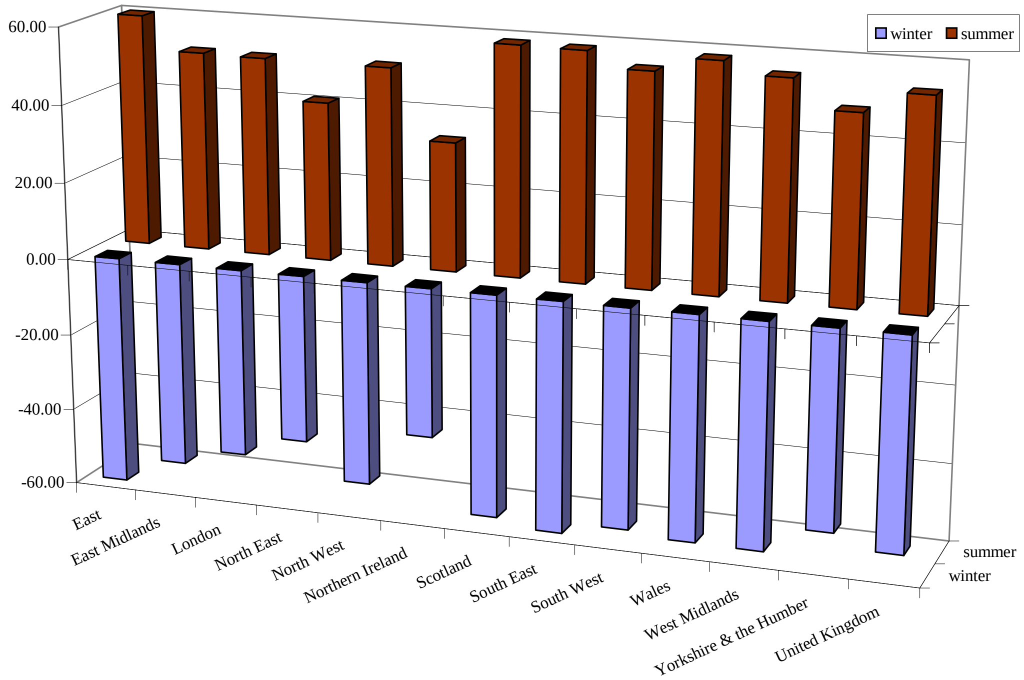


Note: Annualized real price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DCLG, 1983-2007.

Number of Transactions

Seasonal fluctuations in house prices are accompanied by qualitatively similar fluctuations in the number of transactions, proxied here by the number of mortgages. For comparability with the price sample, Figure 3 shows the growth rates in the number of mortgages in the two seasons from 1983 to 2007. (The data, which are compiled by CML, goes back to 1974 for some regions; the patterns are qualitatively similar in the earlier period.) As the Figure shows, the number of transactions increases sharply in the summer term and accordingly declines in the winter term.

Figure 3: Average annualized increases in the number of transactions in summers and winters. CML 1983-2007



Note: Annualized growth rates in the number of transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. CML, 1983-2007.

Statistical Significance of the Differences between Summers and Winters

We test the statistical significance of the differences in growth rates across seasons, $\left[\ln\left(\frac{P_S}{P_W}\right)^2 - \ln\left(\frac{P_{W'}}{P_S}\right)^2\right]$, using a t-test on the equality of means.[13] Tables 1 through 3 report the average difference in growth rates across seasons and standard errors, together with the statistical significance. Table 1 reports the results for prices, both nominal and real, for all regions, using the data from DCLG and Table 2 shows the corresponding information using Halifax. Table 3 shows

[13] The test on the equality of means is equivalent to the t-test on the slope coefficient from a regression of annualized growth rates on a dummy variable that takes value 1 if the observation falls on the second and third quarter and 0 otherwise. The dummy coefficient captures the annualized difference across the two seasons, regardless of the frequency of the data (provided growth rates are annualized). To see this note that the annualized growth rate in, say, the hot season, $\ln\left(\frac{P_S}{P_W}\right)^2$, is equal to the average of annualized quarterly growth rates in the summer term: $\ln\left(\frac{P_S}{P_W}\right)^2 = 2\ln\left(\frac{P_3}{P_1}\right) = \frac{1}{2}\left[4\ln\left(\frac{P_3}{P_2}\right) + 4\ln\left(\frac{P_3}{P_2}\right)\right]$, where the numbers (subindices) inicate the quarter, and, correspondingly, $2\ln\left(\frac{P_{1'}}{P_3}\right) = \frac{1}{2}\left[4\ln\left(\frac{P_{1'}}{P_4}\right) + 4\ln\left(\frac{P_4}{P_3}\right)\right]$. Hence a regression with quarterly (or semester) data on a summer dummy will produce an unbiased estimate of the average difference in growth rates across seasons. We use quarterly data to exploit all the information and gain on degrees of freedom.

the differences in transactions' growth rates.

Table 1: Difference in annualized percentage changes in (nominal and real) house prices between summers and winters in the UK, by region. DCLG.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	6.536*	(3.577)	4.870	(3.461)
East Midlands	8.231**	(3.148)	6.408**	(3.131)
Gr. London	8.788***	(3.273)	6.966**	(3.372)
North East	8.511**	(3.955)	6.845*	(3.915)
North West	13.703***	(3.323)	12.583***	(3.245)
Northern Ireland	4.237	(3.431)	2.415	(3.467)
Scotland	10.393***	(2.793)	8.571***	(2.711)
South East	10.375***	(3.496)	8.709**	(3.301)
South West	11.244***	(3.419)	9.422***	(3.459)
Wales	7.180**	(3.504)	5.358	(3.442)
West Midlands	9.623***	(3.089)	7.801**	(3.070)
Yorkshire & the Humber	10.148***	(3.114)	8.325***	(3.056)
United Kingdom	9.008***	(2.304)	7.185***	(2.314)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Department of Communities and Local Government.

Table 2: Difference in annualized percentage changes in (nominal and real) house prices between summers and winters in the UK, by region. Halifax.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	9.885***	(3.604)	8.081**	(3.706)
East Midlands	10.247***	(3.393)	8.444**	(3.413)
Gr. London	5.696*	(3.048)	3.892	(3.221)
North East	2.197	(2.945)	0.394	(2.864)
North West	8.019***	(2.653)	6.216**	(2.548)
Northern Ireland	6.053*	(3.409)	4.25	(3.494)
Scotland	9.334***	(2.320)	7.530***	(2.272)
South East	7.104**	(3.019)	5.301*	(3.149)
South West	9.258**	(3.474)	7.454**	(3.549)
Wales	7.786**	(3.329)	5.983*	(3.288)
West Midlands	5.987*	(3.540)	4.183	(3.505)
Yorkshire & the Humber	7.253**	(2.892)	5.450*	(2.825)
United Kingdom	7.559***	(2.365)	5.756**	(2.400)

Note: The Table shows the average differences (and standard errors), by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: Halifax.

Table 3: Difference in annualized percentage changes in the volume of transactions between summers and winters in the UK, by region. CML.

Region	Difference	Std. Error
East Anglia	119.420***	(11.787)
East Midlands	104.306***	(11.151)
Gr. London	99.758***	(11.577)
North East	84.069***	(9.822)
North West	103.525***	(8.963)
Northern Ireland	71.466***	(12.228)
Scotland	116.168***	(9.843)
South East	117.929***	(9.710)
South West	110.996***	(8.764)
Wales	115.900***	(13.850)
West Midlands	112.945***	(9.496)
Yorkshire & the Humber	98.904***	(8.192)
United Kingdom	107.745***	(8.432)

Note: The Table shows the average differences (and standard errors) by region for 1983-2007. *Significant at 10%; **significant at 5%; ***significant at 1%.
Source: Council of Mortgage Lenders.

The differences in price increases across seasons are quite sizable for most regions, in the order of 7 to 9 percent on average in nominal terms (depending on whether DCLG or Halifax data are used) and 5.7 to 7 percent in real terms; the results from DCLG appear more significant than those from Halifax from a statistical point of view. For transactions, the differences reach 108 percent for the country as a whole. Put together, the data point to a strong seasonal cycle, with a large increase in transactions and prices during the summer relative to the winter term. Also, seasonal patterns, particularly in transactions, are qualitatively similar across all regions.

Rents and Mortgage Rates Data on rents are not well documented. Only in recent years have data collection efforts started, but there is no long enough time-series to detect seasonality.[14] One source that can serve at least as indicative, is the average registered private rents collected by the UK Housing and Construction Statistics; the data run on a quarterly basis from 1979:01 to 2001:04. We run regressions using as dependent variables both the rent levels and the log of rents on a dummy variable taking value 1 in the second and third quarters and 0 otherwise, detrending the data in different ways. The data showed no deterministic seasonality (regression outcomes available from the authors). This is in line with anecdotal evidence suggesting that rents are fairly sticky. Given the paucity of data on rents, there is little we can say with high confidence. Still, note that for rents to be the driver of price seasonality, one would need an enormous degree of seasonality in rents (as well as a high discount rate), since prices should in principle, according to the standard

[14] See new data produced by the Chartered Institute of Housing since 1999 and ONS since 1996.

asset-pricing approach, reflect the present values of all future rents (in other words, prices should be less seasonal than rents). The lack of even small discernible levels of seasonality in the data suggests that we need alternative explanations for the observed seasonality in prices.

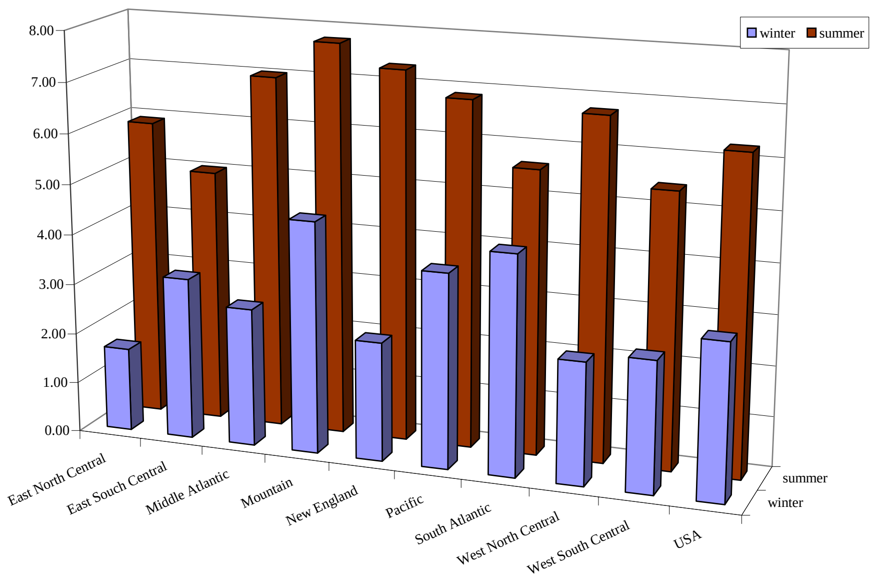
Interest rates in the UK do not exhibit a seasonal pattern, at least in the last four decades of data. We investigated seasonality in different interest rate series provided by the Bank of England: The repo (base) rate; an average interest rate charged by the four UK major banks—before the crisis (Barclays Bank, Lloyds Bank, HSBC, and National Westminster Bank); and a weighted average standard variable mortgage rate from banks and Building Societies. None of the interest rate series displays seasonality (results available from the authors).

Housing Market Seasonality in the US

Nominal and Real House Prices

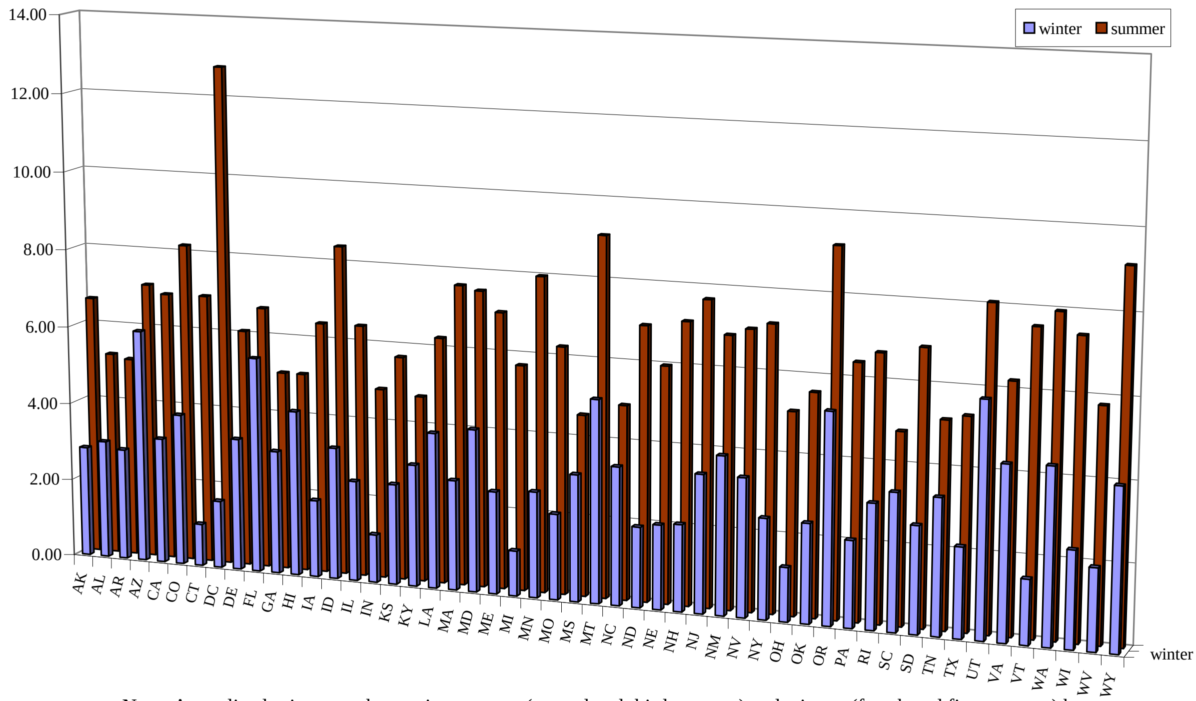
Figure 4 illustrates the annualized nominal house price increases for different regions from OFHEO. Figure 5 shows the corresponding plot for different states, also from OFHEO, and Figure 6 shows the plot using the S&P's Case-Shiller indexes for major cities. One first observation is that for most US regions, states and cities, the seasonal pattern is qualitatively similar to that in the UK, albeit the extent of seasonality is generally smaller. For some of the US major cities, however, the degree of seasonality is comparable to that in the UK.

Figure 4: Average annualized house price increases in summers and winters, by region. OFHEO 1991-2007



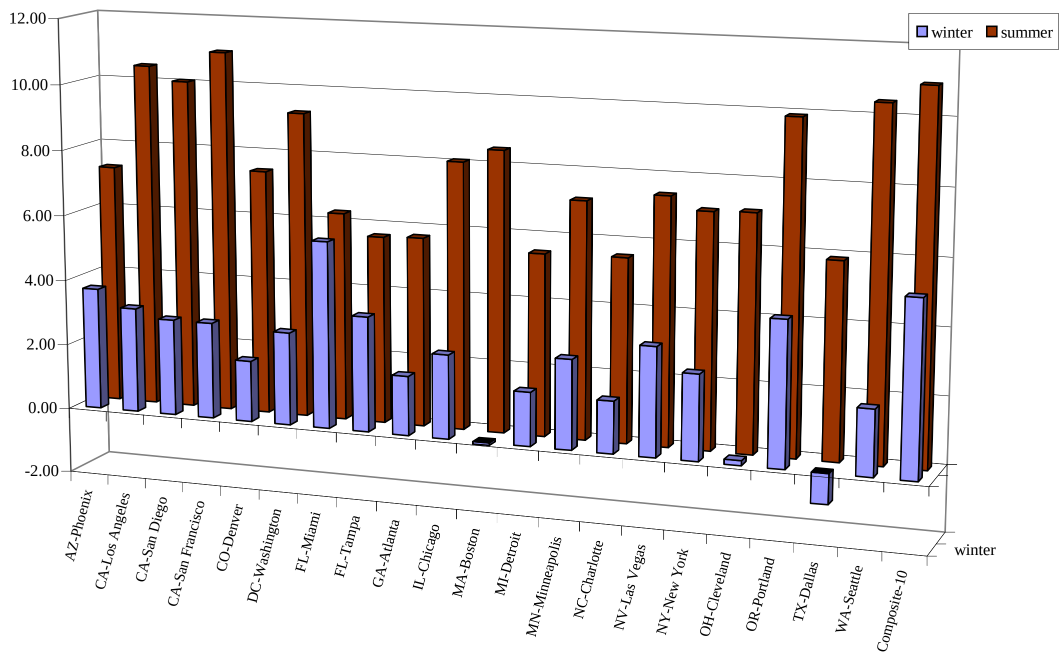
Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. OFHEO, 1991-2007.

Figure 5: Average annualized house price increases in summers and winters by state. OFHEO 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. state. OFHEO, 1991-2007.

Figure 6: Average annualized house price increases in summers and winters by city. S&P's Case-Shiller 1987-2007

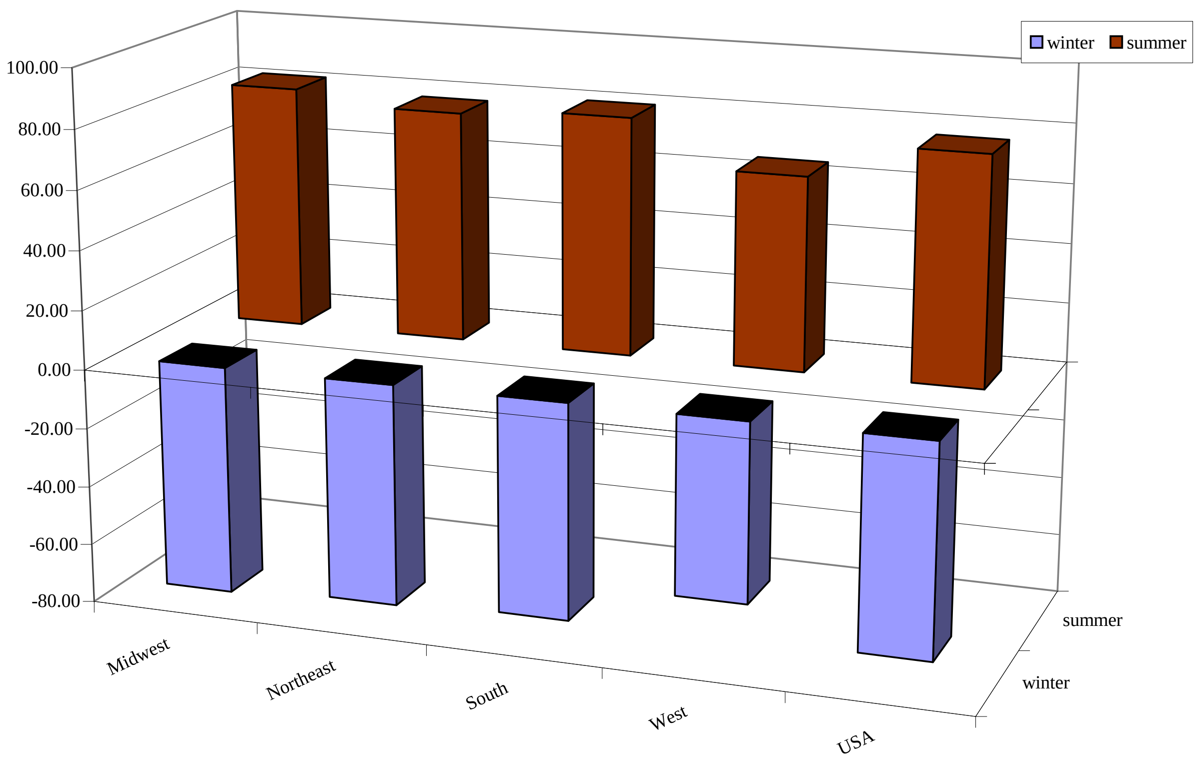


Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) by U.S. city. S&P Case and Shiller, 1987-2007.

The results using real prices (in terms of differences between seasons) are virtually identical to the ones for nominal prices, as CPI inflation rates hardly differ across seasons over the period analyzed and hence the differences in real growth rates across seasons are almost identical to the differences in nominal growth rates. These differences are later summarized in Table 4. (Figures are omitted in the interest of space, but are available from the authors).

Transactions Figure 7 shows the annualized growth rates in the number of transactions from 1991 through to 2007 for main Census regions; the data come from NAR.[15] Seasonality in transactions is overwhelming: The volume of transactions rises sharply in the summer and falls in the winter, by even larger magnitudes than in the UK.

Figure 7: Average annualized increases in the number of transactions in summers and winters. NAR 1991-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.S. and its regions. NAR 1991-2007.

Statistical Significance of the Differences between Summers and Winters We summarize the differences in growth rates across seasons and report the results from a test on mean

[15] The series actually starts in 1989, but we use 1991 for comparability with the OFHEO-census-level division price series; adding these two years does not change the results.

differences in Tables 4 through 7. Table 4 shows the results for prices using OFHEO's Census-division level; Table 5 shows the results using OFHEO's state-level data; Table 6 shows the results using S&P's Case-Shiller city-level data; and Table 7 shows the results for transactions from NAR.

Regarding house prices, for the US as a whole, the differences in annualized growth rates (nominal and real) are in the order of 3 percent. There is considerable variation across regions, with some displaying virtually no seasonality (South Atlantic) and others (East and West North Central, New England and Middle Atlantic) displaying significant levels of seasonality. This variability becomes more evident at the state level. Interestingly, the Case-Shiller index for cities displays higher levels of seasonality, comparable to the levels observed in UK regions. (This will be consistent with our model, which, ceteris paribus, generates more seasonality when the bargaining power of sellers is higher, as it is likely to be the case in cities, where land is relatively scarce.)

The volume of transactions is extremely seasonal in the US, even more than in the UK, with an average difference in growth rates across seasons of 148 percent and the pattern in common to all regions.

Table 4: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) in the US, by region

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East North Central	4.262***	(0.772)	4.106***	(0.924)
East South Central	1.811***	(0.535)	1.654**	(0.701)
Middle Atlantic	4.273**	(1.619)	4.116**	(1.660)
Mountain	3.166**	(1.205)	3.009**	(1.281)
New England	4.980**	(2.081)	4.823**	(2.181)
Pacific	3.010	(2.117)	2.853	(2.195)
South Atlantic	1.281	(1.277)	1.125	(1.370)
West North Central	4.333***	(0.743)	4.176***	(0.872)
West South Central	2.836***	(0.537)	2.679***	(0.650)
USA	3.169***	(0.967)	3.012***	(1.081)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: OFHEO Purchase-only Index.

Table 5: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) by US state.

	Nominal house price		Real house price	
State	Difference	Std. Error	Difference	Std. Error
Alabama	3.812**	(1.400)	3.655**	(1.378)
Alaska	2.189***	(0.692)	2.032**	(0.848)
Arizona	2.263**	(0.848)	2.106**	(0.950)
Arkansas	1.109	(2.586)	0.953	(2.583)
California	3.656	(3.398)	3.499	(3.479)
Colorado	4.285***	(1.323)	4.129***	(1.447)
Connecticut	5.819***	(2.055)	5.662**	(2.133)
District of Columbia	11.040**	(4.229)	10.883**	(4.150)
Delaware	2.687	(1.862)	2.530	(1.925)
Florida	1.185	(2.525)	1.028	(2.571)
Georgia	1.921**	(0.743)	1.764*	(0.887)
Hawaii	0.850	(3.668)	0.693	(3.677)
Idaho	4.440***	(0.615)	4.283***	(0.711)
Illinois	5.035***	(1.659)	4.878***	(1.688)
Indiana	3.864***	(0.755)	3.707***	(0.859)
Iowa	3.621***	(0.768)	3.464***	(0.884)
Kansas	3.134***	(0.709)	2.977***	(0.925)
Kentucky	1.623***	(0.570)	1.466**	(0.707)
Louisiana	2.300***	(0.827)	2.143**	(0.921)
Maine	4.823**	(2.219)	4.666*	(2.339)
Maryland	3.384	(2.341)	3.227	(2.396)
Massachusetts	4.407**	(2.146)	4.250*	(2.231)
Michigan	4.573***	(1.568)	4.416**	(1.698)
Minnesota	5.290***	(1.376)	5.133***	(1.484)
Missouri	4.085***	(0.646)	3.929***	(0.758)
Mississippi	1.379	(1.028)	1.222	(1.108)
Montana	3.957**	(1.469)	3.800**	(1.510)
North Carolina	1.417**	(0.641)	1.260	(0.764)
North Dakota	4.908***	(1.353)	4.751***	(1.423)
Nebraska	3.842***	(1.082)	3.685***	(1.162)
New Hampshire	4.918**	(2.391)	4.761*	(2.463)
New Jersey	4.197*	(2.076)	4.041*	(2.126)
New Mexico	2.857*	(1.560)	2.700	(1.623)
Nevada	3.540	(2.946)	3.383	(3.026)
New York	4.662**	(1.815)	4.505**	(1.872)
Ohio	3.729***	(0.731)	3.572***	(0.911)
Oklahoma	3.095***	(0.477)	2.938***	(0.511)
Oregon	3.903***	(1.380)	3.746***	(1.310)
Pennsylvania	4.226***	(1.317)	4.069***	(1.329)
Rhode Island	3.544	(2.842)	3.388	(2.969)
South Carolina	1.360*	(0.698)	1.203	(0.771)
South Dakota	4.201***	(1.171)	4.044***	(1.248)
Tennessee	1.759**	(0.685)	1.602*	(0.834)
Texas	3.045***	(0.674)	2.888***	(0.763)
Utah	2.204	(1.820)	2.047	(1.803)
Virginia	1.873	(1.758)	1.716	(1.835)
Vermont	5.945**	(2.430)	5.788**	(2.373)
Washington	3.563**	(1.377)	3.406**	(1.377)
Wisconsin	5.007***	(0.738)	4.850***	(0.848)
West Virginia	3.753**	(1.702)	3.596**	(1.765)
Wyoming	5.091***	(1.365)	4.935***	(1.391)

Note: The Table shows the average differences (and standard errors), by state for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: OFHEO Purchase-only Index.

Table 6: Difference in annualized percentage changes in house prices between semesters (second-third quarters vis-à-vis fourth-first quarters) by US city.

	Nominal house price		Real house price	
City	Difference	Std. Error	Difference	Std. Error
AZ-Phoenix	3.571	(3.307)	3.405	(3.357)
CA-Los Angeles	7.273**	(3.478)	6.884*	(3.535)
CA-San Diego	7.107**	(3.204)	6.717**	(3.275)
CA-San Francisco	8.051**	(3.009)	7.662**	(3.045)
CO-Denver	5.576***	(1.599)	5.186***	(1.805)
DC-Washington	6.439**	(2.604)	6.050**	(2.645)
FL-Miami	0.636	(2.744)	0.246	(2.838)
FL-Tampa	2.171	(2.384)	1.781	(2.484)
GA-Atlanta	3.920***	(0.903)	3.763***	(1.042)
IL-Chicago	5.530***	(1.342)	5.141***	(1.459)
MA-Boston	8.560***	(2.091)	8.170***	(2.325)
MI-Detroit	3.864*	(1.909)	3.707*	(2.060)
MN-Minneapolis	4.431***	(1.528)	4.265**	(1.741)
NC-Charlotte	3.968***	(0.721)	3.578***	(0.836)
NV-Las Vegas	4.149	(3.216)	3.76	(3.262)
NY-New York	4.477**	(2.161)	4.087*	(2.342)
OH-Cleveland	6.942***	(0.973)	6.553***	(1.041)
OR-Portland	5.551***	(1.485)	5.161***	(1.388)
TX-Dallas	6.776***	(1.380)	6.138***	(1.823)
WA-Seattle	8.437***	(1.953)	8.175***	(1.942)
Composite-20 cities	6.051***	(2.227)	5.662**	(2.344)

Note: The Table shows the average differences (and standard errors), by region for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: SP's Case-Shiller index.

Table 7: Difference in annualized percentage changes in house transactions between semesters (second-third quarters vis-à-vis fourth-first quarters) by US region.

Region	Coef.	Std. Error
Midwest	159.473***	(6.488)
Northeast	152.551***	(4.918)
South	153.009***	(4.702)
West	124.982***	(6.312)
United States	148.086***	(5.082)

Note: The Table shows the average differences (and standard errors) by region for 1991-2007. *Significant at 10%; **significant at 5%; ***significant at 1%. Source: National Association of Realtors.

Rents and Mortgage Rates As was the case for the UK, the paucity of rent data for the US is regrettable. The Bureau of Labor Statistics (BLS) provides two series that can serve as proxies: One is the NSA series of owner's equivalent rent and the second is the NSA rent of primary residence; both series are produced for the construction of the CPI and correspond to averages over all cities.

For each series, we run regressions using as dependent variables both the rent levels and the log of rents, de-trended in various ways, on a summer-term dummy. The results (available from the authors) yielded no discernible pattern of seasonality. We take this as only suggestive as, of course, the data are not as clean and detailed as we would wish. To reiterate, however, if seasonality in rents were the driver of seasonality in prices, we should observe enormous seasonality in rental flows to generate the observed seasonality in house prices. In the model we present later, we will work under the constraint that rents are aseasonal.

As first documented by Barsky and Miron (1989), interest rates in recent decades do not exhibit seasonality. We investigated in particular data on mortgage rates produced by the Board of Governors of the Federal Reserve, corresponding to contract interest rates on commitments for fixed-rate first mortgages; the data are quarterly averages beginning in 1972; the original data are collected by Freddie Mac. Consistent with the findings of Barsky and Miron (1989) and the evidence from the UK, we did not find any significant deterministic seasonality. (Results available from the authors.)

2.3 Further Discussion

We have argued before that the predictability and size of the seasonal variation in house prices pose a puzzle to models of the housing market relying on a standard asset-market approach. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal service cost. In Appendix 7.2 we assess the extent to which seasonality in service costs might be driving seasonality in prices, given that rental flows do not appear to be seasonal. The exercise makes clear that a standard asset-pricing approach that relies on perfect arbitrage leads to implausibly large levels of required seasonality in service costs.

Our findings suggest that there are important frictions in the market that impair the ability of investors to gain from seasonal arbitrage and therefore call for a deviation from the standard asset-pricing approach.[16] But perhaps a more fundamental reason to deviate is the overwhelming evidence that buying and selling houses involve a non-trivial search process that is not well captured

[16] The need to deviate from the asset-market approach has been acknowledged, in a different context by Stein (1995), among others. While static in nature, Stein's model is capable of generating unexpected booms and busts in prices (and transactions) in a rational-expectation setting. In a dynamic setting with forward-looking agents, however, predictably large changes in prices cannot be sustained: *Expected* price increases in the next season will actually be priced in in the current season. For other modelling frameworks, see Ortalo-Magne and Rady (2005) and Flavin and Nakagawa (2008).

in the standard asset-pricing approach. Furthermore, as is also the case in labor markets (and largely the motivation for the labor-search literature) the coexistence at any point in time of a stock of vacant houses and a pool of buyers searching for houses, suggests a lack of immediate market clearing; explicitly modelling the frictions that impair clearing can help in the understanding of housing market fluctuations. We are of course not the first to use a search-and-matching framework to study housing markets; see for example early work by Wheaton (1990), Williams (1995), Krainer (2001), and Albrecht et al. (2007). Our setup is different from these earlier contributions in that it brings in thick-market effects, which, due to their amplification power, are able to generate large fluctuations in transactions and prices even in the absence of aggregate uncertainty.

3 A Search-and-Matching Model for the Housing Market

In this Section we develop a search-and-matching model for the housing market. The basic premise in the model is that the suitability of a match between a house and a buyer is specific to the pair. For example, a particular house may match a buyer's needs or taste perfectly well, while at the same time being an unsatisfactory match to another buyer. In that context, we introduce the notion that in a market with more houses for sale, a buyer is more likely to find a better match, what we refer to as "thick-market effect."

3.1 The Economy

The economy is populated by a unit measure of infinitely lived agents, who have linear preferences over housing services and a non-durable consumption good. Each period agents receive a fixed endowment of the consumption good which they can either consume or use to buy housing services. An agent can only enjoy housing services from living in one house at a time, that is, he can only be "matched" to one house at a time. Agents who are not matched to a house seek to buy one ("buyers").

There is a unit measure of housing stock. Each period a house can be either matched or unmatched. A matched house delivers a flow of housing services of quality ε to its owner. The quality of housing services ε is match-specific, and it reflects the suitability of a match between a house and its homeowner. In other words, for any vacant house, the quality of housing services is idiosyncratic to the match between the house and the potential buyer. Hence, ε is not the type of house (or of the seller who owns a particular house). This is consistent with our data, which are adjusted for houses' characteristics, such as size and location, but not for the quality of a match.[17] Unmatched houses are "for sale" and are owned by "sellers;" sellers receive a flow u from any unmatched house they own, where the flow u is common to all sellers.

3.2 Seasons and Timing

There are two seasons, $j = s, w$ (for summer and winter); each model period is a season, and seasons alternate. At the beginning of a period j, an existing match between a homeowner and his house breaks with probability $1 - \phi^j$, and the house is put up for sale, adding to the stock of vacant houses v^j. The homeowner whose match has broken becomes simultaneously a seller and a buyer, adding to the pool of buyers b^j. In our baseline model, the parameter ϕ^j is the only (*ex ante*) difference between the seasons.[18] We focus on periodic steady states with constant v^s and v^w. Since a match is between one house and one agent, and there is a unit measure of agents and a unit measure of houses, it is always the case that the mass of vacant houses equals the mass of buyers: $v^j = b^j$.

Each period, each buyer meets with one seller and each seller meets with one buyer. If they agree on a transaction, the buyer pays a price (discussed later) to the seller, and starts enjoying the housing services ε. If not, the buyer looks for a house again next period, the seller receives the flow u, and puts the house up for sale again next period.[19] An agent can hence be a homeowner, a buyer, a seller, both a seller and a homeowner, and both a buyer and a seller. Also, sellers may have multiple houses to sell.

3.3 Match-specific Quality

The model seeks to embed the notion that in a market with many houses for sale, a buyer can see more houses (e.g. by searching online or through newspapers) and hence is more likely to find a better match.[20] We model this idea in the following way. Assume that the potential match quality,

[17] Neither repeat-sale indices nor hedonic price indexes can control for the quality of a match, which is not observed by data collectors.

[18] This difference could be determined, for example, by the school calendar or summer marriages, among other factors, exogenous to our model.

[19] In Section 5.2 we relax the assumption that if the transaction does not go through, buyer and seller need to wait for next period to transact with other agents.

[20] Note that this is different from the stock-flow literature (see e.g. Coles and Smith, 1998), where the stock of old buyers can only draw from the stock of new vacant houses. We do not draw a distinction here between old and new buyers.

ε, between a buyer and a house is drawn from a distribution $F(\varepsilon, v)$, with positive support and finite mean, where v denotes the stock of vacant houses.[21] In that setting, our notion of a "thick-market" is captured by the following assumption:

Assumption 1 $F(., v')$ stochastically dominates $F(., v)$ if and only if $v' > v$.

That is, $F(., v') \leq F(., v)$ if and only if $v' > v$. In words, when the stock of houses v is larger, a random match-quality draw from $F(\varepsilon, v)$ is likely to be higher.[22,23]

The assumption implies that a higher v shifts up the expected surplus of quality above any threshold x. That is,

$$h(x, v) = \int_x (\varepsilon - x)\, dF(\varepsilon, v) \text{ is increasing in } v. \tag{1}$$

To see this, rewrite $h(x, v) = \int_x [1 - F(\varepsilon, v)]\, d\varepsilon$ using integration by parts, which is increasing in v from Assumption 1.

We furthermore assume that the stochastic ordering is "uniform" (see Keilson and Sumita, 1982). Formally,

Assumption 2 $\frac{1-F(\varepsilon,v')}{1-F(\varepsilon,v)}$ is increasing in ε for $v' > v$.

This holds if the thick-market effect is such that the increase in $[1 - F(., v)]$ due to higher v is increasing in ε. Assumption 1 is of course implied by Assumption 2. The first is necessary for our results, the second is only sufficient, as shall become clear later.

Using integration by parts, the conditional surplus can be expressed as:

$$E(\varepsilon - x \mid \varepsilon \geqslant x) = \frac{\int_x [1 - F(\varepsilon, v)]\, d\varepsilon}{1 - F(x, v)},$$

which is increasing in v from Assumption 2.[24]

[21] The stochastic nature of the match-specific quality is closely related to the job matching model of Jovanovic (1979).

[22] This is similar to Diamond (1981)'s labor market model, where he assumes that mobility costs are stochastically lower when there are more firms and workers.

[23] One way to interpret our assumption is to think of order statistics. Suppose the buyer samples n units of vacant houses when the stock of vacancies is v. As long as the number of units sampled n increases in v, the maximum match quality ε in the sample will be "stochastically larger."

[24] Note that first-order stochastic dominance does not guarantee that the conditional surplus is increasing in v, a condition known as *mean residual ordering* (see Shaked and Shanthikumar, 1994). As shown by Shaked and Shanthikumar (1994), Assumption 2 is a sufficient condition for this to hold. They also show that Assumption 2 is a necessary and sufficient condition for *harzard rate ordering*: $\frac{f(.,v)}{1-F(.,v)} \geq \frac{f(.,v')}{1-F(.,v')}$ for $v' \geq v$, where $f(\varepsilon, v)$ is the corresponding probability density function.

3.4 The Homeowner

To study pricing and transaction decisions, we first derive the value of living in a house if a transaction goes through. The value function for a matched homeowner who lives in a house with match quality ε in season s is given by:

$$H^s(\varepsilon) = \varepsilon + \beta\phi^w H^w(\varepsilon) + \beta(1-\phi^w)[V^w + B^w],$$

where $\beta \in (0,1)$ is the discount factor. With probability $(1-\phi^w)$ he receives a moving shock and becomes both a buyer and a seller (putting his house up for sale), with continuation value (V^w+B^w), where V^j is the value of a vacant house to its seller and B^j is the value of being a buyer in season $j = s, w$, defined later. With probability ϕ^w he keeps receiving housing services of quality ε and stays in the house. The formula for $H^w(\varepsilon)$ is perfectly isomorphic to $H^s(\varepsilon)$; in the interest of space we omit here and throughout the paper the corresponding expressions for season w. The value of being a matched homeowner can be therefore re-written as:

$$H^s(\varepsilon) = \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon + \frac{\beta(1-\phi^w)(V^w+B^w) + \beta^2\phi^w(1-\phi^s)(V^s+B^s)}{1-\beta^2\phi^w\phi^s}, \tag{2}$$

which is strictly increasing in ε.

3.5 Market Equilibrium

We focus on the case in which both seller and buyer observe the quality of the match, ε, that is drawn from $F^j(\varepsilon) \equiv F(\varepsilon, v^j)$; we derive the results for the case in which the seller cannot observe ε in Appendix 7.5. If the transaction goes through, the buyer pays a mutually agreed price to the seller, and starts enjoying the housing services flow in the same season j. If the transaction does not go through, the buyer receives zero housing services and looks for a house again next season. This will be the case, for example, if buyers searching for a house pay a rent equal to the utility they derive from the rented property; what is key is that the rental property is not owned by the same potential seller with whom the buyer meets. On the seller's side, when the transaction does not go through, he receives the flow u in season j and puts the house up for sale again next season. The flow u can be interpreted as a net rental income received by the seller. Again, what is key is that the tenant is not the same potential buyer who visits the house.

Let $S_v^s(\varepsilon)$ and $S_b^s(\varepsilon)$ be the surplus of a transaction to the seller and to the buyer, respectively, in season s, when the match quality is ε and the price is $p^s(\varepsilon)$:

$$\begin{aligned} S_v^s(\varepsilon) &\equiv p^s(\varepsilon) - (u + \beta V^w), && (3) \\ S_b^s(\varepsilon) &\equiv H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w. && (4) \end{aligned}$$

Denote the total surplus by:

$$S^s(\varepsilon) \equiv S_v^s(\varepsilon) + S_b^s(\varepsilon) = H^s(\varepsilon) - [\beta(B^w + V^w) + u] \tag{5}$$

Since ε is observable and the surplus is transferrable, a transaction goes through as long as the total surplus $S^s(\varepsilon)$ is positive. Given $H^s(\varepsilon)$ is increasing in ε, a transaction goes through if $\varepsilon \geqslant \varepsilon^s$, where the reservation ε^s is defined by:

$$\varepsilon^s =: H^s(\varepsilon^s) = \beta(B^w + V^w) + u, \tag{6}$$

and $1 - F^s(\varepsilon^s)$ is thus the probability that a transaction is carried out. Since the reservation quality ε^s is related to the total surplus independently of how the surplus is divided between the buyer and the seller, we postpone the discussion of equilibrium prices to Section 4.3.

3.5.1 Reservation Quality

Observe from (5) and (6) that

$$S^s(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1 + \beta\phi^w}{1 - \beta^2\phi^w\phi^s}(\varepsilon - \varepsilon^s), \tag{7}$$

The value functions for being a seller and a buyer in season s are, respectively:

$$\begin{aligned} V^s &= \beta V^w + u + [1 - F^s(\varepsilon^s)] E^s[S_v^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], && (8) \\ B^s &= \beta B^w + [1 - F^s(\varepsilon^s)] E^s[S_b^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], && (9) \end{aligned}$$

where $E^s[.]$ indicates the expectation is taken with respect to the distribution $F^s(.)$. A seller can count on his outside option, $\beta V^w + u$ (the flow u plus the option value of selling next season) and, with probability $[1 - F^s(\varepsilon^s)]$, on the expected surplus from a transaction for sellers. A buyer counts on his outside option, βB^w (the option value of buying next season), and, with the same probability, on the expected surplus for buyers. Therefore,

$$B^s + V^s = H^s(\varepsilon^s) + [1 - F^s(\varepsilon^s)] E^s[S^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s], \tag{10}$$

which is the sum of the housing value $H^s(\varepsilon^s)$ of the marginal transaction and the expected surplus from a transaction with quality ε, above the reservation ε^s. Using the definition of $S^s(\varepsilon)$ and ε^s in (5) and (6), and the expression in (7), the sum of values is:

$$B^s + V^s = \beta\left(B^w + V^w\right) + u + \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left[1 - F^s\left(\varepsilon^s\right)\right] E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right]. \tag{11}$$

Solving this explicitly:

$$B^s + V^s = \frac{u}{1-\beta} + \frac{\left(1+\beta\phi^w\right) h^s\left(\varepsilon^s\right) + \beta\left(1+\beta\phi^s\right) h^w\left(\varepsilon^w\right)}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)}, \tag{12}$$

where $h^s(\varepsilon^s) \equiv h(\varepsilon^s, v^s) = \left[1 - F^s(\varepsilon^s)\right] E\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right]$ is the expected surplus of quality above threshold ε^s as described in (1). Using the definition of ε^s in (6) and expression (2), we derive the reservation quality as:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^s = u - \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^s + V^s\right) + \frac{1-\beta^2\phi^s}{1-\beta^2\phi^w\phi^s}\beta\phi^w\left(B^w + V^w\right). \tag{13}$$

The equilibrium values $\varepsilon^s, \varepsilon^w, (B^s + V^s)$, and $(B^w + V^w)$ in (12) and (13) depend on equilibrium vacancies v^s and v^w, which we now derive.

3.5.2 Stock of vacant houses

In any season s, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$v^s = \left(1-\phi^s\right)\left[v^w\left(1 - F^w\left(\varepsilon^w\right)\right) + 1 - v^w\right] + v^w F^w\left(\varepsilon^w\right)$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = 1 - \phi^s + v^w F^w\left(\varepsilon^w\right)\phi^s. \tag{14}$$

The equilibrium quantities $(B^s + V^s, B^w + V^w, \varepsilon^s, \varepsilon^w, v^s, v^w)$ jointly satisfy equations (12), (13), and (14) together with the isomorphic equations for the other season. They are independent of how the total surplus is shared across buyers and sellers, that is independent of the exact price-setting mechanism. We first discuss seasonality in vacancies and transactions before we specify the particular price-setting mechanism.

4 Model-generated Seasonality

The driver for seasonality in the baseline model is the higher moving probability in the summer: $1 - \phi^s > 1 - \phi^w$.

4.1 Seasonality in Vacancies

Using (14), the stock of vacant houses in season s is given by:

$$v^s = \frac{1-\phi^s+\phi^s F^w(\varepsilon^w)(1-\phi^w)}{1-F^s(\varepsilon^s)F^w(\varepsilon^w)\phi^s\phi^w}. \tag{15}$$

(The expression for v^w is correspondingly isomorphic). The *ex ante* higher probability of moving in the summer $(1-\phi^s > 1-\phi^w)$ clearly has a direct positive effect on v^s, and, as it turns out, this effect also dominates quantitatively when we calibrate the model to match the average duration of stay in a house.[25] Thus, we have $v^s > v^w$.

4.2 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s\left[1-F^s(\varepsilon^s)\right]. \tag{16}$$

(An isomorphic expression holds for Q^w). Seasonality in transactions stems from three sources: First, from (16), it is evident that a bigger stock of vacancies in the summer, $v^s > v^w$, has a direct positive effect on the number of transactions in the summer relative to winter. Second, following from Assumption 1, the thick-market effect shifts up the probability of a transaction for any given reservation level x: $[1-F^s(x)] > [1-F^w(x)]$, which again positively affects Q^s/Q^w. Finally, there is an equilibrium effect through a higher reservation quality in the winter: $\varepsilon^w > \varepsilon^s$, which also leads to lower transactions in the winter.[26] This equilibrium effect turns out to be quantitatively small in our calibrations, as the equilibrium cutoffs are very close in the two seasons. The bulk of

[25] More specifically, the numerator is a weighted average of 1 and $F^w(\varepsilon^w)(1-\phi^w)$, with $1-\phi^s$ being the weight assigned to 1. Since $F^w(\varepsilon^w)(1-\phi^w) < 1$, higher weight on 1, that is, higher weight $(1-\phi^s)$ leads to $v^s > v^w$; this is because $F^w(\varepsilon^w)(1-\phi^w)$ is virtually aseasonal as there are two opposite effects: $F^w(\varepsilon^w) > F^s(\varepsilon^s)$ and $(1-\phi^w) < (1-\phi^s)$ that tend to largely cancel each other.

[26] The outside option for both buyers and sellers in the winter is to wait and transact in the summer, (B^s+V^s), when their expected returns are higher than in the winter, (B^w+V^w). This makes both buyers and sellers more demanding in the winter and hence less likely to transact, yielding an even smaller number of transactions. To see this more explicitly, note first that given $v^s > v^w$, the thick-market effect implies $h^s(\varepsilon^s) > h^w(\varepsilon^w)$ as in (1). It then follows from (12) that $(B^s+V^s) > (B^w+V^w)$. Because the cutoffs are defined by $H^s(\varepsilon^s) = \beta(B^w+V^w)+u$, then $\varepsilon^w > \varepsilon^s$. That is, the *marginal* transaction in the winter has a higher match quality given that the outside option of buyers and sellers (to transact in the summer) is higher. Note, though, that because of the thick-market effect, the *average* transaction in the summer is of higher match quality. As u decreases, the outside option of buyers and sellers decreases and so does the extent of seasonality due to this channel.

the amplification mechanism in the model is due to the second source, the thick-market effect: For a given level of seasonality in vacancies, the thick-market effect through the first-order stochastic dominance of $F^s(.)$ over $F^w(.)$ can generate *higher* seasonality in transactions. Our first result follows:

Amplification: Transactions are more seasonal than vacancies.

4.3 Seasonality in Prices

As discussed earlier, results on seasonality in vacancies and transactions are independent of the exact price-setting mechanism. We now consider the case in which prices are determined by Nash bargaining. The price maximizes the Nash product:

$$\max_{p^s(\varepsilon)} \left[S_v^s(\varepsilon)\right]^{\theta} \left[S_b^s(\varepsilon)\right]^{1-\theta} \quad s.t. \quad S_v^s(\varepsilon), S_b^s(\varepsilon) \geqslant 0;$$

where θ denotes the bargaining power of the seller. The solution implies

$$\frac{S_v^s(\varepsilon)}{S_b^s(\varepsilon)} = \frac{\theta}{1-\theta}, \tag{17}$$

which simplifies to (see Appendix 7.3):

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + (1-\theta)\frac{u}{1-\beta}, \tag{18}$$

a weighted average of the housing value for the matched homeowner and the present discounted value of the flow u. In other words, the price guarantees the seller the proceeds from the alternative usage of the house $(\frac{u}{1-\beta})$ and a fraction θ of the social surplus generated by the transaction $\left[H^s(\varepsilon) - \frac{u}{1-\beta}\right]$.

The average price of a transaction is:

$$P^s \equiv E\left[p^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right] = (1-\theta)\frac{u}{1-\beta} + \theta E\left[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right], \tag{19}$$

which is increasing in the conditional expected surplus of housing services for transactions exceeding the reservation ε^s. Since u is aseasonal, house prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal (as we show). The next result follows:

Seasonality in Prices *When sellers have some bargaining power ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in θ.*

To see this, note first that from (19) the equilibrium price P^s is the discounted sum of the flow value $(\frac{u}{1-\beta})$ plus a fraction θ of the surplus from the sale, $E^s\left[\left(H^s(\varepsilon)-\frac{u}{1-\beta}\right) \mid \varepsilon \geqslant \varepsilon^s\right]$. Because $\frac{u}{1-\beta}$ is constant in the two seasons, the surplus is higher in the summer than in the winter if the average housing value is higher: $E^s\left[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right] > E^s\left[H^w(\varepsilon) \mid \varepsilon \geqslant \varepsilon^w\right]$. Recall from (7) that the average housing value of transacted houses is the sum of two terms:

$$E^s\left[H^s(\varepsilon) \mid \varepsilon \geqslant \varepsilon^s\right] = H^s(\varepsilon^s) + \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} E^s\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right]. \tag{20}$$

The first term, $H^j(\varepsilon^j)$, the housing value of the marginal transaction, tends to reduce the average housing value in the summer, since the cutoff is lower: $\varepsilon^w > \varepsilon^s$. Quantitatively, however, the difference in equilibrium cutoffs across the two seasons turns out to be very small for reasonable parametrizations of the model. The second term, instead, tends to increase the average housing value in the summer for two reasons. First, the probability of stay is higher in the winter, $\phi^w > \phi^s$. Second, and more important, following from Assumption 2, the expected conditional surplus $E^s\left[\varepsilon - x \mid \varepsilon \geqslant x\right]$ is higher in the summer for any given cutoff x; because, as said, the equilibrium cutoffs are quantitatively close, Assumption 2 will in general lead to:

$$E^s\left[\varepsilon \mid \varepsilon \geqslant \varepsilon^s\right] - \varepsilon^s > E^w\left[\varepsilon \mid \varepsilon \geqslant \varepsilon^w\right] - \varepsilon^w \tag{21}$$

More precisely, as shown by Shaked and Shanthikumar (1994), the expected conditional surplus over a cutoff x decreases with the cutoff if the distribution $F(.)$ is log-concave, that is, $E\left[\varepsilon - x \mid \varepsilon \geqslant x\right]$ decreases in x. Thus, it follows from Assumption 2 that condition (21) holds.

Given that θ affects P^s only through the equilibrium mass of vacancies (recall the reservation quality ε^s is independent of θ), it follows that the extent of seasonality in prices is increasing in θ. Since (19) holds independently of the steady state equation for v^s and v^w, this result is independent of what drives $v^s > v^w$. Note finally, that the extent of seasonality in prices is decreasing in the size of the (aseasonal) flow u.

4.4 Quantitative Results

4.4.1 Parameter values

We now calibrate the model to study its quantitative implications. We assume the distribution of match-quality $F(\varepsilon, v)$ follows a uniform distribution on $[0, v]$. When $v^s > v^w$ (which will follow from $\phi^w > \phi^s$), this implies both first-order stochastic ordering, $F^s(.) \leqslant F^w(.)$, and mean residual ordering, $E^s\left[\varepsilon - x \mid \varepsilon \geqslant x\right] \geqslant E^w\left[\varepsilon - x \mid \varepsilon \geqslant x\right]$.

We set the discount factor β so that the implied annual real interest rate is 6 percent.

We calibrate the average probability of staying in the house, $\phi = (\phi^s + \phi^w)/2$, to match survey data on the average duration of stay in a given house, which in the model is given by $\frac{1}{1-\phi}$. The median duration in the US from 1993 through 2005, according to the American Housing Survey, was 18 semesters; the median duration in the UK during this period, according to the Survey of English Housing was 26 semesters. The implied (average) moving probabilities $(1-\phi)$ per semester are hence 0.056 and 0.038 for the US and the UK, respectively. Because there is no direct data on the ex-ante ratio of moving probabilities between seasons, $(1-\phi^s)/(1-\phi^w)$, we use a range of $(1-\phi^s)/(1-\phi^w)$ from 1.1 to 1.5.[27] This implies a difference in staying probabilities between seasons, $\phi^w - \phi^s$, ranging from 0.004 to 0.015 in the UK and 0.005 to 0.022 in the US. One way to pin down the level of $(1-\phi^s)/(1-\phi^w)$ is to use data on vacancy seasonality, which is available for the US from the US Census Bureau (for the UK, data on vacancies only exist at yearly frequency). Seasonality in vacancies in the US was 31 percent during 1991 – 2007.[28] As will become clear from the results displayed below, the ratio that exactly matches seasonality in US vacancies is $(1-\phi^s)/(1-\phi^w) = 1.28$. The reader may want to view this as a deep parameter and potentially use it also for the UK, under the assumption that the extent of seasonality in ex-ante moving probabilities does not vary across countries.

We calibrate the flow value u to match the implied average rent-to-price ratio *received by the seller*. In the UK, the average *gross* rent-to-price ratio is roughly around 5 percent per year, according to *Global Property Guide*.[29] For the US, Davis et al. (2008) argue that the ratio was around 5 percent prior to 1995 when it started falling, reaching 3.5 percent by 2005. In our model, the u/P ratio (where P stands for the average price, absent seasonality) corresponds to the *net* rental flow received by the seller after paying taxes and other relevant costs; it is accordingly lower than the *gross* rent-

[27] The two surveys mentioned also report the main reasons for moving. Around 30 percent of the respondents report that living closer to work or to their children's school and getting married are the main reasons for moving. These factors are of course not entirely exogenous, but they can carry a considerably exogenous component; in particular, the school calendar is certainly exogenous to housing market movements (see Goodman, 1993, and Tucker, Long, and Marx , 1995 on seasonal mobility). In all, the survey evidence supports our working hypothesis that the *ex ante* probability to move is higher in the summer (or, equivalently the probability to stay is higher in the winter).

[28] As a measure of seasonality we use, as before, the difference in annualized growth rates in vacancies between broadly defined summers and winters. The difference is statistically significant at standard levels. Vacancy is computed as the sum of houses for sale at the beginning of the season relative to the stock of houses.

[29] Data for the U.K. and other European countries can be found in http://www.globalpropertyguide.com/Europe/United-Kingdom/price-rent-ratio

to-price ratio. As a benchmark, we choose u so that the net rent-to-price ratio is equal to 3 percent per year (or 1.5 percent per semester), equivalent to assuming a 40 percent income tax on rent).[30] To obtain a calibrated model of u, which, as we said, is aseasonal in the data, we use the equilibrium equations in the model without seasonality, that is, the model in which $\phi^s = \phi^w = \phi$. From (19) and (13), the average price and the reservation quality ε^d in the absence of seasonality are (see Appendix 7.3.2):

$$P = \frac{u}{1-\beta} + \theta \frac{\left[1 - \beta F\left(\varepsilon^d\right)\right] E\left[\varepsilon - \varepsilon^d \mid \varepsilon \geqslant \varepsilon^d\right]}{\left(1-\beta\right)\left(1-\beta\phi\right)}, \tag{22}$$

and

$$\frac{\varepsilon^d}{1-\beta\phi} = \frac{u + \frac{\beta\phi}{1-\beta\phi}\int_{\varepsilon^d} \varepsilon dF\left(\varepsilon\right)}{1-\beta\phi F\left(\varepsilon^d\right)}. \tag{23}$$

We hence substitute $u = 0.015 \cdot P$ in the aseasonal model (equivalent to an annual rent-to-price ratio of 3 percent) for $\theta = 1/2$ (when sellers and buyers have the same bargaining power) and find the equilibrium value of P given the calibrated values for β and $F\left(.\right)$. We then use the implied value of $u = 0.015 \cdot P$ as a parameter.[31]

Finally, in reporting the results for prices, we vary θ, the seller's bargaining power parameter from 0 to 1.

4.4.2 Model-Generated Seasonality

Given the calibrated values of u, β, and ϕ discussed above, Table 8 displays the extent of seasonality in vacancies and transactions generated by the model for different values of the ratio of moving probabilities (recall that seasonality in vacancies and transactions is independent of the bargaining power of the seller, θ). As throughout the paper, our metric for seasonality is the annualized difference in growth rates between the two seasons. Column (1) shows the ratio of moving probabilities, $\frac{1-\phi^s}{1-\phi^w}$. Columns (2) and (5) show the implied difference in moving probabilities between the two seasons for the US and the UK, $[(1-\phi^s) - (1-\phi^w)]$. (Recall that, because the average stay in a house differs across the two countries, a given ratio can imply different values for $\phi^w - \phi^s$, as the average probability of stay ϕ differs.) Columns (3) and (4) show the extent of seasonality in

[30] In principle, other costs can trim down the 3-percent u/P ratio, including maintenance costs, and inefficiencies in the rental market that lead to a higher wedge between what the tenant pays and what the landlord receives; also, it might not be possible to rent the house immediately, leading to lower average flows u. Note that lower values of u/p lead to even higher seasonality in prices and transactions for any given level of seasonality in moving shocks.

[31] We also calibrated the model using different values of u for different θ (instead of setting $\theta = 1/2$), keeping the ratio u/P constant. Results are not significantly different under this procedure, but the comparability of results for different values of θ becomes less clear, since u is not kept fixed.

vacancies and transactions for an average stay of 9 years (as in the US) and Columns (6) and (7) show the corresponding figures for an average stay of 13 years (as in the UK)

Table 8. Seasonality in vacancies and transactions for different $\frac{1-\phi^s}{1-\phi^w}$.

Ratio of moving probabilities between seasons (1)	***Average moving probability: 0.0556 Stay of 9 years (U.S.)***			***Average moving probability: 0.0385 Stay of 13 years (U.K.)***		
	Implied seasonal *difference* in moving probabilities (2)	Vacancies (3)	Transactions (4)	Implied seasonal *difference* in moving probabilities (5)	Vacancies (6)	Transactions (7)
1.10	0.005	12%	49%	0.004	11%	48%
1.20	0.010	23%	94%	0.007	21%	93%
1.30	0.014	33%	136%	0.010	30%	133%
1.40	0.019	42%	174%	0.013	38%	171%
1.50	0.022	51%	211%	0.015	45%	207%

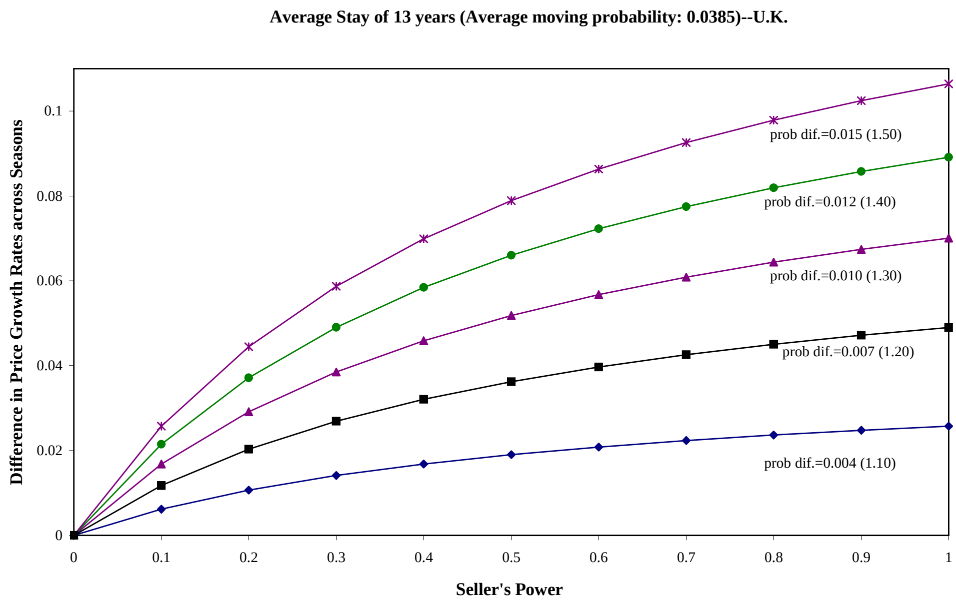
The first point to note is the large amplification mechanism present in the model: For any given level of seasonality in vacancies, seasonality in transactions is at least four times bigger. Second, the Table shows that a small absolute differences in the probability to stay between the two seasons can induce large seasonality in vacancies and transactions. Third, if we constrain ourselves to $\frac{1-\phi^s}{1-\phi^w} = 1.28$ to match the data on vacancies for the US, this implies a level of seasonality in transactions of about 135 percent in the US, very close to the actual 148 percent observed in the data. For the UK, ideally we would like to recalibrate the ratio $\frac{1-\phi^s}{1-\phi^w}$ to match its seasonality in vacancies; however, as said, the data are only available at yearly frequency. Using the same ratio $\frac{1-\phi^s}{1-\phi^w} = 1.28$ as a parameter for the UK would yield a seasonality in vacancies of 29 percent (the difference with the US is due to the longer duration of stay in the UK). This in turn would imply a degree of seasonality in transactions of 131 percent, somewhat above the 108 percent in the data. Note that, for a given ratio $\frac{1-\phi^s}{1-\phi^w}$, the model generates more seasonality in transactions in the US than in the UK (as in the data) because a given ratio implies a higher difference in moving probabilities $[(1-\phi^s)-(1-\phi^w)]$ in the US than in the UK, as the average stay is shorter in the former.

Seasonality in prices, as expressed earlier, depends crucially on the bargaining power of the seller, θ. Figure 8 plots the model-generated seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi^w}$, assuming an average stay of 13 years (as in the UK), and Figure 9 shows the corresponding plot for an average stay of 9 years (as in the US). As illustrated, seasonality increases with both θ and $\frac{1-\phi^s}{1-\phi^w}$. If, as before, we take $\frac{1-\phi^s}{1-\phi^w} = 1.28$ as given, the exercise implies that to match real-price seasonality in the UK (of about 6 percent, averaging between DCLG and Halifax), the bargaining power coefficient

θ needs to be around 75 percent. The corresponding value for the US as a whole, with real-price seasonality just above 3 percent, is 25 percent. For US cities, as noted in Table 6, seasonality is comparable to that in the UK (with an average of 5.7 percent for real prices, using the Case-Shiller composite of cities); the model accordingly suggests that in US cities the bargaining power of sellers is considerably higher than in the economy as a whole.

The question is of course whether large differences in the bargaining power of sellers across the two countries as a whole (and between US cities and the rest of the US) are tenable. There are at least two reasons why we think this is a reasonable characterization. First, population density in the UK (246 inhabitants per km^2) is 700 percent higher than in the US (31 inhabitants per km^2), making land significantly scarcer relative to population in the UK, and potentially conferring home owners more power in price negotiations (this should also be true in denser US cities). Second, anecdotal evidence suggests that land use regulations are particularly stringent in the UK.[32] Indeed in its international comparison of housing markets, the OECD Economic Outlook 2005 highlights the "complex and inefficient local zoning regulations and slow authorization process" in the UK economy, which the report cites as one of the reasons for the remarkable rigidity of housing supply.[33] Restrictions reinforce the market power of owners by reducing the supply of houses.

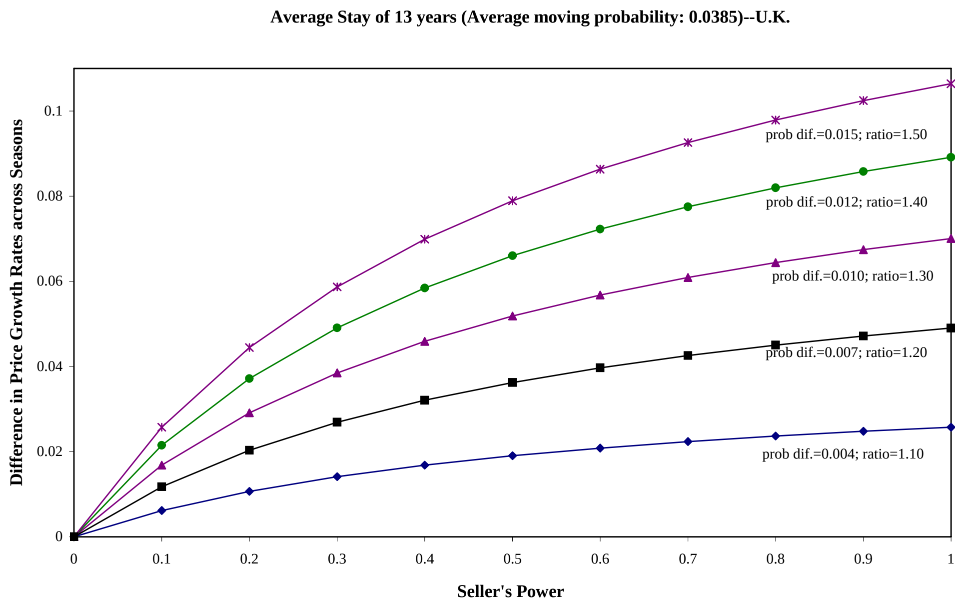
Figure 8: Seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi_w}$. UK.



[32] Again, this is likely to be true also in major cities in the US.

[33] OECD Economic Outlook 2005, Number 78, chapter III, available at http://www.oecd.org/dataoecd/41/56/35756053.pdf

Figure 9. Seasonality in prices for different θ and $\frac{1-\phi^s}{1-\phi_w}$. US.



5 Remarks on the Model

5.1 Efficiency Properties of the model

This Section discusses the efficiency of equilibrium in the decentralized economy. For a complete derivation, see Appendix 7.4. The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The key difference between the planner's solution and the decentralized solution is that the former internalizes the thick-market effect. It is evident that the equilibrium level of transactions in the decentralized economy is not socially efficient because the optimal decision rules of buyers and sellers takes the stock of vacancies in each period as given, thereby ignoring the effects of their decisions on the stock of vacant houses in the following periods. The thick-market effect generates a negative externality that makes the number of transactions in the decentralized economy inefficiently high *for any given stock of vacant houses* (transacting agents do not take into account that, by waiting, they can thicken the market in the following period and hence increase the overall quality of matches).[34]

The efficient level of seasonality in housing markets, however, will depend on the exact distribution of match quality $F(\varepsilon, v)$. Under likely scenarios, the solution of the planner will involve

[34] This result is similar to that in the stochastic job matching model of Pissarides (2000, chapter 8), where the reservation productivity is too low compared to the efficient outcome in the presence of search externalities.

a positive level of seasonality; that is, seasonality can be an efficient outcome. Indeed, in some circumstances, a planner may be willing to completely shut down the market in the cold season, to fully seize the benefits of a thick market.[35] This outcome is not as unlikely as one may a priori think. For example, the academic market for junior economists is extremely seasonal.[36] Extreme seasonality of course relies on the specification of utility—here we simply assume linear preferences; if agents have sufficiently concave utility functions (and intertemporal substitution across seasons is extremely low), then the planner may want to smooth seasonal fluctuations. For housing services, however, the concern of smoothing consumption across two seasons in principle should not be too strong relative to the benefit of having a better match that is on average long lasting (9 to 13 years in the two countries we analyze).

5.2 Model Assumptions

It is of interest to discuss three assumptions in the model. First, we assume that each buyer only meets one seller and each seller meets only one buyer in a given season. We do this for simplicity so that we can focus on the comparison across seasons. One might worry that if the outside option for a buyer is to meet another seller (rather than just renting and searching next period), that might affect the results on price seasonality.[37] This is, however, not the case here. Note first that the seller's outside option is also to sell to another buyer. More formally, the surplus to the buyer if the transaction for her first house goes through is:

$$\tilde{S}_b^s(\varepsilon) \equiv H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \{E^s[S_b^s(\eta)] + \beta B^w\}, \tag{24}$$

where $E^s[S_b^s(\eta)]$ is the equilibrium expected surplus (as defined in (4)) for the buyer if she goes for another house with random quality η. By definition $S_b^s(\eta) \geqslant 0$ (it equals zero when the draw for the second house η is too low). Compared to (4), the outside option for the buyer is higher because of the possibility of buying another house. Similarly, the surplus to the seller if the transaction goes through is:

$$\tilde{S}_v^s(\varepsilon) \equiv \tilde{p}^s(\varepsilon) - \{\beta V^w + u + E^s[S_v^s(\eta)]\}. \tag{25}$$

[35] The same will happen in the decentralized economy when the ratio $(1-\phi^s)/(1-\phi^w)$ is extremely high, e.g. the required ratio is larger than 10 for the calibrated parametrs we use.

[36] And it is perhaps highly efficient, given that it has been designed by our well-trained senior economists.

[37] Concretely, one might argue that the seller of the best house can now only capture part of the surplus of the buyer in excess of the buyer's second-best house. In this case, for the surplus (and hence prices) to be higher in the summer one would need higher dispersion of match quality in the summer. This intuition is, however, incomplete. Indeed, one can show that higher prices are obtained independently of the level of dispersion.

The key is that both buyer and seller take their outside options as given when bargaining. The price $\tilde{p}^s(\varepsilon)$ maximizes the Nash product with the surplus terms $\tilde{S}_b^s(\varepsilon)$ and $\tilde{S}_v^s(\varepsilon)$. The solution implies $(1-\theta)\tilde{S}_v^s(\varepsilon) = \theta\tilde{S}_b^s(\varepsilon)$, but the Nash bargaining for the second house implies that $(1-\theta)E^s[S_v^s(\eta)] = \theta E^s[S_b^s(\eta)]$, so:

$$(1-\theta)[\tilde{p}^s(\varepsilon) - (\beta V^w + u)] = \theta[H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \beta B^w],$$

which has the same form as (17); thus it follows that the equilibrium price equation for $\tilde{p}^s(\varepsilon)$ is identical to (18)—though the actual level of prices is different, as the cutoff match-quality is different. Our qualitative results on seasonality in prices continue to hold as before, and quantitatively they can be even stronger. Recall that in the baseline model we find that seasonality in the sum of buyer's and seller's values tends to reduce the quality of marginal transactions in the summer relative to winter because the outside option in the hot season is linked to the sum of values in the winter season: $B^w + V^w$. Intuitively, allowing the possibility of meeting another party in the same season as an outside option could mitigate this effect and hence strengthen seasonality in prices. To see this, the cutoff quality $\tilde{\varepsilon}^s$ is now defined by: $H^s(\tilde{\varepsilon}^s) = \beta(B^w + V^w) + u + E^s[S^s(\eta)]$. Compared to (6), the option of meeting another party as outside option shows up as an additional term, $E^s[S^s(\eta)]$, which is higher in the hot season.

A second simplification in the model is that buying and selling houses involve no transaction costs. This assumption is easy to dispense with. Let $\bar{\tau}_b^j$ and $\bar{\tau}_v^j$ be the transaction costs associated with the purchase $(\bar{\tau}_b^j)$ and sale $(\bar{\tau}_v^j)$ of a house in season j. Costs can be seasonal because moving costs and repairing costs may vary across seasons.[38] The previous definitions of surpluses are modified by replacing price p^j with $p^j - \bar{\tau}_v^j$ in (3) and with $p^j + \bar{\tau}_b^j$ in (4). The value functions (9) and (8), and the Nash solution (17) continue to hold as before. So, the price equation becomes:

$$p^s(\varepsilon) - \bar{\tau}_v^s = \theta[H^s(\varepsilon) - \bar{\tau}_v^s - \bar{\tau}_b^s] + (1-\theta)\frac{u}{1-\beta}, \tag{26}$$

which states that the net price received by a seller is a weighted average of housing value net of total transaction costs and the present discounted value of the flow value u. And the reservation equation becomes:

$$\varepsilon^s =: H^s(\varepsilon^s) - \left(\bar{\tau}_b^j + \bar{\tau}_v^j\right) = \beta(B^w + V^w) + u. \tag{27}$$

[38]Repair costs (both for the seller who's trying to make the house more attractive and for the buyer who wants to adapt it before moving in) may be smaller in the summer because good weather and the opportunity cost of time (assuming vacation is taken in the summer) are important inputs in construction). Moving costs, similarly, might be lower during vacation (because of both job and school holidays).

The extent of seasonality in transactions depends only on total costs ($\bar{\tau}_b^j + \bar{\tau}_v^j$) while the extent of seasonality in prices depends on the distribution of costs between buyers and sellers. One interesting result is that higher winter costs do not always result in lower winter prices. Indeed, if most of the transaction costs fall on the seller ($\bar{\tau}_v^j$ is high relative to $\bar{\tau}_b^j$), prices could actually be higher in the winter for θ sufficiently high. On the other hand, if most of the transaction costs are bared by the buyer, then seasonal transaction costs could potentially be the driver of seasonality in vacancies (and hence transactions and prices). As said, our theoretical results on seasonality in prices and transactions follow from $v^s > v^w$, independently of the particular trigger (that is, independently of whether it is seasonal transaction costs for the buyer or seasonal moving shocks; empirically, they are observationally equivalent, as they both lead to seasonality in vacancies, which we match in the quantitative exercise[39]).

Finally, the model presented so far assumed observable match-quality. In Appendix 7.5 we derive the case in which the seller cannot observe the match quality ε. We model the seller's power θ in this case as the probability that the seller makes a take-it-or-leave-it offer; $1 - \theta$ is then the probability that the buyer makes a take-it-or-leave-it offer upon meeting.[40] In that setting, $\theta = 1$ corresponds to the case in which sellers always post prices. When ε is observable, a transaction goes through whenever the total surplus is positive. However, when the seller does not observe ε, a transaction goes through only when the surplus to the buyer is positive. Since the seller does not observe ε, the seller offers a price that is independent of the level of ε, which will be too high for some buyers whose $\varepsilon' s$ are not sufficiently high (but whose ε would have resulted in a transaction if ε were observable to the seller). Therefore, because of the asymmetric information, the match is privately efficient only when the buyer is making a price offer. We show that our results continue to hold; the only qualitative difference is that the extent of seasonality in transactions is now decreasing in θ. This is because when ε is unobservable there is a second channel affecting a seller's surplus and hence the seasonality of reservation quality, which is opposite to the effects from the seasonality of outside option described above: When the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions; the higher the seller's power, θ, the more demanding they are and

[39] Furthermore, empirically, we are unaware of data on direct measures of moving costs or propensities to move, much less so at higher frequency.

[40] Samuelson (1984) shows that in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all.

the lower is the seasonality in transaction.

6 Concluding Remarks

This paper documents seasonal booms and busts in housing markets and argues that the predictability and high extent of seasonality in house prices cannot be quantitatively reconciled with models taking a simple asset-pricing approach.

To explain the empirical patterns, the paper presents a search-and-matching model that can quantitatively account for the seasonal fluctuations in prices and transactions observed in the US and the UK. The model sheds new light on interesting mechanisms governing fluctuations in housing markets that are likely to be present at lower frequencies. In particular, the thick-market effect that is at the core of the model's propagation mechanism does not depend on the frequency of the shocks. Lower frequency shocks associated with either business-cycle shocks or with less frequent booms and busts in housing markets could also be propagated through the same thick-market effects, to produce more amplitude in the fluctuations.

7 Appendix

7.1 Aggregate Seasonality (as Reported by the Publishers)

A first indication that house prices display seasonality comes from the observation that most publishers of house price indexes directly report SA data. Some publishers, however, report both SA and NSA data, and from these sources one can obtain a first measure of seasonality, as gauged by the publishers. For example, in the UK, Halifax publishes both NSA and SA house price series. Using these two series we computed the (logged) seasonal component of house prices as the ratio of the NSA house price series, P_t, relative to the SA series, P_t^*, from 1983:01 to 2007:04, $\left\{\ln \frac{P_t}{P_t^*}\right\}$. This seasonal component is plotted in Figure $A.1$. (Both the NSA and the SA series correspond to the UK as a whole.)

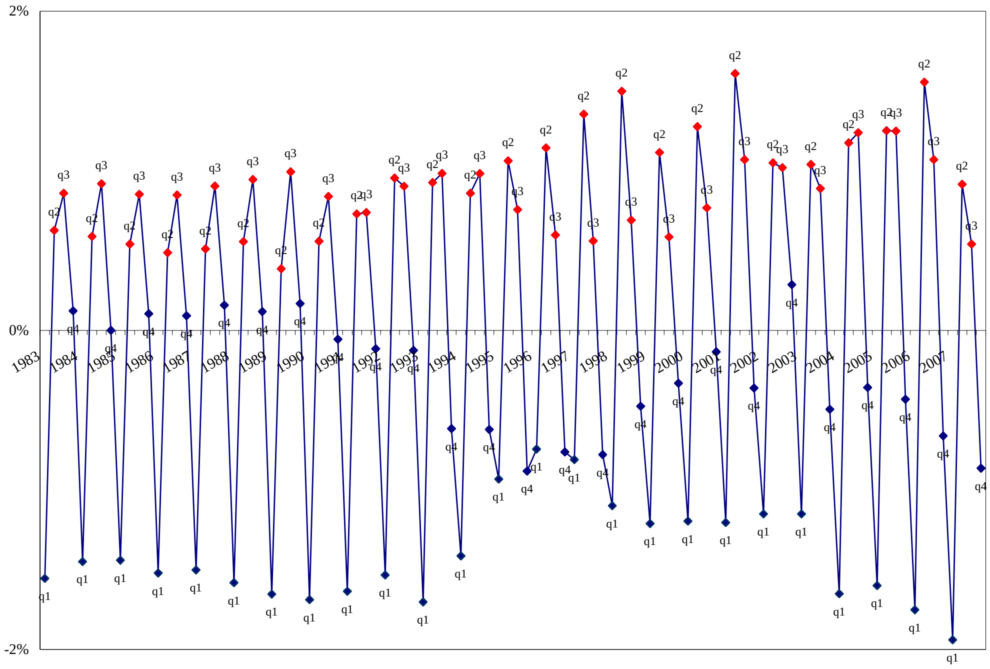
In the US, both the Office of Federal Housing Enterprise Oversight (OFHEO)'s house price index and the Case-Shiller index carried out by Standard & Poor's (S&P) are published in NSA and SA form. Figure $A.2$ depicts the seasonal component of the OFHEO series for the US as a whole, measured as before as $\left\{\ln \frac{P_t}{P_t^*}\right\}$, from 1991:01 through to 2007:04. And Figure $A.3$ shows the corresponding plot for the Case-Shiller index corresponding to a composite of 10 cities, with the data running from 1987:01 through to 2007:04. (The start of the sample in all cases is dictated by data availability.)[41]

All Figures seem to show a consistent pattern: House prices in the second and third quarters tend to rise above trend (captured by the SA series), and prices in the fourth, and particularly in the first quarter, tend to be in general at or below trend. The Figures also make it evident that the extent of price seasonality is more pronounced in the UK than in the US as a whole, though as shown in the text, certain cities in the US seem to display seasonal patterns of the same magnitude as those observed in the UK. (Some readers might be puzzled by the lack of symmetry in Figure $A.2$, as most expect the seasons to cancel out; this is exclusively due to the way OFHEO performs the seasonal adjustment;[42] for the sake of clarity and comparability across different datasets, we base our analysis only on the "raw", NSA series and hence the particular choice of seasonal adjustment by the publishers is inconsequential.)

[41] The original data in S&P's are monthly; we hence take the last month of the quarter—results are virtually identical when taking the average over the quarter.

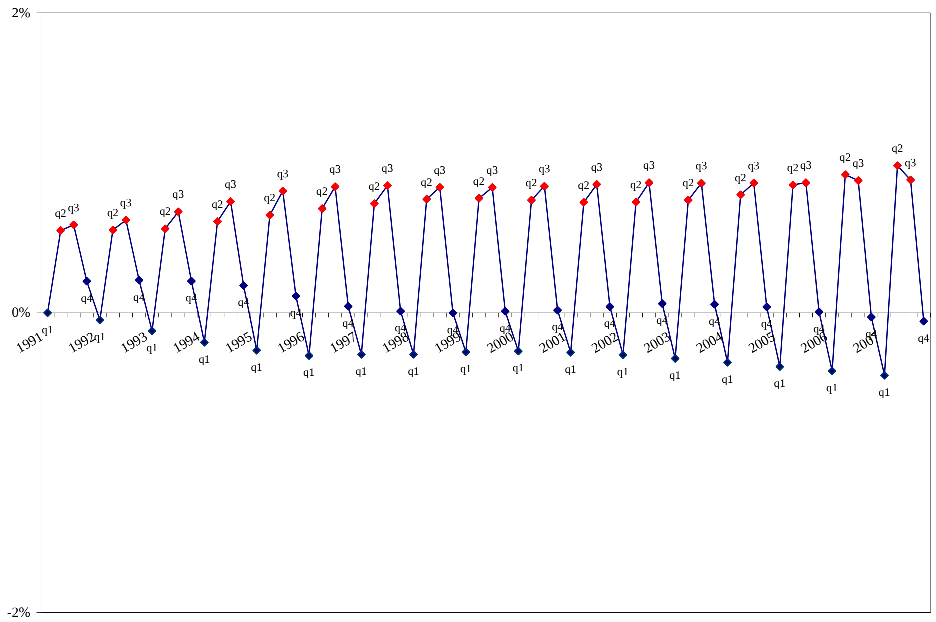
[42] OFHEO uses the Census Bureau's X-12 ARIMA procedure for SA; it is not clear, however, what the exact seasonality structure chosen is.

Figure $A.1$: Seasonal Component of House Prices in the UK 1983-2007.



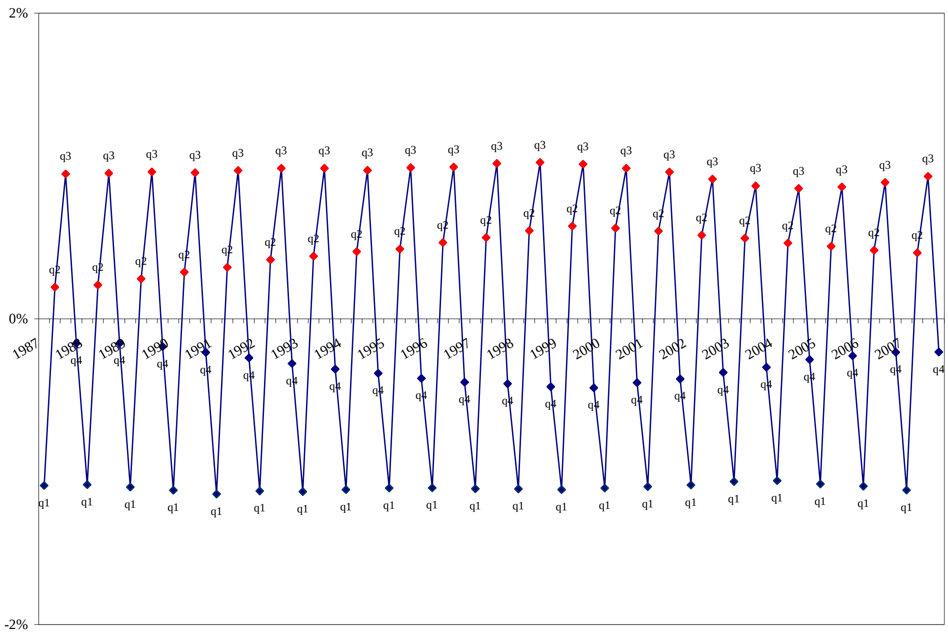
Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index. Source: Halifax.

Figure $A.2$: Seasonal Component of House Prices in the US. 1991-2007.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$; P_t is the NSA and P_t^* the SA index. Source: OFHEO.

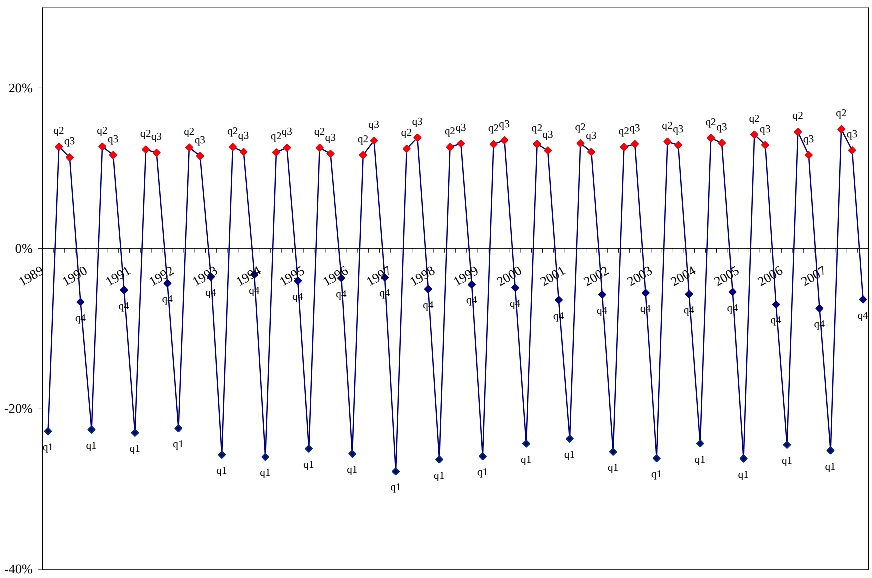
Figure $A.3$: Seasonal Component of House Prices in US cities 1987-2007.



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index. Source: Case-Shiller 10-city composite.

Last, but not least, the US National Association of Realtors (NAR) publishes data on transactions both with and without SA. Figure $A.4$ plots the seasonal component of house transactions, measured (as before) as the (logged) ratio of the (NSA) number of transactions Q_t, divided by the SA number of transactions Q_t^*:$\left\{\ln \frac{Q_t}{Q_t^*}\right\}$.

Figure $A.4$: Seasonal Component of Housing Transactions in the US. 1989-2007.



Note: The plot shows $\left\{\ln \frac{Q_t}{Q_t^*}\right\}$; Q_t is the NSA and Q_t^* the SA number of transactions. Source: NAR.

The seasonal pattern for transactions is similar to that for prices: Transactions surge in the second and third quarters and stagnate or fall in the fourth and first quarters. (In the UK only NSA data for transactions are available from the publishers.)

7.2 A back-of-the-envelope calculation

We argued before that the predictability and size of the seasonal variation in housing prices pose a puzzle to models of the housing market relying on standard asset-market equilibrium conditions. In particular, the equilibrium condition embedded in most dynamic general-equilibrium models states that the marginal benefit of housing services should equal the marginal cost. Following Poterba (1984) the asset-market equilibrium conditions for any seasons $j = s$ (summer), w (winter) at time t is:[43]

$$d_{t+1,j'} + (p_{t+1,j'} - p_{t,j}) = c_{t,j} \cdot p_{t,j} \tag{28}$$

where j' is the corresponding season at time $t+1$, $p_{t,j}$ and $d_{t,j}$ are the real asset price and rental price of housing services, respectively; $c_{t,j} \cdot p_{t,j}$ is the real *gross* (gross of capital gains) $t-$period cost of housing services of a house with real price $p_{t,j}$; and $c_{t,j}$ is the sum of after-tax depreciation, repair costs, property taxes, mortgage interest payments, and the opportunity cost of housing equity. Note that the formula assumes away risk (and hence no expectation terms are included); this is appropriate in this context because we are focusing on a "predictable" variation of prices.[44] As in Poterba (1984), we make the following simplifying assumptions so that service-cost rates are a fixed proportion of the property price, though still potentially different across seasons ($c_{t,j} = c_{t+2,j} = c_j$, $j = s, w$): 1) Depreciation takes place at rate δ_j, $j = s, w$, constant for a given season, and the house requires maintenance and repair expenditures equal to a fraction κ_j, $j = s, w$, also constant for a given season. 2) The income-tax-adjusted real interest rate and the marginal property tax rates (for given real property prices) are constant over time, though also potentially different across seasons; they are denoted, respectively as r_j and τ_j, $j = s, w$ (in the data, as seen, they are actually constant across seasons; we come back to this point below).[45] This yields $c_j = \delta_j + \kappa_j + r_j + \tau_j$, for $j = s, w$.

[43] See also Mankiw and Weil (1989) and Muellbauer and Murphy (1997), among others.

[44] Note that Poterba's formula also implicitly assumes linear preferences and hence perfect intertemporal substitution. This is a good assumption in the context of seasonality, given that substitution across semesters (or relatively short periods of time) should in principle be quite high.

[45] We implicitly assume the property-price brackets for given marginal rates are adjusted by inflation rate, though strictly this is not the case (Poterba, 1984): inflation can effectively reduce the cost of homeownership. This, however, should not alter the conclusions concerning seasonal patterns emphasized here. As in Poterba (1984) we also assume

Subtracting (28) from the corresponding expression in the following season and using the condition that there is no seasonality in rents ($d_w \approx d_s$), we obtain:

$$\frac{p_{t+1,s} - p_{t,w}}{p_{t,w}} - \frac{p_{t,w} - p_{t-1,s}}{p_{t-1,s}} \frac{p_{t-1,s}}{p_{t,w}} = c_w - c_s \cdot \frac{p_{t-1,s}}{p_{t,w}} \tag{29}$$

Using DCLG-based results, *real* differences in house price growth rates for the whole of the UK are $\frac{p_s - p_w}{p_w} \simeq 8.25\%$, $\frac{p_w - p_s}{p_s} \simeq 1.06\%$,[46] the left-hand side of (29) equals $7.2\% \approx 8.25\% - 1.06\% \cdot \frac{1}{1.0106}$. Therefore, $\frac{c_w}{c_s} = \frac{0.072}{c_s} + \frac{1}{1.0106}$. The value of c_s can be pinned-down from equation (28) with $j = s$, depending on the actual rent-to-price ratios in the economy. In Table $A.1$, we summarize the extent of seasonality in service costs $\frac{c_w}{c_s}$ implied by the asset-market equilibrium conditions, for different values of d/p (and hence different values of $c_s = \frac{d_w}{p_s} + \frac{p_w - p_s}{p_s} = \frac{d_w}{p_s} + 0.0106$).

Table $A.1$: Ratio of Winter-To-Summer Cost Rates

(annualized) d/p Ratio	Relative winter cost rates $\frac{c_w}{c_s}$
1.0%	448%
2.0%	334%
3.0%	276%
4.0%	241%
5.0%	218%
6.0%	201%

As the Table illustrates, a remarkable amount of seasonality in service costs is needed to explain the differences in housing price inflation across seasons. Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 334 and 218 percent of those in the summer. Depreciation and repair costs ($\delta_j + \kappa_j$) might be seasonal, being potentially lower during the summer.[47] But income-tax-adjusted interest rates and property taxes ($r_j + \tau_j$), two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the UK is even larger. Assuming, quite conservatively, that the a-seasonal component ($r_j + \tau_j = r + \tau$) accounts for only 50 percent of the service costs in the summer ($r + \tau = 0.5c_s$), then, the formula for relative costs $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + 0.5c_s}{\delta_s + \kappa_s + 0.5c_s}$ implies that the

that the opportunity cost of funds equals the cost of borrowing.

[46] In the empirical Section we computed growth rates using difference in logs; the numbers are very close using $\frac{p_{t+1,j'} - p_{t,j}}{p_{t,j}}$ instead. We use annualized rates as in the text; using semester rates of course leads to the same results.

[47] Good weather can help with external repairs and owners' vacation might reduce the opportunity cost of time—though for this to be true it would be key that leisure were not too valuable for the owners.

ratio of depreciation and repair costs between summers and winters is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = 2\frac{c_w}{c_s} - 1$.[48] For rent-to-price ratios in the range of 2 through 5 percent, depreciation and maintenance costs in the winter should be between 568 and 336 percent of those in the summer. (If the a-seasonal component $(r+\tau)$ accounts for 80 percent of the service costs ($r + \tau = 0.8c_s$), the corresponding values are 1571 and 989 percent). By any metric, these figures seem extremely large and suggest that a deviation from the simple asset-pricing equation is called for. Similar calculations can be performed for different regions in the US; as expressed before, though the extent of price seasonality for the US as a whole is lower than in the UK, seasonality in several US cities is comparable to that in the UK and would therefore also imply large seasonality in service costs, according to condition (28).

7.3 Derivation for the model with observable value

7.3.1 Solving for prices

To derive $p^s(\varepsilon)$ in (18), use the Nash solution (17),

$$[p^s(\varepsilon) - \beta V^w - u](1-\theta) = [H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w]\,\theta,$$

so

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + \beta\left[(1-\theta)V^w - \theta B^w\right] + (1-\theta)u. \tag{30}$$

Using the value functions (8) and (9),

$$(1-\theta)V^s - \theta B^s = \beta\left[(1-\theta)V^w - \theta B^w\right] + (1-\theta)u$$

solving out explicitly,

$$(1-\theta)V^s - \theta B^s = \frac{(1-\theta)u}{1-\beta}$$

substitute into (30) to obtain (18).

7.3.2 The model without seasons

The value functions for the model without seasons are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical

[48] Call λ the aseasonal component as a fraction of the summer service cost rate: $r+\tau = \lambda c_s$, $\lambda \in (0,1)$ (and hence $\delta_s + \kappa_s = (1-\lambda)c_s$). Then: $\frac{c_w}{c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{\delta_s+\kappa_s+\lambda c_s} = \frac{\delta_w+\kappa_w+\lambda c_s}{c_s}$. Or $c_w = \delta_w + \kappa_w + \lambda c_s$. Hence: $\frac{c_w-\lambda v_s}{(1-\lambda)c_s} = \frac{\delta_w+\kappa_w}{(1-\lambda)c_s}$; that is $\frac{\delta_w+\kappa_w}{\delta_s+\kappa_s} = \frac{c_w}{(1-\lambda)c_s} - \frac{\lambda}{1-\lambda}$, which is increasing in λ for $\frac{c_w}{c_s} > 1$.

by simply setting $\phi^s = \phi^w$. Using (7), (12) and (19) to express the average price as:

$$P^s = \frac{u}{1-\beta} + \theta \left[\frac{\beta\left(1+\beta\phi^s\right) h^w\left(\varepsilon^w\right) + \left(1-\beta^2 F^s\left(\varepsilon^s\right)\right)\left(1+\beta\phi^w\right) E\left[\varepsilon - \varepsilon^s \mid \varepsilon \geqslant \varepsilon^s\right]}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)} \right], \tag{31}$$

Using (13),

$$\frac{\varepsilon}{1-\beta\phi} = u + \frac{\beta\phi}{1-\beta\phi}\left(1-\beta\right)\left(V+B\right)$$

and $B+V$ from (12),

$$B+V = \frac{u}{1-\beta} + \frac{1}{1-\beta^2}\left\{\frac{1-F}{1-\beta\phi} E\left[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right] + \beta\frac{1-F}{1-\beta\phi} E\left[\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right]\right\}$$

which reduces to:

$$B+V = \frac{u}{1-\beta} + \frac{1-F\left(\varepsilon\right)}{\left(1-\beta\right)\left(1-\beta\phi\right)} E\left(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right).$$

It follows that

$$\varepsilon = u + \frac{\beta\phi}{1-\beta\phi}\left[1-F\left(\varepsilon\right)\right] E\left(\tilde{\varepsilon} - \varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon\right),$$

and the law of motion for vacancy implies:

$$v = \frac{1-\phi}{1-\phi F\left(\varepsilon\right)}.$$

7.4 Analytical derivations of the planner's solution

The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The interesting comparison is the level of reservation quality achieved by the planner with the corresponding level in the decentralized economy. To spell out the planner's problem, we follow Pissarides (2000) and assume that in any period t the planner takes as given the expected value of the housing utility service per person in period t (before he optimizes), which we denote by q_{t-1}, as well as the beginning of period's stock of vacant houses, v_t. Thus, taking as given the initial levels q_{-1} and v_0, and the sequence $\{\phi_t\}_{t=0\ldots}$, which alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem is to choose a sequence of $\{\varepsilon_t\}_{t=0,..}$ to maximize

$$U\left(\{\varepsilon_t, q_t, v_t\}_{t=0\ldots}\right) \equiv \sum_{t=0}^{\infty} \beta^t \left[q_t + u v_t F\left(\varepsilon_t; v_t\right)\right] \tag{32}$$

subject to the law of motion for q_t :

$$q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF\left(x; v_t\right), \tag{33}$$

the law of motion for v_t (which is similar to the one in the decentralized economy):

$$v_{t+1} = v_t \phi_{t+1} F(\varepsilon_t; v_t) + 1 - \phi_{t+1}, \tag{34}$$

and the inequality constraint:

$$0 \leqslant \varepsilon_t \leqslant \bar{\varepsilon}(v_t), \tag{35}$$

where the upper bound $\bar{\varepsilon}$ can potentially be infinite.

The planner faces two types of trade-offs when deciding the optimal reservation quality ε_t: A static one and a dynamic one. The static trade-off stems from the comparison of utility values generated by occupied houses and vacant houses in period t in the objective function (32). The utility per person generated from vacant houses is the rental income per person, captured by $uv_tF(\varepsilon_t)$. The utility generated by occupied houses in period t is captured by q_t, the expected housing utility service per person conditional on the reservation value ε_t set by the planner in period t. The utility q_t, which follows the law of motion (33), is the sum of the pre-existing expected housing utility q_{t-1} that survives the moving shock and the expected housing utility from the new matches. By increasing ε_t, the expected housing value q_t decreases, while the utility generated by vacant houses increases (since $F(\varepsilon_t)$ increases). The dynamic trade-off operates through the law of motion for the stock of vacant houses in (34). By increasing ε_t (which in turn decreases q_t), the number of transactions in the current period decreases; this leads to more vacant houses in the following period, v_{t+1}, and consequently to a thicker market in the next period. We first derive the case where the inequality constraints are not binding, i.e. markets are open in both the cold and hot seasons.

The Planner's solution when the housing market is open in all seasons

Because the sequence $\{\phi_t\}_{t=0,...}$ alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem can be written recursively. Taking (q_{t-1}, v_t), and $\{\phi_t\}_{t=0,..}$ as given, and provided that the solution is interior, that is, $\varepsilon_t < v_t$, the Bellman equation for the planner is given by:

$$\begin{aligned} W(q_{t-1}, v_t, \phi_t) &= \max_{\varepsilon_t} \left[q_t + uv_tF(\varepsilon_t; v_t) + \beta W\left(q_t, v_{t+1}, \phi_{t+1}\right)\right] \\ s.t. \qquad & q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF(x; v_t), \\ v_{t+1} &= v_t\phi_{t+1}F(\varepsilon_t; v_t) + 1 - \phi_{t+1}. \end{aligned} \tag{36}$$

The first-order condition implies

$$\left(1 + \beta \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial q_t}\right) v_t\left(-\varepsilon_t f(\varepsilon_t; v_t)\right) + \left(\beta \phi_{t+1} \frac{\partial W\left(q_t, v_{t+1}, \phi_{t+1}\right)}{\partial v_{t+1}} + u\right) v_t f(\varepsilon_t; v_t) = 0,$$

which simplifies to

$$\varepsilon_t\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right)=u+\beta\phi_{t+1}\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}. \tag{37}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W\left(q_{t-1},v_t,\phi_t\right)}{\partial q_{t-1}}=\phi_t\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right) \tag{38}$$

and

$$\begin{aligned}\frac{\partial W\left(q_{t-1},v_t,\phi_t\right)}{\partial v_t} &= \left(u+\beta\phi_{t+1}\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}\right)\left(F\left(\varepsilon_t;v_t\right)-v_tT_{1t}\right) \\ &\quad+\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right)\left(\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)+v_tT_{2t}\right)\end{aligned} \tag{39}$$

where $T_{1t}\equiv\frac{\partial}{\partial v_t}\left[1-F\left(\varepsilon_t;v_t\right)\right]>0$ and $T_{2t}\equiv\frac{\partial}{\partial v_t}\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)>0$. In the periodic steady state, the first order condition (37) becomes

$$\varepsilon^j\left(1+\beta\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial q^j}\right)=u+\beta\phi^{j'}\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial v^{j'}} \tag{40}$$

The envelope condition (38) implies

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}=\phi^j\left[1+\beta\left(\phi^{j'}+\beta\phi^{j'}\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}\right)\right]$$

which yields:

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}=\frac{\phi^j\left(1+\beta\phi^{j'}\right)}{1-\beta^2\phi^j\phi^{j'}} \tag{41}$$

Substituting this last expression into (39), we obtain:

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\left(u+\beta\phi^{j'}\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial v^{j'}}\right)A^j+D^j,$$

where

$$A^j\equiv F^j\left(\varepsilon^j\right)-v^jT_1^j;\qquad D^j\equiv\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j}xdF^j\left(x\right)+v^jT_2^j\right), \tag{42}$$

Hence, we have

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\left\{u+\beta\phi^{j'}\left[\left(u+\beta\phi^j\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}\right)A^{j'}+D^{j'}\right]\right\}A^j+D^j,$$

which implies

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\frac{uA^j\left(1+\beta\phi^{j'}A^{j'}\right)+\beta\phi^{j'}D^{j'}A^j+D^j}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}. \tag{43}$$

Substituting (41) and (43) into the first-order condition (40),

$$\varepsilon^j\left(1+\beta\frac{\phi^{j'}\left(1+\beta\phi^j\right)}{1-\beta^2\phi^j\phi^{j'}}\right)=u+\beta\phi^{j'}\frac{uA^{j'}\left(1+\beta\phi^jA^j\right)+\beta\phi^jD^jA^{j'}+D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}$$

simplify to:

$$\varepsilon^j\left(\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\right)=\frac{\left(1+\beta\phi^{j'}A^{j'}\right)u+\beta^2\phi^j\phi^{j'}A^{j'}D^j+\beta\phi^{j'}D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}},\tag{44}$$

and the stock of vacant houses, v^j, $j=s,w$, satisfies (14) as in the decentralized economy.

The thick-market effect enters through two terms: $T_1^j\equiv\frac{\partial}{\partial v^j}\left[1-F^j\left(\varepsilon^j\right)\right]>0$ and $T_2^j\equiv\frac{\partial}{\partial v^j}\int_{\varepsilon^j}^{\bar{\varepsilon}^j}xdF^j\left(x\right)>0$. The first term, T_1^j, indicates that the thick-market effect shifts up the acceptance schedule $\left[1-F^j\left(\varepsilon\right)\right]$. The second term, T_2^j, indicates that the thick-market effect increases the conditional quality of transactions. The interior solution (44) is an implicit function of ε^j that depends on $\varepsilon^{j'}$, v^j, and $v^{j'}$. It is not straightforward to derive an explicit condition for $\varepsilon^j<v^j$, $j=s,w$. Abstracting from seasonality for the moment, i.e. when $\phi^s=\phi^w$, it follows immediately from (14) that the solution is interior, $\varepsilon<v$. Moreover (44) implies the planner's optimal reservation quality ε^p satisfies:

$$\frac{\varepsilon^p}{1-\beta\phi}=\frac{u+\frac{\beta\phi}{1-\beta\phi}\left(\int_{\varepsilon^p}^{\bar{\varepsilon}}xdF\left(x\right)+vT_2\right)}{1-\beta\phi F\left(\varepsilon^p\right)+\beta\phi vT_1}.\tag{45}$$

Comparing (45) with (23), the thick-market effect, captured by T_1 and T_2, generates two opposite forces. The term T_1 decreases ε^p, while the term T_2 increases ε^p in the planner's solution. Thus, the positive thick-market effect on the acceptance rate T_1 implies that the number of transactions is too low in the decentralized economy, while the positive effect on quality T_2 implies that the number of transactions is too high. Since $1-\beta\phi$ is close to zero, however, the term T_2 dominates. Therefore, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome. As discussed in the text, comparing the extent in seasonality in the decentralized equilibrium to the planner's solution depends on the exact distribution $F\left(\varepsilon,v\right)$. We next derive the case in which the Planner finds it optimal to close down the market in the cold season.

The Planner's solution when the housing market is closed in the cold season

Setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$, the Bellman equation (36) can be rewritten as:

$$\begin{aligned} W^s\left(q_{t-1}^w, v_t^s\right) &= \max_{\varepsilon_t^s} \left[\begin{array}{c} \phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + u v_t^s F_t^s(\varepsilon_t^s) \\ +\beta\left(q_{t+1}^w + u\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right]\right) \\ +\beta^2 W^s\left(q_{t+1}^w, v_{t+2}^s\right) \end{array}\right] \\ & \quad s.t. \\ q_{t+1}^w &= \phi^w \left[\phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x)\right], \\ v_{t+2}^s &= \phi^s\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right] + 1 - \phi^s. \end{aligned} \tag{46}$$

Intuitively, "a period" for the decision of ε_t^s is equal to $2t$. The state variables for the current period are given by the vector $\left(q_{t-1}^w, v_t^s\right)$, the state variables for next period are $\left(q_{t+1}^w, v_{t+2}^s\right)$, and the control variable is ε_t^s. The first order condition is:

$$\begin{aligned} 0 &= v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s f_t^s(\varepsilon_t^s) \\ &\quad +\beta\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s \phi^w f_t^s(\varepsilon_t^s)\right) \\ &\quad +\beta^2\left[\frac{\partial W^s}{\partial q_{t+1}^w}\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right)\right) + \frac{\partial W^s}{\partial v_{t+2}^s}\left(\phi^s v_t^s \phi^w f_t^s(\varepsilon_t^s)\right)\right], \end{aligned}$$

which simplifies to:

$$\begin{aligned} 0 &= -\varepsilon_t^s + u + \beta\left(-\phi^w \varepsilon_t^s + u\phi^w\right) \\ &\quad +\beta^2\left[\frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\left(-\phi^w \varepsilon_t^s\right) + \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s\phi^w\right] \end{aligned}$$

and can be written as:

$$\varepsilon_t^s\left[1 + \beta\phi^w + \beta^2\phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right] = (1 + \beta\phi^w)\, u + \beta^2\phi^w\phi^s \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \tag{47}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial q_{t-1}^w} = \phi^s + \beta\phi^w\phi^s + \beta^2\phi^w\phi^s \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}, \tag{48}$$

and

$$\begin{aligned} & \frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\ =\; & (1 + \beta\phi^w)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) + (1 + \beta\phi^w)\, u\left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right] \\ & +\beta^2 \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\phi^w\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) \\ & +\beta^2 \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s\phi^w\left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right], \end{aligned}$$

where $T_{1t}^s \equiv \frac{\partial}{\partial v_t^s}\left[1-F_t^s\left(\varepsilon^s\right)\right]>0$ and $T_{2t}^s \equiv \frac{\partial}{\partial v_t^s}\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s\left(x\right)>0$. Rewrite the last expression as:

$$\begin{aligned} & \frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \qquad (49)\\ = & \left(1+\beta\phi^w+\beta^2\phi^w\frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s\left(x\right)+v_t^s T_{2t}^s\right)\\ & +\left(\left(1+\beta\phi^w\right)u+\beta^2\phi^s\phi^w\frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\right)\left[F_t^s\left(\varepsilon_t^s\right)-v_t^s T_{1t}^s\right]\end{aligned}$$

In steady state, (48) and (49) become

$$\frac{\partial W^s\left(q^w, v^s\right)}{\partial q^w}=\frac{\phi^s\left(1+\beta\phi^w\right)}{1-\beta^2\phi^w\phi^s}, \qquad (50)$$

and

$$\begin{aligned} & \frac{\partial W^s\left(q^w, v^s\right)}{\partial v^s}\left(1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]\right) \qquad (51)\\ = & \left(1+\beta\phi^w+\beta^2\phi^w\frac{\phi^s\left(1+\beta\phi^w\right)}{1-\beta^2\phi^w\phi^s}\right)\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s\left(x\right)+v^s T_2^s\right)\\ & +\left(1+\beta\phi^w\right)u\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right].\end{aligned}$$

Substituting into the FOC (47),

$$\begin{aligned} & \varepsilon^s\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\\ = & \left(1+\beta\phi^w\right)u+\beta^2\phi^w\phi^s\frac{\left(1+\beta\phi^w\right)u\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]+\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s\left(x\right)+v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]}\end{aligned}$$

which simplifies to

$$\frac{\varepsilon^s}{1-\beta^2\phi^w\phi^s}=\frac{u+\frac{\beta^2\phi^w\phi^s}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s\left(x\right)+v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s\left(\varepsilon^s\right)-v^s T_1^s\right]}, \qquad (52)$$

which is similar to the Planner's solution with no seasons in (45), with $\beta^2\phi^w\phi^s$ replacing $\beta\phi$.

7.5 Model with unobservable match quality

Assume that the seller does not observe ε. As shown by Samuelson (1984), in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all. Hence, we adopt a simple price-setting mechanism: The seller makes a take-it-or-leave-it offer p^{jv} with probability $\theta\in[0,1]$ and the buyer makes a take-it-or-leave-it offer p^{jb} with probability $1-\theta$. ($\theta=1$ corresponds to the case in which sellers post prices.) Broadly speaking, we can interpret

θ as the "bargaining power" of the seller. The setup of the model implies that the buyer accepts any offer p^{sv} if $H^s(\varepsilon) - p^{sv} \geqslant \beta B^w$; and the seller accepts any price $p^{sb} \geqslant \beta V^w + u$. Let S_v^{si} and $S_b^{si}(\varepsilon)$ be the surplus of a transaction to the seller and the buyer when the match quality is ε and the price is p^{si}, for $i = b, v$:

$$\begin{aligned} S_v^{si} &\equiv p^{si} - (u + \beta V^w), &(53)\\ S_b^{si}(\varepsilon) &\equiv H^s(\varepsilon) - p^{si} - \beta B^w. &(54)\end{aligned}$$

Note that the definition of S_v^{si} implies that

$$p^{sv} = S_v^{sv} + p^{sb} \tag{55}$$

i.e. the price is higher when the seller is making an offer. Since only the buyer observes ε, a transaction goes through only if $S_b^{si}(\varepsilon) \geqslant 0$, $i = b, v$, i.e. a transaction goes through only if the surplus to the buyer is non-negative regardless of who is making an offer. Given $H^s(\varepsilon)$ is increasing in ε, for any price p^{si}, $i = b, v$, a transaction goes through if $\varepsilon \geqslant \varepsilon^{si}$, where

$$H^s\left(\varepsilon^{si}\right) - p^{si} = \beta B^w. \tag{56}$$

$1 - F^s(\varepsilon^{si})$ is thus the probability that a transaction is carried out. From (2), the response of the reservation quality ε^{si} to a change in price is given by:

$$\frac{\partial \varepsilon^{si}}{\partial p^{si}} = \frac{1 - \beta^2 \phi^w \phi^s}{1 + \beta \phi^w}. \tag{57}$$

Moreover, by the definition of $S_b^{si}(\varepsilon)$ and ε^{si}, in equilibrium, the surplus to the buyer is:

$$S_b^{si}(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1 + \beta\phi^w}{1 - \beta^2\phi^w\phi^s}\left(\varepsilon - \varepsilon^{si}\right). \tag{58}$$

7.5.1 The Seller's offer

Taking the reservation policy ε^{sv} of the buyer as given, the seller chooses a price to maximize the expected surplus value of a sale:

$$\max_p \left\{ [1 - F^s(\varepsilon^{sv})]\,[p - \beta V^w - u] \right\}$$

The optimal price p^{sv} solves

$$[1 - F^s(\varepsilon^{sv})] - [p - \beta V^w - u]\, f^s(\varepsilon^{sv})\, \frac{\partial \varepsilon^{sv}}{\partial p^s} = 0. \tag{59}$$

Rearranging terms we obtain:

$$\underbrace{\frac{p^{sv}-\beta V^{w}-u}{p^{sv}}}_{\text{mark-up}}=\underbrace{\left[\frac{p^{sv}f^{s}\left(\varepsilon^{sv}\right)\frac{\partial\varepsilon^{s}}{\partial p^{s}}}{1-F^{s}\left(\varepsilon^{sv}\right)}\right]^{-1}}_{\text{inverse-elasticity}},$$

which makes clear that the price-setting problem of the seller is similar to that of a monopolist who sets a markup equal to the inverse of the elasticity of demand (where demand in this case is given by the probability of a sale, $1-F^{s}\left(\varepsilon^{s}\right)$). The optimal decisions of the buyer (57) and the seller (59) together imply:

$$S_{v}^{sv}=\frac{1-F^{s}\left(\varepsilon^{sv}\right)}{f^{s}\left(\varepsilon^{sv}\right)}\frac{1+\beta\phi^{w}}{1-\beta^{2}\phi^{w}\phi^{s}}. \tag{60}$$

Equation (60) says that the surplus to a seller generated by the transaction is higher when $\frac{1-F^{s}(\varepsilon^{sv})}{f^{s}(\varepsilon^{sv})}$ is higher, i.e. when the conditional probability that a successful transaction is of match quality ε^{sv} is lower. Intuitively, the surplus of a transaction to a seller is higher when the house is transacted with a stochastically higher match quality, or loosely speaking, when the distribution of match quality has a "thicker" tail.

Given the price-setting mechanism, in equilibrium, the value of a vacant house to its seller is:

$$V^{s}=u+\beta V^{w}+\theta\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}. \tag{61}$$

Solving out V^{s} explicitly,

$$V^{s}=\frac{u}{1-\beta}+\theta\frac{\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right]S_{v}^{wv}}{1-\beta^{2}}, \tag{62}$$

which is the sum of the present discounted value of the flow value u and the surplus terms when its seller is making the take-it-or-leave-it offer, which happens with probability θ. Using the definition of the surplus terms, the equilibrium p^{sv} is:

$$p^{sv}=\frac{u}{1-\beta}+\theta\frac{\left[1-\beta^{2}F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right]S_{v}^{wv}}{1-\beta^{2}}. \tag{63}$$

7.5.2 The Buyer's Offer

The buyer offers a price that extracts all the surplus from the seller, i.e.

$$S_{v}^{sb}=0\Leftrightarrow p^{sb}=u+\beta V^{w}$$

Using the value function V^{w} from (62), the price offered by the buyer is:

$$p^{sb}=\frac{u}{1-\beta}+\theta\frac{\beta^{2}\left[1-F^{s}\left(\varepsilon^{sv}\right)\right]S_{v}^{sv}+\beta\left[1-F^{w}\left(\varepsilon^{wv}\right)\right]S_{v}^{wv}}{1-\beta^{2}}. \tag{64}$$

The buyer's value function is:

$$B^s = \beta B^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] \\ + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right], \tag{65}$$

where $E^s[.]$ indicates the expectation taken with respect to the distribution $F^s(.)$. Since the seller does not observe ε, the expected surplus to the buyer is positive even when the seller is making the offer (which happens with probability θ). As said, buyers receive zero housing service flow until they find a successful match. Solving out B^s explicitly,

$$B^s = \theta\left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] \\ + \beta\left\{\theta\left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S_b^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S_b^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}. \tag{66}$$

7.5.3 Reservation quality

In any season s, the reservation quality ε^{si}, for $i = v, b$, satisfies

$$H^s\left(\varepsilon^{si}\right) = S_v^{si} + u + V^w + \beta B^w, \tag{67}$$

which equates the housing value of a marginal owner in season s, $H^s(\varepsilon^s)$, to the sum of the surplus generated to the seller (S_v^{si}), plus the sum of outside options for the buyer (βB^w) and the seller ($\beta V^w + u$). Using (2), ε^{si} solves:

$$\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si} = S_v^{si} + u + \frac{\beta\phi^w\left(1-\beta^2\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^w + V^w\right) - \frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^s + B^s\right). \tag{68}$$

The reservation quality ε^s depends on the sum of the outside options for buyers and sellers in both seasons, which can be derived from (62) and (66):

$$\begin{aligned} & B^s + V^s \\ = \; & \frac{u}{1-\beta} + \\ & \theta\left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] + \\ & \beta\left\{\theta\left(1 - F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right] + (1-\theta)\left[1 - F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}, \end{aligned} \tag{69}$$

where $S^{si}(\varepsilon) \equiv S_b^{si}(\varepsilon) + S_v^{si}$ is the total surplus from a transaction with match quality ε. Note from (68) that the reservation quality is lower when the buyer is making a price offer: $\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon^{sv} - \varepsilon^{sb}\right) = S_v^{sv}$. Also, because of the asymmetric information, the match is privately efficient when the buyer is making a price offer.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $\left(p^{jv}, p^{jb}, \varepsilon^{jv}, \varepsilon^{jb}\right)$ in season $j = s, w$ through two channels, as shown in (63), (64), and(68)): The conditional density of the distribution at reservation ε^{jv}, i.e. $\frac{f^j\left(\varepsilon^{jv}\right)}{1-F^j(\varepsilon^{jv})}$, and the expected surplus quality above reservation ε^{jv}, i.e. $\left(1 - F^j\left(\varepsilon^{ji}\right)\right) E^j\left[\varepsilon - \varepsilon^{ji} \mid \varepsilon \geqslant \varepsilon^{ji}\right]$, $i = b, v$. As shown in (60), a lower conditional probability that a transaction is of marginal quality ε^{jv} implies higher expected surplus to the seller S_v^{jv}, which increases the equilibrium prices p^{jv} and p^{jb} in (63) and (64). Similarly as shown in (58), a higher expected surplus quality above ε^{jv} (follows from (1)) implies a higher expected surplus to the buyer $\left(1 - F^j\left(\varepsilon^{ji}\right)\right) E^s\left[S_b^{si}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{si}\right]$, $i = b, v$. These two channels affect V^j and B^j in (62) and (66), and as a result affect the reservation qualities ε^{jv} and ε^{jb} in (13).

7.5.4 Stock of vacant houses

In any season s, the average probability that a transaction goes through is $\left\{\theta\left[1 - F^s\left(\varepsilon^{sv}\right)\right] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right]\right\}$, and the average probability that a transaction does not through is $\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}$. Hence, the law of motion for the stock of vacant houses (and for the stock of buyers) is

$$\begin{aligned} v^s &= (1-\phi^s)\left\{v^w\left[\theta\left(1 - F^w\left(\varepsilon^{wv}\right)\right) + (1-\theta)\left(1 - F^w\left(\varepsilon^{wb}\right)\right)\right] + 1 - v^w\right\} \\ &\quad + v^w\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}, \end{aligned}$$

where the first term includes houses that received a moving shock this season and the second term comprises vacant houses from last period that did not find a buyer. The expression simplifies to

$$v^s = v^w \phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\} + 1 - \phi^s, \tag{70}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality $\left(\varepsilon^{wv}, \varepsilon^{wb}\right)$ and on the distribution $F^w(.)$.

An equilibrium is a vector $\left(p^{sv}, p^{sb}, p^{wv}, p^{wb}, B^s + V^s, B^w + V^w, \varepsilon^{sv}, \varepsilon^{sb}, \varepsilon^{wv}, \varepsilon^{wb}, v^s, v^w\right)$ that jointly satisfies equations (63),(66),(68), (69) and (70), with the surpluses S_v^j and $S_b^j(\varepsilon)$ for $j = s, w$, derived as in (60), and (58). Using (70), the stock of vacant houses in season s is given by:

$$v^s = \frac{(1-\phi^w)\phi^s\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\} + 1 - \phi^s}{1 - \phi^w\phi^s\left\{\theta F^s\left(\varepsilon^{sv}\right) + (1-\theta) F^s\left(\varepsilon^{sb}\right)\right\}\left\{\theta F^w\left(\varepsilon^{wv}\right) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}}. \tag{71}$$

Given $1 - \phi^s > 1 - \phi^w$, as in the observable case, it follows that, in equilibrium $v^s > v^w$.

7.5.5 Seasonality in Prices

Let

$$p^s \equiv \frac{\theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] p^{sv} + (1-\theta)\, p^{sb}}{\theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] + 1 - \theta}$$

be the average price observed in season s. Given $p^{sv} = S_v^{sv} + p^{sb}$, we can rewrite it as

$$p^s = p^{sb} + \frac{\theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}}{\theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] + 1 - \theta}$$

using (64)

$$\begin{aligned} p^s &= \frac{u}{1-\beta} + \theta \frac{\beta^2 \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2} + \frac{\theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}}{1 - \theta F^s\left(\varepsilon^{sv}\right)} \\ &= \frac{u}{1-\beta} + \theta \left(\frac{\left[1 - \theta F^s\left(\varepsilon^{sv}\right)\right] \beta^2 + 1 - \beta^2}{\left[1 - \theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)} \right) \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \frac{\theta \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2} \end{aligned}$$

we obtain,

$$p^s = \frac{u}{1-\beta} + \theta \left\{ \frac{\left[1 - \theta \beta^2 F^s\left(\varepsilon^{sv}\right)\right] \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}}{\left[1 - \theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)} + \frac{\beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2} \right\}. \tag{72}$$

Since the flow u is a-seasonal, house prices are seasonal if $\theta > 0$ and the surplus to the seller is seasonal. As in the case with observable match quality, *when sellers have some "market power" ($\theta > 0$), prices are seasonal. The extent of seasonality is increasing in the seller's market power θ.* To see this, note that the equilibrium price is the discounted sum of the flow value (u) plus a positive surplus from the sale. The surplus S_v^{sv}, as shown in (60), is seasonal. Given $v^s > v^w$, Assumption 2 implies hazard rate ordering, i.e. $\frac{f^w(x)}{1-F^w(x)} > \frac{f^s(x)}{1-F^s(x)}$ for any cutoff x, i.e. the thick-market effect lowers the conditional probability that a successful transaction is of the marginal quality ε^{sv} in the hot season, that is, it implies a "thicker" tail in quality in the hot season. In words, the quality of matches goes up in the summer and hence buyers' willingness to pay increases; sellers can then extract a higher surplus in the summer; thus, $S_v^{sv} > S_v^{wv}$. As in the case with observable ε, there is an equilibrium effect through the seasonality of cutoffs. As shown in (68), the equilibrium cutoff ε^{sv} depends on the surplus to the seller (S_v^{sv}) and on the sum of the seller's and the buyer's outside options, while the equilibrium cutoff ε^{sb} depends only on the sum of the outside options. The seasonality in outside options tends to reduce $\varepsilon^{si}/\varepsilon^{wi}$ for $i = b, v$. This is because the outside option in the hot season s is determined by the sum of values in the winter season: $B^w + V^w$, which is lower than in the summer. However, the seasonality in the surplus term, $S_v^{sv} > S_v^{wv}$ (shown before), tends to increase $\varepsilon^{sv}/\varepsilon^{wv}$ (the marginal house has to be of higher quality in order to generate a bigger surplus to the seller). Because of these two opposing forces, the equilibrium effect is likely to be small (even smaller than in the observable case.)

Given that θ affects S_v^{sv} only through the equilibrium vacancies and reservation qualities, it follows that the extent of seasonality in price is increasing in θ.

7.5.6 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[\theta \left(1 - F^w \left(\varepsilon^{wv}\right)\right) + (1-\theta) \left(1 - F^w \left(\varepsilon^{wb}\right)\right)\right]. \tag{73}$$

(An isomorphic expression holds for Q^w). As in the observables case, seasonality in transactions stems from three sources. First, the direct effect from a larger stock of vacancies in the summer, $v^s > v^w$. Second the amplification through the thick-market effects that shifts up the probability of a transaction. Third, there is an equilibrium effect through cutoffs. As pointed out before, this last effect is small. As in the case with observable ε, most of the amplification stems from the thick-market effect. What is new when ε is unobservable is that the extent of seasonality in transactions is decreasing in the seller's market market power θ. This is because higher θ leads to higher surplus in the summer relative to winter, S_v^{sv}/S_v^{wv}, which in turn increases $\varepsilon^{sv}/\varepsilon^{wv}$ and hence decreases Q^s/Q^w); the higher is θ, the stronger is this effect (it disappear when $\theta = 0$).

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