Partial Monitoring and Message Trading

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Abstract

In a random-matching risk-sharing model, the role of public messages is explored when in each pairwise meeting, risk-sharing actions are only monitored by the pair in the meeting (partial monitoring). A risk-sharing outcome and the message on the outcome are determined simultaneously, allowing the message and outcome to be traded with each other (message trading). If agents can commit to not renegotiate a Pareto-dominated trade, then the folk theorem can be established. If there is no such commitment, then the folk theorem can only be established in a special case and, in general, the loss due to renegotitation does not vanish as agents become more patient.

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1 Introduction

The literature on repeated games shows that cooperation can be sustained when people are patient provided that there is suitable monitoring, a renowned result established by various folk theorems.[1] A strand in this literature introduces public messages (about actions) under imperfect private monitoring,

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[1]There is no room to make a complete survey. Mailath and Samuelson [18] provide an excellent reference to this literature. Here we simply note that there are general results for games with perfect public monitoring (c.f. Fudenburg and Levine [6]), and with imperfect public monitoring (c.f. Green and Porter [9], Abreu, Pearce, and Stacchetti [2], and Fudenburg, Levine and Maskin [7]). The results for private monitoring are limited; see Sekiguchi [22] for such a result, and see Kandori [13] for a detailed discussion.

i.e., each agent observes his own imperfect signal resulting from the actions of all agents, and under partial monitoring, i.e., an agent's action is observed by only a subset of the agents. Specifically, after each stage game each agent communicates to the others his signal or observation about actions in the stage game (which creates public messages). The folk theorems are established for some classes of such repeated games; *c.f.* Compte [4] and Kandori and Matsushima [14] for imperfect monitoring, and Ben-Porath and Kahneman [3] for partial monitoring.

In this paper, we investigate public messages under partial monitoring when people are able to exchange messages with other transferable objects of value in *spot* trades—message trading. Message trading is a concern for an elementary reason. Whenever the messages are used to detect and deter defections, agents' future payoffs depend on these messages. With this dependence, defectors have incentives to seek more favorable messages by transferring valuable objects to those who are monitoring their actions. When the creation of public messages is public (as in the example in [3] with centralized meetings and public incrimination), there seems to be no room for message trading. But when the creation of public messages is not public, incentives can naturally lead to trades whose effects should not be overlooked.

As an illustrating example, consider a cake-sharing game between two players (a one-shot event) followed by a reward allocated by an arbitrator. The arbitrator randomly gives the cake to one player, player A say, and announces a reward scheme designed to yield an even split based on *each* player's message about the shares consumed. The arbitrator does not observe the players share and consume the cake. When the creation of messages is public each player writes down his message in front of each other and the arbitrator. When it is not public each player writes down his own message in front of each other but not the arbitrator, and then forwards his message to the arbitrator. Under the public creation of messages, if the reward to player B is a constant, then B does not have an incentive to lie; and given B is telling the truth, if the reward to A suitably varies with B's message, then A will transfer half the cake to B. Under the non-public creation of messages, message trading is problematic for such a scheme, even if the cake is the only valuable object that A can transfer to B. For instance, after A eats two-thirds of the cake but still holds the remaining third, he can offer the remaining third in exchange for a message of B that gives him the highest reward. With the non-public creation of public messages, such a mutually-beneficial trade is difficult to deter, thereby creating a problem for the arbitrator.

We formalize this example into a random-matching risk-sharing model with a large number of agents in which the two agents in a pairwise meeting resemble the players in that example—one is randomly chosen (by nature) to receive some cake, and the rewards derive from future plays. The non-public creation of messages is plausible in large communities in which people's activities are decentralized and people rarely meet in large groups. Matching in pairs is a class approach to partial monitoring—one's action is monitored only by his meeting partner. Moreover, random matching and a large number of agents make messages potentially useful for eliciting cooperation.[2]

In the pairwise meeting, the size of the cake can be a random variable whose realization is observed by the pair (this size is deterministic in the illustrating example). Also, there is a device called a report, in which each agent can input his own message on the risk-sharing outcome and the size of the cake; messages in the report become public to all agents after the meeting is over.[3] One can regard this model as a version of Green [8] in which the endowment realization is not private (it is observed by the pair), the redistribution of resources is decentralized (all transfers are pairwise) and is not public (it is only observed by the pair) but is reported the public. Abstracting from private information helps us concentrate on message trading.

The message-trading process is a game form called a trading mechanism that has trading outcomes and an autarky outcome. A trade consists of a transfer of the cake and messages from the two parties; in autarky each agent inputs his message independently and there is no transfer. We focus on equilibria that are coalition proof or renegotiation proof, where the restriction is only applied to the plays of the two agents in the trading mechanism in each pairwise meeting. Coalition proof requires that the plays in the mechanism must reach the pairwise Pareto frontier, whereas renegotiation proof effectively restricts how two agents split the surplus along the Pareto frontier.

When an equilibrium is coalition proof, the message of the partner must be valuable to both agents to induce an agent to make a transfer, in particular, the partner must be rewarded if his message reveals that a smaller transfer has occurred. The folk theorem holds with the coalition proof equi-

[2]Messages are redundant if two agents stay together forever. Also, with a large number of agents, contagious equilibria (see Kandori [12] and Ellison [5]) are ruled out and some message about the meeting to outsiders is necessary for any positive transfer to occur.

[3]Reports differ from labels in Kandori [12] and status in Okuno-Fujiwara and Postlewaite [21]. A label or a status is a statistic that is updated from with-in meeting actions by an *exogenous* rule.

librium. When an equilibrium is renegotiation proof, the potential for cooperation depends on whether the size of the cake is public information. When the size of the cake in the meeting is public information, the folk theorem holds, but for some preferences it may require a lower bound on the discount factor that is higher than the one with coalition proof equilibrium. When the size of the cake is not public (but subject to pairwise partial monitoring), the folk theorem does not hold and the (welfare) loss does not vanish as the discount factor approaches one. However, the loss vanishes as the size of the cake approaches public information. Moreover, a fairly mild condition ensures that some transfers occur in some meetings.

While our study deals with public messages, it can also be useful for the study of messages created in real-life situations where actions are partially monitored and messages are not public. For instance, a receipt issued by a seller to a buyer, a receipt issued by a creditor to a debtor for a payment, monetary payments in cash, and an individual's credit scores may not freely become public information. In the context of our model, we relate reports to money. In fact, our model shares all the essential ingredients with random matching models of money (e.g., Kiyotaki and Wright [15], Trejos and Wright [24], and Shi [23]). Following convention in those models, we assume each agent's money holdings are only observed by his meeting partner. As it turns out, money is a special report that has a restricted message space. It follows that if an allocation is supported by an equilibrium only with money, then it is supported by an equilibrium only with reports. Hence our results for reports provide a bound on what money can achieve.

Our model has a special structure, but some of the results suggest general findings. Clearly, it is necessary to reward an agent for the agent to reveal another's defection. Also, although the ability to renegotiate may diminish the effectiveness of messages, messages are still useful when the set of trades depends on an action whose outcome is included with the message.

2 The basic model

Time is discrete, dated $t \geq 0$. There is a nonatomic measure of ex-ante identical infinitely lived agents index by I. Each date agents are randomly matched in pairs. The *matching realization*, i.e., the identity of each agent in each meeting, is public information. As is standard, information revealed to all agents is referred to as *public information*.

The sequence of events within a meeting is as follows. At the start of the meeting, by flipping a fair coin, one agent becomes a buyer and the other becomes a seller. This *type realization* is public information. After the type realization, the buyer is endowed with 0 units of a perishable good; the seller's endowment is determined by an i.i.d. shock: he obtains 1 unit of the good with probability $\rho \in (0, 1]$ and 0 with probability $1 - \rho$. When $\rho < 1$ the seller's *endowment realization* is *pairwise public information,* i.e., meeting-specific information only observed by the pair of agents in the meeting. After the endowment realization, the meeting consists of two consecutive stages, stages 1 and 2; the good perishes at the end of stage 2.

Stage 1. The seller chooses to consume part of his endowment.

Stage 2. The buyer and seller trade a report for a transfer of the remaining good (from the seller to buyer) by a given trading mechanism. Details of the report and trading mechanism are described below.

At these two stages, the transfer and consumption of the good are pairwise public information, and they are never revealed to the public. After the meeting, the report becomes public information. Then the date is over.

Preferences of agents are as follows. The buyer's period utility from consuming c at stage 2 is $u(c)$, where $u(0) = 0$, $u' > 0$, and $u'' \leq 0$. The seller's period utility from consuming c_1 at stage 1 and c_2 at stage 2 is $c_1 + c_2$; such linearity is without loss of generality. Each agent maximizes the expected discounted utility *normalized* by $1 - \delta$, where $\delta \in (0, 1)$ is the discount factor. To make a positive transfer ex-ante more desirable than zero transfer, we assume

$$q^* = \arg\max u(q) - q > 0. \tag{1}$$

Details of a report are as follows. A *report*, denoted $r = (r_b^1, r_b^2, r_s^1, r_s^2)$, is an element of the set $\{0, 1\} \times [0, 1] \times \{0, 1\} \times [0, 1]$, where r_b^1 and r_b^2 (r_s^1 and r_s^2, respectively) are the messages of the buyer (the seller, respectively) regarding the seller's endowment realization and the transfer of the good, respectively.

Details of a trading mechanism at stage 2 are as follows. A *trading mechanism*, denoted by T, is an extensive game form which assigns to each terminal node an outcome, which is either a trade or autarky. A *trade*, denoted (y, r), consists of a feasible transfer of y units of the good and a report r; and *autarky* means zero transfer of the good and each agent sending any feasible message. Actions the pair of agents play in T are observed by the pair.

Our definition of trading mechanism follows closely the one in Kocherlakota [16]. Also, following [16], we restrict attention to finite-horizon and

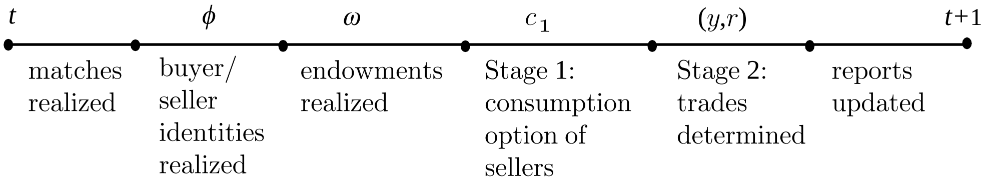

Figure 1: Sequence of Events.

no-commitment mechanisms. *Finite-horizon* means that the game between two agents is over before the next matching process starts.[4] *No-commitment* means that each agent has a sequence of actions leading to autarky, independent of any sequence of actions chosen by his meeting partner (e.g., simultaneous move voting games and ultimatum games).

3 Equilibrium

The matching process, endowment process, the seller's action in stage 1, mechanism in stage 2 of pairwise meetings, and preferences define a game. We restrict our attention to what we call quasi public strategies, which is consistent with the ones adopted by Kocherlakota [16], and the resulting perfect equilibrium is consistent with the class of sequential equilibria constructed by Ben-Porath and Kahneman [3].

To be specific, we first introduce some notation. For $t \geq 0$ and $i \in I$, let $\phi_{i,t}$ be agent i's meeting partner at t; let $\theta_{i,t} = 0$ if i is a buyer at t and $\theta_{i,t} = 1$ if a seller; let $\omega_{i,t}$ be the seller's endowment realization in the meeting between i and $\phi_{i,t}$; and let $r_{i,t} = (r^1_{i,b,t}, r^2_{i,b,t}, r^1_{i,s,t}, r^2_{i,s,t})$ be the report in that meeting. For $x \in \{\phi, \theta, \omega, r\}$, let $x_{i,-1} = i$ and $x_i^{t-1} = (x_{i,-1}, ..., x_{i,t-1})$.[5]

Definition 1 *A strategy σ_k of agent k is a quasi public strategy if in each*

[4] Finiteness can be achieved if T is in a multistage form and consists of a finite number of stages. Even if T has an infinite number of stages, there is a formulation to end the game before the date is over. The key is to let the duration of a stage shrink, and our results seem to apply to such a setup.

[5] By including $x_{i,-1} = i$ in x_i^{t-1}, we effectively associate i's index with any date $\tau \geq 0$ information $x_{i,\tau}$ in x_i^{t-1}. Also, notice that it is public information which part in $r_{i,\tau}$, $(r^1_{i,b,\tau}, r^2_{i,b,\tau})$ or $(r^1_{i,s,\tau}, r^2_{i,s,\tau})$ is the message made by i.

date-t meeting when k moves at an information set, σ_k only conditions on the public information, namely, $\{\gamma_i^t\}_{i\in I}$ with $\gamma_i^t \equiv (\phi_i^t, \theta_i^t, r_i^{t-1})$, the meeting-specific pairwise public information, namely, $\omega_{k,t}$, and actions already taken by agents k and $\phi_{k,t}$ during the meeting.

By Definition 1, if agent k has two information sets in the meeting which differ only in the pairwise public information pertaining to his *previous* meetings, then a public strategy σ_k specifies the same action on those two sets.

Our equilibrium concept is perfect equilibrium.

Definition 2 *Given a trading mechanism T, a profile of strategies $\sigma = \{\sigma_i\}_{i\in I}$ is an equilibrium if $\{\sigma_i\}_{i\in I}$ are quasi public strategies and σ evaluated at any history of the game determines a Nash equilibrium.*

As is shown in the next section, one may embed some threat in T to induce an equilibrium with transfers of goods on the equilibrium path. The basic idea is that T only admits autarky in off-equilibrium path meetings, and, in particular, it does not admit any pairwise efficient trades. For us, this finding identifies a weakness of Definition-1 equilibrium. After all, for the purpose of examining the effect of message trading on the effectiveness of reports, it is not desirable to circumvent message trading by designing a T that rules out mutually desirable trades.

Therefore, we consider two refinements, both having counterparts in the literature on labor and monetary matching and search models, in which how a trade is reached in a pairwise meeting is a critical issue. These two refinements require that, when restricted to stage 2 of a meeting, an equilibrium σ be *coalition proof* or *renegotiation proof*, respectively, regardless of whether some agent in the meeting has deviated from σ in the previous meetings, or whether some agent has deviated from σ in stage 1 of the present meeting.[6]

The coalition-proof restriction is simply the Coase Theorem in our context: when the buyer and seller enter into a pairwise trading process, they reach a pairwise Pareto efficient outcome of the mechanism T. An outcome of T is *pairwise efficient* if it is not pairwise Pareto dominated by any *lottery*

[6]For the counterpart of the coalition-proof refinement in the literature on labor and monetary matching and search models, see, e.g., Hall [10], Hu et al. [11], and Zhu and Wallace [25].; the counterpart of the renegotiation-proof refinenment can be found in papers with generalized Nash bargaining.

over the set of autarky and feasible deterministic trades,[7] and a lottery is pairwise efficient if it is not pairwise Pareto dominated by any other lottery.

Definition 3 *Given a trading mechanism T, an equilibrium σ is a CP equilibrium if in each meeting, following each feasible stage-1 action, any outcome of T implied by σ is pairwise Pareto efficient (i.e., not dominated by any lottery over the set of autarky and feasible deterministic trades).*

The renegotiation-proof restriction starts from the idea that following any feasible stage-1 actions, on any (stage-2) path in which the buyer and seller reach an outcome of the trading mechanism that is Pareto dominated, they will renegotiate to a pairwise efficient outcome;[8] this is the same idea that motivates renegotiation in models in the Nash-implementation literature.

Following a standard treatment in that literature (e.g., Maskin and Moore [20]), we model a renegotiation process as a mapping whose domain consists of all lotteries over the set of autarky and deterministic trades feasible at the stage from which the inefficient outcome of T was reached, and whose range is the subset of all pairwise efficient lotteries in the domain. Following widespread convention, we further restrict attention to a special mapping: it selects a lottery from the domain to maximize the Nash product (with equal weighs on the buyer and seller).

Definition 4 *Given a trading mechanism T, a CP equilibrium σ is a RP equilibrium if in each meeting, following each feasible stage-1 action(s), no agent can improve from any outcome implied by T and σ by any deviation which results in renegotiation (that splits surplus-from-trade by Nash bargaining).*

Some results below use the following notion of equilibrium allocation.

Definition 5 *An allocation, denoted $\{f_{k,t} : k \in I, t \geq 0\}$, is a collection of mappings $\{(\phi_i^t, \theta_i^t, \omega_i^t)\}_{i \in I} \equiv \alpha_t \mapsto f_{k,t}(\alpha_t) \in [0, \omega_{k,t}]$ such that given α_t, $f_{k,t}(\alpha_t)$ is the transfer of goods from the buyer to the seller in the meeting between agents k and $\phi_{k,t}$.*

[7] Recall that in our formulation an outcome of T is a deterministic trade or autarky. Allowing an outcome of T to be a lottery does not change anything in substance. The current presentation simplifies exposition.

[8] In a CP equilibrium, the pair of agents could end up at a Pareto dominated outcome due to some off-equilibrium play in stage 2.

An allocation $\{f_{k,t}\}$ is an equilibrium allocation or supported by an equilibrium if there exists an equilibrium such that for all k, t and α_t, $f_{k,t}(\alpha_t)$ coincides with the transfer of goods specified by on-the-path plays.

In Definition 5, the transfer $f_{k,t}(\alpha_t)$ is completely determined by the history of the physical environment, i.e., the realizations of the matching outcomes, the agents' types, and the sellers' endowments of the whole economy from date 0 to t. (Notice that by definition, $f_{k,t}(\alpha_t) = f_{j,t}(\alpha_t)$, $j = \phi_{k,t}$.)

In our models, actions in the meeting are taken before k and $\phi_{k,t}$ know the reported $\omega_{i,t}$ for $i \neq k, \phi_{k,t}$. Hence the dependence of an equilibrium allocation $f_{k,t}(.)$ on $\{\omega_i^t\}_{i\in I}$ is only on $\{\omega_i^{t-1}\}_{i\in I}$ and $\omega_{k,t}$. Formally, we have the following.

Observation 1 *If $\{f_{k,t}\}$ is an equilibrium allocation, then $f_{k,t}(\alpha_t) = f_{k,t}(\bar{\alpha}_t)$ for $\alpha_t = \{(\phi_i^t, \theta_i^t, \omega_i^t)\}_{i\in I}$ and $\bar{\alpha}_t = \{(\phi_i^t, \theta_i^t, \bar{\omega}_i^t)_{i\in I}\}$ with $\omega_{k,t} = \bar{\omega}_{k,t}$ and $\{\omega_i^{t-1}\}_{i\in I} = \{\bar{\omega}_i^{t-1}\}_{i\in I}$.*

4 Perfect-monitoring benchmark

To examine the message-trading effect, we begin with the following variant of the basic model as the benchmark.

Perfect-monitoring benchmark. Stage 1 is unchanged. In stage 2 there are no reports, but the good can still be transferred. The seller's endowment realization and the transfer of the good become public information after the meeting.

The equilibrium concept is the one in Definition 2 (with T being redundant). At a date-t meeting when agent k moves, his public strategy only conditions on $\{(\phi_i^t, \theta_i^t, \omega_i^{t-1}, q_i^{t-1})\}_{i\in I}$, $\omega_{k,t}$, and actions already taken by agents k and $\phi_{k,t}$ during the meeting, where $q_{i,\tau}$ is the transfer of the good in the meeting between i and $\phi_{i,\tau}$ and $q_i^{t-1} = (q_{i,-1}, ..., q_{i,t-1})$ with $q_{i,-1} = i$. (Notice that Observation 1 applies here.)

The following result compares the set of equilibrium allocations in the basic model with the set of equilibrium allocations in the benchmark.

Proposition 1 *An allocation is an equilibrium allocation in the basic model if and only if it is an equilibrium allocation in the perfect-monitoring benchmark.*

Proof. See the appendix. ■

The "only-if" part of Proposition 1 resembles the money-is-memory result in Kocherlakota [16]. Both of these results concern messages about the actions and the true observations of the actions. Messages about actions are facilitated by money in [16] and reports here, and true observations are facilitated by memory in [16] and perfect monitoring here.

The proofs of both of these results use the following observation. Let an allocation be an equilibrium allocation given the messages of the actions. Given the true observations of the actions, an agent who should surrender some of his own goods is willing to do so because his subsequent continuation value is determined by the allocation, and his continuation value following a deviation is no greater than his continuation value in the corresponding equilibrium for the messages about the actions.

Two remarks about the differences between our result and proof and Kocherlakota's are in order. First, in our proof we do not use any device analogous to the imaginary balance sheet in [16] as an intermediate step. This is because our allocation depends only on the history of the physical environment, but not on the messages (or the method to carry messages) about the actions.

Second, memory in [16] is actually partial monitoring, where the set of agents monitoring an action is a superset of the agents monitoring that action with money. Partial instead of perfect monitoring is necessary for the money-is-memory result only because the agent's knowledge of the matching history with money is different than with memory. As implied by the logic of the proofs discussed above, if agents have the same knowledge of the history of the physical environment under different monitoring assumptions, then an equilibrium allocation given the messages about the actions should also be an equilibrium given the true observations of the actions.

The "if" part of Proposition 1 resembles the converse of the money-is-memory result in Kocherlakota [17]. The proofs of both of these results use the same trick. That is, in some meetings the trading mechanism does not admit any pairwise efficient outcomes. This trick illustrates the weakness of the Definition-1 equilibrium concept indicated in the last section.

To see how the trick works, we sketch our proof as follows. Let $\{f_{k,t}\}$ be an equilibrium allocation in the model with perfect monitoring. Consider the basic model with the following mechanism T: At the start of stage 2, k and $\phi_{k,t} = j$ simultaneously announce a number in $\{1, 0\}$. If both say 1, and if $f_{k,t}(\alpha_t)$ is available, then $y = f_{k,t}(\alpha_t)$ and $r = (\omega_{k,t}, 1, \omega_{k,t}, 1)$; otherwise autarky occurs. In the candidate equilibrium, k and j say 1 if both are non-defectors, and say 0 if either is a defector. In autarky an agent always sends the message $(1, 0)$. When k, as a seller, consumes $c_1 > 1 - f_{k,\tau}(\alpha_\tau)$ at stage 1, at stage 2 a trade with some $y > 0$ and some r with $(r_b^1, r_b^2) = (1, 1)$ Pareto dominates autarky, but it is not admitted by T.[9]

To summarize, the significance of Proposition 1 and its proof is that perfect monitoring sets an upper bound on what reports can achieve, and the refinements in Definitions 3 and 4 should be incorporated into a study of the effects of message trading.

Propositions 1 implies another benchmark result for the basic model, i.e., the folk theorem in our context.

Proposition 2 *Let $\bar{v} = 0.5\rho[u(q^*) + (1 - q^*)]$ (see (1)) and $\underline{v} = 0.5\rho$. Fix $v \in (\underline{v}, \bar{v}]$. Let the allocation $\{f_{k,t}\}$ be such that $f_{k,t}(\alpha_t) = q$, for all k, t, α_t and $\omega_{k,t} = 1$, where $0.5\rho[u(q) + (1 - q)] = v$. Let $\underline{\delta}$ satisfy $\underline{\delta}(v - \underline{v}) = (1 - \underline{\delta})q$. Then $\{f_{k,t}\}$ can be supported by an equilibrium (Definition 2) in the basic model if and only if $\delta \geq \underline{\delta}$.*

Proof. For the "if" part, by Proposition 1, it suffices to show that $\{f_{k,t}\}$ can be supported by an equilibrium (Definition 2) in the model with perfect monitoring. But that is standard. (For a seller, if he transfers q then his (on-path) payoff is $(1 - \delta)(1 - q) + \delta v$. A defector is punished by permanent autarky; that is, the payoff of a deviation is bounded above by $(1 - \delta) \cdot 1 + \delta \underline{v}$.) The "only-if" part is obvious. ■

Given Proposition 2 and the importance of refinements just emphasized, our study below is centered around the folk theorems in CP and RP equilibria.

5 The folk theorem in CP equilibrium

We begin with a general property about CP equilibrium.

[9] In [17], an agent with in-equilibrium money holdings does not trade with an agent with off-equilibrium holdings. As a result, after a meeting the former has off-equilibrium holdings. But when the former is a seller, he desires to trade goods for money with the buyer so that his post-meeting holdings are in path. Such a trade obviously makes both parties better off, but it is not admitted by the trading mechanism.

Lemma 1 *Let σ be a CP equilibrium. Fix γ^t and k and let $\theta_{k,t} = 0$. Suppose that when $\omega_{k,t} = 1$, on-path plays lead to the transfer $q > 0$ and the report r, and that σ specifies $(\hat{y}, \hat{r})$ as the stage-2 outcome following the seller's stage-1 consumption $c_1 > 1 - q$. Denote by w_i and $\hat{w}_i$ agent i's values at the start of $t+1$ implied by r and $\hat{r}$, respectively, $i \in \{k, j = \phi_{k,t}\}$. Then $w_k < \hat{w}_k$ and $w_j > \hat{w}_j$.*

Proof. Notice that $c_1 > 1-q$ implies that $q > 0$ and $\hat{y} < q$. First suppose $w_j \leq \hat{w}_j$. But then j can improve his payoff by consuming c_1 at stage 1 and trading $(\hat{y}, \hat{r})$ at stage-2, a contradiction. Next suppose $w_k \geq \hat{w}_k$. But then given $w_j > \hat{w}_j$, $(\hat{y}, \hat{r})$ is Pareto dominated by $(\hat{y}, r)$, a contradiction. ■

In Lemma 1, the part about w_j and $\hat{w}_j$ is simply the seller's participation constraint; that is, it is necessary to give the seller some incentive to make the purported transfer. The part about w_k and $\hat{w}_k$ says that when the seller deviates to a less transfer, it is necessary to give the buyer an incentive to truthfully report that deviation. Therefore, a rise in the buyer's off-equilibrium continuation value from the in-equilibrium level is associated with a fall in the seller's from the in-equilibrium level.

It is immediate from Lemma 1 that the equilibrium in the proof of the "if" part of Proposition 1 is not a CP equilibrium, and that if the allocation $\{f_{k,t}\}$ in Proposition 2 is supported by a CP equilibrium, then that equilibrium cannot be a global-autarky trigger strategy equilibrium (i.e., if one agent sends a message to reveal his meeting partner's deviation, then no seller makes any transfer in any future meeting).

To support $\{f_{k,t}\}$ in Proposition 2, we consider a CP equilibrium σ with the following features. The set of the continuation values is $[w_0, w_2]$ with $w_0 < w_1 \equiv v < w_2$: if we set $w_k = w_j = v$ in Lemma 1 and let $\hat{r}$ be the report following that the seller consumes 1 at stage 1 when $\omega_{k,t} = 1$, then $\hat{w}_j \in [w_0, w_1)$ and $\hat{w}_k \in (w_1, w_2]$.[10] Because when $\rho < 1$, $\hat{r}$ and the on-path report following $\omega_{k,t} = 0$ imply different continuation values,[11] σ must

[10] Lemma 1 alone requires that the set of the continuation values in σ, denoted W, consists of $\{\hat{w}_l, w_1, \hat{w}_k\}$. If $W = \{\hat{w}_l, w_1, \hat{w}_k\}$, then in some meeting following the seller stage-1 consumption which is sufficiently close to 1, the stage-2 trade specified by σ is Pareto dominated by a lottery, so that σ cannot be a CP equilibrium.

[11] For k and j, $\tilde{r}$ implies the value v, but by the lemma $w_k < \hat{w}_k$ and $w_j > \hat{w}_j$. In fact, $\hat{r}$ cannot be the autarky report when $\rho < 1$ either. For, otherwise, the latter gives k the value $\hat{w}_k > v$; but then, when $\omega_{k,t} = 0$, k is better off by choosing autarky than transfering zero and making the on-path report.

depends on $\omega_{k,t}$. The critical issue is to fulfill a general continuation value $w \in [w_0, w_2]$. Let $y(.)$, $\underline{v}_b(.)$ and $\bar{v}_s(.)$ satisfy

$$\begin{aligned} w &= 0.5\rho[(1-\delta)u(y(w,w_1)) + \delta\underline{v}_b(w,w_1)] + 0.5(1-\rho)\delta w \\ &\quad +0.5\rho[(1-\delta)(1-y(w_1,w)) + \delta\bar{v}_s(w_1,w)] + 0.5(1-\rho)\delta w. \end{aligned} \tag{2}$$

For (2), consider agent k's with the continuation value w at the start of t. With probability one his meeting partner j is with w_1. If the seller in the meeting is not endowed, then w is k's start-of-t+1continuation value. If the seller is endowed, then given the buyer and seller's the start-of-t continuation values (w_b, w_s), $y(w_b, w_s)$ is the transfer of the good and $\underline{v}_b(w_b, w_s)$ and $\bar{v}_s(w_b, w_s)$ are the buyer and seller's start-of-t+1 continuation value, respectively. (If $w = w_1$ then $y(w, w_1) = q$ and $\underline{v}_b(w, w_1) = \bar{v}_s(w_1, w) = w_1$ and, hence, the right side of (2) equals w_1.)

The candidate $\bar{v}_s(.)$, $\underline{v}_b(.)$ and $y(.)$ are given by

$$\bar{v}_s(w_1, w) = w,\ \bar{v}_s(w, w_1) = w_1; \tag{3}$$

$$\underline{v}_b(w_1, w) = w_1,\ \underline{v}_b(w, w_1) = \left\{ \begin{array}{c} w_1, w \geq w_1 \\ w, w < w_1 \end{array} \right\}; \tag{4}$$

and

$$y(w_1, w) = \left\{ \begin{array}{c} q - \frac{(w-w_1)[1-0.5\rho\delta-(1-\rho)\delta]}{0.5\rho(1-\delta)}, w \geq w_1 \\ 0, w < w_1 \end{array} \right\}; \tag{5}$$

$$y(w, w_1) = \left\{ \begin{array}{c} q, w \geq w_1 \\ u^{-1}[u(q) - q - (w_1 - w)(0.5\rho)^{-1}], w < w_1 \end{array} \right\}. \tag{6}$$

That is, when $w < w_1$, if k is the seller then he transfers 0 and his start-of-t+1 value is w; if k is the buyer, then his consumption is less than q and his start-of-t+1 value is w. When $w > w_1$, if k is the seller then he transfers less than q and his start-of-t+1 value is w; if k is the buyer, then his consumption is q and his start-of-t+1 value is w_1.

By Lemma 1, for $y(w_b, w_s)$ to be the equilibrium outcome, there should be some $\bar{v}_b(w_b, w_s) > \underline{v}_b(w_b, w_s)$ and $\underline{v}_s(w_b, w_s) < \bar{v}_s(w_b, w_s)$ such that following the seller's stage-1 consumption $c_1 = 1$, $\bar{v}_b(w_b, w_s)$ and $\underline{v}_s(w_b, w_s)$ are the buyer and seller's start-of-t+1 continuation values at the start of $t+1$, respectively. As it appears, the magnitudes of $\bar{v}_b - \underline{v}_b$ and $\bar{v}_s - \underline{v}_s$ depend on the way that surplus-from-trade is split. Because our purpose is to assure that y, $\underline{v}_b$ and $\bar{v}_s$ are in the meeting-specific Pareto frontier, we can let

$$\begin{aligned} \underline{v}_s(w_b, w_s) &= \bar{v}_s(w_b, w_s) - \delta^{-1}(1-\delta)y(w_b, w_s), && (7) \\ \bar{v}_b(w_b, w_s) &= \underline{v}_b(w_b, w_s) + [\bar{v}_s(w_b, w_s) - \underline{v}_s(w_b, w_s)]. && (8) \end{aligned}$$

If the set of reports can be identified with $[0, 1]$ (by some way) and the value functions on reports can be expressed as

$$\begin{aligned} \kappa_b(x, w_b, w_s) &= \underline{v}_b(w_b, w_s) - [\bar{v}_b(w_b, w_s) - \underline{v}_b(w_b, w_s)]x, && (9) \\ \kappa_s(x, w_b, w_s) &= \underline{v}_s(w_b, w_s) + [\bar{v}_s(w_b, w_s) - \underline{v}_s(w_b, w_s)]x, && (10) \end{aligned}$$

with $x \in [0, 1]$, then the Pareto efficiency is implied by the following.

Lemma 2 *Let $w_1 = v$, and let w_1 and w_0 satisfy $\delta(w_2 - w_1) = 0.5(1-\delta)\rho q$ and $\delta(w_1 - w_0) \geq (1-\delta)q$. Let $\bar{v}_s(.)$, $\underline{v}_b(.)$, $y(.)$, $\bar{v}_b(.)$, $\underline{v}_s(.)$, $\kappa_b(.)$, and $\kappa_s(.)$ be given by (3)-(10). Fix $w_b, w_s \in [w_0, w_2]$ with $w_b = w_1$ or $w_s = w_1$. For $q' \in [0, 1]$, let*

$$(Y(q', w_b, w_s), X(q', w_b, w_s)) = \arg\max U_b(y, x, w_b, w_s) \qquad (11)$$

s.t. $(y, x) \in [0, q'] \times [0, 1]$ and $U_s(y, x, q', w_b, w_s) \geq U_s(0, 0, q', w_b, w_s)$, where

$$\begin{aligned} U_b(y, x, w_b, w_s) &= (1-\delta)u(y) + \delta\kappa_b(x, w_b, w_s), \\ U_s(y, x, q', w_b, w_s) &= (1-\delta)(q' - y) + \delta\kappa_s(x, w_b, w_s), \end{aligned}$$

Then $Y(y(w_b, w_s), w_b, w_s) = y(w_b, w_s)$ and $X(y(w_b, w_s), w_b, w_s) = 1$. Also $S(q') = (1-\delta) + \delta\underline{v}_s(w_b, w_s)$, all q', where $S(q') = (1-\delta)(1-q') + U_s(Y(q', w_b, w_s), X(q', w_b, w_s))$.

Proof. See the appendix. ■

Thus far we have not yet discussed how reports affect agents' continuation values and, in particular, how the value functions on reports in (9)-(10) are possible. This is to be addressed below.

Proposition 3 *Let v and q and $\{f_{k,t}\}$ and $\underline{\delta}$ be the same as in Proposition 2. Then $\{f_{k,t}\}$ can be supported by a CP equilibrium if and only if $\delta \geq \underline{\delta}$.*

Proof. The "only-if" part is obvious. For the "if" part, let $w_1 = v$ and $w_0 = \underline{v}$, and let w_2, $\bar{v}_s(.)$, $\underline{v}_b(.)$, $y(.)$, $\bar{v}_b(.)$, $\underline{v}_s(.)$, $\kappa_b(.)$, and $\kappa_s(.)$ be given by Lemma 2. The proof proceeds by two steps. In step 1, we define a mapping h_t which maps the individual report history r_k^{t-1}, together with all agents' public history γ^{t-1}, into an element in $[w_0, w_2]$. In step 2, we describe the trading mechanism T and σ, and verify that σ is an CP equilibrium.

Step 1. Set $h_0(r_i^0, \gamma^0) = w_1$, all i. Then h_t with $t > 0$ is defined by induction. Fix γ^{t-1}. Fix k and let $w_i = h_{t-1}(r_i^{t-2}, \gamma^{t-2})$, $i \in \{k, j = \phi_{k,t-1}\}$, and $r = r_{k,t-1}$. Set $w_{k,t} = g_b(r, w_k, w_j)$ if $\theta_{k,t} = 0$, and set $w_{k,t} = g_s(r, w_j, w_k)$ if $\theta_{k,t} = 1$, where $g(r, \varsigma) = (g_b(r, \varsigma), g_s(r, \varsigma))$ is defined by

$$g(r,\varsigma) = \left\{ \begin{array}{c} \varsigma,\ \Delta w = 0 \\ \varsigma,\ \Delta w \neq 0 \ \&\ r = \varnothing \\ (w_0, w_0),\ \Delta w \neq 0 \ \&\ r \neq \varnothing \ \&\ r_b^1 r_s^1 = 0 \\ (\kappa_b(r_b^2, \varsigma), \kappa_b(r_b^2, \varsigma)),\ \Delta w \neq 0 \ \&\ r_b^1 r_s^1 = 1 \end{array} \right\}, \tag{12}$$

with $\varsigma = (w_b, w_s)$, $\Delta w = (w_b - w_1)(w_s - w_1)$ and $\varnothing = (0, 0, 0, 0)$.

Now $h_t(r_i^{t-1}, \gamma^{t-1})$ is determined according to the set $A = \{k \in I : w_{k,t} < w_1\}$ as follows.

(i) $\#A \leq 1$. (There is at most one reported defector.) Set $h_t(r_i^{t-1}, \gamma^{t-1}) = w_{i,t}$, all i.

(ii) $\#A = 2$, $A = \{k_1, k_2\}$, $w_{k_1} = w_1$, and $w_{k_2} < w_1$. (There are a new defector, k_1, and an old defector, k_2.) Set $h_t(r_i^t, \gamma^t) = w_1$ if $i = k_2$, and $h_t(r_i^{t-1}, \gamma^{t-1}) = w_{i,t}$ if $i \neq k_2$.

(iii) $\#A = 2$, $A = \{k_1, k_2\}$, $w_{k_1} = w_1$, and $w_{k_2} = w_1$; or $\#A \geq 3$. (There are at least two new reported defectors.) Set $h_t(r_i^{t-1}, \gamma^{t-1}) = w_1$, all i.

Step 2. The trading mechanism T is the following. The buyer and seller simultaneously announce a trade or autarky: if both announce a trade and the two trades are identical then the outcome of T is that trade; otherwise the outcome of T is autarky.

To describe σ, fix $\{\phi_i^t, \theta_i^t, r_i^{t-1}\}_{i \in I}$, k and $\omega_{k,t}$. Let $w_i = h_t(r_i^{t-1}, \gamma^{t-1})$ for $i \in \{k,\ j = \phi_{k,t-1}\}$. Let $\varsigma = (w_k, w_j)$ if $\theta_{k,t} = 0$, and $\varsigma = (w_j, w_k)$ if $\theta_{k,t} = 1$. Let c_1 be the seller's stage-1 consumption. Then σ_k specifies the following actions for k.

Stage 1. If $\theta_{k,t} = 1$ then $c_1 = \min\{\omega_{k,t}, 1 - q\}$.

Stage 2. If $\omega_{k,t} = 0$ then announce $(0, \varnothing)$. If $\omega_{k,t} = 1$ then announce

$$(Y(1 - c_1, \varsigma), r(1 - c_1, \varsigma)) = \arg\max_{(y,r)} V_b(y, r, \varsigma) - V_b(0, \hat{r}, \varsigma) \tag{13}$$

subject to $0 \leq y \leq 1-c_1$ and $V_s(c_1+y, r, \varsigma) \geq V_s(c_1, \hat{r}, \varsigma)$, where $\hat{r} = (1, 0, 1, 0)$ and $V_b(x, r, \varsigma) = (1-\delta)u(x)+\delta g_b(r, \varsigma)$ and $V_s(x, r, \varsigma) = (1-\delta)(1-x)+\delta g_s(r, \varsigma)$. If autarky is reached then send the message $(0, 1)$.

To verify that σ is a CP equilibrium, fix k and let $h_t(r_k^{t-1}, \gamma^{t-1}) = w$. First we consider k's expected payoff when he meets j with the start-of-t value w_1, provided that all agents follow σ.

If j is the seller (i.e., $w_s = w_1$) and endowed, then the actions specified by σ lead to transferring $Y(y(w, w_1), \varsigma)$ and reporting $r(y(w, w_1), \varsigma)$. By Lemma 2, $Y(y(w, w_1), \varsigma) = y(w, w_1)$ and in $r(y(w, w_1), \varsigma)$, $r_b^1 r_s^1 = 1$ and $r_b^2 = 1$; that is, k's payoff is $(1-\delta)u(y(w, w_1)) + \delta \underline{v}_b(w, w_1)$.

Similarly, if k is the seller (i.e., $w_b = w_1$) and endowed, then the actions specified by σ lead to $Y(y(w_1, w), \varsigma) = y(w_1, w)$ and $r(y(w_1, w), \varsigma)$ with $r_b^1 r_s^1 = 1$ and $r_b^2 = 1$; that is, k's payoff is $(1-\delta)(1-y(w_1, w)) + \delta \bar{v}_s(w_1, w)$.

If j is the seller and not endowed or if k is the seller and not endowed, then following the actions specified by σ, k's payoff is w. It follows from (2) that w is k's expected payoff before meeting j.

It remains to verify that that k does not gain by deviating in the date-t meeting provided that j does not deviate. First, when autarky occurs, any message sent by k gives him δw_0, so that there is no gain by deviation. Second, when $\omega_{k,t} = 0$, or when $\omega_{k,t} = 1$ and $\theta_{k,t} = 0$, any deviation gives him δw_0, so that there is no gain by deviation. Third, when $\omega_{k,t} = 1$ and $\theta_{k,t} = 1$, we need to verify that (a) given any c_1, k does not deviate to a trade different from $(Y(1-c_1, \varsigma), r(1-c_1, \varsigma))$, and (b) k does not deviate to $c_1 \neq 1 - y(w_1, w)$. For (a), a deviation gives him δw_0, which is no greater than $\delta \underline{v}_b(w_1, w)$, the payoff from the trade. For (b), no gain by a deviation follows from the constant value of $S(q')$ in Lemma 2. ■

In the above proof, also in the proof of Proposition 4 below, our use of the continuum is that when there is a reported defector, the continuation values for all but two agents (the defector and the partner who reports defection), say, w, are still v. If there are only a finite number of agents, we can adapt our proofs by adjusting w to capture the effect of the positive probability to meet the defector and his partner.

In the above proof, the magnitude of $w_2 - w_1$ does not have any significant role. Indeed, the proof goes through with an arbitrarily small $w_2 - w_1 > 0$. This is not the case in RP equilibrium, which is discussed below.

6 The folk theorem in RP equilibrium: $\rho = 1$

In RP equilibrium, it turns out that whether the seller's endowment realization is truly random makes a big difference. In this section we consider that the seller's endowment is always 1, i.e., $\rho = 1$.

Our first result, applied to both $\rho = 1$ and $\rho < 1$, gives a sufficient and necessary condition for some $q > 0$ to be transferred in a meeting.

Lemma 3 *Fix $q \in (0, 1]$, $d_b > 0$ and $d_s > 0$ with $\delta d_s > (1 - \delta)q$. Let $\iota_b : [0, 1] \to \mathbb{R}$ be weakly decreasing and concave with $\iota_b(0) = 0$ and $\iota_b(1) = -d_b$. Let $\iota_s : [0, 1] \to \mathbb{R}$ be weakly increasing and concave with $\iota_s(0) = 0$ and $\iota_s(1) = d_s$. For $q' \in [0, 1]$, let*

$$(Y(q'), X(q')) = \arg\max[W_b(y, x)]^{1/2}[W_s(y, x)]^{1/2} \tag{14}$$

s.t. $(y, x) \in [0, q'] \times [0, 1]$, where $W_b(y, x) = (1-\delta)u(y)+\delta\iota_b(x)$ and $W_s(y, x) = -(1 - \delta)y + \delta\iota_s(x)$.

(i) If $y(q) = q$, then $u'(q) \geq W_b(q, 1)/W_s(q, 1) \equiv K$.

(ii) If $\iota_b(x) = \left\{ \begin{array}{c} 0,\ x \leq 1 - d_b/(Kd_s) \\ -Kd_sx,\ x > 1 - d_b/(Kd_s) \end{array} \right\}$ and $\iota_s(x) = d_sx$ and $u'(q) \geq K$, then $Y(q) = q$ and $q \in \arg\max W_s(Y(q'), X(q'))$.

Proof. See the appendix. ■

The key point of Lemma 3 (ii) is that if $u'(q) \geq K$ holds, the value functions of reports can be chose to prevent the seller from consuming more than $1 - q$ at stage 1. This is critical for Propositions 4 and 7 below.

Lemma 3 (i), while simple, suggests that the magnitude to reward the buyer may be critical in RP equilibrium. As indicated above, that magnitude is not important in the the proof of Proposition 3. To see the point, identify $w_2 - w_1$ and $w_1 - w_0$ in the proof of Proposition 3 with d_b and d_s in Lemma 3, respectively. Set $d_s = w_1 - w_0$ (i.e. $v - \underline{v}$) and then for the given δ, from

$$u'(q)[-(1 - \delta)q + \delta(w_1 - w_0)] = [(1 - \delta)u(q) - \delta d_b] \tag{15}$$

we obtain the minimal d_b to support $\{f_{k,t}\}$ in RP equilibrium. If the maximal d_b in RP equilibrium to support $\{f_{k,t}\}$ decreases in δ, then we obtain the greatest lower bound to support $\{f_{k,t}\}$.

To see what may restrict on d_b, let

$$\zeta = 0.5[(1 - \delta)u(q) + \delta w_1] + 0.5[(1 - \delta) \cdot 1 + \delta\zeta]. \tag{16}$$

Significance of ζ is in that to fulfill a value $w > \zeta$, in-equilibrium sellers must reward more to a buyer with w (as his start-of-date continuation value) than to in-equilibrium buyers with $w_1 = 0.5[(1-\delta)u(q)+\delta w_1]+0.5[(1-\delta)\cdot(1-q)+\delta w_1]$, which, in turn, requires more incentive to the sellers and hence restricts on w. The next lemma partially addresses the effect of this restriction.

Lemma 4 *Suppose $\rho = 1$. Let v and q and $\{f_{k,t}\}$ be the same as in Proposition 2. Suppose $\{f_{k,t}\}$ is supported by some RP equilibrium σ. Let W be the set of continuation values admitted by σ. If $\sup W \geq \zeta$ (see (16)), then $\sup W \leq \bar{w}(\delta) = \max p(y_0, \eta_1)$ subject to $0 \leq y_0 \leq 1$, $v \leq \eta_1 \leq 0.5[u(y_0)+1]$, and*

$$u'(y_0)[-(1-\delta)y_0 + \delta(p(y_0,\eta_1) - \underline{v})] \geq (1-\delta)u(y_0) - \delta(p(y_0,\eta_1) - \eta_1), \quad (17)$$

where

$$p(y_0,\eta_1) = 0.5(1-0.5\delta)^{-1}[(1-\delta)u(y_0) + \delta\eta_1 + (1-\delta)]. \quad (18)$$

Moreover, $\bar{w}(\delta)$ is weakly increasing in δ.

Proof. See the appendix. ∎

To see a critical feature of (17), suppose $\bar{w}(\delta)$ is attained in σ. In the meeting between buyer k with $\bar{w}(\delta)$ and seller j with w_1, d_s (in term of Lemma 3) can be as large as $\bar{w}(\delta) - \underline{v}$. In contrast, in a meeting between two agents with w_1, d_s can only be $v - \underline{v}$. The possibility to use a larger d_s (i.e., a larger incentive to an in-equilibrium seller) in an off-equilibrium meeting is the key to fulfill a continuation value greater than ζ in (16)

Now we are ready to establish the main result of this section.

Proposition 4 *Suppose $\rho = 1$. Let v and q and $\{f_{k,t}\}$ and $\underline{\delta}$ be the same as in Proposition 2. Let $d_b(\delta)$ be the d_b satisfying (15) and let $\bar{\delta} \in (\underline{\delta}, 1)$ satisfy $d_b(\bar{\delta}) = 0$. There exists a unique $\hat{\delta} \in [\underline{\delta}, \bar{\delta})$ such that $\{f_{k,t}\}$ can be supported by a RP equilibrium if and only if (a) $\delta > \hat{\delta}$, (b) $\delta = \hat{\delta}$ and $\bar{w}(\delta) = p(y_0, \eta_1)$ with $\eta_1 < 0.5[u(y_0)+1]$, or (c) $\delta = \hat{\delta} = \underline{\delta}$ and $\bar{w}(\delta) > d_b(\delta)$.*

Proof. First we define $\hat{\delta}$. If $\bar{w}(\underline{\delta}) \geq \underline{\delta}^{-1}(1-\underline{\delta})u(q) + v$, then let $\hat{\delta} = \underline{\delta}$. If $\bar{w}(\underline{\delta}) < \underline{\delta}^{-1}(1-\underline{\delta})u(q) + v$, then let $\hat{\delta}$ be defined by $d_b(\hat{\delta}) + v = \bar{w}(\hat{\delta})$. This is well defined because $d_b(\delta)$ is decreasing in δ, and by Lemma 4 $\bar{w}(\delta) > v$ is weakly increasing in δ.

Next we turn to the "only-if" part of the proposition. First set $\delta < \hat{\delta}$. By Lemmas 3 (i) and 4, $\{f_{k,t}\}$ cannot be supported by a RP equilibrium. Next set $\delta = \hat{\delta}$, and suppose either $\delta > \underline{\delta}$, or $\delta = \underline{\delta}$ and $\bar{w}(\delta) = d_b(\delta)$, and $\bar{w}(\delta) = p(y_0, \eta_1)$ with $\eta_1 = 0.5[u(y_0) + 1]$. Suppose by contrary that $\{f_{k,t}\}$ is supported by a RP equilibrium σ. Then by Lemmas 3 (i) and 4, $\bar{w}(\delta)$ is the maximal continuation value in σ. Now $\eta_1 = 0.5[u(y_0)+1]$ implies $\eta_1 = \bar{w}(\delta)$. That is, for a buyer with $\bar{w}(\delta)$ as his start-of-t continuation value, in meeting a seller with v, he consumes y_0 and his start-of-t+1 value is $\bar{w}(\delta)$. By $\bar{w}(\delta) > \underline{v}$, we have $y_0 > 0$, which by Lemma 1 implies that there exists a continuation value $w > \bar{w}(\delta)$ in σ, a contradiction.

Next we turn to the "if" part. Let $w_1 = v$, $w_0 = \underline{v}$ and $w_2 = \bar{w}(\delta)$. As in the proof of Proposition 3, we now introduce the objects $\bar{v}_s(.)$, $\underline{v}_b(.)$, $y(.)$, $\bar{v}_b(.)$, $\underline{v}_s(.)$, $\kappa_b(.)$ and $\kappa_s(.)$.

For $\bar{v}_s(.)$, $\underline{v}_b(.)$ and $y(.)$, if $w < \zeta$ then maintain the definitions in (3)-(7). If $w \geq \zeta$ then let

$$\bar{v}_s(w_1, w) = w,\ \underline{v}_b(w_1, w) = w_1,\ y(w_1, w) = 0; \tag{19}$$

also let $\bar{v}_s(w, w_1) \geq w_0$, $\underline{v}_b(w, w_1) \geq w_1$ and $y(w, w_1)$ satisfy

$$w(1 - 0.5\delta) = 0.5[(1-\delta)u(y(w, w_1)) + \delta\underline{v}_b(w, w_1) + (1-\delta)] \tag{20}$$

and

$$u'(y(w, w_1))[-(1-\delta)y(w, w_1) + \delta(w_2 - w_0)] \geq (1-\delta)u(y) - \delta(w_2 - w_1). \tag{21}$$

Existence of of $\bar{v}_s(.)$, $\underline{v}_b(.)$ and $y(.)$ satisfying (20)-(21) is verified in the appendix.

For $\bar{v}_b(.)$ and $\underline{v}_s(.)$, let $\varsigma = (w_b, w_s)$ and we require

$$\begin{aligned} & u'(y(\varsigma))[-(1-\delta)y(\varsigma) + \delta(\bar{v}_s(\varsigma) - \underline{v}_s(\varsigma))] \\ \geq\ & (1-\delta)u(y(\varsigma)) - \delta(\bar{v}_b(\varsigma) - \underline{v}_b(\varsigma)). \end{aligned}$$

Existence of $\bar{v}_b(.)$ and $\underline{v}_s(.)$ is ensured by the above construction of $\bar{v}_s(.)$, $\underline{v}_b(.)$ and $y(.)$.

For $\kappa_b(.)$ and $\kappa_s(.)$, let $\varsigma = (w_b, w_s)$ and we have

$$\kappa_b(x, \varsigma) = \bar{v}_b(\varsigma) + \iota_b(x),\ \kappa_s(x, \varsigma) = \underline{v}_s(\varsigma) + \iota_s(x), \tag{22}$$

where $\iota_b(x)$ and $\iota_s(x)$ are those in Lemma 3 (ii) with $d_b = \bar{v}_b(\varsigma) - \underline{v}_b(\varsigma)$ and $d_s = \bar{v}_s(\varsigma) - \underline{v}_s(\varsigma)$.

Now we apply the two-step proof of Proposition 3 with the following modifications. In step 1, redefine $g(.) = (g_b(.), g_s(.))$ by

$$g(r, \varsigma) = \left\{ \begin{array}{c} \varsigma, \ w_b \text{ and } w_s \neq w_1 \\ \kappa_b(r_b^2, \varsigma), \kappa_b(r_b^2, \varsigma)), \ w_b \text{ or } w_s = w_1 \end{array} \right\}.$$

That is, if w_b or $w_s = w_1$ then the values of a report for the buyer and seller depend only on the buyer's message of the transfer. In step 2, remove the part related $\omega_{k,t} = 0$ and redefine

$$\begin{aligned} & (Y(1-c_1, \varsigma), r(1-c_1, \varsigma)) && (23) \\ = \ & \arg\max_{(y,r)} [V_b(y, r, \varsigma) - V_b(0, \hat{r}, \varsigma)]^{1/2} [V_s(y, r, \varsigma) - V_s(0, \hat{r}, \varsigma)]^{1/2}. \end{aligned}$$

with the same constraints as those in the problem in (13). Moreover, in autarky let k send the message $(1, 0)$ (instead of $(0, 1)$). In verifying that k does not gain by deviating in the date-t meeting when he is the seller, we apply Lemma 3. ■

Proposition 4 is of less interest if $\hat{\delta} = \underline{\delta}$ for any (u, q). Also, the distinct between $\eta_1 = 0.5[u(y_0) + 1]$ and $\eta_1 < 0.5[u(y_0) + 1]$ is redundant if only one relationship actually holds for any (u, q). Neither is the case by the following observation.

Observation 2 *Let $u(c) = \gamma c$ with $\gamma > 1$.*

(i) If $\gamma^2 - \gamma^3 + \gamma + 1 > 0$, $\gamma < 2$ and $q = 1$, then $\hat{\delta} = \underline{\delta}$ and $\bar{w}(\hat{\delta}) = p(y_0, \eta_1)$ with $\eta_1 < 0.5[u(y_0) + 1]$.

(ii) If $\gamma^2 > 2$ and $q = 1$, then $\hat{\delta} > \underline{\delta}$ and $\bar{w}(\hat{\delta}) = p(y_0, \eta_1)$ with $\eta_1 = 0.5[u(y_0) + 1]$.

Proof. First notice that $\underline{\delta}^{-1} = 0.5(\gamma + 1)$. For part (i), check that at $\delta = \underline{\delta}$, $y_0 = 1$ and $\eta_1 = 0.5[u(y_0) + 1]$ satisfy (17) and $p(y_0, \eta_1) > \delta^{-1}(1 - \delta)u(q) + 0.5\delta v$. For part (ii), to see $\hat{\delta} > \underline{\delta}$, check that $0.5[u(1) + 1] < \delta^{-1}(1 - \delta)u(q) + 0.5\delta v$ at $\delta = \underline{\delta}$. Then at $\delta = \hat{\delta}$, it follows from $d_b(\delta) + v = \bar{w}(\delta)$ and $\gamma > 1$ that if $p(y_0, \eta_1) = \bar{w}(\delta)$, then the inequality in (17) is strict. But then $\eta_1 = 0.5[u(y_0) + 1]$; otherwise η_1 can be increased and so can $\bar{w}(\delta)$. ■

Thus far we consider symmetric Nash bargaining. With a general bargaining power of buyers $\lambda \in (0, 1]$, replace $[W_b(y, x)]^{1/2}[W_s(y, x)]^{1/2}$ in (14) with $[W_b(y, x)]^{\lambda}[W_s(y, x)]^{1-\lambda}$. Then Lemma 3 is valid if $W_b(q, 1)/W_s(q, 1)$ is replaced with $[(1-\lambda)W_b(q,1)]/[\lambda W_s(q,1)]$ if $\lambda < 1$ and with 0 if $\lambda = 1$. This leads to the generalizations of (15) and (17) as

$$\lambda u'(q)[-(1-\delta)q + \delta(v - \underline{v})] = (1-\lambda)[(1-\delta)u(q) - \delta d_b] \tag{24}$$

and

$$\lambda u'(y_0)[-(1-\delta)y_0 + \delta(p(y_0,\eta_1) - \underline{v})] \geq (1-\lambda)[(1-\delta)u(y_0) - \delta(p(y_0,\eta_1) - \eta_1)]. \tag{25}$$

Let $d_b(\delta,\lambda)$ be the d_b satisfying (24) if $\lambda < 1$, and let $d_b(\delta,\lambda) = 0.5\delta(1-\delta)q$ if $\lambda = 1$. Let $\bar{\delta}(\lambda) \in (\underline{\delta}, 1)$ satisfy $d_b(\bar{\delta},\lambda) = 0$. Also, let $\bar{w}(\delta,\lambda) = \max p(y_0,\eta_1)$ (see (18)) subject to $0 \leq y_0 \leq 1$, $v \leq \eta_1 \leq 0.5[u(y_0)+1]$ and (25). By straightforward adaptation of the proof of Proposition 4 (e.g. replace the power terms in (23) with λ and $1-\lambda$), we have the following.

Corollary 1 *Suppose $\rho = 1$. Let v and q and $\{f_{k,t}\}$ and $\underline{\delta}$ be the same as in Proposition 2. Suppose the bargaining power of buyers in renegotiation is $\lambda \in (0,1]$. There exists a unique $\hat{\delta}(\lambda) \in [\underline{\delta}, \bar{\delta}(\lambda))$ such that $\{f_{k,t}\}$ can be supported by a RP equilibrium if and only if (a) $\delta > \hat{\delta}(\lambda)$, (b) $\delta = \hat{\delta}(\lambda)$ and $\bar{w}(\delta) = p(y_0,\eta_1)$ with $\eta_1 < 0.5[u(y_0)+1]$, or (c) $\delta = \hat{\delta} = \underline{\delta}$ and $\bar{w}(\underline{\delta},\lambda) > d_b(\underline{\delta},\lambda)$. Moreover, $\hat{\delta}(\lambda)$ is weakly decreasing in λ, $\hat{\delta}(1) = \underline{\delta}$, and $\lim_{\lambda\to 0}\hat{\delta}(\lambda) = 1$.*

7 The folk theorem in RP equilibrium: $\rho < 1$

Our first result here is a general property about RP equilibrium when $\rho < 1$, which is due to Lemma 1 and renegotiation.

Lemma 5 *Suppose $\rho < 1$. Let σ be a RP equilibrium. Fix γ^t and k and let $\omega_{k,t} = 0$. Suppose that when $\omega_{k,t} = 1$ on-path plays lead to the transfer $q > 0$ and the report r_1, and that when $\omega_{k,t} = 0$ on-path plays lead to the report r_0. Denote by $w_{n,i}$ agent i's end-of-t continuation values implied by r_n, $n \in \{0,1\}$ and $i \in \{k, j = \phi_{k,t}\}$. Then $w_{1,k} - w_{0,k} < 0$ and $w_{1,j} - w_{0,j} \geq \delta^{-1}(1-\delta)q$.*

Proof. Let $\hat{r}$ be the report specified by σ when $\omega_{k,t} = 1$ but j consumes $c_1 = 1$ at stage 1, and let $\hat{w}_i$ be i's end-of-t continuation values implied by $\hat{r}$, $i \in \{k,j\}$. Because both outcomes $(0,r_0)$ and $(0,\hat{r})$ survive renegotiation if renegotiation is triggered by a stage-2 action (e.g., one agent takes an action that leads to autarky), it follows that $\{0,1\} \in \arg\max_{a\in[0,1]}[aw_{0,k} + (1-$

$a)\hat{w}_k][aw_{0,j} + (1-a)\hat{w}_j]$. Hence $w_{0,k} = \hat{w}_k$ and then by Pareto efficiency of $(0, r_0)$ and $(0, \hat{r})$, $w_{0,j} = \hat{w}_j$. By Lemma 1, $w_{1,k} < \hat{w}_k$ and $w_{1,j} > \hat{w}_j$, and, moreover, if $\delta(w_{1,j} - \hat{w}_k) < (1-\delta)q$ then j is better off by consuming $c_1 = 1$ at stage 1 when $\omega_{k,t} = 1$. Now the two inequalities in the lemma follow as desired. ■

Lemma 5 makes a simple point: To prevent the seller from deviation when endowed, the seller is punished (and also the buyer is rewarded) when not endowed. This need not be the case in CP equilibrium. Indeed, in the CP equilibrium in Proposition 3 , r_1 and r_0 imply the same continuation values. This lemma implies the following negative results in RP equilibrium.

Proposition 5 *Suppose $\rho < 1$. Let σ be a CP equilibrium.*

(i) The $\{f_{k,t}\}$ in Proposition 2 (associated with v there) cannot be supported by σ.

(ii) The average start-of-0 continuation value in σ is bounded above by some $\bar{w}_\rho < \bar{v}_\rho$, where $\bar{v}_\rho$ is the $\bar{v}$ in Proposition 2.

Proof. For part (i), suppose by converse that $\{f_{k,t}\}$ is supported by σ. Then in Lemma 5, let r_i^{t-1} be generated by on-path plays, $i \in \{k, j\}$. But then $w_{1,k} = w_{0,k} = w_{1,j} = w_{0,j} = v$, a contradiction. For part (ii), we obtain the bound $\bar{w}_\rho$ by considering a program which takes (part of) the second inequality in Lemma 5 as a constraint. The program is as follows: (i) At each t maximize the mass m_t of meetings, constrained by that $1 - m_t$ is no less than the mass of agents in the m_{t-1} mass of meetings at t-1 who are sellers but not endowed; (ii) In the mass a_t of meetings, maximize each agent's unconstrained pre-meeting period payoff, denoted p_{1t}, which apparently is $(1-\delta)\bar{v}_\rho$ (i.e., the transfer is y^* if the seller in the meeting is endowed); (iii) In the mass $1 - m_t$ of meetings, maximize each agent's pre-meeting period payoff, denoted p_{0t}, constrained by that $p_{0t} \leq p_{1t} - (1-\delta)\delta^{-1}q^*$. The sequence of $\{m_t\}$ is determined by $m_t = 1 - 0.5(1-\rho)m_{t-1}$ with $m_0 = 1$ and for $t > 0$. Now let $\bar{w}_{\rho,\delta} = \sum_{t=0}^{\infty} \delta^t (1-\delta)[(\bar{v}_\rho - \delta^{-1}q^*)(1-m_t) + \bar{v}_\rho m_t]$. Because $\{m_t\}$ is strictly decreasing, $\bar{v}_\rho - \bar{w}_{\rho,\delta} = \delta^{-1}q^*[1 - (1-\delta)\sum_{t=0}^{\infty} \delta^t m_t] > q^*(1-m_1)$. Then we can set $\bar{w}_\rho = \bar{v}_\rho - q^*(1-m_1)$. ■

With reference to Proposition 3, Proposition 5 says that when $\rho < 1$ there is a loss due to renegotiation, and the loss does not vanish as $\delta \to 1$. In the proof, with our choice of $\bar{w}_\rho$, $\bar{v}_\rho - \bar{w}_\rho$ only serves the purpose of showing this latter point. Evidently, $\bar{w}_\rho$ is above the least upper bound of the average start-of-0 continuation values in RP equilibria. While it is of great interest to find this upper bound, we have not been able to do so. As implied by Lemma 5, in any non-autarky RP equilibrium for any t, the end-of-t distribution of the individual continuation values is not degenerate, which makes analysis much more difficult.

One way to deal with the distribution is to make it simple. As an application, we construct an equilibrium to show that positive transfers can occur in some meetings under a fairly mild condition. In that equilibrium, the distribution has a two-point support $\{w_0, w_2\}$ with equal mass. where $w_0 < w_2$ satisfies

$$w_0 = 0.25\rho(1-\delta) + (1-0.25\rho)\delta w_0 + 0.25\rho[(1-\delta)(1-y) + \delta w_2] \quad (26)$$

$$w_2 = 0.5\rho(1-\delta) + (1-0.25\rho)\delta w_2 + 0.25\rho[(1-\delta)u(y) + \delta w_0] \quad (27)$$

for some $y > 0$. That is, when two agents meet at t and their start-of-t continuations values are in the support, there is a positive transfer if and only if the buyer's start-of-t value is w_2, the seller's is w_0, and the seller is endowed. With the transfer, each agent's start-of-t+1 value is the same as his meeting partner's start-of-t value. With no transfer, each agent's start-of-t+1 value is the same as his start-of-t value.

Proposition 6 *Suppose $\rho \leq 1$ and assume*

$$\lim_{c\to 0} \frac{[1+u'(c)][u(c)/c - 1]}{u'(c) + u(c)/c} > \frac{1-\delta}{0.25\rho\delta}. \quad (28)$$

Then there exists an RP equilibrium in which in each period in the 0.25ρ proportion of meetings the transfer is some $y > 0$ that satisfies (26)-(29).

Proof. By Lemma 3, y and w_0 and w_2 must satisfy

$$u'(y)[-(1-\delta)y + \delta(w_2 - w_0)] \geq [(1-\delta)u(y) - \delta(w_2 - w_0)]. \quad (29)$$

Given (28), there exist (w_0, w_2) and $y > 0$ that satisfies (26)-(29). Then we apply the proof of the "if" part of Proposition 4 with the following modifications.

First, we reconstruct the objects $\bar{v}_s(.)$, $\underline{v}_b(.)$, $y(.)$, $\bar{v}_b(.)$, $\underline{v}_s(.)$, $\kappa_b(.)$ and $\kappa_s(.)$. Because there are two mass points in the distribution of the continuation values, those objects should be defined for a generic $\varsigma = (w_b, w_s)$ with either w_b or $w_s \in \{w_0, w_2\}$. In addition, an extra issue is that now $\bar{v}_b(\varsigma)$ and $\underline{v}_s(\varsigma)$ not just provide incentives for $y(\varsigma)$ to be transferred, but also may directly be a component of one's continuation value. Details of the construction are in the appendix.

Next, we only need to redefine h_t as follows. Set $h_0(r_i^0, \gamma^0) = w_n$, all $i \in I_{0,n}$, where $I = I_{0,0} \cup I_{0,2}$ and the measure of $I_{0,n}$ is 0.5. Then define $h_t(r_i^{t-1}, \gamma^{t-1})$ with $t > 0$ by induction. Fix γ^{t-1}. Fix k and set $w_i = h_{t-1}(r_i^{t-2}, \gamma^{t-2})$ for $i \in \{k, j = \phi_{k,t-1}\}$, and $r = r_{k,t-1}$. Set $w_{k,t} = g_b(r, w_k, w_j)$ if $\theta_{k,t} = 0$, and set $w_{k,t} = g_s(r, w_j, w_k)$ if $\theta_{k,t} = 1$, where $g(r, \varsigma)$ is defined

$$g(r, \varsigma) = \left\{ \begin{array}{c} (w_b, w_s),\ w_b \text{ and } w_s \notin \{w_0, w_2\} \\ \kappa_b(r_b^2, \varsigma), \kappa_b(r_b^2, \varsigma)),\ w_b \text{ or } w_s \in \{w_0, w_2\} \end{array} \right\}$$

with $\varsigma = (w_b, w_s)$.

Now $h_t(r_i^{t-1}, \gamma^{t-1})$ is determined according to the sets $A = \{k : w_0 < w'_k < w_1\}$ and $B_n = \{k : w'_k = w_n\}$, $n = 0, 2$, as follows.

(i) $\#A \leq 1$ and the measure of B_n is π_n, all n. Set $h_t(r_i^{t-1}, \gamma^{t-1}) = w'_i$, all i.

(ii) $\#A = 2$, $A = \{k, k^*\}$, $w_{k^*} = w_1$, $w_k < w_1$, and the measure of B_n is π_n, all n. Set $h_t(r_i^t, \gamma^t) = w_1$ if $i = k^*$, and $h_t(r_i^{t-1}, \gamma^{t-1}) = w'_i$ if $i \neq k^*$.

(iii) $\#A = 2$, $A = \{k, k^*\}$, $w_{k^*} = w_1$, and $w_k = w_1$; or $\#A \geq 3$; or the measure of B_n is not π_n, some n. Set $h_t(r_i^{t-1}, \gamma^{t-1}) = w_1$, all i. ■

In the proof of Proposition 6, our use of a continumm of agents is substantial. For our analysis depends on the explicit relationships of (w_0, w_2) given in (26)-(27), which are built on that the masses of w_0 and w_2 are equal. But the equal masses, in turn, appeal to the law of the large number implied by the continuum of agents.

A remaining question is whether the welfare loss due to renegotiation given in Proposition 5 vanishes as $\rho \to 1$. For this question, the nondegenerate distributions impose a bigger challenge. We manage to construct an equilibrium for ρ close to 1 when agents are patient, and this equilibrium shows that the welfare loss in discussion vanishes as $\rho \to 1$. In this equilibrium, there is a mass of $\pi_n > 0$ agents whose continuation values are w_n at the start of each t, $n \in \{0, 1, 2\}$. The masses (π_0, π_1, π_2) satisfy $\pi_0 + \pi_1 + \pi_2 = 1$ and

$$\begin{aligned} 0.5\pi_1(\pi_1 + \pi_2)(1 - \rho) &= z\pi_0, & (30) \\ 0.5\pi_1(\pi_1 + \pi_2)(1 - \rho) &= \pi_2, & (31) \end{aligned}$$

where $z \in (0, 1]$ is a parameter explained below.

Letting $q = q^*$, the values (w_0, w_1, w_2) satisfy $0 < w_0 < w_1 < w_2$ and

$$w_0 = 0.5\rho(1-\delta) \cdot 1 + z\delta w_1 + (1-z)\delta w_0, \tag{32}$$

$$\begin{aligned} w_1 = {} & \pi_0\{0.5\rho(1-\delta) \cdot 1 + \delta w_1\} \\ & +\pi_1\{0.5\rho(1-\delta)[u(q) + 1 - q] + 0.5\rho\delta w_1 + 0.5(1-\rho)\delta(w_2 + w_0)\} \\ & +\pi_2\{0.5\rho(1-\delta)(1-q) + 0.5\rho\delta w_1 + 0.5(1-\rho)\delta(w_1 + w_0)\}, \end{aligned} \tag{33}$$

$$\begin{aligned} w_2 = {} & \pi_0\{0.5\rho(1-\delta) \cdot 1 + \delta w_1\} \\ & +\pi_1\{0.5\rho(1-\delta)[u(q) + 1] + 0.5\rho\delta w_1 + (1-\rho)0.5\delta(w_1 + w_0)\}, \\ & +\pi_2\{0.5\rho(1-\delta) \cdot 1 + 0.5\rho\delta w_1 + (1-\rho)\delta w_1\}. \end{aligned} \tag{34}$$

To understand this equilibrium, consider agent k's with the continuation value w_k at the start of t. Equation (33) pertains to $w_k = w_1$. When meeting a buyer with w_1 or w_2, if endowed then k stays in w_1 (by transferring q) at $t+1$, and if not endowed the he switches to w_0 at $t+1$. When meeting a seller with w_1, if the seller is endow then k stays in w_1 (and receiving q), and if the seller is not endowed k then k switches to w_2. When meeting a seller with w_2, k stays in w_1 (and there is no transfer). When meeting an agent with w_0, k stays in w_1 (and there is no transfer).

Equation (32) pertains to $w_k = w_0$. Agent k is punished (compared to one with w_1) by autarky at t. With probability z he switches to w_1 at $t+1$ while with probability $1-z$, he stays in w_0 at $t+1$. More about this switch probability is addressed later.

Equation (34) pertains to $w_k = w_2$. Agent k is rewarded (compared to one with w_1) in that when he is the seller, he switches to w_1 but there is no transfer. When meeting a seller with w_1, if the seller is endowed then k receives q and switches to w_1, and if the seller is not endowed k stays in x_2. When meeting a seller with $w \neq w_1$, there is no transfer and k stays in w_1.

Equations (30) and (31) are implied by the above switching probabilities. First, in the end of t the outflow of the mass for agents with w_0 is $z\pi_0$, while the inflow is $0.5\pi_1(\pi_1 + \pi_2)(1-\rho)$ (a seller with w_1 meets a buyer with w_1 or w_2 and the seller is not endowed). This gives rise to (30). Next, the outflow of the mass for agents with w_2 is π_2, while the inflow is $0.5\pi_1(\pi_1 + \pi_2)(1-\rho)$ (a buyer with w_2 meets a seller with w_1 or w_2 and the seller is not endowed). This gives rise to (31).

To ensure q being transfer when two agents with w_1 meet and the seller is endowed, we can set

$$u'(q)[-(1-\delta)q+\delta(w_1-w_0)] = [(1-\delta)u(q)-\delta(w_2-w_1)]. \qquad (35)$$

To proceed, we have to deal with existence of (π_0, π_1, π_2) and (w_0, w_1, w_2). As is typical, this may not be resolved by a standard fixed-point approach. In our case, as illustrated in the following lemma, this existence can be simplified to existence of a number solving a single equation.

Lemma 6 *If $\delta < 1$ exceeds a lower bound, then when ρ is close to 1, there exist (π_0, π_1, π_2) and (w_0, w_1, w_2) which satisfy $\pi_0 + \pi_1 + \pi_2 = 1$, $0 < w_0 < w_1 < w_2$, and (30) to (35). Moreover, $\sum w_n \pi_n = w(\rho) \to 0.5[u(q^*) + 1 - q^*]$ as $\rho \to 1$.*

Proof. First, observe that (π_0, π_1, π_2) is completely determined by z. By (30) and (31), $\pi_0 = \pi_2/z$. Then by $\pi_0 + \pi_1 + \pi_2 = 1$, $\pi_2 = (1-\pi_1)z/(z+1)$. This and (30) imply $\pi_1 = \pi_1(z)$, where

$$\pi_1(z) = \frac{-[1+0.5(1-\rho)]z + \sqrt{[1+0.5(1-\rho)]^2 z^2 + 2(1-\rho)z}}{(1-\rho)}. \qquad (36)$$

Notice that $0 < \pi_1(z) < 1$ and hence $0 < \pi_0, \pi_2 < 1$. Also, $\pi_1(z) \to 1$ as $\rho \to 1$. Next, observe that w_0 in (32) and w_2 in (34) can be explicitly expressed as functions of w_1 and π_1. Hence w_1 in (33) can be explicitly expressed as a function of z, and so are $w_1 - w_0$ and $w_2 - w_1$. Substituting those functions into (35), we see that the existence comes down to the existence of z satisfying the transformed (35). Existence of such z and $\lim_{\rho \to 1} w(\rho) = 0.5[u(q^*) + 1 - q^*]$ are delegated to the appendix, where the role of δ and ρ being close to 1 becomes clear. ■

Now we address two questions related to the switching probability z: why it is needed and how the switch is carried out. The former is related to existence. As an alternative to keep punishing an agent with w_0 by autarky with probability $1 - z$, the agent may be punished by N dates; that is, starting from the first date the agent is in w_0, he stays in autarky for N consecutive dates. With this alternative, we can transform existence of suitable (π_0, π_1, π_2) and (w_0, w_1, w_2) to existence of suitable N. But because N is an integer, existence becomes a problem. Regarding how the switch can be carried out, we introduce some public random devices as follows.

(RD) Before each date's matching, each agent makes an independent draw from the uniform distribution over $[0,1]$, whose realization is public information.

Letting $z_{i,t}$ be agent i's date-t draw, $z_{i,-1} = i$ and $z_i^{t-1} = (z_{i,-1}, ..., z_{i,t-1})$, then under (RD), z_i^t is a component of γ_i^t (see Definition 1), where $z_{i,t}$ is agent i's date-t draw, $z_i^{t-1} = (z_{i,-1}, ..., z_{i,t-1})$, and $z_{i,-1} = i$. Now we are ready to establish the following.

Proposition 7 *Suppose $\rho < 1$ and assume (RD). If $\delta < 1$ exceeds a lower bound, then for ρ close to 1 there exists a RP equilibrium. Moreover, the equilibrium average start-of-0 continuation value converges to* $0.5[u(q^*) + 1 - q^*]$ *as ρ converges to 1.*

Proof. See the appendix. ■

8 Reports and money

In this section, we relate reports to money. To this end, we introduce the following variant of the basic model.

The model with money. Stage 1 is unchanged. In stage 2 agents trade the good with a durable and intrinsically useless object called money. Each agent is assigned some initial money holdings in $M \subset \mathbb{R}_+$, and the average holdings are less than $\max M$ if $\max M$ exists. The trading mechanism is the same as in Section 2 except that here a trade (y, l) consists of a feasible transfer of y units of the good and a feasible transfer of l units of money (from the buyer to seller), and autarky means zero transfer of the good and each agent being able to dispose of his own money. In each meeting each agent's money holdings and the transfer of money are pairwise public information. For $t \geq 0$, the start-of-t distribution of money across agents, denoted $\pi_{m,t}$, is public information.[12]

The equilibrium concepts are those in Definitions 2-4. Specifically, let $m_{i,t}$ be agent i's money holdings at the start of t. Then at a date-t meeting

[12] We can weaken this assumption by assuming that only the initial distribution of money is public information, and that, as is standard in monetary matching models, at the start of each date $t > 0$, each agent has a belief on the distribution of money that is consistent with the equilibrium strategy profile σ and the initial distribution of money.

when agent k moves, his public strategy conditions on $\{\phi_i^t, \theta_i^t\}_{i\in I}$, $\omega_{k,t}$, $m_{i,t}$ for $i \in \{k, j = \phi_{k,t}\}$, $\pi_{m,\tau}$ for $\tau \leq t$, and actions already taken by agents k and j during the meeting.

Proposition 8 *If $\{f_{k,t}\}$ is an equilibrium allocation in the model with money, then it is an equilibrium allocation in the basic model.*

Proof. Let $\gamma : M \to [0,1]$ be strictly increasing. Let $\beta : [0,1] \to M$ be such that $\beta(x) = \gamma^{-1}(x)$ if $x \in \{\beta(m) : m \in M\}$, and $\beta(x) = 0$ otherwise. Let $m_{i,0}$ be i's initial money holdings in the model with money. In the basic model, set $x_{i,0} = m_{i,0}$, and given $\{\phi_i^t, \theta_i^t, r^{t-1}\}_{i\in I}$ with $t > 0$, define $x_{i,\tau}$ for $\tau \in \{1, ..., t\}$ by induction: $x_{i,\tau} = x_{i,\tau-1} + r_{i,s,\tau}^1 \beta(r_{i,b,\tau}^2)$ if $\theta_{i,\tau} = 1$ and $x_{i,\tau} = x_{k,\tau-1} + r_{i,s,\tau}^1 \beta(r_{i,b,\tau}^2)$ if $\theta_{k,\tau} = 0$. Suppose $\{f_{k,t}\}$ is supported by an equilibrium σ in the model with money under some T. In the basic model, consider the following T' and σ'.

Fix $\{\phi_i^t, \theta_i^t, r^{t-1}\}_{i\in I}$, k and $\omega_{k,t}$. Denote by $\pi_{x,t}$ the start-of-t distribution of the above-defined statistics x across agents. Under T', k and $j = \phi_{k,t}$ have the same actions as under T given $\{\phi_i^t, \theta_i^t\}_{i\in I}$, $\pi_{m,\tau} = \pi_{x,\tau}$ for $\tau \leq t$, $m_{i,t} = x_{i,t}$ for $i \in \{k, j\}$, and $\omega_{k,t}$. A sequence of actions that leads to autarky under T leads to autarky under T'. A sequence of actions that leads to a trade (y, l) under T leads to (y, r) under T', where $r = (\omega_{k,t}, \gamma(l), \omega_{k,t}, \gamma(l))$.

The strategy σ_k' specifies the same actions as σ_k specifies in the model with money given $\{\phi_i^t, \theta_i^t\}_{i\in I}$, $\pi_{m,\tau} = \pi_{x,\tau}$ for $\tau \leq t$, $m_{i,t} = x_{i,t}$ for $i \in \{k, j\}$, and $\omega_{k,t}$. When autarky is reached, σ_k' specifies that k enters $(1, 0)$ if $\theta_{k,t} = 0$, and $(1, 1)$ if $\theta_{k,t} = 1$. Provided that σ is an equilibrium (CP equilibrium, RP equilibrium, respectively), such a defined σ' is an equilibrium (CP equilibrium, RP equilibrium, respectively). ■

By Proposition 8 and its proof, money can be regarded as a special form of reports, special in that money supports a more restrictive information content. In the proof, the function γ maps the set of the individual money holdings to a subset of the set of reports. As a result, the information content ς (on an information set) to which σ in the model with money responds can be identified as a component of the information content ς' (on an information set) to which σ' in the basic model responds. When the response of σ' to ς' is set to be the response of σ to ς, we obtain an equilibrium σ' (given that σ is an equilibrium).

This proof differs from the proofs of the "only-if " part of Proposition 1 and the money-is-memory result in Kocherlakota [16]. There is no inclusion relationship between the information contents in the basic model and the model with perfect monitoring, nor between the information contents in the model with money and the model with memory in [16]. Indeed, as discussed above, Proposition 1 and the money-is-memory result concern messages about the actions and the true observations of the actions. Proposition 8 concern different methods to carry messages about the actions with different information content.

Given Proposition 8, a natural question is whether reports, with the more information content, can support more allocations than money. While a thorough study on this issue is beyond the scope of this paper, we have a result pertaining to RP equilibrium when $\rho = 1$.

Proposition 9 *Suppose $\rho = 1$ in the model with money. Let v and q and $\{f_{k,t}\}$ be the same as in Proposition 2. If buyers have all the bargaining power in renegotiation and $\delta q > 2(1-\delta)u(q)$, then $\{f_{k,t}\}$ cannot be supported by a RP equilibrium.*

Proof. See the appendix. ■

We suspect that Proposition 9 is valid with a general bargaining power, but our proof only applies to the extreme bargaining power. Two further remarks about Proposition 9 are in order.

First, if we do not use any refinement in the model with money, then $\{f_{k,t}\}$ in Proposition 9 can be supported by an equilibrium when $\delta \geq \underline{\delta}$. The argument is essentially the same one for the "only-if" part of Proposition 2. Let $m_{i,0} = 1$, all i. Let $x_{i,0} = 1$ and define by induction $x_{i,t+1} = x_{i,t} + 10^{-(t+1)^2}$ if $\theta_{i,t} = 1$, and $x_{i,t} = x_{i,t} - 10^{-(t+1)^2}$ if $\theta_{i,t} = 0$. By T, when k meets j at t, they simultaneously announce a number in $\{1, 0\}$: the outcome is $(q, 10^{-(t+1)^2})$ if the seller's stage-1 consumption $c_1 = 0$ and both say 1, and the outcome is autarky otherwise. By σ, at stage 1 of the meeting, $c_1 = 0$; at stage 2, each agent says 1 if $m_{i,t} = x_{i,t}$, and says 0 otherwise. Given $\delta \geq \underline{\delta}$, such a defined σ is an equilibrium.

Second, that an agent's money holdings are not public information is essential for Proposition 9. To see this, assume instead each agent's money holdings at the start of a date is public information, so that at a date-t meeting when agent k moves, his public strategy conditions on $\{\phi_i^t, \theta_i^t, m_i^t\}_{i \in I}$,

$\omega_{k,t}$, and actions already taken by agents k and $j = \phi_{k,t}$ during the meeting, where $m_i^t = (m_{i,0}, ...m_{i,t})$. In the proof of Proposition 4, only the component r_b^2 in r is relevant. Given $\{\phi_i^t, \theta_i^t\}_{i \in I}$, we can map the relevant information in r_i^{t-1} one-to-one into m_i^t as follows. Let $m_{i,0}(r_i^{-1}) = 1$ and for $\tau > 0$, define by induction $m_{i,\tau}(r_i^{\tau-1}) = m_{i,\tau-1}(r_i^{\tau-2}) + r_{i,\tau-1,b}^2 10^{-\tau}$ if $\theta_{\tau-1,i} = 1$ and $m_{i,\tau}(r_i^{\tau-1}) = m_{i,\tau-1}(r_i^{\tau-2}) - r_{i,\tau-1,b}^2 10^{-\tau}$ if $\theta_{\tau-1,i} = 0$. It follows that there is a counterpart of Corollary 1 in the model with money.

9 The concluding remarks

We have not characterized the optimal allocation with renegotiation when the seller's endowment realization is random and subject only to partial monitoring (even when agents are patient). Such a characterization should reveal more about the consequences of decentralization, partial monitoring and message trading, in particular when compared with characterizations of the optimal allocations in standard centralized risk-sharing models where private information is the key problem (e.g. Atkeson and Lucas [1] and Green [8]).

In this paper, we assume that outcomes of actions and the creation of messages about the outcomes occur simultaneously. Perceivably, there are situations where outcomes precede message creation. Such a situation can be represented in the two-player example in the introduction as follows. Players share and consume the cake in room 1; then they move to room 2 where each player writes down his own message in front of each other but not the arbitrator, and then forwards his message to the arbitrator. Because there is no cake in room 2, player A must use some other valuable object (e.g., transferable utility which does not exist in the original example and model) to exchange with B's message. For this modified example, we can modify our original model such that the transfer and consumption of the endowed good occurs in stage 1 and reporting and message trading occurs in stage 2, and there seems to be no difficulty to establish the counterpart of Proposition 3. The problem is renegotiation. Indeed, with renegotiation, there cannot be any transfer in stage 1 because the set of trades in stage 2 does not depend on the stage 1 transfer. How may messages still be useful? This question is relevant in part because many real-life messages are created after outcomes of actions are realized (e.g., credit scores). One possibility is that there also exists hard evidence to confirm a transfer (which is not in the original example and model) so that the stage 1 transfer can affect the set of trades in stage 2. But if there is hard evidence, then why do messages matter in the first place? This seems interesting enough to be pursued by a separated study. If messages do matter, then there seems to be no substantial difficulty to establish counterparts of propositions in Sections 6 and 7.

In the model with money, it is best to interpret money as cash when the agents' payment histories are not public. When those histories are public (see the end of the last section), the cash interpretation is not suitable. In any case, money creates messages about partially-monitored actions. This common function of money under different interpretations suggests a common approach to formulating different payment methods, cash or noncash. Outside this common function, different payment methods can differ in many aspects. For example, for checks (or credit cards) to be used, peoples' payment histories ought to be made public for some finite cost—otherwise a payer is not to pay. Hence checks may create richer information content than cash. Also paying \$100,000 by check may incur less physical effort and time than paying \$100,000 in cash; checks require costly clearing systems but cash does not. Whatever differences among payment methods are, a cashless economy under this common approach is not frictionless—partial monitoring is a friction faced by all payment methods.

Appendix

The proof of Proposition 1

Proof. For the "only-if" part, let $\{f_{k,t}\}$ be an equilibrium allocation in the basic model. Now consider the ideal of perfect monitoring. To describe the candidate equilibrium σ, fix $\alpha_t \cup \{(q_i^{t-1})_{i\in I}\}$. First set $x_{i,-1} = 1$ and define $x_{i,\tau}$ for $\tau \in \{0, ..., t\}$ by induction as follows: when $x_{i,\tau-1} = 1$, $x_{i,\tau} = 0$ if $\theta_{i,\tau} = x_{h,\tau-1} = 1$ where $h = \phi_{i,\tau}$ and $q_{i,\tau} < f_{i,\tau}(\alpha_\tau)$, and $x_{i,\tau} = 1$ otherwise; when $x_{i,\tau-1} = 0$, set $x_{i,\tau} = 0$. Let X_{t-1} be the measure of the set $\{i : x_{i,t-1} = 1\}$. Now fix k and let $j = \phi_{k,t}$. In the date-t meeting, σ_k specifies the following actions. At stage 1, when $\theta_{k,t} = 1$, transfer $y_1 = 0$ to j, and consume $c_1 = 0$; when $\theta_{k,t} = 0$, consume the transfer from j. At stage 2, when $\theta_{k,t} = 1$, transfer $y_2 = f_{k,t}(\alpha_t) - y_1$ to j if $X_{t-1} = x_{k,t-1} = x_{j,t-1} = 1$ and $y_1 \leq f_{k,t}(\alpha_t)$ and $1 - c_1 \leq f_{k,t}(\alpha_t)$, and transfer 0 otherwise; when $\theta_{k,t} = 0$, consume the transfer from j.

To verify that such a defined $\{\sigma_i\}_{i\in I}$ is an equilibrium, it suffices to show that in the date-t meeting, k does not deviate from σ_k if j does not deviate. The only nontrivial case is when $\theta_{k,t} = x_{k,t-1} = x_{j,t-1} = X_{t-1} = 1$, and k does not gain by choosing $y_1 + y_2 < f_{k,t}(\alpha_t)$. Notice that k's continuation value v following $y_1 + y_2 = f_{k,\tau}(\alpha_\tau)$ is completely determined by the allocation $\{f_{k,t}\}$, and that k's continuation value v' following $y_1 + y_2 < f_{k,t}(\alpha_t)$ is the stay-in-autarky-forever value. But in the basic model, given the same public information, by following the equilibrium strategy the seller k transfers $f_{k,\tau}(\alpha_\tau)$ and his continuation value is the one determined by $\{f_{k,t}\}$ (and α_t), while after deviating to a transfer less than $f_{k,\tau}(\alpha_\tau)$ his continuation value cannot be lower than the stay-in-autarky-forever value. Given that k does not deviate in the basic model, he does not deviate in the ideal.

For the "if" part, let $\{f_{k,t}\}$ be an equilibrium allocation in the ideal. Now consider the basic model with the following T and σ. Fix $(\gamma_i^t)_{i\in I}$. Set $x_{i,-1} = 1$ and then define $x_{i,\tau}$ for $\tau \in \{0, ..., t\}$ by induction: when $x_{i,\tau-1} = 1$, $x_{i,\tau} = 0$ if $\theta_{i,\tau} = x_{h,\tau-1} = 1$ where $h = \phi_{i,\tau}$ and $r^2_{i,b,\tau-1} \neq 1$, and $x_{i,\tau} = 1$ otherwise; when $x_{i,\tau-1} = 0$, set $x_{i,\tau} = 0$. Let X_{t-1} be the measure of the set $\{i : x_{i,t-1} = 1\}$. Now fix k and let $j = \phi_{k,t}$. By T, in the date-t meeting, k and j simultaneously announce a number in $\{1, 0\}$. If both say 1 and there is neither a transfer nor consumption at stage 1, then the outcome is $q = f_{k,\tau}(\alpha_\tau)$ and $r = (\omega_{k,t}, 1, \omega_{k,t}, 1)$; otherwise the outcome is autarky. By σ_k, k's actions in the meeting are as follows. At stage 1, when $\theta_{k,t} = 1$, transfer $y_1 = 0$ to j, and consume $c_1 = 0$; when $\theta_{k,t} = 0$, consume 0. At stage 2, say 1 if $X_{t-1} = x_{k,t-1} = x_{j,t-1} = 1$ and there is neither a transfer nor consumption at stage 1, and say 0 otherwise. In autarky, send the message $(1, 0)$.

To verify that such a defined $\{\sigma_i\}_{i\in I}$ is an equilibrium, it suffices to show that in the date-t meeting, k does not deviate from σ_k if j does not deviate. The only nontrivial case is when $\theta_{k,t} = \theta_{k,t} = x_{k,t-1} = x_{j,t-1} = X_{t-1} = 1$ and there is neither a transfer nor consumption at stage 1. In this case, k cannot not gain by saying 0 at stage 2. For, if he says 1 then his continuation value v is completely determined by the allocation $\{f_{k,t}\}$, and if he says 0 then his continuation value v' is the stay-in-autarky-forever value. But in the ideal, given the same public information, by following the equilibrium strategy the seller k transfers $f_{k,\tau}(\alpha_\tau)$ and his continuation value is the one determined by $\{f_{k,t}\}$ (and α_t), while after deviating to a transfer less than $f_{k,\tau}(\alpha_\tau)$ his continuation value cannot be lower than the stay-in-autarky-forever value. Given that k does not deviate in the ideal, he does not deviate in the basic model. ■

The proof of Lemma 2

Proof. If $q' \le y(w_b, w_s)$, then by $y(w_b, w_s) \le q$, $u'(q) \ge 1$, and $\bar{v}_b(w_b, w_s) - \underline{v}_b(w_b, w_s) \le \bar{v}_s(w_b, w_s) - \underline{v}_s(w_b, w_s)$ (implied by $w_2 - w_1 < w_1 - w_0$), the first order conditions to (11) imply $Y(q', w_b, w_s) = q'$ and $X(q', w_b, w_s)[\bar{v}_s(w_b, w_s) - \underline{v}_s(w_b, w_s)] = (1-\delta)q'$; in particular $Y(q', w_b, w_s) = y(w_b, w_s)$ and $X(q', w_b, w_s) = 1$ for $q' = y(w_b, w_s)$. If $q' > y(w_b, w_s)$, then $Y(q', w_b, w_s) = y(w_b, w_s)$ and $X(q', w_b, w_s) = 1$. It follows that $S(q') = (1-\delta) + \delta \underline{v}_s(w_b, w_s)$, all q'. ■

The proof of Lemma 3

Proof. For part (i), $u'(q) \ge W_b(q, X(q))/W_s(q, X(q)) \ge K$, where the first inequality is necessary for $Y(q) = q$, and the second follows from the hypotheses of ι_b and ι_s. For part (ii), by $u'(q) \ge K = -\iota_b'(1)/\iota_s'(1)$, we have $(Y(q), X(q)) = (q, 1)$ so that $W_s(y(q), x(q)) = W_s(q, 1)$. For $q' > q$ (in case $q < 1$), it is clear that $W_s(q, 1) > W_s(Y(q'), X(q'))$. For $q' < q$, we have $Y(q') = q'$ and $x(q') = (1-\delta)q[u(q') + Kq'](2\delta K d_s)^{-1}$. It follows that $W_s(Y(q'), X(q')) - W_s(q, 1) \ge 0 \Leftrightarrow [u(q) - u'(q)](q - q')^{-1} \ge K$, but the latter follows from $u'(q) \ge K$. ■

The proof of Lemma 4

Proof. Choose $\{w_n\}$ in W which converges to $\tilde{w} \equiv \sup W$. For each w_n, there exist some $y_{b,0}^n$, $y_{s,0}^n \in [0,1]$, and $v_{b,1}^n, \eta_{s,1}^n \in [\underline{v}, \tilde{w}]$ such that

$$w_n = 0.5[(1-\delta)u(y_{b,0}^n) + \delta\eta_{b,1}^n)] + 0.5[(1-\delta)(1-y_{s,0}^n) + \delta\eta_{s,1}^n]$$

where $(y_{b,0}^n, \eta_{b,1}^n)$ satisfies

$$u'(y_{b,0}^n)[-(1-\delta)y_{b,0}^n + \delta(\eta_s^n - \underline{v})] \ge (1-\delta)u(y_{b,0}^n) - \delta(\eta_b^n - \eta_{b,1}^n)$$

with some $\eta_s^n \in (v, \tilde{w}] \cap W$ and $\eta_b^n \in (\eta_{b,1}^n, \tilde{w}] \cap W$. Assume that $(y_{b,0}^n, \eta_{b,1}^n, y_{s,0}^n, \eta_{s,1}^n)$ converge to some $(y_{b,0}, \eta_{b,1}, y_{s,0}, \eta_{s,1})$ (if not then pass to some convergent subsequence) so that

$$\tilde{w} = 0.5[(1-\delta)u(y_{b,0}) + \delta\eta_{b,1})] + 0.5[(1-\delta)(1-y_{s,0}) + \delta\eta_{s,1}].$$

Notice that $(y_{b,0}, \eta_{b,1})$ satisfies

$$u'(y_{b,0})[-(1-\delta)y_{b,0} + \delta(\eta_s - \underline{v})] \ge (1-\delta)u(y_{b,0}) - \delta(\eta_b - \eta_{b,1}) \tag{37}$$

where $\eta_s \in [v, \tilde{w}]$ and $\eta_b \in [\eta_{b,1}, \tilde{w}]$. Because $(1-\delta)(1-y_{s,0}) + \delta\eta_{s,1} \le (1-\delta) + \delta\tilde{w}$, $\eta_s - v \le \tilde{w} - v$, and $\eta_b - \eta_{b,1} \le \tilde{w} - \eta_{b,1}$, it follows that $\tilde{w} \le \tilde{w}(\delta)$, where $\tilde{w}(\delta) = \max\, p(y_0, \eta_1)$ subject to $0 \le y_0 \le 1$, $\underline{v} \le \eta_1 \le 0.5[u(y_0) + 1]$, and (17).

Comparing $\tilde{w}(\delta)$ and (16) lead to $(1-\delta)u(y_{b,0})+\delta\eta_{b,1} \geq (1-\delta)u(q)+\delta v$. When $\eta_{b,1} < v$, replacing $(y_{b,0}, \eta_{b,1})$ with $(y'_{b,0}, v)$ such that $(1-\delta)[u(y_{b,0}) - u(y'_{b,0})] = \delta(v - \eta_{b,1})$ does not affect the constraint (37). Hence $\tilde{w} \leq \bar{w}(\delta)$. The proof of weak monotonicity of $\bar{w}(.)$ in the appendix.

Now we show that $\bar{w}(.)$ is weakly increasing. Fix an arbitrary δ and let $p(y_0, \eta_1) = \bar{w}(\delta)$. If $\eta_1 = 0.5[u(y_0) + 1]$, then because (17) must holds with $\delta' > \delta$ for (y_0, η_1), it follows that $\bar{w}(\delta') \geq 0.5[u(y_0) + 1] = \bar{w}(\delta)$. So suppose $\eta_1 < 0.5[u(y_0) + 1]$. Now for $\delta' > \delta$ in a neighborhood of δ, we can find $\eta_1 < \eta'_1 < 0.5[u(y_0) + 1]$ such that

$$u'(y_0)[-(1-\delta')y_0 + \delta'(p' - \underline{v})] \geq (1-\delta')u(y_0) - \delta'(p\prime - \eta_1)$$

and $p' = (1-\delta')[u(y_0) + 1] + \delta'\eta'_1 > \bar{w}(\delta)$. It follows that $\bar{w}(\delta') > \bar{w}(\delta)$. ■

Completion of the proof of Proposition 4

Proof. For existence of $\bar{v}_s(.)$, $\underline{v}_b(.)$ and $y(.)$ satisfying (20)-(21), denote $\bar{w}(\delta)$ by η_0. By definition, $\eta_0 = 0.5(1-0.5\delta)^{-1}[(1-\delta)u(y_0)+\delta\eta_1+(1-\delta)]$ for some (y_0, η_1) satisfying $u'(y_0)[-(1-\delta)y_0+\delta(\eta_0 - w_0)] \geq (1-\delta)u(y_0)-\delta(\eta_0 - \eta_1)$. We proceed by induction in η_n to generate a sequence $\{(y_n, \eta_{n+1})\}$ until $\eta_{n+1} \leq \zeta$. For $\eta_n > \zeta$ and define x_{n+1} by $\eta_n = 0.5[(1-\delta)u(y_{n-1}) + \delta x_{n+1}] + 0.5[(1-\delta) + \delta\eta_n]$. If $x_{n+1} \geq w_1$, then set $y_n = y_{n-1}$ and $\eta_{n+1} = x_{n+1}$. If $x_{n+1} < w_1$, then by the definition of ζ, $(1-\delta)[u(y_{n-1})-u(q)] > \delta(w_1 - x_{n+1})$. So let $\eta_{n+1} = w_1$ and let $y_n \in (q, y_{n-1})$ be defined by $(1-\delta)u(y_{n-1})+\delta x_{n+1} = (1-\delta)u(y_n) + \delta w_1$. It follows from $u'(y_{n-1})[-(1-\delta)y_{n-1} + \delta(\eta_0 - w_0)] \geq (1-\delta)u(y_{n-1}) - \delta(\eta_n - x_{n+1})$ that $u'(y_n)[-(1-\delta)y_n + \delta(\eta_0 - w_0)] \geq (1-\delta)u(y_n) - \delta(\eta_n - w_1)$. Therefore the sequence $\{(y_n, \eta_{n+1})\}$ satisfies

$$\eta_n = 0.5[(1-\delta)u(y_n) + \delta\eta_{n+1})] + 0.5[(1-\delta) + \delta\eta_n] \tag{38}$$

and

$$u'(y_n)[-(1-\delta)y_n + \delta(\eta_0 - w_0)] \geq (1-\delta)u(y_n) - \delta(\eta_n - \eta_{n+1}). \tag{39}$$

Now for $w > \zeta$, let $n \geq 0$ be such that $w \in (\eta_{n+1}, \eta_n]$. Then we can choose $y(w, w_1) \in [y_{n+1}, y_n]$ and $\underline{v}_b(w, w_1) \in [\eta_{n+1}, \eta_n]$ and $\bar{v}_s(w_1, w) \leq \eta_0$ that satisfy (20)-(21). ■

Completion of the proof of Lemma 6

Proof. Taking the difference between (32) and (33) and the difference between (34) and (33), we have

$$\begin{aligned}
\frac{w_1 - w_0}{1-\delta} &= \frac{w_1 - 0.5\rho \cdot 1}{1-(1-z)\delta}, && (40)\\
\frac{w_2 - w_1}{1-\delta} &= \frac{0.5\rho u(q)\pi_1 + 0.5\rho \cdot 1}{1 - 0.5(1-\rho)\delta\pi_1} - w_1. && (41)
\end{aligned}$$

Rewriting (33) and substituting $w_1 - w_0$ and $w_2 - w_1$ from (34) and (33), we have

$$w_1 = \frac{\frac{K_1}{1-0.5(1-\rho)\delta\pi_1} - 0.5\rho q(\pi_1 + \pi_2) + 0.5\rho\frac{0.5(1-\rho)\delta(\pi_1+\pi_2)}{1-(1-z)\delta}}{1 + \frac{K_2}{1-(1-z)\delta}}, \tag{42}$$

where

$$\begin{aligned}
K_1 &= 0.5\rho u(q)\pi_1 + 0.5\rho,\\
K_2 &= 0.5(1-\rho)\delta\pi_1[1-\delta(1-z)] + 0.5(1-\rho)\delta(\pi_1+\pi_2).
\end{aligned}$$

Further substituting (42) into (40) and (41), we have

$$\begin{aligned}
\frac{w_1 - w_0}{1-\delta} &= \frac{\frac{K_1}{1-0.5(1-\rho)\delta\pi_1} + 0.5\rho q(\pi_1+\pi_2) - 0.5\rho \cdot 1 \cdot [1 + 0.5(1-\rho)\delta]}{1-(1-z)\delta + K_2}\\
\frac{w_2 - w_1}{1-\delta} &= \frac{\frac{K_1 K_2}{1-0.5(1-\rho)\delta\pi_1} + 0.5\rho q - 0.5\rho \cdot 1 \cdot 0.5(1-\rho)\delta(\pi_1+\pi_2)}{1-(1-z)\delta + K_2},
\end{aligned}$$

and substituting those equations into (35), we have

$$\begin{aligned}
C_1 K_3 &= \frac{K_1(K_2 + C_2)}{1-0.5(1-\rho)\delta\pi_1} - 0.5\rho q(\pi_1+\pi_2)[(1-z)\delta + C_2 - 1] && (43)\\
&\quad -0.5\rho \cdot 1 \cdot [C_2 + 0.5(1-\rho)\delta(C_2\pi_1 + \pi_1 + \pi_2)],
\end{aligned}$$

where $C_1 = u(q) + C_2 q$ and $C_2 = u'(q)$ and

$$K_3 = [1-(1-z)\delta][\delta^{-1} + 0.5(1-\rho)\pi_1] + 0.5(1-\rho)(\pi_1+\pi_2).$$

Now choose some $z_0 > 0$. Then choose ρ_0 and δ_0 sufficiently close to 1 so that K_3 sufficiently close to z_0. Suppose, without loss of generality, that the left side of (43) is greater than the right side with $(z, \rho, \delta) = (z_0, \rho_0, \delta_0)$ (and with Ks and πs defined by (z_0, ρ_0, δ_0) accordingly). Then fix δ_0, but decrease

z_0 and if necessary increase ρ_0. Because for any $z > 0$, $\pi(z) \to 1$ as $\rho \to 1$, this process assures that given δ_0, there is some (z_0, ρ_0) such that (43) holds with $(z, \rho, \delta) = (z_0, \rho_0, \delta_0)$.

Fix this triple (z_0, ρ_0, δ_0) and consider two exercises.

(a) Fix $\rho_1 > \rho_0$. Then there exists $z_1 > z_0$ such that (43) holds with $(z, \rho, \delta) = (z_1, \rho_1, \delta_0)$. By the fact that $\pi(z) \to 1$ as $\rho \to 1$, when ρ_1 is sufficiently close to 1, then (i) $w_0 < w_1 < w_2$, (ii) $u'(q)[-(1-\delta)q + \delta(w_1 - w_0)] \geq [(1-\delta)u(q) - \delta(w_2 - w_1)]$, and (iii) $\sum w_n \pi_n$ is sufficiently close to $0.5\rho_1[u(q) + 1 - q]$.

(b) Fix $\delta_1 > \delta_0$. Then there exists $z_1 > z_0$ such that (43) holds with $(z, \rho, \delta) = (z_1, \rho_0, \delta_1)$. ■

Completion of the proof of Proposition 7

Proof. First, we reconstruct the objects $\bar{\upsilon}_s(.)$, $\underline{\upsilon}_b(.)$, $y(.)$, $\bar{\upsilon}_b(.)$, $\underline{\upsilon}_s(.)$, $\kappa_b(.)$ and $\kappa_s(.)$. To begin with, we define $y_s(.)$ and $y_b(.)$ on $[w_0, w_2]$ by

$$y_s(w) = \frac{w_2 - w}{w_2 - w_0} y$$

and

$$\begin{aligned} 4w &= \rho[(1-\delta) + \delta w] + \rho[(1-\delta)(1 - y_s(w)) + \delta w_2] \\ &\quad + \rho[(1-\delta)u(y_b(w)) + \delta w_0] + \delta w + 3(1-\rho)\delta w. \end{aligned} \tag{44}$$

Next we construct $\bar{\upsilon}_s(.)$, $\underline{\upsilon}_b(.)$, $y(.)$, $\bar{\upsilon}_b(.)$ and $\underline{\upsilon}_s(.)$ by three exclusive and exhaustive cases about $\varsigma = (w_b, w_s)$.

Case 1. $w_b = w_0$ or $w_s = w_2$. Then $y(\varsigma) = 0$ and $\underline{\upsilon}_b(\varsigma) = w_b$ and $\bar{\upsilon}_s(\varsigma) = w_s$. Also, $\bar{\upsilon}_b(\varsigma) = \underline{\upsilon}_b(\varsigma)$ and $\underline{\upsilon}_s(\varsigma) = \bar{\upsilon}_s(\varsigma)$.

Case 2. $w_b = w_2$ and $w_s \in (w_0, w_2)$. Then $y(\varsigma) = y_s(w_s)$ and $\underline{\upsilon}_b(\varsigma) = w_s$ and $\bar{\upsilon}_s(\varsigma) = w_b$. Also, $\underline{\upsilon}_s(\varsigma) = w_s$ and $\bar{\upsilon}_b(\varsigma) = w_b$.

Case 3. $w_b \in (w_0, w_2)$ and $w_s = w_0$. Then $y(\varsigma) = y_b(w_b)$ and $\underline{\upsilon}_b(\varsigma) = w_s$ and $\bar{\upsilon}_s(\varsigma) = w_b$. Also, $\underline{\upsilon}_s(\varsigma) = w_s$ and $\bar{\upsilon}_b(\varsigma) = w_b$.

By construction, for an agent with w, his expected payoff before matching is w (see (44)).

For $\kappa_b(.)$ and $\kappa_s(.)$, we can use the same definition as in (22). ■

The proof of Proposition 8

Proof. Let (w_0, w_1, w_2) and (π_0, π_1, π_2) satisfy (30)-(34). Then we apply the proof of the "if" part of Proposition 4 with the modifications similar to (but more complicate than) those in the proof of Proposition 6.

First, we reconstruct the objects $\bar{\upsilon}_s(.)$, $\underline{\upsilon}_b(.)$, $y(.)$, $\bar{\upsilon}_b(.)$, $\underline{\upsilon}_s(.)$, $\kappa_b(.)$ and $\kappa_s(.)$. To begin with, we define some functions on $[w_0, w_2]$ or part of the interval. For $w \geq w_0$, let $p_0(w)$ be defined by

$$p_0(w) = 0.5\rho(1-\delta) + \delta w.$$

For $w \in (w_0, w_1)$, let $\bar{\upsilon}_b(w)$ be defined by

$$\bar{\upsilon}_b(w) = w_0 + \frac{w - w_0}{w_1 - w_0}(w_1 - w_0)$$

and let $p_2(w)$ be defined by

$$p_2(w) = 0.5\rho(1-\delta) + \delta w;$$

then let $y_b(w)$ and $p_1(w)$ be defined by

$$p_1(w) = 0.5\rho(1-\delta)[u(y_b(w)) + 1] + \rho\delta w + (1-\rho)0.5\delta[\bar{\upsilon}_b(w) + w]$$

and

$$w = \pi_0 p_0(w) + \pi_1 p_1(w) + \pi_2 p_2(w);$$

and then let $\bar{\upsilon}_s(w)$ satisfy

$$u'(y_b(w)) \geq \frac{(1-\delta)u(y_b(w)) + \delta[w - \bar{\upsilon}_b(w)]}{-(1-\delta)y_b(w) + \delta[\bar{\upsilon}_s(w) - w_0]}.$$

For $w \geq w_1$, let $y_s(w)$ and $\underline{\upsilon}_s(w)$ be defined by

$$\begin{aligned} y_s(w) &= \frac{w_2 - w}{w_2 - w_1} q, \\ \underline{\upsilon}_s(w) &= w_0 + \frac{w - w_1}{w_2 - w_1}(w_1 - w_0); \end{aligned}$$

and then let $p_1(w)$ and $p_2(w)$ be defined by

$$\begin{aligned} p_1(w) &= 0.5\rho(1-\delta)[u(q) + 1 - y_s(w)] + 0.5\rho\delta w_1 + 0.5(1-\rho)\delta[w_2 + \underline{\upsilon}_s(w)], \\ p_2(w) &= 0.5\rho(1-\delta)[1 - y_s(w)] + 0.5\rho\delta w_1 + (1-\rho)\delta[w_1 + \underline{\upsilon}_s(w)]. \end{aligned}$$

Next we construct $\bar{\upsilon}_s(.)$, $\underline{\upsilon}_b(.)$, $y(.)$, $\bar{\upsilon}_b(.)$ and $\underline{\upsilon}_s(.)$ by three cases about $\varsigma = (w_b, w_s, z_b, z_s)$. Throughout those cases, set $\bar{\upsilon}_b(\varsigma) = \underline{\upsilon}_b(\varsigma)$ and $\underline{\upsilon}_s(\varsigma) = \bar{\upsilon}_s(\varsigma)$ whenever $y(\varsigma) = 0$.

Case 1. $w_b = w_0$ or $w_s = w_0$. Then $y(\varsigma) = 0$ and

$$\underline{v}_b(\varsigma) = \left\{ \begin{array}{c} w_0,\ w_b = w_1 \ \&\ z_b > z \\ w_1,\ w_b = w_0 \ \&\ z_b \leq z \\ w_b,\ w_b > w_0 \end{array} \right\},\ \bar{v}_s(\varsigma) = \left\{ \begin{array}{c} w_0,\ w_s = w_1 \ \&\ z_s > z \\ w_1,\ w_s = w_0 \ \&\ z_s \leq z \\ w_s,\ w_s > w_0 \end{array} \right\}$$

Case 2. $w_b \in \{w_1, w_2\}$ and $w_s \in (w_0, w_1)$, or $w_b \in (w_0, w_1)$ and $w_s \in \{w_1, w_2\}$. Then

$$y(\varsigma) = \left\{ \begin{array}{c} 0,\ w_b \in \{w_1, w_2\} \\ 0,\ w_s = w_2 \\ y_b(w_b),\ w_s = w_1 \end{array} \right\},\ \underline{v}_b(\varsigma) = w_b,\ \bar{v}_s(\varsigma) = \left\{ \begin{array}{c} w_s,\ w_b \in \{w_1, w_2\} \\ w_1,\ w_s = w_2 \\ \bar{v}_s(w_b),\ w_s = w_1 \end{array} \right\}.$$

If $w_s = w_1$, then $\bar{v}_b(\varsigma) = \bar{v}_b(w_b)$ and $\underline{v}_s(\varsigma) = w_0$.

Case 3. $w_b \in \{w_1, w_2\}$ and $w_s \geq w_1$, or $w_b \geq w_1$ and $w_s \in \{w_1, w_2\}$. Then

$$y(\varsigma) = \left\{ \begin{array}{c} q,\ w_s = w_1 \\ 0,\ w_s = w_2 \\ y_s(w_s),\ w_b \in \{w_1, w_2\} \end{array} \right\},\ \underline{v}_b(\varsigma) = w_1,\ \bar{v}_s(\varsigma) = w_1.$$

If $w_s = w_1$, then $\bar{v}_b(\varsigma) = w_2$ and $\underline{v}_s(\varsigma) = w_0$. If $w_b \in \{w_1, w_2\}$, then $\underline{v}_s(\varsigma) = \underline{v}_s(w)$ and let $\bar{v}_b(\varsigma)$ satisfy

$$u'(y_s(w)) \geq \frac{(1-\delta)u(y_s(w)) + \delta[w_1 - \bar{v}_b(\varsigma)]}{-(1-\delta)y_s(w) + \delta[w - \underline{v}_s(w)]}.$$

By construction, for an agent with w_0, his expected payoff before matching is w_0 (case 1). Also, for an agent with $w > w_0$, his payoffs from meeting an agent with w_0, w_1 and w_2, respectively, are $p_0(w)$, $p_1(w)$ and $p_2(w)$, and $w = \pi_0 p_0(w) + \pi_1 p_1(w) + \pi_2 p_2(w)$.

For $\kappa_b(.)$ and $\kappa_s(.)$, we can use the same definition as in (22).

Although (z_b, z_s) are the component of ς, for $a \in \{b, s\}$, $\bar{v}_a(\varsigma)$, $\underline{v}_a(\varsigma)$ and $\kappa_a(\varsigma)$ depend on z_a if and only if $w_a = w_0$ (and they never depend on w_{-a} with $\{-a\} = \{b, s\} \backslash \{a\}$), and $y(\varsigma) = 0$ whenever $w_a = w_0$. Hence we limit the use of the public random draw to the purpose stated in the main text.

Now we redefine h_t as follows. Set $h_0(r_i^0, \gamma^0) = w_n$, all $i \in I_{0,n}$, where $I = I_{0,0} \cup I_{0,1} \cup I_{0,2}$ and the measure of $I_{0,n}$ is π_n. Then define $h_t(r_i^{t-1}, \gamma^{t-1})$ with $t > 0$ by induction. Fix γ^{t-1}. Fix k and set $w_i = h_{t-1}(r_i^{t-2}, \gamma^{t-2})$ and $z_i = z_{i,t-1}$ for $i \in \{k, j = \phi_{k,t-1}\}$, and $r = r_{k,t-1}$. Set $w_{k,t} = g_b(r, w_k, w_j, z_k, z_j)$

if $\theta_{k,t} = 0$, and set $w_{k,t} = g_s(r, w_j, w_k, z_j, z_k)$ if $\theta_{k,t} = 1$, where $g(r, \varsigma)$ with $\varsigma = (w_b, w_s, z_b, z_s)$ is defined

$$g(r,\varsigma) = \left\{ \begin{array}{c} (w_b, w_s),\ w_b \text{ and } w_s \notin \{w_0, w_1, w_2\} \\ \kappa_b(r_b^2, \varsigma), \kappa_b(r_b^2, \varsigma)),\ w_b \text{ or } w_s \in \{w_0, w_1, w_2\} \end{array} \right\}.$$

Now $h_t(r_i^{t-1}, \gamma^{t-1})$ is determined according to the sets $A = \{k : w_0 < w'_k < w_1\}$ and $B_n = \{k : w'_k = w_n\}$, $n = 0, 1, 2$, as follows.

(i) $\#A \leq 1$ and the measure of B_n is π_n, all n. Set $h_t(r_i^{t-1}, \gamma^{t-1}) = w'_i$, all i.

(ii) $\#A = 2$, $A = \{k, k^*\}$, $w_{k^*} = w_1$, $w_k < w_1$, and the measure of B_n is π_n, all n. Set $h_t(r_i^t, \gamma^t) = w_1$ if $i = k^*$, and $h_t(r_i^{t-1}, \gamma^{t-1}) = w'_i$ if $i \neq k^*$.

(iii) $\#A = 2$, $A = \{k, k^*\}$, $w_{k^*} = w_1$, and $w_k = w_1$; or $\#A \geq 3$; or the measure of B_n is not π_n, some n. ■

The proof of Proposition 9

Proof. Suppose the contrary and let $\hat{t}$ be such that

$$1 + 0.5[u(q) + 1] \sum_{t=1}^{\hat{t}} \delta^t > (1 - q) + 0.5\delta(1 - \delta)^{-1}[u(q) + (1 - q)]. \qquad (45)$$

Fix agent k, t, and a history of realizations of his endowment and matching. Let $m_{k,t}$ be his in-equilibrium holding at the start of t. Conditional on that he is the seller at t, define $L_{k,t} = \sum_{\tau=1}^{\hat{t}} l_{k,t+\tau}$, a statistic depending on the realization of meetings from $t+1$ to $t+\hat{t}$, where $l_{k,t+\tau}$ is k's the in-equilibrium spending of money when he is a buyer at $t+\tau$, and as convention $l_{k,t+\tau} = 0$ when he is a seller at $t+\tau$.

Claim (i): If $L_{k,t} > 0$ (i.e., agent k is a buyer for at least one meeting from $t+1$ to $t+\hat{t}$), then $L_{k,t} > m_{k,t}$.

Claim (ii): If $L_{k,t} > 0$, then there must exist some $1 \leq \tau \leq \hat{t}$ such that $l_{k,t+\tau} > m_{k,t}/\hat{t}$.

For claim (i), suppose by contrary that $L_{k,t} = m_{k,t}$. Then k is better off by deviating to autarky in the date-t meeting. To see this, notice that from $t+1$ to $t+\hat{t}$, k can obtain q as a buyer and stay in autarky as a seller. To obtain q as a buyer, he just needs to pay the seller the same amount he would pay as a non-defector, for he has all the bargaining power. Because $L_{k,t} = m_{k,t}$, he can finance his spending with $m_{k,t}$ when he is in autarky as a seller. Then (45) implies that the deviation is beneficial. Claim (ii) follows from claim (i) immediately.

Because claims (i) and (ii) apply for arbitrary k and t, it follows that for any $m > 0$, when t is sufficiently large, there is a history such that agent k has an in-equilibrium holding $m_{k,t} > m$. (One such history is as follows. Agent k is a seller from 0 to t. At each $\tau \in \{1, \hat{t}-1\}$, k's partner j is a buyer from 0 to $\tau - 1$, and at each $0 \leq \tau' < \tau$ meets a seller who has the same holding as j at τ'. At each $\tau \in \{\hat{t}+1, 2\hat{t}-1\}$, k's partner j is a seller 0 to $\hat{t}-1$, is a buyer from $\hat{t}$ to $\tau - 1$, and at each $\hat{t} \leq \tau' < \tau$ meets a seller who has the same holding as j at τ'. And so on.)

Now choose a sufficiently large m so that ψ_m, the measure of agents with holdings no greater than $m/\hat{t}$, is sufficiently close to unity. Let $m_{k,t} > m$ and let k be the seller at t. The contradiction is drawn by showing that k is better off by deviating to autarky in a date $t+\tau$ meeting when he is supposed to spend $l_{k,t+\tau} > m_{k,t}/\hat{t}$.

With the deviation, k's current loss is $(1-\delta)u(q)$, but he has at least $m_{k,t}/\hat{t}$ more units of money at the start of $t + \tau + 1$. We claim that following the deviation, with a probability sufficiently close to unity and with a sufficiently large $\hat{\tau}$, k can obtain at least q as a buyer but also 1 as a seller in meetings from $t + \tau + 1$ to $t + \hat{\tau}$ (following the deviation). It follows from this claim that the lower bound on k's gaining from holding $m_{k,t}/\hat{t}$ more units of money at the start of $t+\tau+1$ can be sufficiently close to $0.5\delta[1-(1-q)]$. Therefore, i is better off because $\delta q > 2(1 - \delta)u(q)$.

It remains to verify the last claim. First, because i has all the bargaining power as a buyer, he can obtain q as long as he pays the seller the same amount he would pay as a non-defector. Second, because ψ_m is sufficiently close to 1, the holdings of most buyers are far less than i's large savings at $t + \tau$. Therefore, he can finance his spending with the savings even if he always chooses autarky when he is a seller (so that without any inflow of money) until some $t + \hat{\tau}$. ∎

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