Energy Tax and Equilibrium Indeterminacy

Yan Zhang*

Antai College of Economics and Management

Shanghai Jiao Tong University

August 11, 2010

Abstract

We study the effect of energy taxes (or tariffs) in a standard neoclassical growth model with imported energy in production. We find that (1) the model may exhibit local indeterminacy and sunspots when energy tax rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures (or lump-sum tansfer), and (2) indeterminacy disappears if the government finances endogenous public spending and transfers with fixed tax rates. Under the first type of balanced budget formulation, we provide numerical (calibration) examples to illustrate that the government should not distort the energy price paid by firms with energy taxes in order to avoid aggregate instability. Under the second type of balanced budget formulation, we prove that the economy exhibits equilibrium uniqueness regardless of the existence of lump-sum transfers.

*This is a shortened version of my dissertation at New York University, "Tariff and Equilibrium Indeterminacy". I would like to thank my advisers, Jess Benhabib and Jushan Bai, for their guidance. I likewise thank Boyan Jovanovic, Luis Aguiar-Conraria, Wen Yi, Zhou Lin, Pierpaolo Benigno, Paul Dower, Martin Uribe, Vivian Yue, Alain Venditti, Zheng Xu, Chen Yan, Lu Lei, Viktor Tsyrennikov and seminar participants at the 8th Journees Louis-Andre Gerard-Varet conference in public economics held in Marseille in June 2009 for their valuable comments. Finally, the author acknowledges the financial support of New York University (Henry M. MacCracken Fellowship) and Shanghai Jiao Tong University. Address correspondence to Zhang Yan, Department of Economics, Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, 200052, China. Tel and Fax: 86-21-52302560; E-mail address: laurencezhang@yahoo.com (Y. Zhang).

Key Words : Indeterminacy, Endogenous Energy Tax Rate, Balanced-budget Rule

JEL Classification Number: Q43, F41

1. Introduction

A large body of literature on optimal taxation has recently suggested that energy taxes (or tariffs) have a high efficiency cost when the government needs to raise revenue by using tax instruments. Although much of the early research was concerned with determinacy cases in dynamic stochastic general equilibrium (DSGE) models with energy in production [Rotemberg and Woodford (1994); de Miguel and Manzano (2006)], energy taxes on intermediate goods (say, imported energy), which act like a tax on the returns to factors of production, may generate indeterminacy in a way similar to factor income taxes.

The pioneer work of Schmitt-Grohe and Uribe (1997, henceforth SGU) shows that, in a Ramsey model with a pre-set level of government expenditures (or lump-sum transfers), if the labor income tax rates are endogenously determined by a balanced budget rule, the model can exhibit indeterminacy for empirically plausible values of income tax rates. Guo and Harrison (2004, henceforth GH) further show that constant income tax rates can not be *per se* a source of local indeterminacy in a Ramsey model.

In this paper, we extend their analysis, studying the role of public spending financed by energy taxes (or tariffs) on the emergence of local indeterminacy, within a standard Ramsey model with energy in production, for the same class of fiscal policy rules.[1] We find that energy taxes and labor income taxes are equivalent in generating local indeterminacy when fiscal increasing returns arise. More precisely, we consider a countercyclical (flat) energy tax policy in which constant (endogenous) government expenditures (or lump-sum transfers) are financed by endogenous (exogenous) tax rates.

[1] For simplicity, we assume that the government does not impose consumption taxes on the tradable goods or factor income taxes on the production factors. The energy tax revenue in this model can also be interpreted as tariff revenue.

It turns out that a countercyclical energy tax policy is required in generating indeterminacy in this model, while a flat energy tax policy can make the economy immune to indeterminacy regardless of the existence of lump-sum transfers. In addition, under the balanced-budget rule with a countercyclical tax policy, for empirically plausible values of energy tax rates, we provide calibration examples to illustrate that the government should not distort the energy (oil) price paid by firms with energy taxes in order to avoid aggregate instability.

A novel feature of this model is that the mechanism behind our indeterminacy result is essentially the same as that in SGU. If the representative agent expects future energy tax rates to increase, future imports of foreign inputs and the marginal product of capital (for any given capital stock) will be lower. It implies that the current demand for foreign input will be lower, thus leading to a fall in total output. If the energy tax rate is regressive with respect to the output, the tax rate today will increase, thus validating the agent's initial expectations. In contrast, the mechanism described above does not work in the case of a constant energy tax rate because constant rates with the diminishing marginal products of input can reduce the higher anticipated returns, thus making indeterminacy hard to arise. Calibrated examples show that when we use Aguiar-Conraria and Wen's (2005, 2007, 2008; henceforth ACW) estimation of the imported energy share in Denmark and the Netherlands, the high tax rates on energy (oil) in these countries can lead them into destabilization.[2]

Another novel feature of this model is that the larger the energy share in gross domestic product (GDP), the easier it is for the economy to be subject to multiple equilibria. Here, we consider the case where expectations of a future tax increase shift the labor supply curve up. We find that the larger the energy share is, the larger the decline in employment is because the slope of the labor demand curve decreases in absolute value as the energy share increases. Therefore, the larger the increase in energy tax rate required to balance the budget is, the larger the energy share is. Being

[2] Although throughout the paper, we analyze the model for the developed countries, the result also holds for less-developed countries whose productions are dependent on the imported factors.

aware of this mechanism existing in this model is crucial to understand why reliance on foreign energy can increase the likelihood of aggregate instability.

Since these properties described above are obtained under the assumption that the energy is a capital substitute, a relevant issue is to understand whether indeterminacy can easily arise when the energy is a labor substitute. In this paper, we briefly discuss this issue and find that multiple equilibria become more likely to occur under the capital substitute assumption than under the labor substitute assumption because the absolute value of the slope of the labor demand curve under the capital substitute assumption is smaller than that obtained under the labor substitute assumption, the increase in energy tax rate required to balance the budget under the former assumption should be larger than that obtained under the latter assumption when expectations of a future tax increase shift the labor supply curve up.

In the rest of this paper, we present the model in Section 2, that extends the SGU (1997) framework to allow for public spending, which is financed by energy taxes instead of labor taxes. We obtain the model dynamics and discuss in detail our indeterminacy results, providing numerical examples. In Section 3, we compare our model with the Benhabib and Farmer (1994), SGU, and ACW models and prove several properties of the indeterminacy results. Finally, we conclude the paper in Section 4.

2. An Economy with Energy Taxes

This paper incorporates two different formulations of government budget constraints into a standard neoclassical growth model that incorporates foreign energy as a third production factor. We assume that labor is indivisible (as in Hansen (1985)), and the only source of government revenue is an energy tax. In particular, the balanced-budget rule consists of exogenous (and/or endogenous) government purchases (and/or transfers) and endogenous (and/or exogenous) tax rates levied on the imported input.

2.1. Firms

We introduce government tax policy into the continuous time neoclassical growth model with productive imported input. A continuum of identical competitive firms exists, with the total number normalized to one. The single good is produced by the representative firm with constant returns to scale Cobb-Douglas production function:

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0}, \tag{1}$$

where y_t is the total output, k_t is the aggregate stock of capital, n_t is the aggregate labor supply, $a_k + a_n + a_0 = 1$, and the third factor in the production, say oil (o_t), is imported. Assuming that firms are price takers in the factor markets, profit maximization by each firm leads to the following first-order conditions:

$$w_t = a_n \frac{y_t}{n_t}, \tag{2}$$

$$r_t = a_k \frac{y_t}{k_t}, \tag{3}$$

and

$$p^o(1 + \tau_t) = a_0 \frac{y_t}{o_t}, \tag{4}$$

where r_t denotes the marginal product of capital, w_t denotes the real wage, p^o denotes the real price of oil, and τ_t is the tax rate levied on the imported oil and is uniform to all firms. We should emphasize that (1) p^o is the relative price of the foreign input in terms of the single good, which is the numeraire and tradable, and (2) the variable τ_t represents the endogenous (or exogenous) tax rate levied on the foreign input, and we require that $\tau_t \geq 0$ to rule out the existence of import subsidies.

Since we assume that the economy is open to importing energy, the agent can use the tradable good to buy the foreign input. The energy price is assumed to be exogenous, and the foreign input is assumed to be perfectly elastically supplied.[3] These imply that the energy price, p^o, is independent of the factor demand for o_t and determined on a world market that is not influenced by the domestic economy. Hence, by substituting o_t in the production function with $o_t = a_0 \frac{y_t}{p^o(1+\tau_t)}$, we can obtain the following reduced-form production function:

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}. \tag{5}$$

The term $A_t = [\frac{a_0}{p_0(1+\tau_t)}]^{\frac{a_0}{1-a_0}}$ acts as the "Solow residual" in a neoclassical growth model, which is inversely related to the foreign factor price and τ_t. In this reduced-form production function, the "effective returns to scale" are measured by

$$\frac{a_k + a_n}{1 - a_0} = 1. \tag{6}$$

2.2. Households

The economy is populated by a unit measure of identical, infinitely lived households. Each household is endowed with one unit of time and maximizes the intertemporal utility function:

$$\int_0^\infty e^{-\rho t}(\log c_t - b n_t) dt,\ b > 0, \tag{7}$$

[3] The model is based on the standard DSGE models that incorporate foreign energy as a third production factor. This class of models (such as those of Kim and Loungani (1992), Finn (2000), Rotemberg and Woodford (1996), Wei (2003), and ACW (2005, 2007, and 2008)) has been used widely to study the business-cycle effects of oil price shocks. The empirical justification for the exogeneity of p^o is provided by Hamilton (1983, 1985).

where c_t and n_t are individual household's consumption and hours worked, and $\rho \in (0, 1)$ is the subjective discount rate. We assume that there are no intrinsic uncertainties present in the model.

The budget constraint of the representative agent is given by

$$\dot{k}_t = (r_t - \delta)k_t + w_t n_t - c_t + T_t,\ k_0 > 0 \text{ given}, \tag{8}$$

where $\dot{k}_t$ denotes the net investment, $\delta \in (0, 1)$ denotes the depreciation rate of capital, and $T_t \geq 0$ is the lump-sum transfers.

The first-order conditions for the household's problem are

$$\frac{1}{c_t} = \Lambda_t, \tag{9}$$

$$b = \Lambda_t w_t, \tag{10}$$

$$\dot{\Lambda}_t = (\rho + \delta - r_t)\Lambda_t, \tag{11}$$

where Λ_t denotes the marginal utility of income. Equations (9) and (10) require that the household's marginal rate of substitution between consumption and leisure be equal, that is, $b = \frac{w_t}{c_t}$. Equations (9) and (11) imply the consumption Euler equation.

2.3. Government

The government chooses the tax/transfer policy $\{\tau_t, T_t\}$, and balances its budget in each period. At each point in time, the budget constraint of the government can be stated as follows:

$$p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)} = G_t + T_t, \tag{12}$$

where $G_t \geq 0$ represents government expenditures. Finally, market clearing requires that aggregate demand equal aggregate supply:

$$c_t + G_t + \dot{k}_t + \delta k_t + o_t p^o = y_t. \tag{13}$$

Note that international trade balances each period. Equation (13) shows that domestic production is divided among consumption, investment, imports, and government expenditures ($c_t + i_t + p^o o_t + G_t = y_t$, $i_t = \dot{k}_t + \delta k_t$). In other words, in order to receive energy, the domestic firm pays the amount $p^o o_t$ in terms of output to foreign country. And the tax revenue $p^o \tau_t o_t$ is divided between government expenditures and lump-sum transfers.

2.4. Analysis of the model dynamics

Similar to GH (2004), we assume that tax revenues can either be consumed by the government (i.e., $G_t \geq 0$ for all t) or returned to households as transfers (i.e., $T_t \geq 0$, for all t). Verifying that the economy in which the government finances endogenous public spending and/or transfers with fixed tax rates is immune to indeterminacy is trivial. This is due to the following proposition:

Proposition 1. *If the tax rate is exogenous, production does not exhibit increasing returns to scale since the A_t term is a constant for all t. (In this case, government expenditures are endogenous under the balanced budget rule.) Therefore, the economy exhibits saddle path stability regardless of the existence of lump-sum transfers.*

GH prove that under perfect competition and constant returns-to-scale, if the government finances endogenous public spending and transfers with fixed income tax rates, a one-sector real business cycle model exhibits determinacy regardless of the existence of lump-sum transfers. In this model, we have the same result. With constant tax rates, the model does not display increasing returns to scale. Therefore, indeterminacy cannot arise.

To remain comparable with SGU's analysis, we focus on the cases where the government either consumes all tax revenues (i.e., $T_t = 0$) or transfers the revenue to the household in a lump-sum way (i.e., $G_t = 0$). In the following sections, we mainly discuss the case where $T_t = 0$ holds for all t. Under this specific assumption $T_t = 0$, we replace consumption with $\frac{1}{\Lambda_t}$ and transform the wage rate and the rental rate into functions of capital and labor. Then the equilibrium conditions can be reduced to the following five equations:

$$b = \Lambda_t a_n A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1}, \tag{14}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho + \delta - a_k A_t k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}}, \tag{15}$$

$$\dot{k}_t = (1 - \frac{a_0}{1+\tau_t}) y_t - \delta k_t - \frac{1}{\Lambda_t} - G_t, \tag{16}$$

$$G_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)}, \tag{17}$$

and

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}. \tag{18}$$

First, we claim that for a given tax rate, a unique interior steady state exists in the the dynamical system.

Lemma 1. *The dynamical system possesses a unique interior steady state when the government consumes all tax revenues, and the tax rate is exogenous, that is, $\tau_t = \tau$, for all t. (In this case, $A_t(\tau_t) = A(\tau)$ holds for all t.)*

Proof. To find the unique steady state, we first set $\dot{\Lambda}_t$ in (15) equal to zero and solve the capital/labor ratio in the steady state. We find that $(\frac{k}{n})_{ss} = [\frac{\rho+\delta}{a_k A(\tau)}]^{\frac{1-a_0}{-a_n}}$ is dependent on the energy tax rate and unique for the given tax rate. Second, a unique and positive value of Λ in the steady state, which is $\Lambda_{ss} = \frac{b}{a_n A(\tau)}[\frac{\rho+\delta}{a_k A(\tau)}]^{a_k/a_n}$, can be derived from equation (14). Using the steady state value of Λ, the government budget constraint (17), and the steady state condition $\dot{k}_t = 0$, the market-clearing condition (16) can be written as

$$(1-a_0)k_{ss}[A(\tau)(\frac{k}{n})_{ss}^{\frac{-a_n}{1-a_0}} - \delta] = \frac{a_n A(\tau)}{b}[\frac{\rho+\delta}{a_k A(\tau)}]^{-a_k/a_n}. \tag{kss}$$

Since $(\frac{k}{n})_{ss}$ is known given the tax rate, we can find that k_{ss} (the steady state value of the capital stock) is unique and positive. Since the steady state values of the capital stock and the capital/labor ratio are positive and unique, n_{ss} (the steady state value of the labor supply) is also positive and unique. Finally, the steady state level of government purchases given by (17) is also unique. Straightforward computations show that it can be written as

$$G_{ss} = \frac{\tau}{1+\tau}a_0 A(\tau)k_{ss}(\frac{k}{n})_{ss}^{-\frac{a_n}{1-a_0}}, \tag{g}$$

where k_{ss} is the solution to the (kss) equation. Therefore, G_{ss} is continuous in τ.

From (kss) and (g), one can show that when τ is equal to zero, G_{ss} is also equal to zero because k_{ss} is positive and finite in this case. If the tax rate is exogenous, we can prove that there exists a unique tax rate that maximizes G_{ss}. It is $\tau_m = \frac{a_n}{a_0}$. $\blacksquare$

Second, for a given level of government expenditures, a (steady-state) Laffer curve-type relationship between the tax rate and tax revenue exists, which means that the number of steady state tax rates that generates enough revenue to finance the pre-set level of government purchases will generally be either 0 or 2. We prove this in the following lemma:

Lemma 2. *When tax rates are endogenously determined by a balanced-budget rule with a constant level of government expenditures, the steady state in the dynamical system that consists of (14)-(18) may exist, and the number of steady state tax rates (τ_{ss}) that generates enough revenue to finance the pre-set level of government purchases will generally be either 0 or 2. If two steady states exist in the model, we only focus on the steady state associated with the low steady state tax rate since the steady state associated with the high steady state tax rate is always locally determinate.*

Proof. We derive the steady state values of the variables $\frac{k}{n} = [\frac{\rho+\delta}{a_k A(\tau_{ss})}]^{\frac{1-a_0}{-a_n}}$, $\Lambda = \frac{b}{a_n A(\tau_{ss})}[\frac{\rho+\delta}{a_k A(\tau_{ss})}]^{\frac{a_k}{a_n}}$, and $k = \frac{\frac{a_n A(\tau_{ss})}{b}[\frac{\rho+\delta}{a_k A(\tau_{ss})}]^{-\frac{a_k}{a_n}}}{[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$, where $A(\tau_{ss})$ denotes the steady state value of A_t as τ_t is equal to its steady state value τ_{ss}. We find that in the steady state, $G = \frac{\tau_{ss}}{(1+\tau_{ss})^{\frac{a_n+a_0}{a_n}}}$constant$\equiv F(\tau_{ss})$ holds, and the constant is $\frac{(\frac{a_0}{p^o})^{\frac{a_0}{a_n}} a_0(\rho+\delta)a_n(\frac{\rho+\delta}{a_k})^{-\frac{a_k}{a_n}}}{a_k b[\frac{1-a_0}{a_k}(\rho+\delta)-\delta]}$. $F(\tau_{ss})$ is therefore clearly non-monotone, and the number of positive steady state tax rates that generates enough revenue to finance a pre-set level of government purchases will generally be either 0 or 2.

The second interesting finding is that if τ is exogenous as in the above lemma, $\frac{\partial G_{ss}}{\partial \tau} = 0$ implies that there exists a unique exogenous tax rate that maximizes G_{ss}. Its value is $\frac{a_n}{a_0}$. This is due to G_{ss} being equal to $\frac{\tau}{(1+\tau)^{\frac{a_n+a_0}{a_n}}}$constant. $\blacksquare$

Insert Figure 1 here. Figure caption: Steady-state Laffer curve.

Third, we show that when government expenditures are exogenous, the tax rate is countercyclical with respect to the tax base or the output under the balanced budget rule. The following proposition is the key to indeterminacy in this model:

Proposition 2. *If government expenditures are exogenous, the tax rate is regressive with respect to the tax base ($p^o o_t$) or the output under the balanced budget rule, that is, $\frac{\partial \tau_t}{\partial y_t} < 0$. The regressive (countercyclical) tax rate ($\frac{\partial \tau_t}{\partial y_t} < 0$) can induce increasing returns to scale with respect to capital and labor.*

Proof. $p^o \tau_t o_t = \frac{\tau_t a_0 y_t}{(1+\tau_t)} = G$ implies that $\frac{\partial \tau_t}{\partial y_t} < 0$. Considering the log-linearization of the following equations around the steady state $G = \frac{\tau_t a_0 y_t}{(1+\tau_t)}$, $A_t = [\frac{a_0}{p_0(1+\tau_t)}]^{\frac{a_0}{1-a_0}}$, and $y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}$, it is easy to verify that $\hat{y}_t = \frac{a_k}{1-a_0(1+\tau_{ss})}\hat{k}_t + \frac{a_n}{1-a_0(1+\tau_{ss})}\hat{n}_t$, where $\hat{k}_t$, $\hat{n}_t$, and $\hat{y}_t$ denote the log deviations of k_t, n_t, and y_t from their respective steady states (i.e., k_{ss}, n_{ss} and y_{ss}). This means that production exhibits increasing returns to scale with respect to capital and labor, that is, $\frac{a_k+a_n}{1-a_0(1+\tau_{ss})} > 1$. Thus, an endogenous energy tax rate could be a source of fiscal increasing returns. ■

GH illustrate that in a standard neoclassical growth model, SGU's indeterminacy result depends on a fiscal policy rule in which the tax rate decreases with household's taxable income. In this model, we obtain a similar result that requires the countercyclical rate to generate indeterminacy.

To facilitate the analysis of model dynamics, we consider the log linear approximation of the equilibrium conditions around the steady state. Let λ_t, $\hat{k}_t$, $\hat{\tau}_t$, and $\hat{n}_t$ denote the log deviations of Λ_t, k_t, τ_t, and n_t from their respective steady states. The dynamics of the economy can be summarized by the system of differential equations

$$\begin{bmatrix} \dot{\lambda}_t \\ \dot{\hat{k}}_t \end{bmatrix} = \begin{bmatrix} -(\rho+\delta)\frac{a_n}{a_k - a_0\tau_{ss}} & (\rho+\delta)\frac{-\tau_{ss}a_0}{a_k - a_0\tau_{ss}} \\ (\rho+\delta)\frac{(1-a_0)}{a_k}\frac{1-(\tau_{ss}+1)a_0}{a_k - a_0\tau_{ss}} - \delta & (\rho+\delta)\frac{1-a_0}{a_k - a_0\tau_{ss}} - \delta \end{bmatrix} \begin{bmatrix} \lambda_t \\ \hat{k}_t \end{bmatrix}. \tag{19}$$

Note that the trace and the determinant of the Jacobian matrix that determines the local dynamics around the steady state (denoted by J) are stated as follows:

$$tr(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho+\delta) - \delta, \tag{20}$$

$$\det(J) = \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}\{\delta(a_n - a_0\tau_{ss}) - \frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}[a_n(1-a_0) - a_0\tau_{ss}\frac{1-a_0}{a_k}(1-a_0(1+\tau_{ss}))]\} \tag{21}$$

Proposition 3. *The necessary and sufficient condition for the model to exhibit indeterminacy is* $tr(J) < 0 < \det(J)$, *or,* $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$.

Since the capital stock (k_t) is a predetermined variable, the model is indeterminate if and only if both eigenvalues of the Jacobian matrix have negative real parts. This is equivalent to requiring that the determinant be positive and the trace, negative. It is easy to verify that $tr(J) = \frac{a_k}{a_k - a_0\tau_{ss}}(\rho + \delta) - \delta < 0$ if and only if $\tau_{ss} > \frac{a_k}{a_0}$. If the trace condition is satisfied, the term $\frac{(\rho+\delta)}{a_k - a_0\tau_{ss}}$ on the right side of the determinant is negative. $det(J) > 0$ if and only if $G(\tau_{ss}) = [\frac{(\rho+\delta)a_0^2(1-a_0)}{a_k} - \delta a_0^2]\tau_{ss}^2 - \tau_{ss}[\frac{(\rho+\delta)a_0(1-a_0)^2}{a_k} - \delta a_0(1-a_0)] + [(\rho+\delta)a_n(1-a_0) - \delta a_n a_k] < 0$. Further, showing that $G(\frac{a_k}{a_0}) = 0$ and $G(0) > 0$ is trivial. Then the necessary and sufficient condition for the model to exhibit indeterminacy is equivalent to $G < 0$, or, $\frac{a_k}{a_0} < \tau_{ss} < \tau^*$, where $\tau^* = \frac{[(\rho+\delta)a_n(1-a_0) - \delta a_n a_k]}{[(\rho+\delta)a_0(1-a_0) - \delta a_0 a_k]} = \frac{a_n}{a_0} > \frac{a_k}{a_0}$.

Similar to SGU, if the set of tax rates satisfying the necessary and sufficient condition is not empty, a sufficient condition is that the labor share should be larger than the capital share (i.e., $a_n > a_k$). If the steady-state tax rates are smaller than $\frac{a_k}{a_0}$ or greater than $\frac{a_n}{a_0}$, the determinant is negative, and therefore, the model exhibits local determinacy. That SGU shows that the revenue maximizing tax rate is the least upper bound of the set of tax rates for which the model is indeterminate should be emphasized; this property also holds in our case.

The intuition behind the indeterminacy result is quite straightforward. If agents expect future tax rates to increase, then for any given capital stock, future imports will be lower. Since the marginal product of capital increases in the oil input, the rate of return on capital will be lower as well. The decrease in the expected rate of return on capital may lower the current oil demand, leading to a decrease in current output. Since the tax rate is countercyclical ($\frac{\partial \tau_t}{\partial y_t} < 0$), budget balance can cause the current tax rate to increase, thus validating agents' initial expectations.

To help understand the intuition better, we consider the consumption Euler equation (in discrete time for ease of interpretation).

$$\frac{c_{t+1}}{c_t} = \beta(1 - \delta + a_k \frac{y_{t+1}}{k_{t+1}}) = \beta[1 - \delta + (1 + \tau_{t+1})^{-\frac{a_0}{1-a_0}} r_{t+1}^{bt}], \tag{22}$$

where β denotes the discount factor, $r_{t+1}^{bt} \equiv a_k (\frac{a_0}{p_0})^{\frac{a_0}{1-a_0}} k_{t+1}^{\frac{a_k}{1-a_0}-1} n_{t+1}^{\frac{a_n}{1-a_0}}$ the before-tax return on capital and τ_{t+1} the tax rate in period $(t+1)$. When the agent's optimistic expectations lead to a higher investment, the left-hand side of this equation will increase but the before-tax return on capital r_{t+1}^{bt} will be lower due to the diminishing marginal products. The countercyclical tax rate can increase the right-hand side of the equation, thus making the initial expectations become self-fulfilling. If the tax rate is a constant under the fiscal policy rule with endogenous public spending and/or transfers, the right-hand side of (22) falls. As a consequence, indeterminacy cannot arise in the latter case regardless of the existence of lump-sum transfers.

Capital accumulation is crucial in generating indeterminacy in this economy. It is easy to show that without capital accumulation, the equilibrium is locally determinate.[4] In addition, we can extend the basic model to consider the case where $G_t = 0$ and $T_t =$constant hold for all t. It can be shown that considering a constant level of lump-sum transfers does not alter our main result, and a similar condition to that obtained in proposition 3 on endogenous tax rates is all that is needed for indeterminacy. In that case, we cannot explicitly derive the necessary and sufficient condition

[4]Without capital, (14) becomes $b/\Lambda_t = a_n[\frac{a_0}{p^o(1+\tau_t)}]^{\frac{a_0}{1-a_0}}$, (16) becomes $1/\Lambda_t = (1 - \frac{a_0}{1+\tau_t})y_t - G$, and (17) becomes $G/n_t = \tau_t(\frac{1}{p^o})^{\frac{a_0}{1-a_0}}(\frac{a_0}{1+\tau_t})^{\frac{1}{a_n}}$. From these equations, locally unique solutions for τ_t, n_t, and Λ_t exist.

for the balanced budget rule to generate indeterminacy. This is because relaxing the assumptions of public spending will make the determinant of the Jacobian matrix become more complicated up to a third-order polynomial. However our indeterminacy result is robust to this extension.

2.5. Calibrated Examples

In this section, following SGU (1997), we calibrate the model using structural parameters that are standard in the real business cycle literature. We set the time period in the model to be one year, the annual real interest rate $\rho = 0.04$, and the annual depreciation rate $\delta = 0.1$. Estimates of the labor income share (a_n) for the Netherlands economy range from 0.684 to 0.71 according to OECD. Stat. Based on input-output tables from OECD (1995) reports, ACW (2005) estimate imported energy's share in GDP for the Netherlands economy to be about 0.21 (a_o). This implies that the capital share (a_k) ranges from 0.08 to 0.106. Therefore, the lower bound of the indeterminacy region ranges from 38.1% to 50.5%. The energy tax rates reported by the International Energy Agency (1998) for the Netherlands economy range from 61.3% to 74.9%, which clearly fall within the indeterminacy region.[5]

Next, we consider the energy tax policy that is implemented in Denmark. As the optimal tariff argument, the energy taxes are relatively high in this country.[6] From 1990 to 1995, the energy tax rates on leaded and unleaded gasoline range from 60.3% to 72.2%.[7] We calibrate the model's structural parameters following SGU (1997) and ACW (2005). We set the time period in the model to be one year, the annual real interest rate $\rho = 0.04$, the energy share $a_o = 0.2$, and the annual depreciation rate $\delta = 0.1$. Estimates of the labor income share in Denmark (a_n) range from 0.662 to

[5] From 1990 to 1996, the energy tax rates for the leaded and unleaded gasoline range from 61.3% to 74.9% (see Tables 8, 10 and 11, pp. 295-298, 4th Quarter 1998, Energy prices and taxes). The parameter value of a_n is taken from 1993 to 1997. As in SGU, if the only source of government revenues is an endogenous consumption tax, the model will exhibit local determinacy. Similar to Giannitsarou (2007), we conjecture that the possibility of aggregate instability caused by energy taxes is reduced if we add an endogenous consumption tax into this model.

[6] The energy tax revenue is overwhelmingly oil tax revenue in some EU countries, see Newbery (2005).

[7] See Tables 8, 9, and 10, pp. 295-297, 4th Quarter 1998, Energy prices and taxes.

0.684, which implies that the capital share in Denmark (a_k) ranges from 0.116 to 0.138. The oil tax rates in Denmark fall within the indeterminacy region.[8]

Insert Figure 2 here. Figure caption: Energy tax rates and labor share.

3. Comparison with the Benhabib and Farmer, SGU, and ACW Models

In this section, we first show that a close correspondence exists between the indeterminacy condition of our model and that of the Benhabib and Farmer (1994) model with productive increasing returns. That is, the necessary condition for local indeterminacy is that the "equilibrium labor demand schedule" can be upward sloping and steeper than the labor supply schedule. Unlike the model of Benhabib and Farmer (1994), this model does not rely on increasing returns in production to make the "equilibrium labor demand schedule" upward sloping. In fact, the equilibrium labor demand schedule in our model is upward sloping because increases in employment can decrease the equilibrium tax rates and increase the after-tax return on labor. To see this, we write the after-tax labor demand function as follows (in log deviations from the steady state):

$$\hat{w}_t = \frac{a_k}{1-a_0}\hat{k}_t - \frac{a_k}{1-a_0}\hat{n}_t - \frac{a_0}{1-a_0}\frac{\tau_{ss}}{1+\tau_{ss}}\hat{\tau}_t, \tag{23}$$

where $\hat{w}_t = \hat{w}_t^{bt} - \frac{a_0}{1-a_0}\frac{\tau_{ss}}{1+\tau_{ss}}\hat{\tau}_t$ is the log deviation of the after-tax wage rate from the steady state.[9] We see that the firm's labor demand schedule is a decreasing function of $\hat{n}_t$. However, when we replace $\hat{\tau}_t$ with $\hat{k}_t$ and $\hat{n}_t$ using the balanced-budget equation, obtaining the equilibrium labor demand schedule is trivial:

[8] The parameter value of a_n is also taken from OECD. Stat from 1993 to 1997. The values of δ and ρ are taken from SGU (1997). The lower bound of the indeterminacy region ranges from 0.58 to 0.69.

[9] $\hat{w}_t^{bt}$ denotes the log deviation of the before-tax wage rate from the steady state.

$$\hat{w_t} = \frac{a_k}{1 - a_0(1 + \tau_{ss})}\hat{k_t} + \frac{-(a_k - a_0\tau_{ss})}{1 - a_0(1 + \tau_{ss})}\hat{n_t}. \tag{24}$$

As $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$, the equilibrium labor demand function is upward sloping since $\frac{-(a_k - a_0\tau_{ss})}{1 - a_0(1+\tau_{ss})} > 0$. In our case, $\hat{w_t} = \hat{c_t}$, so the aggregate labor supply is horizontal and the labor demand schedule will be steeper than the labor supply schedule if $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$. That our economy can easily be shown to be equivalent to the SGU model must be emphasized because in both cases, the price-to-cost markup is countercyclical with respect to the output, which is the key to generating indeterminacy (see the Appendix).

Second, we compare our model with the SGU model. SGU prove that in a Ramsey model, when the government relies on changes in labor income taxes to balance the budget, this fiscal policy rule can realize expectations of higher tax rates. If the import factor is assumed to be a labor substitute, the endogenous tax rate levied on the imported oil may not make indeterminacy arise more easily. Although in the above sections, we follow ACW to assume that the imported factor is mainly a substitue for capital, we cannot eliminate the possibility that the imported factor is a substitute for labor.

We thus come up with the following proposition:

Proposition 4. *If we assume that the imported factor is mainly a labor substitute instead of a capital substitute, which means that we fix a_k at a given level (say, $a_k = 0.3$), and let a_0 vary in the interval $(0, 1 - a_k - a_n)$, indeterminacy may not easily arise under the labor substitute assumption.*[10]

Proof. A formal proof can be stated as follows. We consider an economy with capital share (a) and labor share ($1 - a$). When we introduce the foreign input with share b as a labor substitute into the model, the indeterminacy region becomes $\frac{a}{b} < \tau_{ss} < \frac{1-a-b}{b}$. When we introduce the

[10] *Under the capital substitute assumption, we fix a_n at a given level, and let a_0 vary in the interval $(0, 1 - a_k - a_n)$.*

foreign input with share b as a capital substitute into the model, the indeterminacy region becomes $\frac{a-b}{b} < \tau_{ss} < \frac{1-a}{b}$. The lower bound of the region under the labor substitute assumption is clearly larger than that obtained under the capital substitute assumption. ■

From this proposition, we find that although energy taxes share a similar mechanism for indeterminacy with factor income taxes, they have different implications in generating indeterminacy. That is, the "equivalence" relationship between them only holds through fiscal increasing returns by endogenizing rates and making government revenue exogenous. ACW (2008, p. 721) find that if the imported factor is a substitute for labor, a larger oil share (a_0) implies a smaller threshold value of the production externality although the reduction in the latter is less dramatic. In this model, we find that for the same oil share (a_0), under the labor substitute assumption, the threshold value of the (steady state) tax rate needed to generate indeterminacy (i.e., the lower bound of the indeterminacy region) can be larger than that obtained under the capital substitute assumption.

We provide the economic intuition behind this result by considering the equilibrium condition in the labor market. Suppose that the expectations of a future tax increase shift the labor supply schedule up (since the firm will import more oil today to produce more output). The slope of the labor demand schedule is equal to $-\frac{a_k}{1-a_0}$, so the absolute value of the slope under the capital substitute assumption ($|-\frac{a-b}{1-b}| = \frac{a-b}{1-b}$) is smaller than that obtained under the labor substitute assumption ($|-\frac{a}{1-b}| = \frac{a}{1-b}$). This implies that the decline in employment under the capital substitute assumption should be larger than that obtained under the labor substitute assumption. As a result, the increase in energy tax rate required to balance the budget under the capital substitute assumption should be larger than that obtained under the labor substitute assumption. Hence, multiple equilibria become more likely to occur under the capital substitute assumption than under the labor substitute assumption (see Fig. 3a, where LS represents the aggregate labor supply, LD represents the aggregate labor demand under the capital substitute assumption, LD* represents the aggregate labor demand under the labor substitute assumption, and WH represents the equilibrium wage-hour locus).

Insert Figure 3. here. Figure caption: Indeterminacy and the labor market.

Finally, we compare our model with the ACW model. ACW (2008) show that heavy reliance on imported energy can induce aggregate instability in the presence of increasing returns to scale: the larger the imported energy share in GDP, the easier it is for the economy to be indeterminate and unstable. We have a similar proposition as follows:

Proposition 5. *We fix a_n at a given level (say, $a_n = 0.7$), which implies that the imported input is a capital substitute (i.e., $a_k + a_0 = (1 - a_n)$ is fixed). The larger the imported energy share in GDP, the easier it is for the economy to be indeterminate. The lower bound of the indeterminacy region $\frac{a_k}{a_0} < \tau_{ss} < \frac{a_n}{a_0}$ decreases as a_0 increases, so indeterminacy occurs more easily in the range of empirical tax rates the larger a_0 is.*

When the energy share (a_0) increases, the lower bound of the steady state tax rate that generates indeterminacy decreases (given that a_n is fixed). We provide the economic intuition by considering the equilibrium condition in the labor market. If the agent expects that there is a future tax increase, this will shift the labor supply schedule up. The slope of the labor demand schedule is $-\frac{a_k}{1-a_0}(= -\frac{1}{1+\frac{a_n}{a_k}})$. Therefore, the smaller a_k is, the larger the decline in employment is (since the slope of the labor demand schedule decreases in absolute value as a_k decreases). As a consequence, in order to balance the budget, the required increase in tax rate is larger the smaller a_k is; hence, it is easier for multiple equilibria to occur the larger a_0 is (see Fig. 3b, where LS represents the aggregate labor supply, LD represents the aggregate labor demand with a large energy share, LD' represents the aggregate labor demand with a small energy share, and WH represents the equilibrium wage-hour locus).

4. Conclusion

We explore the "channel equivalence" between factor income taxes and energy taxes to generate indeterminacy.[11] The channel is through fiscal increasing returns by endogenizing rates and making the government spending (or lump-sum transfers) exogenous. We show that, in the presence of fiscal increasing returns caused by endogenous energy taxes, indeterminacy easily occurs in open economies that import foreign energy. The required steady state energy tax rates can be empirically realistic. An implication of this paper is that economies largely dependent on non-reproducible natural resources may be vulnerable to sunspot fluctuations if the government finances public spending with endogenous energy taxes.

One future research direction is to determine under what circumstances, energy taxes and capital income taxes are equivalent in generating indeterminacy since the essential element for indeterminacy in the SGU model is an endogenous labor income tax rate.

5. Appendix:

We summarize the equilibrium conditions of the model with a balanced-budget rule, endogenous energy tax rates, and constant government purchases shown in this paper. We consider the discrete time case of an energy tax. The balanced-budget rule is as follows

$$G = \frac{\tau_t a_0 y_t}{(1 + \tau_t)}.$$

The following equilibrium conditions hold for all t,

[11] In the working paper, Zhang (2009) briefly discusses the robustness of our indeterminacy result in the economy with endogenously determined energy tax rates. As in SGU, income-elastic government spending and more general preferences are allowed. In addition, we think that allowing for public debt and predetermined tax rates will not change our main results.

$$U_c(c_t, n_t) = \theta_t,$$

$$U_n(c_t, n_t) = w_t\theta_t,$$

$$\mathsf{Y}_t = c_t + k_{t+1} - (1-\delta)k_t,$$

and

$$1 = \beta\frac{\theta_{t+1}}{\theta_t}(1 - \delta + r_{t+1}),$$

where θ_t is the Lagrangian multiplier of the budget constaint of the agent. In this model, disposable income, Y_t, is

$$\mathsf{Y}_t = (1 - a_0)y_t = y_t - p^0 o_t - G,$$

where G denotes a fixed cost that guarantees that the domestic firms do not earn pure profits in the long run (given that the foreign firms take away their payments). The after-tax wage rate w_t, and the after-tax rental rate r_t are

$$r_t^{bt} = a_k(\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}-1} n_t^{\frac{a_n}{1-a_0}} = \mu_t r_t,$$

and

$$w_t^{bt} = a_n(\frac{a_0}{p^o})^{\frac{a_0}{1-a_0}} k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}-1} = \mu_t w_t,$$

where r_t^{bt} and w_t^{bt} denote the before-tax rental rate and the before-tax wage rate respectively. In this model, μ_t denotes the wedge between marginal product and after-tax factor prices. It is easy to verify that the markup μ_t is countercyclical with respect to y_t since

$$\mu_t = (1+\tau_t)^{\frac{a_0}{1-a_0}} = (1 - \frac{1-a_0}{a_0}\frac{G}{\mathsf{Y}_t})^{-\frac{a_0}{1-a_0}} = \mu(\frac{G}{\mathsf{Y}_t}).$$

References

[1] Aguiar-Conraria, L., Wen, Y., 2005. Foreign trade and equilibrium indeterminacy. Federal Reserve bank of St. Louis, 2005–041a.

[2] Aguiar-Conraria, L., Wen, Y., 2007. Understanding the large negative impact of oil shocks. Journal of Money, Credit, and Banking 39, 925–944.

[3] Aguiar-Conraria, L., Wen, Y., 2008. A note on oil dependence and economic instability. Macroeconomic Dynamics 12, 717–723.

[4] Benhabib, J., Farmer, R.E., 1994. Indeterminacy and increasing returns. Journal of Economic Theory 63, 19–41.

[5] De Miguel, C., Manzano, B., 2006. Optimal oil taxation in a small open economy. Review of Economic Dynamics 9, 438–454.

[6] Finn, M., 2000. Perfect competition and the effects of energy price increases on economic activity. Journal of Money, Credit, and Banking 32, 400–417.

[7] Giannitsarou, C., 2007. Balanced-budget rules and aggregate instability: the role of consumption taxes. The Economic Journal 117, 1423–1435.

[8] Guo, J.T., Harrison, S.G., 2004. Balanced-budget rules and macroeconomic (in)stability. Journal of Economic Theory 119, 357–363.

[9] Hamilton, J., 1983. Oil and the macroeconomy since World War II. Journal of Political Economy, 91:2, 228–248.

[10] Hamilton, J., 1985. Historical causes of postwar oil shocks and recessions. Energy Journal, 6, 97–116.

[11] Hansen, G., 1985. Indivisible labor and the business cycles. Journal of Monetary Economics 16, 309–327.

[12] International Energy Agency, 1998. Energy prices and taxes, 4th Quarter.

[13] Kim I.M., Loungani, P., 1992. The role of energy in real business cycle models. Journal of Monetary Economics 29, 173–189.

[14] Newbery, D., 2005. Why tax energy? towards a more rational policy. The Energy Journal 26 (3), 1–39.

[15] OECD, 1995. The OECD input-output database. OECD Publication and Information Center.

[16] Rotemberg, J., Woodford, M., 1994. Energy taxes and aggregate economic activity. In J. Poterba (ed.), Tax Policy and the Economy, vol. 8, Cambridge, MA: MIT Press. pp. 159–195.

[17] Rotemberg, J., Woodford, M., 1996. Imperfect competition and the effects of energy price increases on economic activity. Journal of Money, Credit, and Banking 28, 549-577.

[18] Schmitt-Grohe, S., Uribe, M., 1997. Balanced-budget rules, distortionary taxes, and aggregate instability. Journal of Political Economy 105, 976-1000.

[19] Wei, C., 2003. Energy, the stock market, and the putty-clay investment model. American Economic Review 93, 311-323.

[20] Zhang, Y., 2009. Tariff and equilibrium indeterminacy. Mimeo, New York University. http://mpra.ub.uni-muenchen.de/13099/.