Volatility Spillover in the US and European Equity Markets: Evidence from Ex-ante and Ex-post Volatility Indicators

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ABSTRACT

This article utilizes the multiplicative error model to analyze and compare the volatility spillover effect based on two volatility measures, namely, the volatility index and the price range. We find that the lead-lag relationships are similar based on these two volatility measures, and that there exists a structural break when the subprime mortgage crisis occurred. The results based on both the volatility index and price range measures indicate that there are dual relationships between the U.S. and Europe. Furthermore, we measure the economic value of the volatility spillover effect and find that a maximum benefit of 20.06 annualized basis points is yielded in terms of the out-of-sample results. An investor with higher risk aversion will give rise to a lower performance fee. In addition, the volatility forecasts based on the price range are found to perform better than those based on the volatility index.

KEY WORDS: Economic Value; Price Range; Volatility Index; Volatility Spillover

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1. Introduction

Over the last few decades, many financial and economic crises have occurred, such as the Mexican economic crisis in 1994, the Asian financial crisis in 1997, the Russian financial crisis in 1998, the Argentine economic crisis in 2001 and the U.S. subprime mortgage crisis in 2007 and so on. Because of financial globalization, these crises have not only affected the local economy, but have also affected other related economies. Financial globalization causes the transmission of information to play an important role in financial studies. Hence, if investors can understand the information transmission between international financial markets before making their decisions, this can help them with their plans for asset diversification and dynamic hedging strategies, i.e., asset allocation decisions and risk management.

Many studies have indicated that not only return but also volatility can be used to examine the transmission of information across different markets, since volatility is regarded as a measure of risk. The topics, which are relevant to risk, are still important in financial studies. Hamao et al. (1990) found volatility spillover from the U.S. and the U.K. markets to the Japanese market, but they did not find any significant spillover from the Japanese market to the other markets. Martens and Poon (2001) found volatility spillover from the U.S. to the U.K, and from the U.K. to the U.S, i.e., their relationships were dual. Baele (2005) and Christiansen (2007) investigated the volatility spillover effects between the U.S. and individual local European markets (as global effects), and between the aggregate European market and these individual local European markets (as regional effects). These two studies focused on stock markets and bond markets, respectively. They found evidence of volatility spillover from the aggregate European and U.S. markets to local European markets. There also exist many studies on volatility spillover such as Bekaert and Harvey (1997), Ng (2000), Miyakoshi (2003), Worthington and Higgs (2004), Skintzi and Refenes (2006), and more recently Badshah (2009), Diebold and Yilmaz (2009), McMillian and Speight (2010), Singh, Kumar and Pandey (2010), and so on. In fact, financial volatility is difficult to observe, and can only be estimated using a certain known market price process. In addition, the U.S. and Europe are the major economies of the world, and the subprime mortgage crisis that occurred in the U.S. would first have attacked the U.S. and European economies. This causes us to be particularly interested in the lead-lag relationships between these two regions.

In previous studies, different measures for volatility, such as return-based volatility, implied volatility, and the high-low price range have been used. Here, we focus on the implied volatility and price range measures, which can be regarded as ex-ante and ex-post volatility indicators, respectively. Implied volatility employs the forward-looking concept which consists of both historical information and market expectations, and hence it can be thought of as an ex-ante volatility measure. In 1993, the volatility index was introduced by the Chicago Board Options Exchange (CBOE). The CBOE volatility index, VIX, which is based on the Black-Scholes model, is used to calculate the implied volatility of the S&P 100 index. In fact, VIX is used to measure the market's expectation regarding the next 30 days' volatility. If the VIX is higher, then it indicates that the investors expect the stock index to exhibit higher volatility in the future. Therefore, the volatility index can be thought of as the "investor fear gauge". In 2003, the CBOE proposed a new method to compute the volatility index without any of the assumptions of the option-pricing model. That is, the new VIX is a model-free implied volatility. Besides, the new VIX measures the implied volatility of the S&P 500 index, which is more closed to the U.S. market than the S&P 100 index[1]. Other markets have followed the U.S. market to issue their own volatility index.

In addition, a number of studies have confirmed that implied volatility has better forecasting ability than the return-based volatility measure. Christensen and Prabhala (1998) explained that the reason why the implied volatility is biased and inefficient relative to the return-based volatility measure in previous studies is due to the overlapping data problem. Hence they used the non-overlapping (monthly) data of the S&P 100 index option and proved that the implied volatility outperforms the return-based volatility. Fleming (1998) also used the S&P 100 index option to confirm that implied volatility has better forecasting ability than historical volatility, i.e., the range-based model of Parkinson (1980). Blair et al. (2001) used the S&P100 and VIX indices to compare the incremental information using historical volatility, low- and high-frequency return-based volatility, and implied volatility. They showed that the VIX index provides more accurate forecasts than the other indices. Li and Yang (2009) further used the S&P/ASX 200 index options, which are traded infrequently and with a long maturity cycle, and found that implied volatility outperforms historical volatility.

On the other hand, the price range is defined as the difference between the highest price and lowest price at a fixed time interval, for instance, daily, weekly or monthly. Since the price range is calculated using the realized data, it can be thought as an ex-post volatility measure. Because the return-based volatility is only computed by the

[1] The S&P 100 index is a subset of the S&P 500. In addition, the old VIX is renamed the VXO index.

close-to-close price return, it may ignore the information regarding price changes and thus generate inappropriate volatility estimators. Parkinson (1980) was the first to use the price range as a proxy variable for measuring volatility, and confirmed that the price range is a more efficient proxy variable of volatility than the return-based volatility estimator. Chou (2005) proposed a conditional autoregressive range (CARR) model which is a range-based volatility model. The CARR model is more efficient in analyzing the volatility structure than the GARCH model. Moreover, many studies, such as Brandt and Jones (2006), Martens and Dijk (2007), and Chou et al. (2009), confirmed that the range-based volatility model is more powerful for forecasting than the return-based volatility model. According to the above studies, we think that the range-based volatility estimator may be more accurate than the return-based volatility estimator.

In this paper, we utilize the multiplicative error model (MEM) to examine and compare the relationships between the U.S. and European stock volatilities based on ex-ante and ex-post volatility measures, and whether there were breaks in the relationships when the subprime mortgage crisis occurred. This model has the property of satisfying the non-negative process which is the main difference with the other models. Another advantage is that this model provides unbiased predictions without transforming forecasts. Although early models such as the ARCH/GARCH model can deal with the autocorrelation and volatility clustering properties, these models cannot satisfy the positive process. We use the volatility index and price range data, and find that the relationships between the two measures are similar. The results of the two volatility measures show that the U.S. and European indices are characterized by dual relationships. For the volatility index, the VXD and VXN are the major indices which affect the European indices, and the VFTSE is the major European index which affects the U.S. indices. The results for the price range show that the DJIA and FTSE 100 are the major indices which affect the European and U.S. indices, respectively.

In addition, the structural change phenomenon is significant except for France based on the price range measure. In the pre-crisis period, the spillover effects of the volatility indices from the U.S. to the European indices are those of the VIX and VXN, and none of the European indices affects the U.S. indices. However, in the post-crisis period, the VXD becomes the main index which affects the European indices, and the European VFTSE index is found to have a significant influence on the U.S. indices. However, the results of the price range are the opposite of the results of the volatility index for the two sub-periods. Before the crisis, dual relationships are found to exist for the U.S. and

Europe. Following the subprime mortgage crisis, the relationships between the U.S. and Europe become one-way, i.e., the volatility spillovers were found to exist from the U.S. to Europe. The France indices, i.e., VCAC and CAC 40, became independent of other indices in the post-crisis period.

Furthermore, in order to explore whether the volatility spillover effect can benefit an investor, we evaluate the economic value of the volatility spillover effect by means of an asset allocation strategy. We find that the volatility spillover effect is economically significant in most cases and can yield a maximum fee of 6.02 basis points based on the volatility index measure and that of 20.06 basis points based on the price range measure. An investor with a higher relative risk level will give rise to a lower performance fee. In addition, the volatility forecasts based on the price range measure perform better than those based on the volatility index measure.

The remainder of this paper is organized as follows. In Section 2, we introduce the multiplicative error model and its properties. Section 3 describes the data set and the empirical estimation results. Section 4 presents an economic evaluation methodology to value the volatility spillover effect. Finally, Section 5 summarizes our conclusions.

2. The Multiplicative Error Model

The multiplicative error model is extended from the ARCH/GARCH model (Engle (1982) and Bollerslev (1986)) and is designed to satisfy a non-negative value process. Furthermore, contrary to taking logarithms for dealing with the data, the MEM provides unbiased predictions without transforming forecasts. In previous studies, we know that the financial time series, such as the stock return, trading volume, interest rate and so on, has the leptokurtic and volatility clustering properties. By means of time series plots of stock returns (e.g., Figure 3), we can simply observe that the results of large changes generally follow large changes, and small changes generally follow small changes. The ARCH/GARCH model can almost explain these two properties. In fact, we know that some financial series are positive processes, but the ARCH/GARCH model does not take the positive property into consideration. Therefore, Engel (2002) modified the GARCH model and proposed the MEM, and Engle and Gallo (2006) further extended the MEM model by using a Gamma distribution. The MEM(p,q) based on the Gamma distribution can be expressed as

$$y_t \mid I_{t-1} = \mu_t \varepsilon_t \tag{1}$$

$$\varepsilon_t \mid I_{t-1} \sim \; i.i.d \;\; Gamma\left(\phi, 1/\phi\right) \tag{2}$$

where I_{t-1} is the information set at time $t-1$, μ_t is the conditional expectation of y_t, i.e., $\mu_t = E\left(y_t \mid I_{t-1}\right)$, which depends on a vector of unknown parameters θ, that is

$$\mu_t = \mu_t\left(\theta\right) = w_i + \sum_{k=1}^{q} \alpha_k y_{t-k} + \sum_{s=1}^{p} \beta_s \mu_{t-s} \tag{3}$$

and ε_t is an i.i.d. innovation term with $E\left(\varepsilon_t \mid I_{t-1}\right) = 1$ and $V\left(\varepsilon_t \mid I_{t-1}\right) = 1/\phi$. Based on the expression in Equations (1) and (2), we have

$$y_t \mid I_{t-1} \sim Gamma\left(\phi, \mu_t/\phi\right) \tag{4}$$

with $E\left(y_t \mid I_{t-1}\right) = \mu_t$ and $V\left(y_t \mid I_{t-1}\right) = \mu_t^2/\phi$.

Here we consider two cases of the conditional mean equation based on MEM(1,1). The difference between the two cases concerns whether we take the volatility spillover effects into consideration. The MEM model can be extended for the analysis of more than one index. This can help us to examine the relationships between different assets. In addition, we consider the leverage effects, and the conditional mean equation of the extended MEM(1,1) is written as

$$\mu_{i,t} = \omega_i + \alpha_{i,i} y_{i,t-1} + \beta_i \mu_{i,t-1} + \sum_{j \neq i} \alpha_{i,j} y_{j,t-1} + d_i I\left(r_{i,t-1} < 0\right) y_{i,t-1} \tag{5}$$

where $r_{i,t-1}$ is the return of stock index *i* at time *t-1*, and $I\left(r_{i,t-1} < 0\right)$, a dummy variable to test the leverage effect, is defined as

$$I\left(r_{i,t-1} < 0\right) = \begin{cases} 1, & \text{if } r_{i,t-1} < 0 \\ 0, & \text{if } r_{i,t-1} \geq 0 \end{cases}. \tag{6}$$

The extended MEM(1,1) reduces to the base MEM(1,1) when all parameters $\alpha_{i,j} = 0$ for all $i \neq j$. Hence, we can test the null hypothesis: $\alpha_{i,j} = 0$ for $i \neq j$. If the result rejects null hypothesis, then we know that there exists a spillover effect between assets *i* and *j*.

From Equation (4), the log-likelihood function can be written as

$$\ln L_t\left(\Theta\right) = T\phi \ln \phi - T \ln \Gamma\left(\phi\right) + \left(\phi - 1\right) \sum_{t=1}^{T} \ln y_t - \phi \sum_{t=1}^{T} \left(\ln \mu_t\left(\theta\right) + \frac{y_t}{\mu_t\left(\theta\right)} \right) \tag{7}$$

Thus, we can get the ML estimators $\hat{\theta}$ and $\hat{\phi}$. The details of the discussions and extensions regarding the properties of MEM can be found in Engle (2002), Engel and Gallo (2006), and Cipollini et al. (2006).

3. Empirical Results

In this section, we present our data and the results of the volatility spillover examination based on two types of volatility measures, the volatility index and the price range.

3.1 Data and Preliminary Statistics

The data set contains six stock indices, which are three U.S. and three European indices, six volatility indices corresponding to the stock indices, and two risk-free rates based on the U.S. and European markets. The stock indices are the S&P 500 index (the U.S.), DJIA index (the U.S.), NASDAQ 100 index (the U.S.), DAX 30 index (Germany), FTSE 100 index (U.K.), and CAC 40 index (France). In addition, the corresponding volatility indices[2] are the VIX, VXD, VXN, VDAXNEW, VFTSE and VCAC, respectively. The six volatility indices are computed using a similar algorithm to that for VIX[3]. All of these series are downloaded from the Datastream database. The price range is calculated by the difference between the highest price and the lowest price, $100 \times \left(\ln\left(P_{t,high}\right) - \ln\left(P_{t,low}\right)\right)$. Although the low frequency (weekly or monthly) data have less noise, the markets' efficiency indicates that the information is quickly and efficiently incorporated into the stock markets. Therefore, the low frequency data ignore some information compared with the high frequency data. In addition, the convenience brought about by communications technology, such as the Internet and cell phones, enables the information to be transmitted everywhere more rapidly. As a result, we adopt the daily data to examine our questions. Each series has 2,197 daily frequency observations in the sample period from February 2001 to January 2010, and these time series are shown in Figures 1 to 4. In addition, in order to evaluate the economic value of the volatility spillover effect, we use the USD Libor and Euribor 3-month interbank interest rates as the U.S. and European risk-free rates.

Table 1 reports the descriptive statistics for the stock returns. In terms of the Jarque-Bera[4] statistic, we can test whether the series is normally distributed. We find that none of the index returns follow a normal distribution. In addition, we know that these series have the property of leptokurtosis from the kurtosis coefficient. From the

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[2] See Appendix B for the information regarding the volatility index.

[3] See the CBOE website for the white paper: http://www.cboe.com/micro/vix/vixwhite.pdf

[4] $\text{Jarque-Bera} = \frac{T}{6}(S^2 + \frac{1}{4}(K-3)^2) \sim \chi^2(2)$, where T denotes the size of the sample, S denotes the skewness of the series and K denotes the kurtosis of the series.

Ljung-Box Q[5] statistic, we observe that not all of the series are random. This indicates that these series exhibit autocorrelation in their lagged terms. In particular, from the Ljung-Box Q statistic for the squared return series, we know that each return series has the heteroskedasticity property. That is, the variance for each return is time-varying and exhibits clustering behavior.

【Insert Table 1】

Tables 2 and 3 show the descriptive statistics for the volatility indices and high-low price ranges of the stock indices, respectively. From the kurtosis, Jarque-Bera and Ljung-Box Q statistics, we know that the indices are not normally distributed and they also exhibit autocorrelation. Because of the autocorrelation and non-negativity properties of the series, we adopt the MEM model to analyze the spillover effects.

【Insert Tables 2&3】

【Insert Figures 1&2&3&4】

3.2 Results of the MEM Estimation

In this section, our main purpose is to examine whether volatility spillover effects exist between different stock markets and whether a structural break exists between the pre- and the post-subprime mortgage crisis. We separately use the volatility index and price range as a stock volatility proxy to test the above questions.

Tables 4 and 5 show the base and extended MEM results for the volatility index and price range over the entire period. From the base MEM(1,1), we know that both the volatility index and the price range depend on their past values. This is consistent with the results of the Ljung-Box portmanteau test in Tables 2 and 3. We then use the likelihood ratio (LR) test[6] statistics to examine the volatility transmission across different markets and find that the volatility index is significantly associated with other indices except for VIX and VXN. Furthermore, the volatility spillover effects based on the price range are all significant.

In order to facilitate observation, the spillover effects of the extended MEM(1,1) are marshaled in Panel A of Figures 5 and 6. First, if we only focus on the relationship between the U.S. and Europe, we can find that the results of the two volatility measures are similar, i.e., these two regions are interdependent. Second, if we observe the

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[5] $Q(p) = T(T+2)\sum_{i=1}^{p}\frac{\rho^2(i)}{T-i} \sim \chi^2(p)$, where $\rho(i)$ denotes the *i*th autocorrelation.

[6] $LR = -2(\ln L_{Base} - \ln L_{Extended}) \sim \chi^2(n=5)$, where *n* denotes the difference in the number of parameter estimates between the base and extended model.

relationships among these stock indices, we can find that the lead-lag relationships are a little different for the two volatility measures. The results of the volatility index show that the VXD and VXN, and VFTSE are the major indices which affect the European and U.S. indices, respectively. The volatility spillover effects exist from VIX and VXN to VXD, from VXD to VDAXNEW and VCAC, from VDAXNEW to VCAC, and from VFTSE to VIX, VDAXNEW and VCAC. The VXN and VFTSE, and VXD and VFTSE are interdependent relationships. Now, we shall look at the other volatility measure, namely, the high-low price range. The DJIA and FTSE 100 are also the major indices which affect the European and U.S. indices, respectively, but some relationships between indices are different from the relationships based on the volatility index. The volatility spillover effect exists from the S&P 500 to the NASDAQ 100, from the DJIA to the DAX 30 and CAC 40, from the DAX 30 to the CAC 40, and from the FTSE 100 to the DJIA, S&P 500 and CAC 40. The interdependent relationships between the indices disappear. The relationships between the three European indices are similar to the relationships among the volatility indices. The only difference is that the volatility spillover from the U.K. to Germany disappears. The dissimilar results between the volatility measures may result from the various properties of the two proxies, i.e., the volatility index is the expected volatility of the stock index in the future, but the price range uses the historical data. In addition, the results for the leverage effects are dissimilar to those for the two volatility measures. The leverage effects based on the price range are all significant, but the effects based on the volatility indices are not significant except for the VCAC. This may be due to the volatility indices computed by the option prices being thought of as the average of future one-month volatility, and hence the short-run impulse does not have an apparently asymmetric impact on this average future volatility.

【Insert Tables 4&5】

Tables 6 and 7 also report the base and extended MEM(1,1) results for the two volatility measures. However, the major purpose of the two tables is to examine whether there exists a structural break between the pre- and the post-subprime mortgage crisis for these indices. We choose July 2007 as the break-point. The pre-subprime period extends from February 2001 to June 2007 and the post-subprime from July 2007 to January 2010. We use the LR test statistics based on the method of Dias and Embrechts (2004)[7] to

[7] The Dias and Embrechts (2004) LR test statistics can be derived as a generalized LR test. After we choose the break-point *t**, the LR test can be expressed as $LR = 2[\ln L_1 + \ln L_2 - \ln L_{total}] \sim \chi^2(n)$, where L_1, L_2

examine our question. From the LR test statistics, we know that there exist structural breaks between the pre- and post-subprime crisis periods based on the volatility index. For the other volatility measure, the price range, the structural breaks exist over time except for the CAC40 index.

【Insert Tables 6&7】

【Insert Figures 5-6】

The results of the extended MEM(1,1) are marshaled in Panels B and C of Figures 5 and 6. First, if we focus on relationships between the two regions (the U.S. and Europe), we can find that the relationships are different based on various volatility measures and sub-periods. For the volatility index, the results show that the U.S. is the leader compared with Europe in the pre-subprime period. The dual relationships exist in the post-subprime period. The results of the price range are opposite to the results of the volatility index. Second, we can find that the relationships between various indices are almost all different between the entire, pre- and post-subprime periods without regard to the volatility index or the price range. The differences between the pre- and the post-crisis period based on the volatility index are that, in the pre-crisis period, the VIX and VXN are the major indices which affect the European indices, and none of the European indices affect the U.S indices. The spillover effects exist from VXN to VIX, VXD and VCAC, from VIX to VXD, VCAC and VFTSE, and from VDAXNEW and VFTSE to VCAC. However, in the post-crisis period, VXD becomes the major index which affects the European indices, and the European VFTSE index significantly affects the U.S. indices, and VCAC becomes independent. The spillover effects exist from VIX to VXN, from VXD to VXN, VDAXNEW and VFTSE, and from VFTSE to VIX and VXN. The VFTSE and VDAXNEW are interdependent.

Based on the price range measure, the differences between the pre- and post-crisis periods are that, in the pre-crisis period, the S&P 500 and DJIA are the major indices which affect the European indices, and the DAX 30 is the major index which affects the U.S. indices. The DJIA and DAX 30, NASDAQ 100 and DJIA are interdependent, respectively. The spillover effects exist from the S&P 500 to the NASDAQ 100 and FTSE 100, from the DJIA to the CAC 40 and FTSE 100, and from the DAX 30 and FTSE 100 to the CAC 40. In the post-crisis period, the spillover effects exist from the

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and L_{total} denote the likelihood for the first t^* observations, the remnant observations from t^*+1 to T and for all observations, respectively. Furthermore, the LR test statistics follow the χ^2 distribution with the number of degrees of freedom equal to the number of parameter estimates in the model. Here n=9.

DJIA to the NASDAQ 100 and DAX 30. The DAX30 and FTSE 100 are interdependent. In addition, the S&P 500 and CAC 40 become independent of other indices.

4. An Economic Evaluation of Volatility Spillover

We have used statistical methods to examine the volatility relationship among different stock markets, but this does not determine whether investors can gain a substantial benefit by understanding the volatility spillover effect. In this section, we implement an asset allocation exercise and use a quadratic utility function to evaluate the economic value of the volatility spillover effect based on the two volatility measures, respectively.

4.1 Optimal Portfolio Weight

We consider that an investor with constant relative risk aversion can dynamically allocate his wealth between a risky asset and a risk-free asset. The investor can choose the optimal portfolio weight by maximizing an expected quadratic utility function at each time t, with the optimization problem being given by

$$\begin{aligned}\max_{w_{t+1}} \quad E_t\left(U_{t+1}\right) &= E_t\left(W_t\left(r_{p,t+1}-\frac{\gamma}{2(1+\gamma)}r_{p,t+1}^2\right)\right) \\ &= W_t\left(\mu_{p,t+1}-\frac{\gamma}{2\left(1+\gamma\right)}\left(\mu_{p,t+1}^2+\sigma_{p,t+1}^2\right)\right)\end{aligned} \tag{8}$$

where W_t denotes the investor's wealth at time t and γ denotes the coefficient of relative risk aversion, $r_{p,t+1}=w_{t+1}r_{t+1}+\left(1-w_{t+1}\right)r_{f,t+1}$ is the portfolio return at time $t+1$, and $\mu_{p,t+1}$ and $\sigma_{p,t+1}^2$ are the conditional mean and variance of $r_{p,t+1}$, respectively.

By solving the optimization problem, the optimal risky asset weight is given as follows:

$$w_{t+1}^{*}=\frac{\left(\mu_{t+1}-r_{f,t+1}\right)\left(1-\frac{\gamma}{1+\gamma}r_{f,t+1}\right)}{\frac{\gamma}{1+\gamma}\left(\sigma_{t+1}^2+\left(\mu_{t+1}-r_{f,t+1}\right)^2\right)} \tag{9}$$

In order to match the realized market, we consider two constraints. By assuming that short selling and borrowing at the risk-free asset rate are disallowed, the optimal weight should lie in the interval $[0, 1]$ as

$$w_{t+1}^{**}=\begin{cases} 0, & \text{if } w_{t+1}^{*}\le 0 \\ 1, & \text{if } w_{t+1}^{*}\ge 1 \\ w_{t+1}^{*}, & \text{if otherwise}\end{cases} \tag{10}$$

4.2 Modeling Returns and Volatilities

In this section, we describe our forecast model whose estimates can be used to evaluate the economic value of the volatility spillover effect. Here, we use the AR(1) model to capture the autocorrelation behavior of stock returns and follow Brownlees and Gallo (2010) by assuming that the return volatility is a linear function of the volatility measures. Assuming that y_t denotes the volatility measures, volatility index or price range, then the return and volatility processes can be expressed as

$$r_t = \varpi_0 + \varpi_1 r_{t-1} + \varepsilon_t,\ \ \varepsilon_t = \sigma_t z_t,\ \ z_t \sim skewed - t\left(z_t \mid \lambda, \varphi\right) \tag{11}$$

$$\sigma_t = \delta_0 + \delta_1 y_{t-1|t} = \delta_0 + \delta_1 \hat{\mu}_t \tag{12}$$

where $y_{t-1|t} = \hat{\mu}_t$ is the one-step-ahead forecast at time t and y_t follows the base or extended MEM(1,1) based on the volatility measures, which are the volatility index or price range in this study. The innovation term z_t follows Hansen's skewed-t distribution, whose density function is defined as:

$$skewed - t(z|\lambda, \varphi) = \begin{cases} bc\left(1 + \dfrac{1}{\lambda - 2}\left(\dfrac{bz + a}{1 - \varphi}\right)^2\right)^{-(\lambda+1)/2} & , z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{\lambda - 2}\left(\dfrac{bz + a}{1 + \varphi}\right)^2\right)^{-(\lambda+1)/2} & , z \geq -\dfrac{a}{b} \end{cases} \tag{13}$$

where $2 < \lambda < \infty$ and $-1 < \varphi < 1$ are, respectively, the kurtosis and asymmetric coefficients, respectively, and the values of a, b, and c are defined as $a \equiv 4\varphi c \dfrac{\lambda - 2}{\lambda - 1}$, $b \equiv 1 + 3\varphi^2 - a^2$, and $c \equiv \Gamma\left(\dfrac{\lambda+1}{2}\right) \Big/ \left(\sqrt{\pi(\lambda - 2)} \cdot \Gamma\left(\dfrac{\lambda}{2}\right)\right)$.

4.3 Performance Evaluation

When the above assumption holds, we follow West et al. (1993) to use the average realized utility, $\overline{U}(\cdot)$, to consistently estimate the expected utility with a given initial wealth, W_0,

$$\overline{U}(\cdot) = W_0 \sum_{t=0}^{T-1} \left(r_{p,t+1} - \frac{\gamma}{2(1+\gamma)} r_{p,t+1}^2 \right) \tag{14}$$

Then, by following Fleming et al. (2001), we can compare the performance of the two volatility forecasts by estimating a maximum performance fee, Δ, which an investor would be willing to pay to switch from one strategy to another strategy. Assuming that the initial wealth is equal to one, we can estimate the performance fee by equating the average utilities of two alternative strategies, which may be expressed as

$$\sum_{t=0}^{T-1}\left[\left(r_{p,t+1}^{A}-\Delta\right)-\frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^{A}-\Delta\right)^{2}\right]=\sum_{t=0}^{T-1}\left[r_{p,t+1}^{B}-\frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^{B}\right)^{2}\right] \tag{15}$$

where $r_{p,t+1}^{A}$ and $r_{p,t+1}^{B}$ are the portfolio returns of strategy *A* and strategy *B*, respectively. If the fee, Δ, is positive, this means that the strategy *A* is better than strategy *B*. Finally, we calculate the annualized performance fees for a different relative risk aversion level, γ.

4.4 Economic Evaluation Results

In this section, we compare the economic values for switching between different volatility models, namely, the base MEM(1,1) and extended MEM(1,1), based on the two volatility measures. A rolling window method with a fixed window size, equal to 1,443 observations, is used to forecast the one-step-ahead mean and variance. Hence the sample period can be separated into in- and out-of-sample forecasts. The in-sample sub-period extends from February 2001 to December 2006 (1,443 observations), and the out-of-sample sub-period from January 2007 to January 2010 (754 observations).

Figure 7 shows the out-of-sample one-step-ahead forecasts of volatility for four different models, namely, the base MEM(1,1) based on the volatility index and the price range, and the extended MEM(1,1) based on the volatility index and the price range. We can find that the trends of the volatility estimates are similar, but there also exist some differences. First, the volatility forecasts based on the price range are higher than those based on the volatility index when the stock market is turbulent, especially during the period from September 2008 to February 2009. On the contrary, we can find that the volatility forecasts based on both measures are overlapping, or that the volatility forecasts based on the volatility index are higher than those based on the price range during the calm periods. In addition, the differences in volatility forecasts between these two measures are only slight before October 2008, while they become large after October 2008. Second, when we take the volatility spillover effects into consideration, we find that the results are different for the two volatility proxies. The base MEM and extended MEM are almost overlapping based on the volatility index. However, the volatility forecasts based on the price range differ between the base and extended MEM. Larger volatility forecasts are generated when the volatility spillover effect is considered, especially during the turbulent periods.

【Insert Figure 7】

In addition, in order to assess the economic value of the volatility spillover effect, we implement an asset allocation problem under the mean-variance framework. Table 8 shows the out-of-sample average utility and annualized performance fees, Δ, which an investor would be willing to pay for switching from the volatility model without a spillover effect to that with a spillover effect, i.e., switching from the base MEM(1,1) to the extended MEM(1,1). The performance fees are calculated for three different relative risk aversion levels, equal to 1, 5 and 10, based on two volatility measures, namely, the volatility index and price range. Panels A and B report the results of the forecasts without and with short sale constraints, respectively.

【Insert Table 8】

The results for the performance fees without constraints (Panel A) show that the extended MEM (1,1) is superior to the base MEM(1,1) based on two volatility measures, with the exception of the NASDAQ 100. By taking the volatility spillover effect into account, we can gain a maximum extra benefit of 6.02 basis points (bps) based on the volatility index measure and of 20.06 bps based on the price range measure. That is, no matter which kinds of volatility measure are used, the volatility spillover effect is valuable in the asset allocation strategy. Panel B also shows similar results to Panel A, suggesting that the volatility spillover effect is economically profitable, even if short sales are not allowed. Maximum performance fees of 10.52 bps and 16.35 bps are yielded by the volatility spillover effect based on the volatility index and price range measures, respectively. In addition, we can find that the volatility forecasts based on the price range perform better than those based on the volatility index. Even if we consider the constraints in order to match the real market, the results are robust. Moreover, an investor with a more risk-averse level will lead to a lower performance fee to switch his dynamic strategy from the volatility model without a spillover effect to that with a spillover effect.

5. Conclusions

This article utilizes the multiplicative error model to analyze the volatility transmission mechanisms between the U.S. and European stock markets over the period from February 2001 to January 2010 based on two volatility measures, namely, the volatility index and the price range. We then examine whether the subprime mortgage crisis causes the relationships between the different indices to break. In addition, an asset allocation strategy is implemented to assess the economic value of the volatility spillover effect. The main findings can be summarized as follows:

First, if we simply think of these indices as two regions, namely, the U.S. and Europe, we find that the results of the two volatility proxies are similar. The U.S and Europe are interdependent. Second, both volatility measures indicate that similar relationships exist between European indices. The volatility spillovers exist from Germany and the U.K. to France. The only difference is that the volatility spillover exists from the U.K. to Germany based on the volatility index measure. The reasons for the different relationships based on two volatility measures may be due to the properties of two volatility measures. The volatility index can be viewed as an ex-ante volatility measure, but the price range can be viewed as an ex-post volatility measure. We further find that a structural break really exists between the pre- and the post-subprime crisis periods except for France based on the price range measure. One deserves to be mentioned. After the crisis, France becomes independent of other countries based on both the volatility index and price range measures.

Finally, a maximum benefit of 6.02 bps and 20.06 bps is yielded by the volatility spillover effect based on the volatility index and price range measures, respectively, suggesting that the volatility spillover effect is economically significant. Furthermore, the volatility forecasts based on the price range measure exhibit superior performance to those based on the volatility index measure.

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Table 1 Descriptive Statistics for the Stock Index Return

	S&P500	DJIA	NAS100	DAX30	FTSE100	CAC40
Mean	-0.013	-0.006	-0.016	-0.005	-0.009	-0.017
Median	0.056	0.037	0.081	0.062	0.026	0.003
Maximum	10.957	10.508	11.849	10.797	9.384	10.595
Minimum	-9.470	-8.201	-11.115	-7.336	-9.266	-9.472
Std. Dev.	1.409	1.327	1.967	1.694	1.359	1.587
Skewness	-0.123	0.055	0.063	0.172	-0.066	0.155
Kurtosis	11.312	11.122	6.791	7.212	9.499	8.212
Jarque-Bera	6329.915	6040.237	1316.994	1634.739	3868.141	2495.890
Q(12)	59.437	58.594	43.437	32.790	81.968	65.252
$Q^2(12)$	2533.121	2212.606	1722.827	1578.223	1911.558	1685.060

Note: The table reports the descriptive statistics for the stock index returns, including the S&P500, DJIA, NAS100, DAX30, FTSE100 and CAC40 indices for the sample period from February 2001 to January 2010. The sample size is 2197. Q(12) and $Q^2(12)$ report the Ljung-Box portmanteau test statistics with 12 lags for the return and square return, respectively. The critical values of the Jarque-Bera and the Ljung-Box statistics at the 5% level are 5.991 and 21.026, respectively.

Table 2 Descriptive Statistics for the Volatility Index

	VIX	VXD	VXN	VDAXNEW	VFTSE	VCAC
Mean	21.950	20.590	30.359	26.352	21.807	24.443
Median	19.890	18.940	25.680	23.200	19.503	22.100
Maximum	80.860	74.600	80.640	83.230	75.540	78.050
Minimum	9.890	9.280	12.610	11.650	9.099	9.240
Std. Dev.	10.429	9.652	14.075	11.793	10.335	10.768
Skewness	1.855	1.719	0.955	1.388	1.550	1.449
Kurtosis	7.857	7.126	2.952	4.792	6.035	5.269
Jarque-Bera	3419.171	2639.524	334.057	998.938	1723.297	1239.677
Q(12)	23603.406	23781.503	24384.138	23484.341	22841.017	23088.376

Note: The table reports the descriptive statistics for the volatility indices, including VIX, VXD, VXN, VDAXNEW, VFTSE and VCAC for the sample period from February 2001 to January 2010. The sample size is 2197. Q(12) reports the Ljung-Box portmanteau test statistics with 12 lags for the volatility index series. The critical values of the Jarque-Bera and the Ljung-Box statistics at the 5% level are 5.991 and 21.026, respectively.

Table 3 Descriptive Statistics for the Price Range

	S&P500	DJIA	NAS100	DAX30	FTSE100	CAC40
Mean	24.339	24.139	34.423	31.994	25.449	27.362
Median	19.159	19.334	28.088	25.623	20.334	22.032
Maximum	173.098	192.953	177.160	176.863	170.705	147.009
Minimum	3.928	3.130	6.959	4.065	3.687	4.720
Std. Dev.	18.661	18.208	22.636	22.720	18.411	18.756
Skewness	3.084	3.218	1.883	1.975	2.537	2.034
Kurtosis	17.858	19.669	7.956	8.370	13.069	8.876
Jarque-Bera	23690.769	29225.099	3546.558	4068.616	11639.258	4674.825
Q(12)	10783.828	10410.741	10992.559	11611.930	10159.710	10174.375

Note: The table reports the descriptive statistics for the annualized high-low price range of stock indices, including the S&P500, DJIA, NAS100, DAX30, FTSE100 and CAC40 for the sample period from February 2001 to January 2010. The sample size is 2197. Q(12) reports the Ljung-Box portmanteau test statistics with 12 lags for the price range series. The critical values of the Jarque-Bera and the Ljung-Box statistics at the 5% level are 5.991 and 21.026, respectively.

Table 4 MEM Estimation Results for the Volatility Index – Entire Period

Parameters	VIX		VXD		VXN		VDAXNEW		VFTSE		VCAC	
	Base	Extended	Base	Extended	Base	Extended	Base	Extended	Base	Extended	Base	Extended
w	0.009**	0.010**	0.009***	0.008**	0.011***	0.013***	0.011**	0.006	0.008**	0.001	0.010***	0.015***
	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.003)	(0.004)	(0.004)	(0.004)
μ_{t-1}	0.096***	0.087***	0.125***	0.100***	0.091***	0.100***	0.001	0.038	0.102***	0.053**	0.161***	0.044
	(0.027)	(0.029)	(0.026)	(0.030)	(0.025)	(0.027)	(0.031)	(0.026)	(0.032)	(0.026)	(0.024)	(0.033)
VIX_{t-1}	0.900***	0.895***		0.101***		-0.047		-0.004		0.033		-0.045
	(0.028)	(0.031)		(0.028)		(0.107)		(0.032)		(0.031)		(0.028)
VXD_{t-1}		-0.012	0.867***	0.737***		0.031		0.093**		0.076**		0.138***
		(0.030)	(0.027)	(0.038)		(0.139)		(0.037)		(0.039)		(0.033)
VXN_{t-1}		0.006		0.009***	0.902***	0.895***		-0.003		-0.007*		-0.006
		(0.004)		(0.003)	(0.026)	(0.027)		(0.004)		(0.003)		(0.004)
$VDAXNEW_{t-1}$		-0.010		0.004		-0.003	0.994***	0.927***		-0.012		0.116***
		(0.013)		(0.012)		(0.021)	(0.032)	(0.028)		(0.014)		(0.015)
$VFTSE_{t-1}$		0.032**		0.035***		0.051***		-0.046**	0.892***	0.851***		0.072***
		(0.014)		(0.014)		(0.017)		(0.018)	(0.033)	(0.025)		(0.016)
$VCAC_{t-1}$		-0.002		-0.006		-0.032		0.006		0.014	0.829***	0.689***
		(0.020)		(0.020)		(0.025)		(0.025)		(0.021)	(0.026)	(0.032)
d	-0.005	-0.005	0.001	-0.000	0.003	0.004	-0.004	-0.003	0.001	-0.002	0.008**	0.000
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)
ϕ	268.620***	269.120***	278.671***	280.514***	422.085***	421.575***	363.880***	372.824***	271.181***	280.864***	238.170***	307.880***
	(8.101)	(8.117)	(8.405)	(8.460)	(12.732)	(12.717)	(10.976)	(11.199)	(8.179)	(8.330)	(8.541)	(9.173)
Ln-likelihood	2512.831	2516.991	2692.320	2706.386	2312.904	2317.411	2431.191	2457.885	2544.438	2582.963	2314.815	2400.092
LR-Test	8.32		28.132***		9.014		53.388***		77.05***		170.554***	

Note: The table shows the maximum likelihood estimates of the MEM model, which are based on the volatility index for the entire period from February 2001 to January 2010. The numbers in the parentheses are standard deviations. The model is described as follows: $y_{i,t} = \mu_{i,t}\varepsilon_{i,t}$, $\varepsilon_{i,t} \sim Gamma(\phi_i, 1/\phi_i)$, and the mean equation of extended MEM(1,1) is: $\mu_{i,t} = w_i + \alpha\mu_{i,t-1} + \beta_1 VIX_{t-1} + \beta_2 VXD_{t-1} + \beta_3 VXN_{t-1} + \beta_4 VDAXNEW_{t-1} + \beta_5 VFTSE_{t-1} + \beta_6 VCAC_{t-1} + d_i I\left(r_{i,t-1} < 0\right) y_{i,t-1}$. If it is the base MEM(1,1), then only the lag term of the same asset exists in the mean equation. The last row reports the results of the likelihood ratio test statistics from imposing zero constraints on the interaction coefficients. *, ** and *** denote significance at the 10%, 5% and 1% levels for 2-tailed tests, respectively.

Table 5 MEM Estimation Results for the Price Range of the Stock Index – Entire Period

Parameters	S&P500		DJIA		NAS100		DAX30		FTSE100		CAC40	
	Base	Extended	Base	Extended	Base	Extended	Base	Extended	Base	Extended	Base	Extended
w	0.022***	0.021***	0.021***	0.023***	0.019***	0.015***	0.030***	0.022***	0.020***	0.016***	0.022***	0.019***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)	(0.006)
μ_{t-1}	0.876***	0.868***	0.874***	0.860***	0.897***	0.895***	0.857***	0.852***	0.847***	0.831***	0.858***	0.830***
	(0.011)	(0.013)	(0.011)	(0.014)	(0.010)	(0.011)	(0.014)	(0.015)	(0.014)	(0.016)	(0.013)	(0.017)
$S\&P500_{t-1}$	0.056***	0.010		-0.001		-0.061**		-0.034		0.003		-0.024
	(0.010)	(0.021)		(0.020)		(0.025)		(0.027)		(0.023)		(0.028)
$DJIA_{t-1}$		0.025	0.061***	0.039*		0.043		0.065**		0.033		0.060**
		(0.022)	(0.010)	(0.023)		(0.027)		(0.029)		(0.024)		(0.030)
$NAS100_{t-1}$		-0.003		0.001	0.057***	0.054***		-0.000		-0.003		0.002
		(0.002)		(0.003)	(0.007)	(0.011)		(0.003)		(0.003)		(0.004)
$DAX30_{t-1}$		-0.009		0.002		-0.012	0.076***	0.070***		-0.014		0.033***
		(0.008)		(0.008)		(0.009)	(0.013)	(0.017)		(0.009)		(0.012)
$FTSE100_{t-1}$		0.024***		0.029***		0.014		0.006	0.092***	0.075***		0.048***
		(0.011)		(0.011)		(0.013)		(0.015)	(0.014)	(0.017)		(0.015)
$CAC40_{t-1}$		0.015		-0.002		0.022		-0.011		0.020	0.078***	0.000
		(0.015)		(0.015)		(0.019)		(0.022)		(0.017)	(0.012)	(0.021)
d	0.105***	0.114***	0.101***	0.107***	0.075***	0.080***	0.105***	0.103***	0.093***	0.092***	0.101***	0.103***
	(0.009)	(0.009)	(0.009)	(0.009)	(0.007)	(0.007)	(0.008)	(0.009)	(0.009)	(0.009)	(0.009)	0.009
ϕ	6.664***	6.752***	7.023***	7.092***	7.607***	7.666***	7.427***	7.478***	7.518***	7.577***	7.486***	7.619***
	(0.196)	(0.198)	(0.198)	(0.234)	(0.151)	(0.225)	(0.219)	(0.221)	(0.222)	(0.223)	(0.221)	(0.225)
Ln-likelihood	-1622.983	-1607.917	-1548.432	-1537.224	-2417.661	-2408.725	-2194.078	-2186.085	-1577.679	-1568.632	-1992.032	-1972.180
LR-Test	30.132***		22.416***		17.872***		15.986***		18.094***		39.704***	

Note: The table shows the maximum likelihood estimates of the MEM model, which are based on the price range of the stock index for the entire period from February 2001 to January 2010. The numbers in the parentheses are standard deviations. The model is described as follows: $y_{i,t} = \mu_{i,t}\varepsilon_{i,t}$, $\varepsilon_{i,t} \sim Gamma(\phi_i, 1/\phi_i)$, and the mean equation of the extended MEM(1,1) is: $\mu_{i,t} = w_i + \alpha\mu_{i,t-1} + \beta_1 S\&P500_{t-1} + \beta_2 DJIA_{t-1} + \beta_3 NAS100_{t-1} + \beta_4 DAX30_{t-1} + \beta_5 FTSE100_{t-1} + \beta_6 CAC40_{t-1} + d_i I\left(r_{i,t-1} < 0\right) y_{i,t-1}$. If it is the base MEM(1,1), then only the lag term of the same asset exists in the mean equation. The last row reports the results of the likelihood ratio test statistics from imposing zero constraints on the interaction coefficients. *, ** and *** denote significance at the 10%, 5% and 1% levels for 2-tailed tests, respectively.

Table 6 MEM Estimation Results for the Volatility Index – Sub-periods

Parameters	Pre-subprime Crisis, 2001/02-2007/06						Post-subprime Crisis, 2007/07-2010/01					
	VIX	VXD	VXN	VDAXNEW	VFTSE	VCAC	VIX	VXD	VXN	VDAXNEW	VFTSE	VCAC
w	0.027***	0.013**	0.015***	0.000	0.000	0.000	0.006	0.003	0.030	0.075***	0.089***	0.100
	(0.006)	(0.005)	(0.006)	(0.007)	(0.006)	(0.006)	(0.022)	(0.019)	(0.019)	(0.024)	(0.020)	(0.024)
μ_{t-1}	0.002	0.017	0.023	0.002	0.068**	0.041	0.050	0.132***	0.074	0.000	0.033	-0.061
	(0.036)	(0.036)	(0.020)	(0.044)	(0.033)	(0.033)	(0.050)	(0.049)	(0.047)	(0.051)	(0.043)	(0.052)
VIX_{t-1}	0.917***	0.131***	-0.016	0.042	0.075**	0.093**	0.726***	-0.167	-0.236***	0.088	0.042	0.030
	(0.050)	(0.043)	(0.058)	(0.057)	(0.038)	(0.039)	(0.115)	(0.107)	(0.078)	(0.115)	(0.089)	(0.178)
VXD_{t-1}	-0.027	0.752***	-0.010	0.045	0.008	0.013	0.171	0.969***	0.275***	0.189*	0.241***	0.304
	(0.049)	(0.046)	(0.062)	(0.066)	(0.039)	(0.041)	(0.126)	(0.137)	(0.070)	(0.101)	(0.093)	(0.206)
VXN_{t-1}	0.027***	0.017***	0.975***	-0.009	-0.006	-0.019***	0.011	0.018	0.863***	-0.025	0.021	0.038
	(0.006)	(0.005)	(0.020)	(0.006)	(0.005)	(0.006)	(0.056)	(0.044)	(0.066)	(0.057)	(0.045)	(0.065)
$VDAXNEW_{t-1}$	0.019	0.019	0.010	0.948***	0.003	0.101***	0.045	0.002	-0.009	0.835***	-0.121***	0.131
	(0.018)	(0.016)	(0.018)	(0.039)	(0.016)	(0.018)	(0.044)	(0.038)	(0.038)	(0.065)	(0.035)	(0.052)
$VFTSE_{t-1}$	-0.002	0.014	0.021	-0.019	0.857***	0.093***	0.079*	0.026	0.063*	-0.184***	0.678***	0.022
	(0.018)	(0.017)	(0.020)	(0.024)	(0.032)	(0.018)	(0.044)	(0.038)	(0.035)	(0.046)	(0.046)	(0.050)
$VCAC_{t-1}$	0.016	0.009	-0.020	0.015	-0.001	0.706***	-0.057	0.033	-0.011	0.073	0.075	0.503***
	(0.024)	(0.023)	(0.026)	(0.031)	(0.022)	(0.031)	(0.057)	(0.050)	(0.045)	(0.059)	(0.050)	(0.087)
d	-0.003	0.002	0.004*	-0.005	0.000	-0.001	-0.019***	-0.012*	-0.006	0.000	-0.006	-0.004
	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.007)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
ϕ	327.853***	347.984***	561.954***	408.594***	319.668***	406.502***	196.200***	199.28***	269.606***	335.121***	234.246***	210.57***
	(11.722)	(12.442)	(20.096)	(14.610)	(11.428)	(14.535)	(11.019)	(11.192)	(15.145)	(18.828)	(13.157)	(11.827)
Ln-likelihood	2151.272	2276.894	1920.687	1925.044	2139.381	2066.022	417.913	483.857	474.877	570.723	488.430	437.894
LR-Test	104.388***	108.73***	156.306***	75.764***	89.696***	207.648***						

Note: The table shows the maximum likelihood estimates of the MEM model, which are based on the volatility indices for two sub-periods. We divide the entire period into two sub-periods, namely, the pre- and the post-subprime financial crisis periods. We choose July 2007 as the break-point. The numbers in the parentheses are standard deviations. The model is described as follows: $y_{i,t} = \mu_{i,t}\varepsilon_{i,t}$, $\varepsilon_{i,t} \sim Gamma(\phi_i, 1/\phi_i)$, and the mean equation of the extended MEM(1,1) is: $\mu_{i,t} = w_i + \alpha\mu_{i,t-1} + \beta_1 VIX_{t-1} + \beta_2 VXD_{t-1} + \beta_3 VXN_{t-1} + \beta_4 VDAXNEW_{t-1} + \beta_5 VFTSE_{t-1} + \beta_6 VCAC_{t-1} + d_i I\left(r_{i,t-1} < 0\right) y_{i,t-1}$. The last row reports the results of the likelihood ratio test statistics to examine the structural breaks between the two sub-periods for each index. *, ** and *** denote significance at the 10%, 5% and 1% levels for 2-tailed tests, respectively.

Table 7 MEM Estimation Results for the Price Range – Sub-periods

Parameters	Pre-subprime Crisis, 2001/02-2007/06						Post-subprime Crisis, 2007/07-2010/01					
	S&P500	DJIA	NAS100	DAX30	FTSE100	CAC40	SP500	DJIA	NAS100	DAX30	FTSE100	CAC40
w	0.031***	0.033***	0.019***	0.015**	0.023***	0.014**	0.056**	0.060***	0.081***	0.120***	0.119***	0.120***
	(0.006)	(0.006)	(0.005)	(0.007)	(0.006)	(0.007)	(0.022)	(0.021)	(0.019)	(0.029)	(0.028)	(0.033)
μ_{t-1}	0.887***	0.877***	0.919***	0.873***	0.829***	0.844***	0.807***	0.822***	0.856***	0.806***	0.764***	0.758***
	(0.017)	(0.018)	(0.011)	(0.016)	(0.021)	(0.022)	(0.025)	(0.025)	(0.024)	(0.032)	(0.039)	(0.037)
$S\&P500_{t\text{-}1}$	0.000	-0.037	-0.119***	-0.058	-0.058*	-0.034	0.039	-0.026	-0.065	0.049	0.101	0.030
	(0.027)	(0.027)	(0.037)	(0.037)	(0.031)	(0.039)	(0.059)	(0.051)	(0.055)	(0.058)	(0.065)	(0.070)
$DJIA_{t-1}$	-0.010	0.027	0.071**	0.074**	0.066**	0.059*	0.085	0.140**	0.164***	0.108*	0.006	0.102
	(0.024)	(0.027)	(0.034)	(0.035)	(0.029)	(0.034)	(0.065)	(0.058)	(0.064)	(0.065)	(0.071)	(0.077)
$NAS100_{t-1}$	0.004	0.008**	0.045***	-0.001	0.005	0.004	-0.019	-0.019	0.000	-0.042	0.015	-0.010
	(0.003)	(0.003)	(0.012)	(0.004)	(0.004)	(0.005)	(0.028)	(0.026)	(0.031)	(0.030)	(0.032)	(0.035)
$DAX\,30_{t-1}$	0.009	0.018*	-0.002	0.047***	0.001	0.033**	0.002	0.013	0.009	0.087**	-0.056*	0.033
	(0.008)	(0.009)	(0.010)	(0.018)	(0.011)	(0.014)	(0.031)	(0.028)	(0.028)	(0.042)	(0.032)	(0.037)
$FTSE100_{t-1}$	0.005	0.012	0.002	0.019	0.062***	0.041***	0.012	0.019	-0.010	-0.076*	0.012	0.002
	(0.010)	(0.011)	(0.013)	(0.016)	(0.020)	(0.017)	(0.043)	(0.039)	(0.040)	(0.043)	(0.050)	(0.048)
$CAC\,40_{t-1}$	0.013	-0.000	0.026	-0.001	0.019	0.000	-0.008	-0.035	-0.033	-0.030	0.057	0.000
	(0.014)	(0.015)	(0.018)	(0.022)	(0.018)	(0.023)	(0.057)	(0.052)	(0.053)	(0.061)	(0.058)	(0.065)
d	0.110***	0.110***	0.071***	0.098***	0.096***	0.101***	0.127***	0.125***	0.127***	0.123***	0.087***	0.101***
	(0.010)	(0.010)	(0.008)	(0.010)	(0.011)	(0.011)	(0.022)	(0.021)	(0.018)	(0.019)	(0.020)	(0.020)
ϕ	7.163***	7.556***	8.057***	7.726***	7.745***	7.884***	6.119***	6.303***	7.179***	7.182***	7.557***	7.152***
	(0.250)	(0.263)	(0.277)	(0.270)	(0.269)	(0.276)	(0.335)	(0.344)	(0.394)	(0.385)	(0.416)	(0.393)
Ln-likelihood	-860.874	-838.476	-1621.220	-1424.463	-850.283	-1201.847	-731.428	-686.105	-768.752	-748.525	-701.353	-763.086
LR-Test	31.23***	25.286***	37.506***	26.194***	33.992***	14.494						

Note: The table shows the maximum likelihood estimates of the MEM model, which are based on the price range of the stock index for two sub-periods. We divide the entire period into two sub-periods, namely, the pre- and the post-subprime financial crisis periods. We choose July 2007 as the break-point. The numbers in the parentheses are standard deviations. The model is described as follows: $y_{i,t} = \mu_{i,t}\varepsilon_{i,t}$, $\varepsilon_{i,t} \sim Gamma(\phi_i, 1/\phi_i)$, and the mean equation of the extended MEM(1,1) is: $\mu_{i,t} = w_i + \alpha\mu_{i,t-1} + \beta_1 S\&P500_{t-1} + \beta_2 DJIA_{t-1} + \beta_3 NAS100_{t-1} + \beta_4 DAX\,30_{t-1} + \beta_5 FTSE_{t-1} + \beta_6 CAC40_{t-1} + d_i I\left(r_{i,t-1} < 0\right) y_{i,t-1}$. The last row reports the results of the likelihood ratio test statistics to examine the structural breaks between the two sub-periods for each index. *, ** and *** denote significance at the 10%, 5% and 1% levels for 2-tailed tests, respectively.

Table 8 Out-of-Sample Economic Value of the Selected Volatility Models

γ		Volatility Index			Price Range		
		Utility		Δ	Utility		Δ
		Base	Extended		Base	Extended	
Panel A: Out-of-sample Comparison of the Volatility Timing Values without Short Sale Constraints							
1	S&P500	0.0242	0.0244	3.32	0.0264	0.0271	20.06
	DJIA	0.0209	0.0209	1.56	0.0219	0.0226	17.68
	NAS100	0.0165	0.0165	-0.31	0.0162	0.0161	-0.12
	DAX30	0.0110	0.0112	5.99	0.0112	0.0117	12.32
	FTSE100	0.0135	0.0137	6.02	0.0148	0.0152	11.24
	CAC40	0.0167	0.0169	5.65	0.0182	0.0184	4.71
5	SP500	0.0190	0.0191	1.98	0.0203	0.0208	11.95
	DJIA	0.0170	0.0171	0.93	0.0177	0.0181	10.51
	NAS100	0.0144	0.0144	-0.19	0.0142	0.0142	-0.05
	DAX30	0.0118	0.0119	3.57	0.0120	0.0122	7.36
	FTSE100	0.0133	0.0134	3.59	0.0141	0.0143	6.73
	CAC40	0.0152	0.0153	3.37	0.0161	0.0162	2.90
10	SP500	0.0184	0.0185	1.81	0.0195	0.0199	10.94
	DJIA	0.0166	0.0166	0.85	0.0171	0.0175	9.62
	NAS100	0.0142	0.0142	-0.17	0.0140	0.0140	-0.04
	DAX30	0.0119	0.0120	3.27	0.0120	0.0123	6.74
	FTSE100	0.0133	0.0134	3.29	0.0140	0.0142	6.16
	CAC40	0.0150	0.0151	3.09	0.0159	0.0160	2.68
Panel B: Out-of-sample Comparison of the Volatility Timing Values with Short Sale Constraints							
1	S&P500	0.0177	0.0178	1.59	0.0194	0.0193	-2.20
	DJIA	0.0154	0.0154	0.14	0.0166	0.0168	5.66
	NAS100	0.0134	0.0134	0.25	0.0152	0.0150	-4.92
	DAX30	0.0100	0.0104	8.69	0.0101	0.0107	16.35
	FTSE100	0.0069	0.0072	9.02	0.0093	0.0099	15.67
	CAC40	0.0100	0.0105	10.52	0.0114	0.0117	7.99
5	SP500	0.0152	0.0152	0.94	0.0162	0.0161	-1.32
	DJIA	0.0138	0.0138	0.08	0.0145	0.0146	3.36
	NAS100	0.0126	0.0126	0.15	0.0137	0.0136	-2.95
	DAX30	0.0112	0.0114	5.18	0.0113	0.0117	9.75
	FTSE100	0.0094	0.0096	5.38	0.0108	0.0112	9.36
	CAC40	0.0113	0.0115	6.27	0.0121	0.0123	4.83
10	SP500	0.0148	0.0149	0.86	0.0158	0.0157	-1.21
	DJIA	0.0135	0.0136	0.08	0.0142	0.0144	3.07
	NAS100	0.0125	0.0125	0.13	0.0135	0.0134	-2.70
	DAX30	0.0114	0.0116	4.74	0.0114	0.0118	8.93
	FTSE100	0.0097	0.0099	4.92	0.0110	0.0113	8.57
	CAC40	0.0114	0.0116	5.74	0.0122	0.0123	4.43

Note: The table reports the out-of-sample average utility of the selected volatility models and the annualized fees (bps), Δ, which are calculated by switching from the base MEM to the extended MEM, both without and with restrictions on the portfolio weight. We use the estimates of different volatility models to calculate the daily portfolio weights and the utility. By means of Equation (15), we solve for the performance fee under different relative risk aversion levels, γ, which is equal to 1, 5 and 10. Furthermore, we divide the sample period into in-sample and out-of-sample sub-periods. The in-sample period extends from February 2001 to December 2006 (1,443 observations), and the out-of-sample period extends from January 2007 to January 2010 (754 observations).

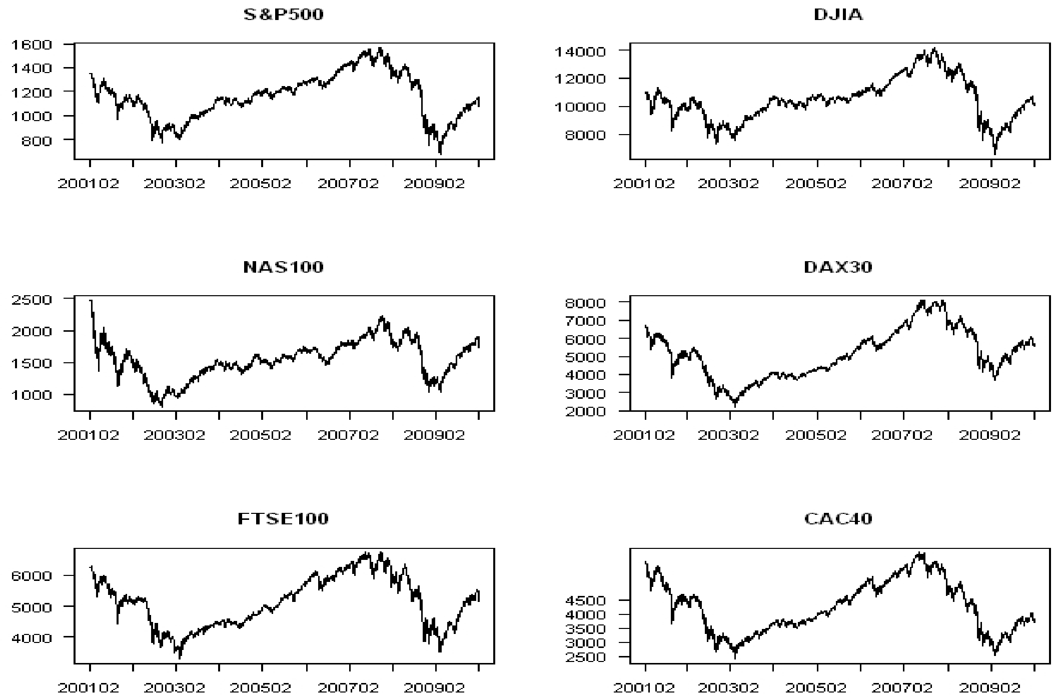


Figure 1 Time Series Plot – Stock Indices. This figure shows the time series for various stock indices, including those for the S&P500, DJIA, NAS100, DAX30, FTSE100 and CAC40, from February 2001 to January 2010.

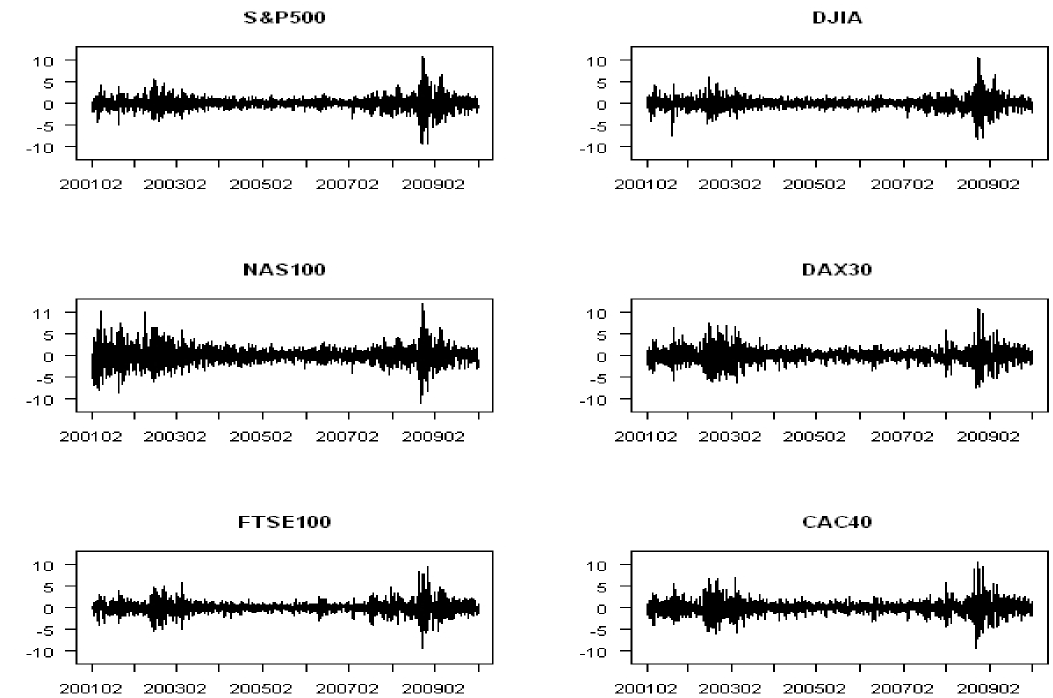


Figure 2 Time Series Plot – Stock Returns. This figure shows the time series for various stock returns, including those for the S&P500, DJIA, NAS100, DAX30, FTSE100 and CAC40, from February 2001 to January 2010.

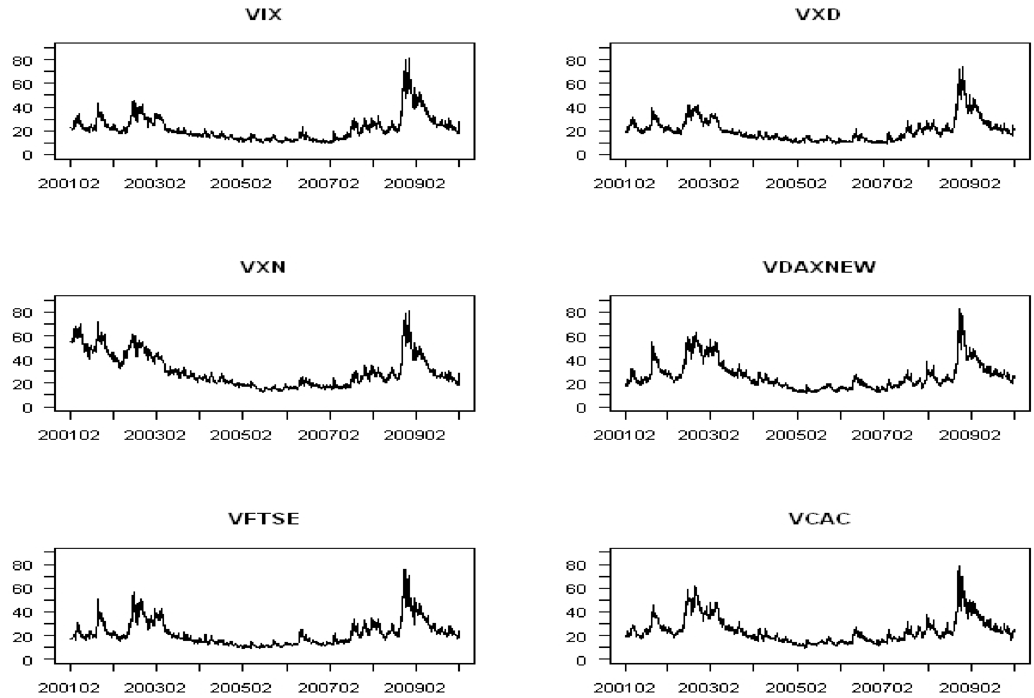


Figure 3 Time Series Plots –Volatility Indices. This figure shows the time series of various volatility indices, including those for the VIX, VXD, VXN, VDAXNEW, VFTSE and VCAC, from February 2001 to January 2010.

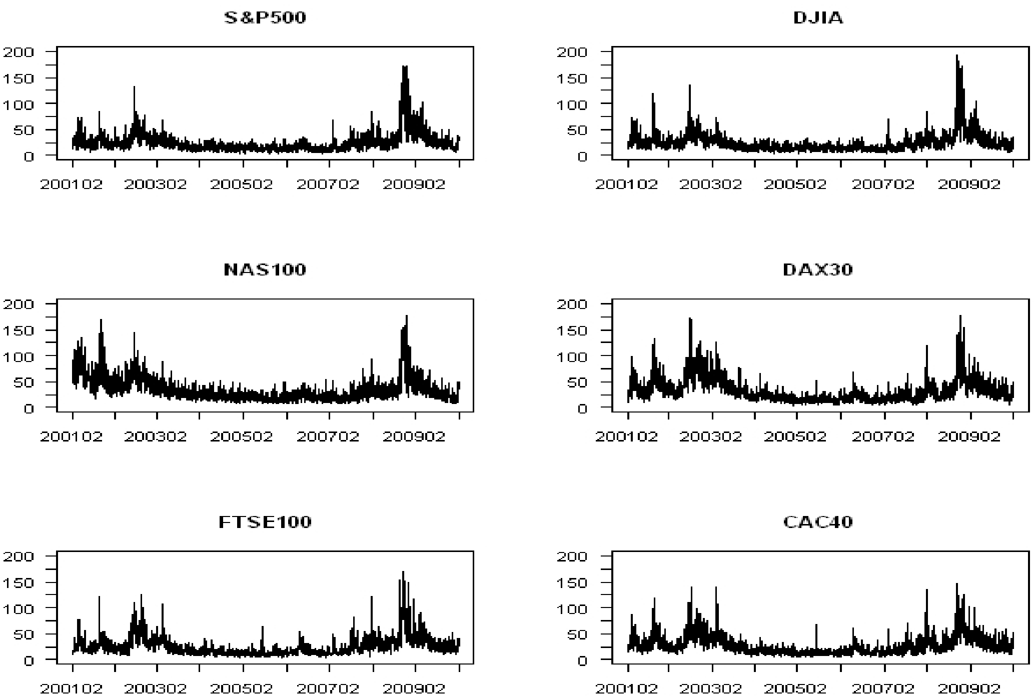


Figure 4 Time Series Plots – Price Ranges of Stock Indices. This figure shows the time series of various annualized high-low price ranges, including those for the S&P500, DJIA, NAS100, DAX30, FTSE100 and CAC40, from February 2001 to January 2010.

Panel A: Entire Period **Panel B: Pre-subprime Crisis, 2001/02-2007/06** **Panel C: Post-subprime Crisis, 2007/07-2010/01**

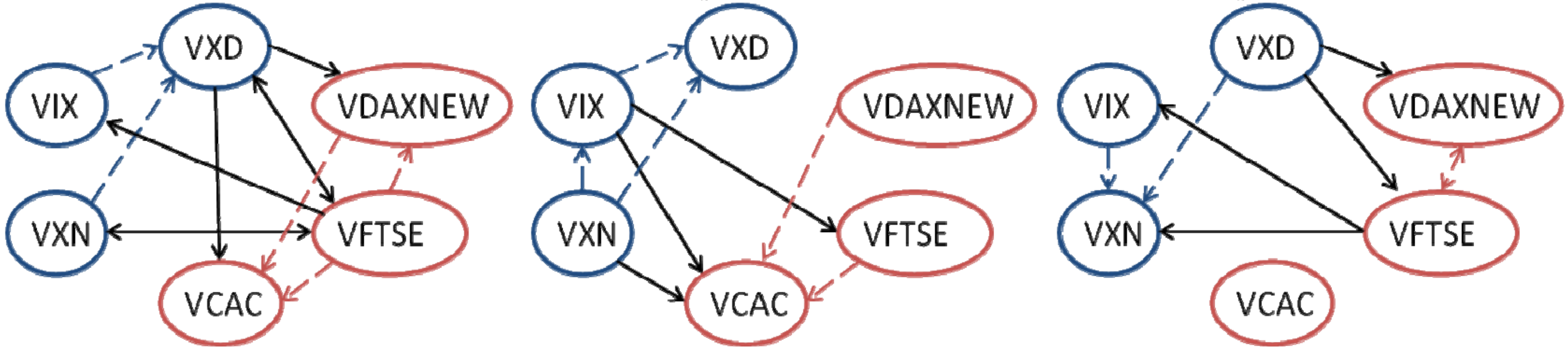


Figure 5 The Relationships between the U.S. and European Volatility Indices. The figure shows the relationships between different volatility indices. The dashed lines represent the relationships within the same region. The solid lines represent the relationships between different regions.

Panel A: Entire Period **Panel B: Pre-subprime Crisis, 2001/02-2007/06** **Panel C: Post-subprime Crisis, 2007/07-2010/01**

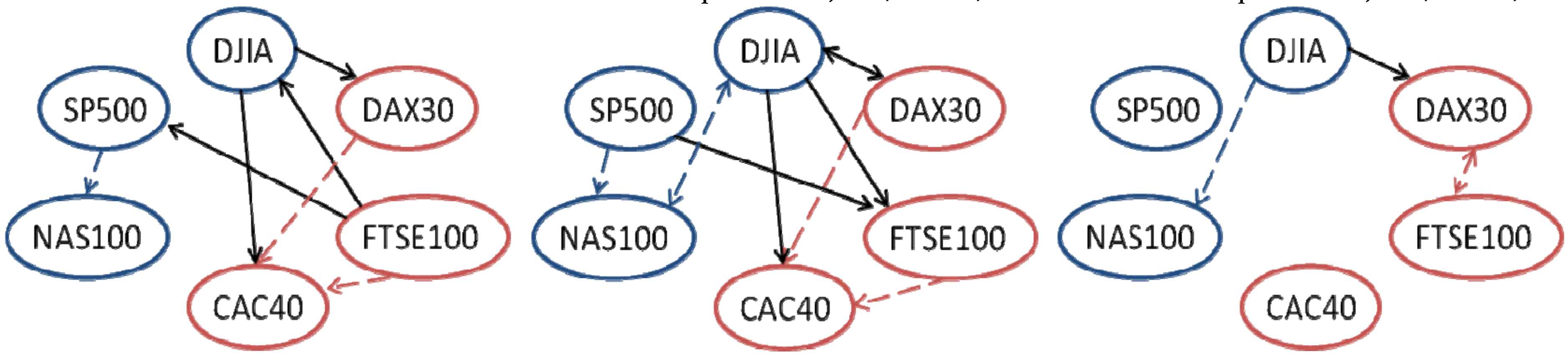
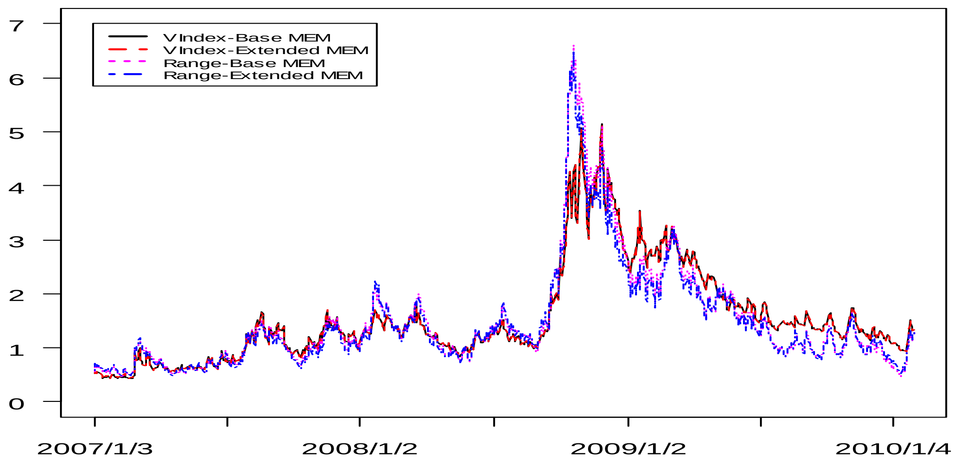
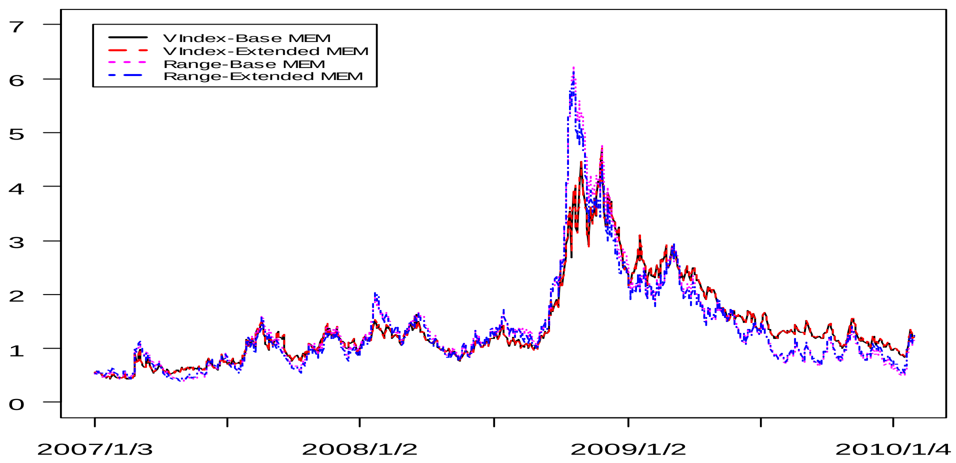


Figure 6 The Relationships between the U.S. and European Stock Price Ranges. The figure shows the relationships between different volatility indices. The dashed lines represent the relationships within the same region. The solid lines represent the relationships between different regions.

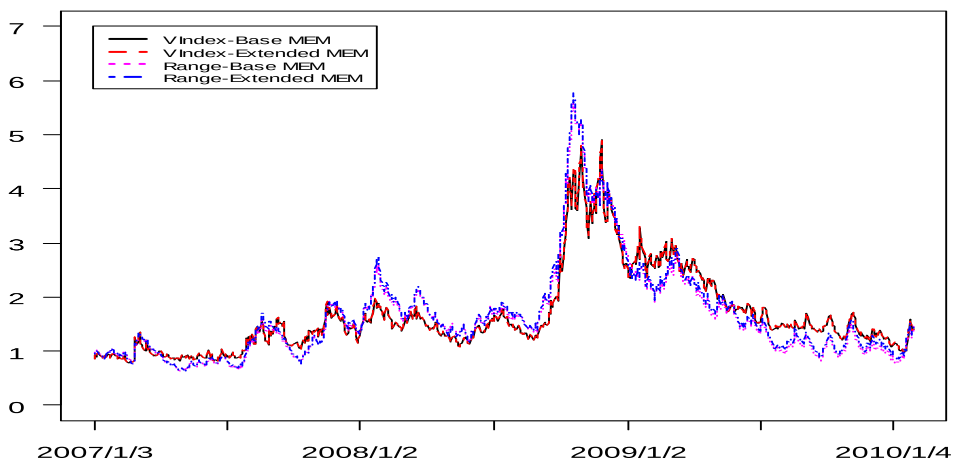
Panel A: S&P500



Panel B: DJIA



Panel C: NAS100



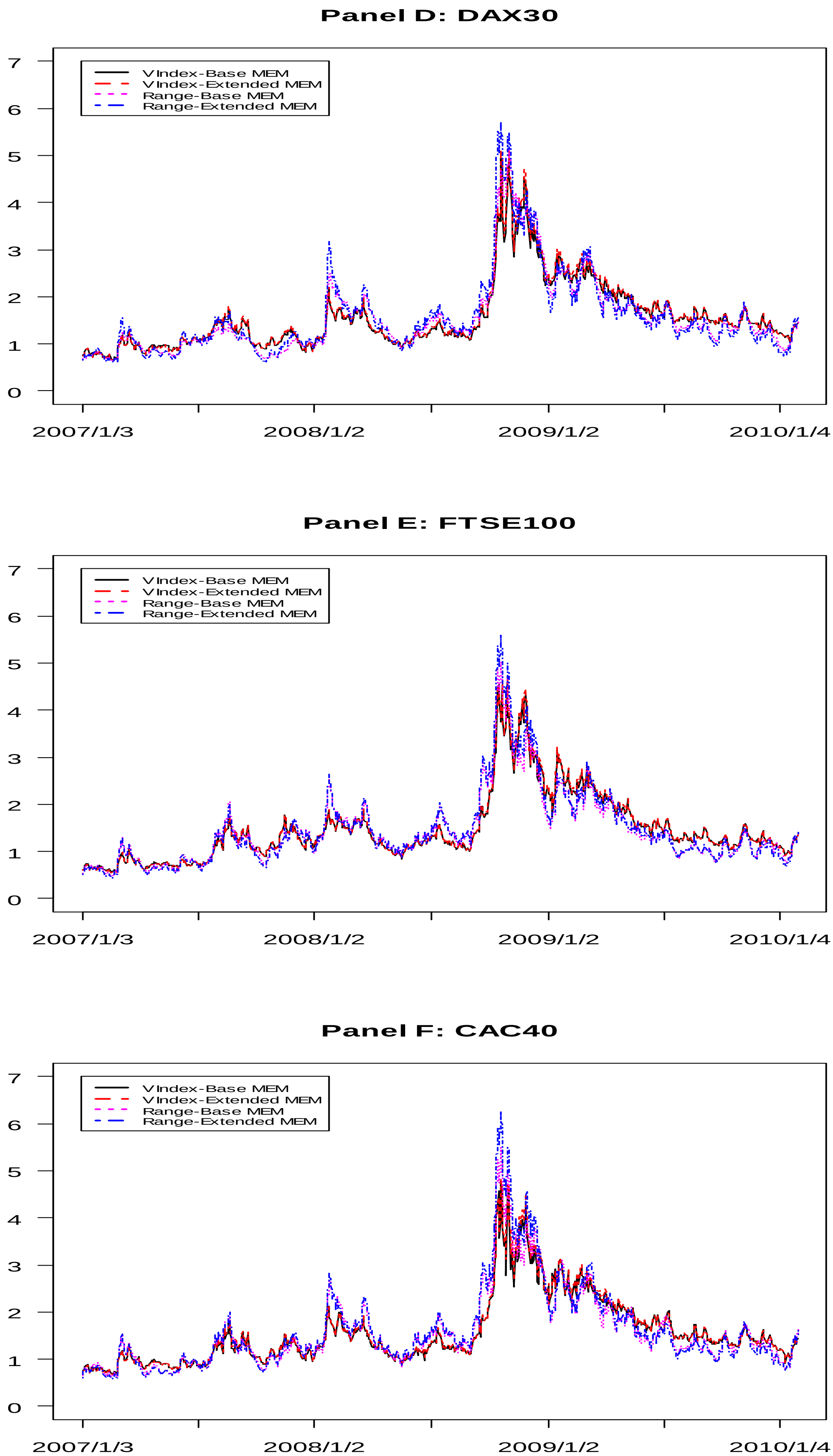


Figure 7 Out-of-Sample Volatility Forecasts Based on Selected Model

Appendix

List of Volatility Indices

Volatility Index (DS Mnemonic)	Launch Date Started Date	Market	Methodology	Underlying Index (DS Mnemonic)
VIX (CBOEVIX)	2003/09/22 1990/01/02	United States	Model-free	S&P 500 (S&PCOMP)
VXD (CBOEVXD)	2005/04/25 1997/10/06	United States	Model-free	DJIA (DJINDUS)
VXN (CBOEVXN)	2003/09/22 2001/02/02	United States	Model-free	NASDAQ 100 (NASA100)
VDAXNEW (VDAXNEW)	2005/04/20 1992/01/02	Germany	Model-free	DAX30 (DAXINDX)
VFTSE (VFTSEIX)	2008/06/23 2000/01/04	United Kingdom	Model-free	FTSE 100 (FTSE100)
VCAC (CACVOLI)	2007/09/03 2000/01/03	France	Model-free	CAC40 (FRCAC40)

Note: The table presents the information regarding the volatility index. The DS Mnemonic refers to the code for Datastream.