Intergenerational Transmission of Human Capital and Wage Inequality

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Abstract

Parents with more human capital spend more time teaching and taking care of their children, in spite of the higher opportunity cost. Why is this? How does that affect intergenerational mobility and wage inequality? What are the implications on policies meant to provide equal opportunity through public schooling? We develop and estimate a theoretical model to answer these questions, in the light that parental time investment is a powerful means to transmit human capital inter-generationally.

Keywords: Human capital production, Parental investment, Time investment Wage inequality, Public schooling;

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1 Introduction

Empirical studies reveal a strong positive correlation between parental education/wage and parental time with children. Hill and Stafford (1974) show that the time spent in preschool child are by high socio-economic status mothers is two to three times as much as that by ow socio-economic status mothers. More recently, Kimmel and Connelly (2007), based on 2003-2004 American Time Use Survey, find that womans predicted wage is positively correlated to the time women allocate to child care. Using 2003-2006 waves of the American Time Use Survey, Guryan et al. (2008) document a positive education/income gradient in child care which holds true for both non-working and working mothers and working fathers. It also holds true for various sub-categories of child care – higher wage/educated parents spend more time in basic child are, education and recreational child care, and travel related to child care. In addition, the positive correlation is also found in countries other than the U.S. (Gauthier and Frank F. Furstenberg (2004), Guryan et al. (2008)), including developed countries such as Canada, France, Germany, Italy, Norway and UK, as well as developing countries such as Chile, Estonia, South Africa and Palestine.

Why should higher wage/educated parents spend more time with their children despite the higher opportunity cost? How much does it contribute to inter-generational earning persistence and wage inequality? What are the implications on the effect of public investment in education? We address these issues by developing and estimating a model of inter-generational transmission of human capital. We view parental time with children as an investment made by parents in their children's human capital on the ground of two data facts. First, the amount of time allocated to home production and to leisure **falls** sharply as education and income rise(Aguiar and Hurst (2007),Kimmel and Connelly (2007), Guryan et al. (2008)). Therefore it is unreasonable to categorize parental time with children as either leisure or home production from the viewpoint of a standard Beckerian time allocation model. Secondly and more importantly, parental time exhibits a positive effect on children's outcome (Cooksey and Fondell (1996),Li et al. (2005)).

The model developed in this paper has a number of distinctive features. It requires parents to make both time investment and goods investment to produce children's human capital. This allows for rich dynamics among goods allocation and time allocation. Another feature is that it allows for various channels through which parents pass their human capital on to children, such that the model is able to quantify the relative importance of time investment. Furthermore, our model can be solved analytically, which not only makes the mechanisms at work transparent, but also facilitates the estimation of the model. We estimate 7 parameters via simulated method of moments, which is extremely time-consuming if the model had to be solved numerically. The estimation results show that the model captures quite well a set of important data observations regarding wage rate, time allocation, goods allocation and inter-generational earning persistence[1].

Within our framework, parents invest in children's human capital altruistically. Those with more human capital makes more goods investment because they are richer. They also make more time investment because it complements goods investment. Therefore the complementarity between time investment and goods investment is the key to understand the positive correlation between parents' human capital and time investment[2].

In our model, three channels potentially lead to the inter-generational persistence in earning: i) intergenerational persistence of endowed learning ability (ii) parental investment in child's human capital, including goods investment and time investment; (iii) higher teaching productivity of parents with more human capital. The estimation results show little support for the third channel to be at work. This is consistent with the findings of [citations needed, perhaps Behrman, J. R. and Rosenzweig, M. R. (AER2002) and E Plug - American Economic Review, 2004 "Estimating the effect of mother's schooling on children's schooling"]. If we following the literature and term channel (i) as nature effect and channel (ii) as nurture effect, nature effect accounts the larger portion of intergenerational persistence and income inequality, consistent with the empirical findings in Behrman and Taubman (1989), Sacerdote (2002), and Plug and Vijverberg (2003)[3]. Among nurture effect, time investment is slightly more

[1] Earnings exhibit considerable intergenerational persistence According to Solon (1992), the intergenerational persistence of earnings in U.S., measured as the slope coefficient obtained by regressing children's log earnings when adults against parent's log earnings, is around 0.45. We use this as a moment in the estimation

[2] Ramey and Ramey (2009) offer an alternative explanation – increased competition for college admissions.

[3] Huggett et al. (2009) study a human capital model in which agents are heterogeneous in the ability to accumulate human capital. They also find that learning ability differences constitute an important part of the rise in earnings dispersion over the lifetime.

important than goods investment.

The model has strong policy implications. Namely, public investment in education, at a moderate level, can advance both equity and efficiency, even in the absence of market friction and informational friction. When the government levies income tax to provide public schooling, parents find it optimal to increase time investment to compliment the increased total goods investment, which leads to increased human capital accumulation. Once again, the complementarity between goods and time investment plays a critical role. In contrast, in a model that ignores time investment, taxation and public schooling leads to reduced work hour and less human capital accumulation, resulting in the classical trade-off between equity and efficiency. On the other hand, when tax rate is too high, work hour is greatly reduced which harms consumption and utility. We compute the optimal income tax that is about 6.7% of income.

Existing studies on human capital transmission and inequality have focused on the fact that richer parents make more financial investment on children's education[4]. For example, Glomm and Ravikumar (2003) and Glomm and Ravikumar (1992) study the effect of intergenerational transmission of human capital on income inequality, focusing on the roles played by public schooling. Restuccia and Urrutia (2004) study intergenerational human capital transmission considering three sources: innate ability, early education, and college education. They find that about half of the intergenerational correlation in earnings is accounted for by parental investment especially in early education. Our paper emphasize the role played by parental time investment.

The rest of the paper is organized as follows. Section 2 lays out the baseline model. The model can be solved analytically. In section 3 we estimate the model parameters. Section 4 provides the quantitative results. Section 5 concludes. And finally, regression results and description of the data are left to the Appendix part.

[4]The most notable example is that high-earning parents send their children to expensive private schools for better quality of education. Acemoglu and Pischke (2001) note that a 10 percent increase in family income is associated with a 1.4 percent increase in the probability of attending a four-year college.

2 The baseline model

Parents receive labor income which is either used to purchase their own consumption good c_t, or to invest in the production of their children's human capital h_{t+1}. Parents maximize the utility from consumption plus the utility from leisure and the utility from bequest in the form of their children's human capital. Specifically, parents of family i solve the following optimization problem:

$$\max \ln c_{i,t} + \gamma \ln n_{i,t} + \beta \ln h_{i,t+1}$$

subject to

$$c_{i,t} + I_{i,t} = (1 - e_{i,t} - n_{i,t}) w_{i,t} \tag{1}$$

$$w_{i,t} = h_{i,t} \tag{2}$$

$$h_{i,t+1} = z_{i,t} A[\alpha(h_{i,t}^{\delta} e_{i,t})^{\sigma} + (1-\alpha) I_{i,t}^{\sigma}]^{\frac{1}{\sigma}} \tag{3}$$

Equation 1 and 2 are parents' budget constraint. We assume parents invest $I_{i,t}$ units of goods and $e_{i,t}$ units of time to produce children's human capital. $n_{i,t}$ is the leisure time. The remaining time, $1 - e_{i,t} - n_{i,t}$, is used to work with wage rate $w_{i,t}$. The wage rate is simply the human capital $h_{i,t}$. The only source of heterogeneity among families in our model is the learning ability $z_{i,t}$ which evolves according to the following equation.

$$\ln z_{i,t} = \rho \ln z_{i,t-1} + \epsilon_{i,t} \tag{4}$$

In equation 4, $\epsilon_{i,t}$ is a random shock drawn from normal distribution with mean zero and variance ν^2. Learning ability of generation t in family i, $z_{i,t}$, is correlated with the learning ability of previous generations unless $\rho = 0$.

Equation 3 is the human capital production function. In this formulation, the amount of human capital to be owned by children is entirely chosen by parents. The production function takes a constant elasticity of substitution form. As will be clear later, the parameter that governs the elasticity between goods investment $I_{i,t}$ and time investment $e_{i,t}$ will play a key role in our analysis. It is reasonable to assume that the time investment of parents with more human capital is more productive. We capture that by multiplying $e_{i,t}$ by $h_{i,t}^{\delta}$ where $h_{i,t}$ is the stock of human capital owned by parents.

Our model has a number of nice features. First of all, it is simple but captures some key data facts. For example, parents with more human capital (higher education attainment) invest more goods and time in the formation of children's human capital. Another example is that wage inequality is not explosive. Secondly it has an analytical solution – all the endogenous variables can be expressed as explicit functions of current state variables $h_{i,t}$ and $\epsilon_{i,t}$. Furthermore, there exists a steady state in which time investment, goods investment, wage rate, leisure and consumption converge to $\tilde{e}_i$, $\tilde{I}_i$, $\tilde{w}_i$, $\tilde{n}_i$, $\tilde{c}_i$. We use these steady state values to estimate our model parameters, then carry out policy analysis around the steady state.

2.1 Solution to the baseline model

For simplicity we omit family subscript i in the rest of the paper. The following equations present the optimal level of time and goods investment and leisure[5].

$$I_t = \frac{h_t}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}+1\right]} \tag{5}$$

$$e_t = \frac{\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}+1\right]} \tag{6}$$

$$n_t = \frac{\gamma}{1+\gamma+\beta} \tag{7}$$

In the data, more educated parents spend more time teaching their children. Therefore it is necessary for $\frac{\partial e_t}{\partial h_t} > 0$ to hold. This requires just on condition in our model $\sigma < 0$. Intuitively, $\sigma < 0$ means goods input and time input are complements in the production of human capital. Therefore when parents with higher wage rates spend more money on children's education, they need to spend more time teaching children due to the complementarity.

$$\frac{\partial I_t}{\partial h_t} = \frac{\left(\frac{\sigma\delta-1}{\sigma-1}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}+1}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}+1\right]^2} \tag{8}$$

[5] Following Becker (1981),a strand of literature emerged discussing whether parents choose to investment more in the human capital of abler children or not. From the equation below it can be seen that out model implicitly assume parental investment is independent of children's ability.

$$\frac{\partial e_t}{\partial h_t} = \frac{\left(\frac{(1-\delta)\sigma}{\sigma-1}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{1-\delta\sigma}{\sigma-1}}}{\left(\frac{1+\gamma}{\beta}+1\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}+1\right]^2} \tag{9}$$

From equation (8) and (9), the importance of σ is evident. If $\sigma < 0$ both $\frac{\partial I_t}{\partial h_t} > 0$ and $\frac{\partial e_t}{\partial h_t} > 0$. Intuitively, if $\sigma < 0$, e_t and I_t are complements, more monetary investment must be matched up with more time investment in order to effectively pass parental human capital on. A number of empirical works (e.g., Kimmel and Connelly (2007) and Guryan et al. (2008) provide evidence that parents with greater human capital indeed invest more time and resource in children's education. Therefore in the rest of this paper we focus on the situation of $\sigma < 0$.

Plug the solution for I_t and e_t into the equation 3, we have the dynamics of human capital accumulation.

$$h_{t+1} = \left(\frac{\beta z_t A(1-\alpha)^{\frac{1}{\sigma}}}{1+\gamma+\beta}\right)\left[\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\sigma}} h_t^{\frac{(1-\delta)\sigma}{\sigma-1}}+1\right]^{\frac{1-\sigma}{\sigma}} h_t \tag{10}$$

We abstract from growth since the focus is on inter-generational mobility, wage inequality and the relevant policy implication[6]. The model has a steady state to which human capital, consumption and investment converge in the long run. The convergence of wage (human capital) is consistent with data observation that all the earnings advantages and disadvantages are of the ancestors are wiped out within a few generations (Becker and Tomes (1986)).

Setting $h_{t+1} = h_t$ and $z_t = 1$, we derive steady state human capital stock.

$$\tilde{h} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\sigma(1-\delta)}}\left[\left(\frac{1}{1-\alpha}\right)^{\frac{1}{1-\sigma}}\left(\frac{1+\gamma+\beta)}{\beta A}\right)^{\frac{\sigma}{1-\sigma}}-1\right]^{\frac{\sigma-1}{\sigma(1-\delta)}}; \tag{11}$$

With $\tilde{h}$ solved, it is straightforward to derive $\tilde{e}$ and $\tilde{I}$.

$$\tilde{e} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\sigma-1}}\left(\frac{1+\gamma+\beta}{\beta\alpha A^{\sigma}}\right)^{\frac{1}{\sigma-1}}\left[\left(\frac{1+\gamma+\beta}{\beta A}\right)^{\frac{\sigma}{1-\sigma}}(1-\alpha)^{\frac{1}{\sigma-1}}-1\right] \tag{12}$$

$$\tilde{I} = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\sigma(1-\delta)}}\left(\frac{1+\gamma+\beta}{\beta(1-\alpha)A^{\sigma}}\right)^{\frac{1}{\sigma-1}}\left[\left(\frac{1+\gamma+\beta}{\beta A}\right)^{\frac{\sigma}{1-\sigma}}(1-\alpha)^{\frac{1}{\sigma-1}}-1\right]^{\frac{\sigma-1}{\sigma(1-\delta)}} \tag{13}$$

[6]The model can be easily extended to encompass growth by viewing A as technology which has a time trend.

Table 1: Model Parameters

β	degree of altruism toward children
α	relative importance of time investment
σ	substitutability between time and goods investment
δ	efficacy of parents' human capital in children's human capital production
γ	relative importance of leisure in preference
ρ	intergenerational persistency of learning ability
ν	standard deviation of shocks to learning ability

3 Estimation

The model outlined above has 8 parameters: A, β, α σ, δ, γ, ρ, ν. A is a scaling parameter that shifts the level of human capital stock and other variables. We normalize the steady state human capital stock to one, which implies

$$A = \frac{1+\gamma+\beta}{\beta} \frac{\left[1+\left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\sigma)}\right]^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)^{\frac{1}{\sigma}}}$$

Therefore,once we estimate the rest of 7 parameters, A is pinned down[7]. Table 1 recapitulates the meaning of the 7 parameters to be estimated.

3.1 Data moments

We estimate these parameters using Simulated Method of Moments as formalized in Ingram and Lee (1991). This is basically a moment-matching procedure. Seven moments are considered that capture the key feature in the data. We take six of them from the data – American Time Use Survey and Consumer Expenditure Survey, both from 2003. The remaining moment, intergenerational persistence of earning, is taken from Solon (1992). Table 2 explains the sources of moments and how they are normalized.

The first three moments are the mean levels of consumption, goods investment and time investment. These moments pin down the allocation of parents' re-

[7] Based on the estimation results, A is around 11.5 which is not reported in the table of results.

Table 2: Data moments

	moments	std. error	sources
average consumption	0.469*	0.003	CEX 2003
average goods investment	0.025*	0.016	CEX 2003
avarage time investment	0.086**	0.002	ATUS 2003
wage elasticity of goods investment	1.067	0.371	CEX 2003
wage elasticity of time investment	0.206	0.077	ATUS 2003
intergeneration earning correlation	0.449	0.095	Solon (1992)
coefficient of variation of wage	0.663	0.021	ATUS 2003

* relative to wage/(1-saving rate)

** a fraction of total time available

courses. In addition they are informative about the relative importance of time investment in human capital production and the degree of altruism toward the next generation.

The fourth and fifth moments are wage elasticities of parental investment on children's education, that is, the rate of change of goods and time investment with respect to parents' wage rate. This captures the salient data feature that parents with more human capital (wage rate) not only invest more money in children's education, but also more time. Using ATUS data, we regress the logarithm of time investment on the logarithm of wage rate which is instrumented by respondents' educational attainment. The regression coefficient is the wage elasticity. In the model counterpart, we simply calculate $\frac{\partial E}{\partial H} \times \frac{H}{E}$. The same steps are taken to obtain wage elasticity of goods investment. For either moments, the change of rate is fairly accurately measured with t-statistics around two. Table 10 and table 11 report the regression results.

The sixth moment is the intergenerational correlation in earnings. According to Solon (1992), correlation is around 0.45. This is obtained by regressing the logarithm of children's earnings on those of parents and set of control variables. We do the same regressions in the model counterpart, in which earning is calculated from $h_{i,t} \times (1 - e_{i,t} - n_{i,t})$. This moment is particularly informative on ρ, the persistence of learning ability over generations.

The last moments is the coefficient of variation in wage rate. This measures the overall wage inequality in the economy and pins down the size of random shocks to learning ability. From ATUS 2003, the number is around 0.66. The literature on economic inequality use the variance of logarithm of wage rate as a typical measure of inequality. Since wage rate in the model is much smaller in scale than in the data, we use coefficient of variation so that the model moment is comparable to data moment.

3.2 Estimation results

We minimize the distance between data moments and model moments computed from simulated data. The distance is weighted by the inverse of variance of each data moments[8]. First, we estimate a full set of seven parameters, with the results presented in the upper-panel of Table 3. The model moments are fairly close to the data counterparts, and the weighted distance is 0.460. However, three parameters, namely σ, δ and ρ, are imprecisely measured. More importantly, δ is estimated to be 0.011. Recall that a small δ means parents with more human capital have similar productivity in teaching children.

Since the estimate of δ is indeed tiny with large standard errors, we postulate that parental human capital does not affect teaching productivity in reality. Therefore we set $\delta = 0$ and estimate the remaining 6 parameters using the same moments. The results are reported in the lower panel of table 3. Compared with the results in the upper panel, parameter estimates as well as model moments are changed only slightly. However, now the parameters are very precisely measured. This supports our hypothesis that teaching productivity has little importance in the inter-generational transmission of human capital.

Overall our model delivers the stylized facts in the data fairly well. Moments from simulated data match those from real data almost perfectly, except that the wage elasticities of time/goods investments are a little lower in the model than in the real data.

As predicted, σ is negative, which means time investment and goods investment are compliments rather than substitutes. Leisure turns out to be important with $\gamma = 1.502$. This implies that previous work that assume $\gamma = 1$ may produce biased quantitative results.

In the model as well as in real data, coefficient of variation of wage rate is 0.663. Recall in the model, wage rate is simply the stock of human capital whose

[8]Ideally we should have used the optimal weighting matrix which is the inverse of the variance-covariance matrix of the data moments.However the off-diagonal components are not available from the data, due to the lack of panel data and the fact that moments are taken from different sources

Table 3: Estimation Results

	average goods invest.	average time invest.	average consump.	$\frac{\partial I}{\partial H} \times \frac{H}{I}$	$\frac{\partial E}{\partial H} \times \frac{H}{E}$	inter-gen earning corr.	wage ineq.
data moments	0.025	0.086	0.469	1.067	0.207	0.449	0.663
model moments	0.026	0.086	0.469	0.744	0.188	0.449	0.633
parameter	β	σ	α	δ	γ	ρ	ν
estimates	0.246	-0.909	0.893	1e-005	0.883	0.315	0.522
standard errors	0.005	5.498	0.582	3.150	0.048	2.351	0.016
distance between model and data moments:				0.821			

Estimation results when δ is set to zero

	average goods invest.	average time invest.	average consump.	$\frac{\partial I}{\partial H} \times \frac{H}{I}$	$\frac{\partial E}{\partial H} \times \frac{H}{E}$	inter-gen earning corr.	wage ineq.
data moments	0.025	0.086	0.469	1.067	0.207	0.449	0.663
model moments	0.026	0.086	0.469	0.744	0.188	0.449	0.633
parameter	β	σ	α	γ	ρ	ν	
estimates	0.246	-0.909	0.893	0.883	0.315	0.522	
standard errors	0.005	0.006	0.041	0.049	0.053	0.017	
distance between model and data moments:				0.821			

steady state value is normalized to 1. Therefore in the model, cross-sectional standard deviation of wage rate is 0.663. On the other hand, the standard deviation of learning ability shocks (ν) is estimated to be 0.522. Clearly, the only source of heterogeneity in our model, the heterogeneity in learning ability, is magnified, which leads to greater heterogeneity in wage rate. In addition, the persistence of learning ability shocks is $\rho = 0.378$, while the inter-generational persistence of earning is 0.449. It is the mechanisms we proposed in the beginning of the paper that generate the greater heterogeneity and more persistence in earning than in learning ability shocks. One of the main objectives of our paper is to quantitatively assess the effects of these mechanisms.

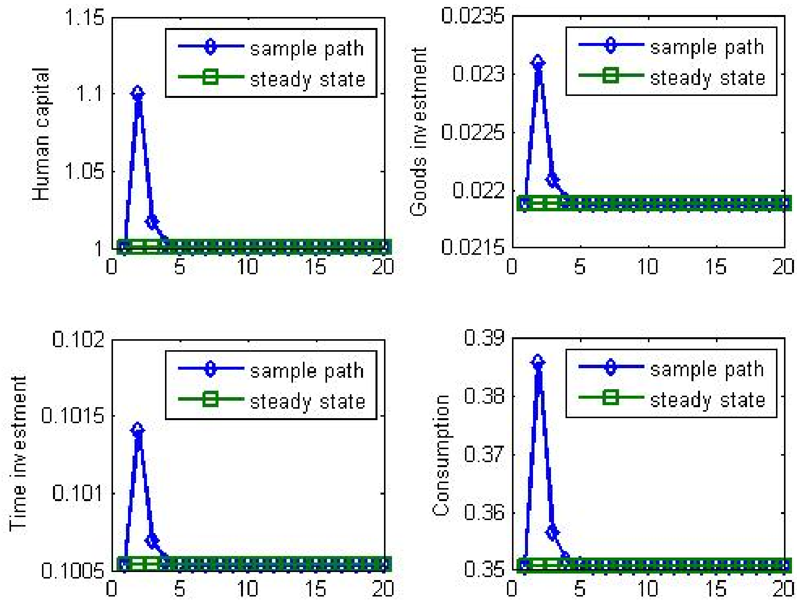

Figure 1: Impulse response of the variables:feed in the model a learning ability shock of one standard deviation

4 Inter-generational persistence and wage inequality

Becker and Tomes (1986) find that almost all the earnings advantages or disadvantages of ancestors are wiped out in three generations. As figure 1 illustrates, this is captured in our model. Figure 2 and Figure 3 show how different factors affect persistence of wage rate. Figure 2 plots impulse response of wage rate for the full model, nature effect only and nurture effect only. The effect of nature on persistence of wage exceeds the effect of nurture. Figure 3 shows splitting nurture effect into time investment and goods investment.

The model produces intergenerational correlation in income as 0.45. In order to asses the role of nature and nurture effects we decompose the overall persistence in earnings in the model into nature and nurture effects components. Then only nature (we simulate the model setting I and e at the mean levels, $\bar{I}, \bar{e}$) generates income persistence as 0.34. If we shut down the nature effect (we simulate the model, setting $\rho = 0$) then nurture effects generate in-

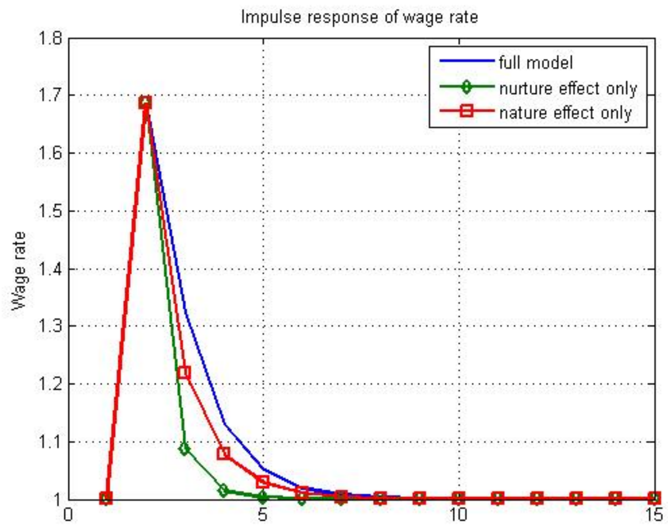

Figure 2: Impulse response of wage rate. Relative importance of nature effect and nurture effect

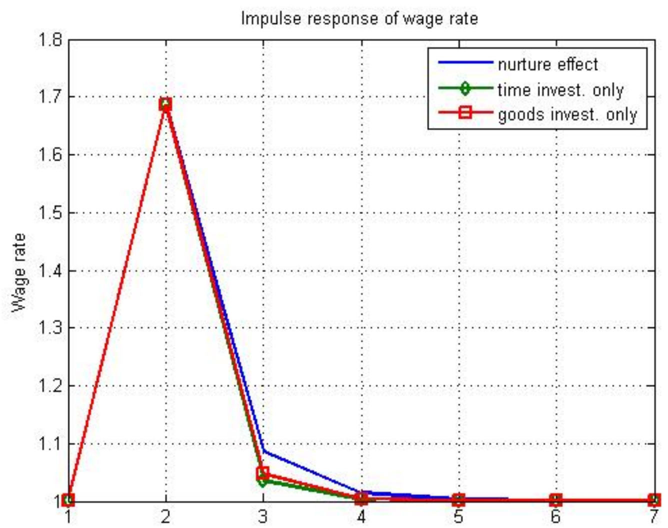

Figure 3: Impulse response of wage rate: relative importance of time investment and goods investment

Table 4: Intergenerational Persistence of Earnings

	$log(w_{t+1}) = \beta_0 + \beta_1 log(w_t) + \varepsilon$
Full Model	0.45
Nurture only	0.15

Nurture only: set $\rho = 0$

Table 5: Intergenerational Persistence of Earnings

	$log(w_{t+1}) = \beta_0 + \beta_1 log(w_t) + \varepsilon$
Full Model	0.45
Nurture only	0.34

Nature only: set $I = \bar{I}$ and $e = \bar{e}$

tergenerational correlation in income as 0.15. Table 4 and table 5 summarize our findings. Nature accounts for a large portion of correlation in earnings.

We also asses the role of nature and nurture on wage inequality. We simulate the model for 1000 families for 50 periods. If we shut down nature then $var(\log(w))$ is 0.57, but if we shut down nurture $var(\log(w))$ is 0.61 hence nature accounts for a large portion of wage inequality.

Table 6: Wage inequality

	$var(\log(w))$
Full Model	0.67
Nurture only	0.57

Nurture only: set $\rho = 0$

Table 7: Wage inequality

	$var(\log(w))$
Full Model	0.67
Nature only	0.61

Nature only: set $I = \bar{I}$ and $e = \bar{e}$

4.1 Wage inequality and parental time investment

In our model, the only source of heterogeneity among families is the random learning ability. When a family receives a good shock, it passes it on to the next generation by making more time and goods investment. Since $\delta < 1$, the rich families do not remain rich forever. If δ is large, parents with more human capital are much more effective in time investment and are able to pass much of the human capital down. A δ smaller than one means the effectiveness of transmission decreases marginally. Intuitively, a larger δ should be associated with greater wage inequality. But we find little support for the mechanism that parents with more human capital are more productive in teaching children.

5 A model with public investment

In this section we introduce public investment in education into our model. Assume government levies proportional tax and use the tax revenue to provide public schooling. We also assume a balanced budget for the government. Therefore given a tax rate τ, tax revenue P_t changes over time because households' wage rates change due to the random shocks to learning ability.

We assume that public investment and private investment are perfectly substitutable. Now the parent's problem becomes

$$\max \ln c_{i,t} + \gamma \ln n_{i,t} + \beta \ln h_{i,t+1}$$

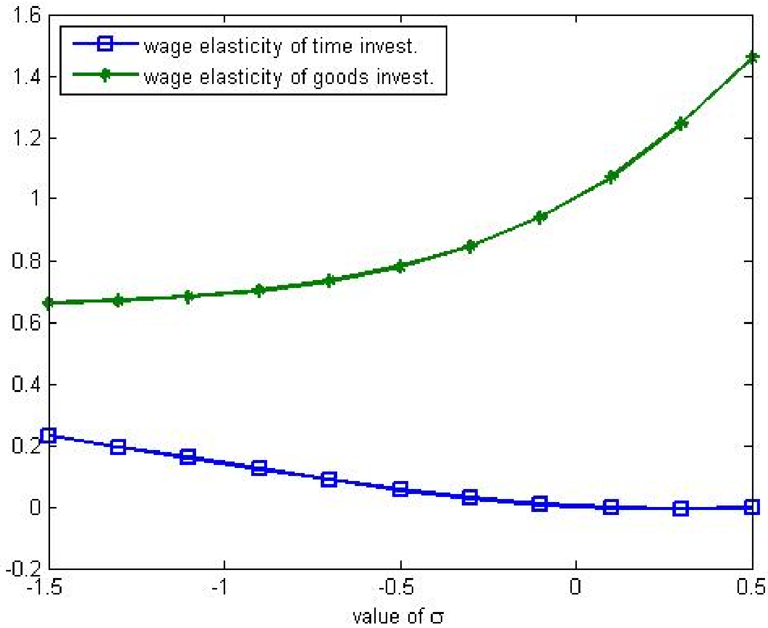

Figure 4: The wage elasticity of time investment and goods investment

subject to

$$c_{i,t} + I_{i,t} = (1 - \tau_t)(1 - e_{i,t} - n_{i,t})w_{i,t} \tag{14}$$

$$w_{i,t} = h_{i,t} \tag{15}$$

$$h_{i,t+1} = z_{i,t}A[\alpha(h_{i,t}^{\delta}e_{i,t})^{\sigma} + (1-\alpha)(I_{i,t} + P_t)^{\sigma}]^{\frac{1}{\sigma}} \tag{16}$$

$$I_{i,t} \geq 0 \tag{17}$$

Notice that the condition $I_{i,t} \geq 0$ can be binding – when public investment P_t is large enough, household has incentive to make negative goods "investment".

5.1 Solution to the model with public investment

When the nonnegativity constraint for good investment is not binding, the model with public investment can again be solved analytically. h_{t+1}, I_t, e_t and n_t as functions of the state variables $\{h_t, z_t\}$ are shown in the following equations.

$$I_t = \frac{\beta(1-\tau)h_t + \beta P_t}{(1+\gamma+\beta)\left[1 + \left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}}[(1-\tau)h_t]^{\frac{\sigma}{\sigma-1}}\right]} - P_t \tag{18}$$

$$e_t = \frac{\beta\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}}\left[(1-\tau_t)h_t^{1-\sigma\delta}\right]^{\frac{1}{\sigma-1}}\left[(1-\tau_t)h_t + P_t\right]}{(1+\gamma+\beta)\left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}}\left[(1-\tau_t)h_t^{1-\delta}\right]^{\frac{\sigma}{\sigma-1}}+1\right]} \tag{19}$$

$$n_t = \frac{\gamma\left[(1-\tau_t)h_t + P_t\right]}{(1+\gamma+\beta)(1-\tau_t)h_t} \tag{20}$$

$$h_{t+1} = \left(\frac{\beta z_t A(1-\alpha)^{\frac{1}{\sigma}}}{1+\gamma+\beta}\right)\left[\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\sigma-1}}\left[(1-\tau_t)h_t^{1-\delta}\right]^{\frac{\sigma}{\sigma-1}}+1\right]^{\frac{1-\sigma}{\sigma}}\left[(1-\tau_t)h_t+P_t\right] \tag{21}$$

If the government makes excessive public investment, the nonnegativity constraint for I_t becomes binding. It is easy to show that, when a household makes zero goods investment in the education of next generation (i.e.,I=0), the optimal time investment satisfies the following equation

$$\alpha\gamma\left(1+\frac{\beta}{1+\gamma}\right)e_t^{\sigma} - \frac{\alpha\beta\gamma}{1+\gamma}e_t^{\sigma-1} + (1-\alpha)\gamma P_t^{\sigma} = 0 \tag{22}$$

In addition, the household's time allocation follows the simple rule given in the equation below.

$$n_t = \frac{\gamma}{1+\gamma}(1-e_t) \tag{23}$$

5.2 Definition of equilibrium and model solution

In the model with public investment, we assume there exists a unit measure of families, differentiated by parents' stock of human capital $h_{i,t}$. The subscript i denotes i^{th} family. For any given tax rate τ, an equilibrium is the sequence of $\{h_{i,t}\ C_{i,t},\ e_{i,t},n_{i,t},\ I_{i,t},\ P_t\ \}_{t=0}^{\infty}$ such that

1. $h_{i,t}\ C_{i,t},\ e_{i,t},\ I_{i,t},\ n_{i,t}$ solves parent's optimization problem.
2. Government budget is balanced. i.e.,$\int \tau(1-e_{i,t}-n_{i,t})h_t di = P_t\ \forall\ t$.

For a given τ, we solve for the equilibrium paths for the variable outlined above numerically based on the estimated parameter values. Specifically, we take the following steps

1. Draw random shocks to leaning ability for H families for T periods, denoted $\{\{\epsilon_{i,t}\}_{i=1}^{H}\}_{t=1}^{T}$.

2. Using $lnz_{i,t} = \rho lnz_{i,t-1} + \epsilon_{i,t}$ to obtain the paths of stochastic learning abilities for all the families. For the i^{th} family, the path is $\{z_{i,t}\}_{t=1}^{T}$

3. Guess a sequence of public investment in education $\{P_t\}_{t=1}^{T}$

4. For each family, given τ, $\{P_t\}_{t=1}^{T}$, $\{z_{i,t}\}_{t=1}^{T}$, derive the optimal paths of human capital $\{h_{i,t}\}_{t=1}^{T}$, time investment $\{e_{i,t}\}_{t=1}^{T}$ and leisure $\{n_{i,t}\}_{t=1}^{T}$. Whenever analytical solutions are not available (i.e., whenever the non-negativity condition for goods investment binds), we resort to numerical solutions. Then we calculate tax obligation for the i^{th} family as $\Phi_{i,t} = (1 - e_{i,t} - n_{i,t})\tau h_{i,t}$

5. Integrate over all the family to calculate the path of aggregate tax revenue $\{\Phi_t\}_{t=1}^{T}$

6. Compare tax revenue with public investment in education. Specifically, if the L-infinity norm $|x|_\infty = max_t|\{\Phi_t\}_{t=1}^{T} - \{P_t\}_{t=1}^{T}|$ is greater than a pre-set tolerance level, find a new guess of $\{P_t\}_{t=1}^{T}$ and repeat step (3)-(5). To find the new guess, whenever tax revenue is less than expenditure, P_t is reduced, otherwise P_t increased.

7. When $|x|_\infty$ is less than tolerance level, the corresponding $\{P_t\}_{t=1}^{T}$ satisfies the balanced budget condition. Then we solve for the corresponding $\{h_{i,t}$ $C_{i,t}$, $e_{i,t}$,$n_{i,t}$, $I_{i,t}$, $\}_{t=0}^{\infty}$ for each of the family.

5.3 Effects of public investment

To examine the effect public investment in education. We simulate the 60000 families for 10 generations in an economy in which the government taxes 5% of households labor income to provide public schooling. Then we compare average time investment, goods investment, consumption, wage rate and wage inequality to those obtained from the baseline model. The results are shown in figure 5.

The upper-left panel of the figure plots the percentage change of average wage rate in the economy with public schooling relative to the wage rate in the baseline model. Let $w_{0,t}$ be the average wage rate of generation t from the baseline model, and w_t is the average wage rate with public schooling. Then this panel plots $\frac{w_t - w0,t}{w_{0,t}}$ for 6 generations since the enforcement of public schooling. Wage rate increases rather rapidly in the first three generations after public schooling is in place. Basically, at 5% tax rate, wage rate will be raised

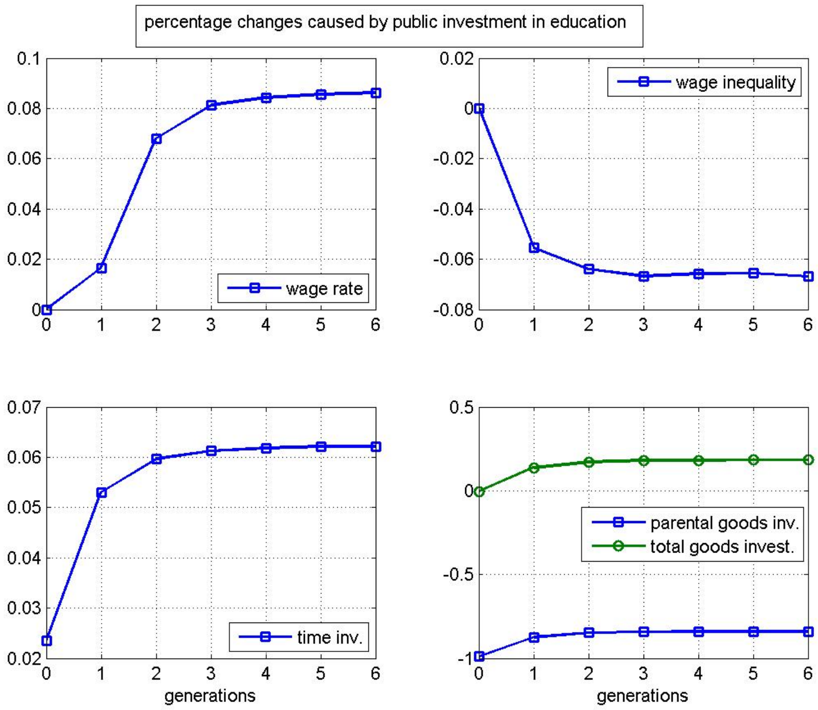

Figure 5: The effects of public schooling when $\tau = 0.05$. Each panel plots the percentage change of a particular variable relative to that from the baseline model after the enforcement of public investment. Tax rate is 5% of labor income.

by more than 8%. In the upper-right panel, we plot the percentage in wage inequality caused by public schooling. Wage inequality declines quickly in the first few generations. Overall, wage inequality is reduced by over 6% due to public schooling financed by the 5% tax on labor income.

Generally speaking, the purpose of public investment in education is to provide equal opportunity to children and reduce wage inequality. In light of the observation that goods investment is one of the key components in children's human capital formation, public schooling should reduce inter-generational correlation in wage and thereby reduce wage inequality. On the other hand, public investment is supported by tax revenue, and economic theory generally suggests that taxation can leads to inefficiency due to the distortion in resource allocation. Regarding human capital accumulation, taxation on labor income should reduce labor supply, which might reduce total goods investment in human capital formation, due to reduced output. Therefore a potential cost of public investment in education is a decline in human capital stock and wage rate.

However, this classical trade-off between efficiency and equity[9] does not appear in our model. We observe gain in both equity and efficiency even though we do not assume any market/informational incompletion, as shown in figure 5. The lower-left panel of the figure sheds light on this seemingly non-intuitive result. It plots the percentage change in parental time investment caused by public schooling. In the presence of public investment, parents find it optimal to increase time investment, since goods investment and parental time investment are complements. In addition, labor income tax reduces the opportunity cost of parental time, which encourages parents to make more time investment on their children. Consequently, while public investment in education reduces wage inequality, it also boosts parental time investment and thereby human capital accumulation, hence higher wage rate.

Finally the lower-right panel of figure 5 shows the dynamics of percentage changes in goods investment. After the enforcement of public schooling, parents reduce private goods investment almost by 100% initially. Then next-generations parents increase private investment slightly. Within a few generations, parental goods investment stabilizes at a level that is about 82% lower than that in the baseline model. However, total goods investment (public investment + private investment) increases about 20% after public investment is in place. Taken together, the lower panels of figure 5 show that public schooling leads to more investment in human capital formation – both time investment and goods investment, which in turn leads to higher wage rate.

Although public schooling gains both equality and efficiency, it is not free lunch. Figure 6 depicts the percentage change in consumption caused by public schooling. It is clear that consumption falls for the first two generations after the implementation of public schooling, then rises. The reduction in consumption for the first generation is over 2%. Therefore, public investment in education is not an pareto-improving policy. As a matter of fact, it is the generation that initiates public public schooling that suffers from reduced consumption. This implies that it might be hard for an economy to increase public investment in education if it entails majority voting.

[9] Mirrlees (1971) is one of the seminal articles. For more recent works, see Conesa and Krueger (2006) and the references therein.

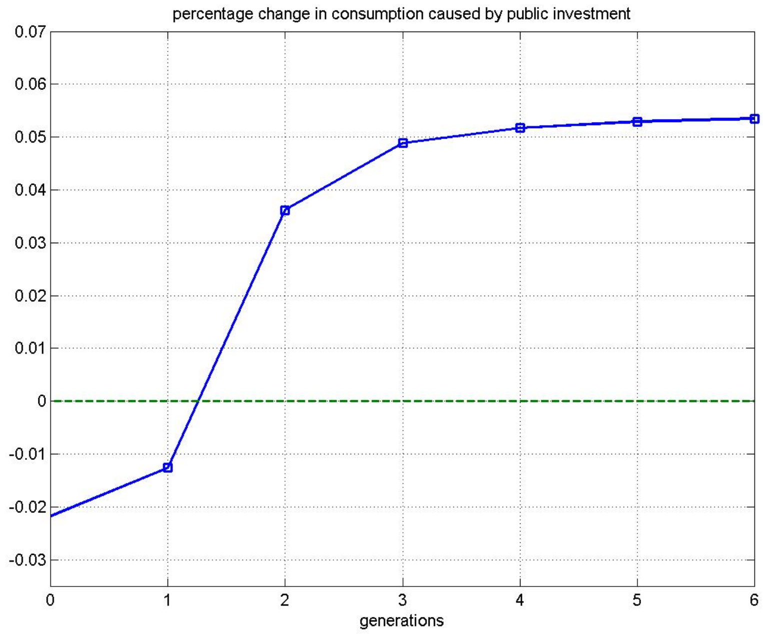

Figure 6: Evolution of average consumption after the enforcement of public schooling with $\tau = 0.05$. Consumption falls for the first two generations, then it rises by up to 5.3% of its original level in the baseline model.

5.4 Optimal tax rate

Having studied the benefits and costs of public schooling financed through labor income tax, a natural question to ask is: what are the optimal tax rate from the social planner's point of view? Since human capital accumulation involves benefiting future generations at the cost of current generation, a social planner should consider the weighted sum of each generation's utility. It is reasonable to assume that social planner discount future utility at a certain rate. Therefore one key element in conducting welfare analysis is to choose the appropriate discount factor. We begin with the estimate of β – the degree of altruism toward children. Then we compute the optimal tax rate at different discount rates.

Another important issue in welfare analysis is how to measure the welfare gain from reduced wage inequality. The gain stems from the concavity of utility

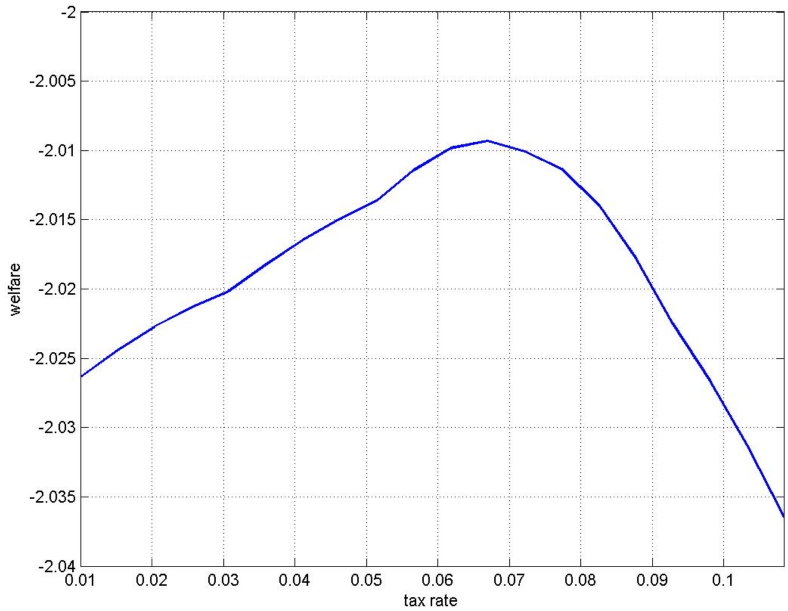

Figure 7: Optimal tax rate. When discount rate equals β, optimal tax rate is 6.7% of labor income.

function from social planner's point of view. A natural candidate is log utility which is also the assumed utility for individuals in the model. We define the planner's value to be the sum of discounted average utility. Mathematically, value $= \sum_{t=0}^{\infty} \beta^t [\int_i (logc_{i,t} + \gamma logn_{i,t}) di]$.

Figure 7 shows how the welfare (value) perceived by social planner changes with respect to tax rate when discount rate is β. In practice we simulate 50 generations instead of infinity, and 1000 households to compute welfare. It is clear from the figure that social welfare increases gradually with the tax rate initially, then declines quickly as the tax rate increases further. The optimal tax rate is 6.7%.

To better understand why the social planner's value exhibit a hump-shape as show in figure 7, we examine how consumption, goods investment, time allocation, human capital stock and wage inequality change with tax rate. Figure 8 depicts the averages of these variables against tax rate. Here the average is both across households and over time. For example, Goods investment displayed in the upper-left panel is defined as $\frac{1}{NT} \sum_{t=0}^{T} \sum_{i=1}^{N} c_{i,t}$, with T=50 and N=1000[10].

[10] We also tried to use β to discount future consumption and other variables. The resulting

The two panels on the top plot consumption and leisure against tax rate. Both are hump-shaped, which inevitably leads to the hump-shaped welfare profile in figure 7. Why does leisure exhibit a hump-shaped profile? A further look at time allocation reveals that as tax rate increases, parental time investment increases while work hour decreases. At relatively low tax rate, the decline in work hour outweighs the rise in parental time investment, causing leisure to increase with tax rate. When tax rate is high (above 5% of wage), the rise of time investment outweighs the decline of work hour, causing leisure to fall with tax rate. Obviously, parents make more time investment because higher tax rate reduces the opportunity cost. In addition, public investment increases with tax rate, which raises the return to time investment due to the complementarity between time and goods investment.

To see why consumption profile is hump-shaped, it is helpful to recall the budget constraint $c = (1 - e - n)(1 - \tau)w - I$. Here for simplicity we omit the subscripts i and t. We need to examine how work hour (1-e-n), wage rate and parental goods investment change with tax rate. It is already known that work hour decreases with tax rate, affecting consumption negatively. This negative pressure on consumption is outweighed by the positive effects of higher wage rate (the "human capital" panel) and reduced goods investment. As tax rate rises further, goods investment no longer decrease because it hits the zero lower-bound. In addition, wage rate increases with tax rate at a reduced pace. Therefore the negative effect of declining work hour becomes dominating, causing consumption to fall the tax rate.

Figure 8 also shows how wage inequality change with tax rate. As tax rate rise from zero to about 7%, wage inequality (coefficient of variation of cross sectional wage rate) is reduced from 0.663 to 0.60 – a reduction rate of over 10%. However, as shown in the figure, increasing tax rate further has little effect on wage inequality. Recall that our model allows for two major sources of wage inequality: random learning ability (nature effect) and parental investment in children's human capital (nurture effect). Public schooling has no impact on nature effect at all. Public schooling cannot completely eliminate nurture completely either – parents can transmit human capital through time investment effectively. Finally, from the viewpoint of inter-generational mobility, public schooling reduces persistence from 0.449 to 0.28 at 10% tax rate. Again,

profiles are basically the same.

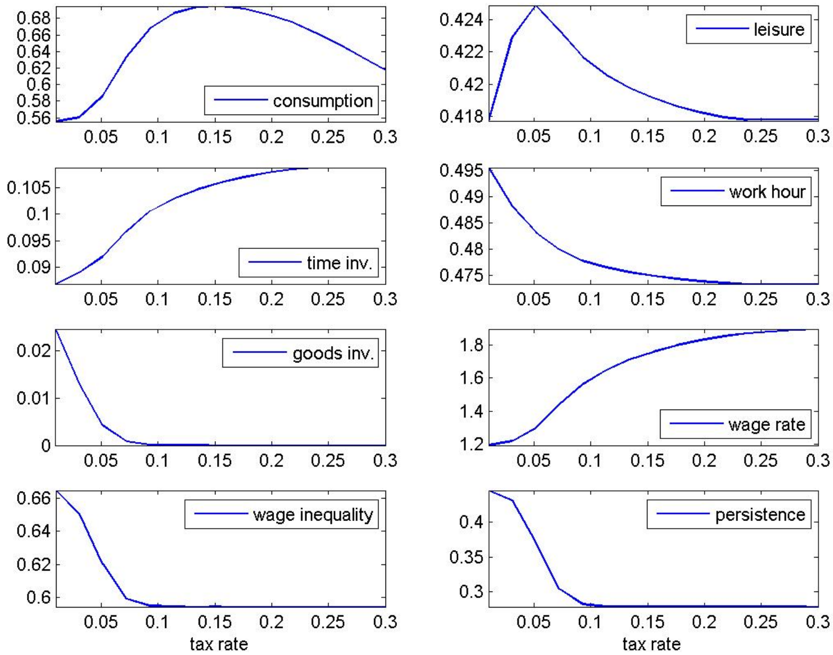

Figure 8: Tax rate and variables of interest

increasing tax rate further has little effect on inter-generational wage persistence. This is shown in the lower-right panel of figure 8

Thus far our analysis is based on the assumption that planner's discount rate equals β, parents' degree of altruism toward their children. It is interesting to know how the optimal tax rate and social welfare depend on planner's discount rate. Under different discount rate, we compute optimal tax rate and the associated welfare gain relative to the baseline case. The results are presented in table 8. From the first column of the table it is clear that optimal tax rate increases with discount rate. This is quite intuitive – public schooling through taxation benefits future generations at the cost of current generation, so it is more beneficial from the planner's point of view if the discount rate is larger. The second column shows that welfare gain at optimal tax rate is also increasing with discount rate. When discount rate equals β, public schooling at optimal tax rate increases social welfare by 1%. When discount rate rises to 0.35, social welfare gain is 2.5%.

Finally, table 8 shows how inter-generational earning correlation, wage rate

Table 8: Optimal tax rate under different discount rate

discount rate	optimal tax rate	welfare gain	inter-generation corr.	wage ineq.	average wage
0.1	0.041	0.4%	0.406	0.636	1.249
0.15	0.046	0.5%	0.390	0.628	1.270
0.2	0.057	0.6%	0.353	0.613	1.329
0.246	**0.067**	1%	**0.318**	**0.603**	**1.401**
0.3	0.077	1.7%	0.295	0.597	1.472
0.35	0.082	2.5%	0.288	0.596	1.504
0.4	0.082	3.4%	0.288	0.596	1.504
0.45	0.088	4.3%	0.285	0.595	1.534
0.5	0.103	5.3%	0.280	0.595	1.606

and wage inequality change with discount rate. Higher discount rate, at optimal taxation, leads to lower inter-generation correlation, lower wage inequality and higher average wage rate. Obviously, the more patient is the social planner, the greater value is public schooling.

5.5 Comparison with the goods-investment-only model

Throughout the paper we have emphasized the role of parental investment in human capital accumulation. In this section, we show that ignoring time investment leads to totally different policy implications. To do so, we consider a simplified version of the full model outlined above. First, we assume human capital production needs goods investment only. Human capital accumulation takes the form of

$$h_{i,t+1} = z_{i,t}A[B + (1-\alpha)(I_{i,t} + P_t)^{\sigma}]^{\frac{1}{\sigma}}$$

Obviously, we replaced αe^{σ} in the full model with a constant B. In addition, since parents no longer make time investment, they allocate time between work and leisure. Therefore income of i^{th} household in period t is $(1-\tau)(1-n_{i,t})h_{i,t}$. Every other aspects of the full model is preserved. In spirit this simplified version of the model is very close to that in Restuccia and Urrutia (2004), although the latter is richer.

We also require the simplified model to capture the key data feature. For

	average goods invest.	average consumption	$\frac{\partial I}{\partial H} \times \frac{H}{I}$	inter-gen earning corr.	wage ineq.
data moments	0.025	0.469	1.067	0.449	0.663
model moments	0.025	0.471	0.265	0.440	0.661
	β	σ	γ	ρ	ν
parameters	0.24	-0.87	1.07	0.38	0.53

Table 9: Calibration of the simplified model

this purpose it is necessary to reconsider the parameter values. Parameter A in the simplified model is still so chosen that human capital is one on average. We take α to be the same as in the full model, and set $B = \alpha * E_{ss}^{\sigma}$ where E_{ss} is the steady state time investment in the baseline model. The remaining parameters are calibrated by choosing values that bring the models moments as close to the data moments as possible. The same set of moments are used except that now we do not consider average time investment and wage elasticity of time investment. We find that the calibrated parameter values are very closed to those estimated in the full model. Table 5.5 shows the parameter values and corresponding model moments. It should be emphasized that results in this section hold qualitatively even if we use the same parameter value as estimated from the full model.

In figure 9, we compare the percentage changes caused by public investment in the model model versus in the simplified model when tax rate is $\tau = 0.02$. The difference is quite striking. In the simplified model where time investment plays no role, public schooling leads to lower wage rate and only moderately reduced wage inequality. The lower panels depicts changes made to variables that are welfare-related – consumption and leisure. With the simplified model, consumption is always reduced by public schooling. Households take more leisure in both models, but the effect is stronger in full model.

Intuitively, households have less incentive to work when their income is taxed, this lead to reduced consumption as well as reduced goods investment, which in term leads to less human capital accumulation. Perhaps it is surprising to see that public schooling is more effective in reducing wage inequality in the full model as apposed to the simplified model – after all time investment is an

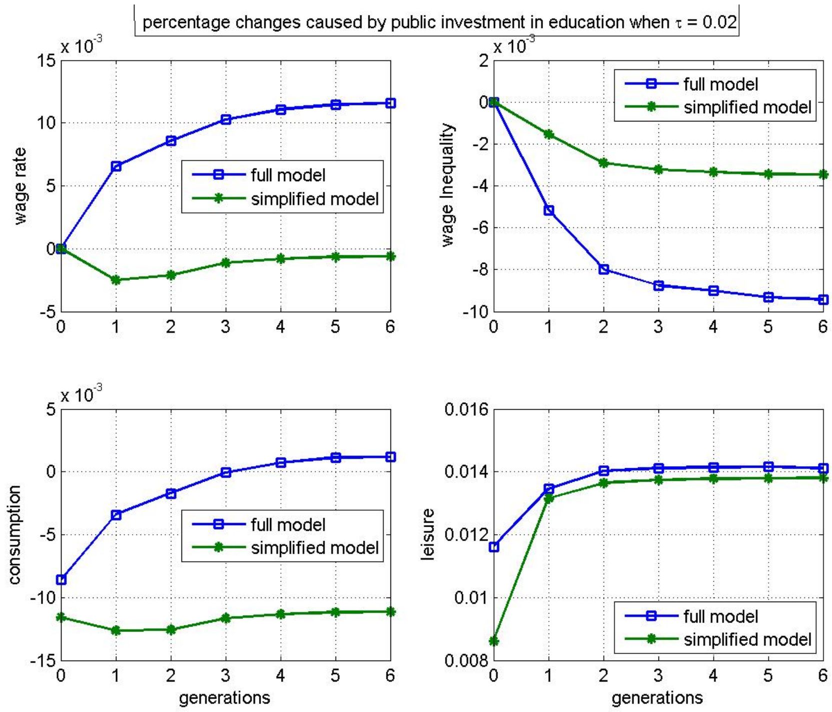

Figure 9: Comparing the effects of public schooling on wage level, wage inequality, consumption and leisure in the full model versus in the simplified model.

channel for parents to pass on their earning ability, hence it should be harder to reduce wage inequality in a model with time investment. The caveat is, the same tax rate leads to different tax revenue in the two models.

Figure 10 compares the tax revenue in the full model with that in the simplified model when tax rate is $\tau = 0.02$. In the full model, tax revenue is significantly higher because public investment in education stimulates parental time investment for two resounding reasons (i) time investment is less expensive due to labor income tax, and (ii) time investment is more effective due to public goods investment. On the other hand, such stimulation does not exist in the simplified model. Therefore, in the full model, public schooling leads to higher wage rate, which leads to higher tax revenue, which in turn leads to more reduction in wage inequality.

When tax rate is high, households are constrained by the zero goods investment lower bound. In this case social planners are forcing households to consume less and accumulate more human capital for the future generations.

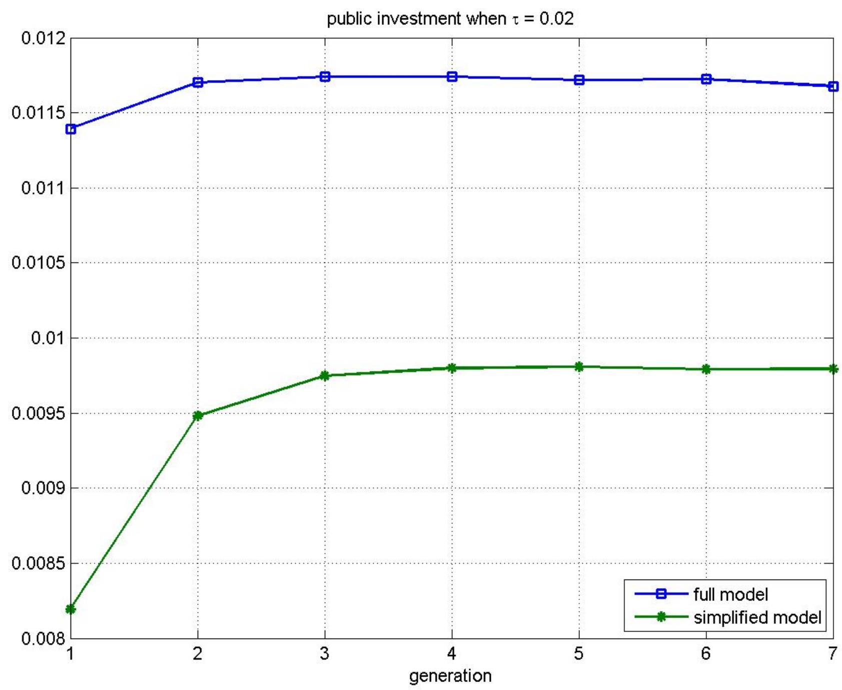

Figure 10: Comparing tax revenue in the full model versus in the simplified model.

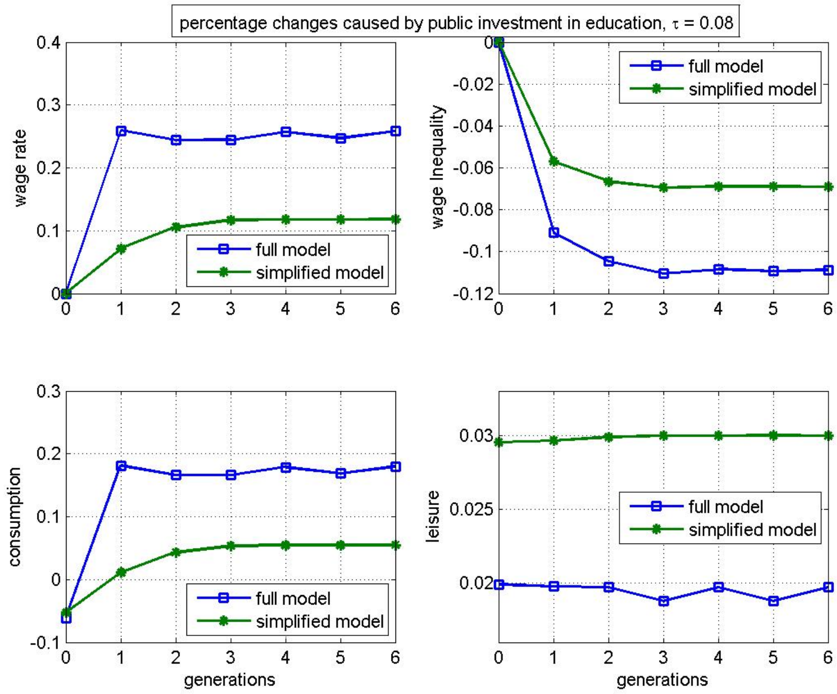

Figure 11: Comparing effects of public schooling when tax rate is high.

Therefore wage rate is bound to rise, even in the simplified model. With high tax rate, the full model again predict more desirable effects of public schooling than the simplified model. This is shown in figure 11. Compared to the simplified model, in the presence of time investment as a choice variable, public schooling lead to much higher wage rate and consumption, and much lower wage inequality. Finally, the increase in leisure is less in the full model, because much of the time is shifted to parental time investment.

6 Conclusion

We develop a model of inter-generational transmission of human capital in which human capital transmission takes three forms, intergenerational persistence of endowed learning ability (nature), time investment and goods investment and higher teaching productivity of parents with more human capital (nurture). Our model has some nice properties, it captures some data facts and it is solved analytically. Model parameters are estimated by Simulated Method of Moments.

We quantify the relative importance of three mechanisms of intergenerational correlation. In addition we are using previously unexplored time investment channel as one component of nurture. Additionally, we quantify the relative importance of two contributors in wage inequality: biological persistence in learning ability and intergeneration effect (time and goods investment). Our results indicate that nature accounts for a large portion of earning persistence and income inequality. We introduce public investment into the model and study the effect of public investment in education. Next we will find optimal level of public investment. This topic is left for the future research.

Limitations of our paper: we don't consider the effect of borrowing constraints on intergenerational mobility; Parents can not bequest in the form of financial wealth; Individual effort plays no role in human capital formation.

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7 Appendix

7.1 Data Appendix

In this paper we estimate model parameters by matching seven key moments in the model to their data counterparts. Except for the moment of intergenerational correlation in earnings which is taken from Solon (1992), all the moments are obtained from the 2003 waves of American Time Use Survey and Consumer Expenditure Survey.

American Time Use Survey is conducted by the U.S. Bureau of Labor Statistics (BLS) and the data is available from BLS website. Using the time use taxonomy introduced in Aguiar and Hurst (2007), we consider three categories of parental time with children: basic child care, teaching and play. Teaching children includes activities such as reading to/with children, talking/listening to children, and helping children with their homeworks. Clearly these should be regarded as time investment. Much of the time categorized as basic child care and playing with children is beneficial or auxiliary to the development of human capital. For example, "basic child care" includes activities such as picking up and dropping of children from school, attending children's events; "playing with children" includes playing sports with children, arts and crafts with children, etc. Therefore we take the sum of the three categories as the proxy for parental time investment. We proxy leisure with the time spent on activities related to the following: lawn, garden and houseplants, animals and pets, socializing and relaxing, sports, exercise and recreation, telephone calls, household and personal mail, travel related to these events, eating, sleeping and personal care. This definition of leisure is precisely "leisure measure two" in Aguiar and Hurst (2007)

Consumer Expenditure Survey data is publicly available from NBER collection. Educational expenditure has three categories in the data. The first one, higher education, is the tuition for college. The second one is Nursery, Elementary, and Secondary Education. This category includes tuition for elementary and high school, payment for private school bus, and other expenses for day care centers and nursery schools. The third category, other education service, includes tuition for other schools, rental of books and equipment and other school-related expenses, and contributions to educational organizations. We take the summation of the three as the proxy for goods investment in children's human capital. The survey provides no information on whether these expenditure are for children or not, therefore we delete the observations from families whose head of household is younger than 25 or not married or have no children. Consumption is measured by the summation of expenditure on nondurable non-educational goods and services.

We need to calculated educational expenditure normalized by wage rate. We also need to regress educational expenditure on wage rate to compute the response of goods investment with respect to parents' wage rate. Since the data set provides only family level expenditures, the corresponding wage rate should also be at family level. Therefore we take wage rate to be the sum of both spouse's wage divided by the sum of work hours. i.e., $w = \frac{w_{husband}+w_{wife}}{hours_{husband}+hours_{wife}}$. Note that for both spouse, CEX provides information about annual wage, weeks worked in the year and hours worked in each week.

In both ATUS and CEX, we delete observations that have one of the following characteristics: (1) no complete information on age, education attainment, number of children, age of children, wage rate, labor income, marital status, child care time, leisure, work hours; (2) Respondent(in ATUS) or househead(in

CEX) has zero wage; (3) not married; (4) have no children; In ATUS we also drop respondents whose are younger than 21 or older than 65. This is because we focus on parents in their working age who allocate time among work, leisure and child care.

Table 10: Regression of Time Investment

	coefficient	std.error
constant	1.7125	0.3608
wage	0.1988	0.1224
spouse years of schooling =12	-0.0491	0.115
spouse years of schooling > 12, $<= 16$	-0.1128	0.1276
spouse years of schooling > 16	0.0088	0.1595
number of children = 2	0.1803	0.0639
number of children = 3	0.3446	0.0832
number of children = 4	0.2993	0.1317
number of children = 5	-0.0588	0.2563
number of children = 6	-0.1546	0.5356
number of children =7	0.8368	0.8085
number of children = 8	0.0408	0.6425
number of children $>= 9$	0.0944	1.209
age of youngest child = 1	-0.0407	0.1594
age of youngest child = 2	-0.3075	0.1662
age of youngest child = 3	-0.5552	0.157
age of youngest child = 4	-0.9221	0.176
age of youngest child = 5	-0.1397	0.1704
age of youngest child = 6	-0.3841	0.1802
age of youngest child = 7	-0.0185	0.5057
age of youngest child = 8	-0.4214	0.1672
age of youngest child = 9	-0.1989	0.4632
age of youngest child = 10	-0.5064	0.1697
age of youngest child = 11	-0.235	0.1999
age of youngest child = 12	-0.4341	0.4814
age of youngest child =13	-0.2844	0.3631
age of youngest child = 14	-0.0183	0.2944
age of youngest child$>= 15$	0.0184	0.5823
spouse unemployed	0.2491	0.0639

Regressing time investment on wage rate and demographics. Wage rate is instrumented by educational attainment.

Table 11: Regression of Goods Investment

	coefficient	std.error
constant	2.4907	0.9813
wage	0.8785	0.337
spouse years of schooling =12	0.3053	0.3412
spouse years of schooling $> 12, <= 16$	0.2862	0.3786
spouse years of schooling > 16	0.7579	0.4735
female head of household	0.0244	0.1422
number of children = 2	0.0434	0.1687
number of children = 3	-0.1247	0.2129
number of children = 4	0.3655	0.3352
number of children = 5	-1.1209	0.5782
number of children = 6	-0.4256	0.6643
number of children =7, 8	0.9721	0.4425
number of children > 8	1.3039	0.4354
age of youngest child = 1	1.5105	0.4281
age of youngest child = 2	1.2629	0.4575
age of youngest child = 3	1.3718	0.4743
age of youngest child = 4	0.2693	0.4884
age of youngest child = 5	1.2593	0.5251
age of youngest child = 6	0.9949	0.5223
age of youngest child = 7	0.0387	0.6088
age of youngest child = 8	0.448	0.4507
age of youngest child = 9	0.562	0.4851
age of youngest child = 10	1.7295	0.5004
age of youngest child = 11	0.7525	0.4857
age of youngest child = 12	0.6672	0.4658
age of youngest child $>= 13$	1.3086	0.3706

Regressing goods investment on wage rate and demographics. Wage rate is instrumented by head of household's educational attainment