Market Structure, Factor Endowment and Industrial Upgrading*

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Abstract

We develop a simple dynamic general equilibrium model to study the industrial upgrading process, where the market structure may change endogenously depending on when the first mover chooses to start operating the new technology, which is more capital-intensive and initially privately accessible and becomes publicly available one period after its initial operation. We show that the endowment structure (capital-labor ratio in the aggregate economy) plays a very important role. It determines the magnitude of the monopoly rent, therefore affects whether or not and when the new technology will be implemented. In particular, when the capital endowment is sufficiently small or sufficiently large, the equilibrium allocation always achieves Pareto efficiency, no matter whether the endogenous market structure is monopoly or perfectly competitive. When the endowment structure falls into some intermediate range, inefficiency arises: Old technology (industry) survives for too long, the industrial upgrading is delayed, and the aggregate output is depressed. We also show that there may exist multiple dynamic equilibria, which can be Pareto-ranked. In addition, a limited improvement in the initial endowment structure may sometimes delay the adoption of capital-intensive technology. Welfare-enhancing industrial policies are discussed.

Key Words: Market Structure, Industrial Policies, Structural Transformation, Industrial Upgrading, Capital Accumulation

JEL Codes: L50, O14, O25, O41

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1 Introduction

Economic development is a process of technological innovation and structural transformation. New industries emerge with the adoption of new technologies and old industries decline. To understand how industrial upgrading kicks start in developing economies, we must study the incentive and behavior of the first entrant (or first mover thereafter) to the new industry, which is the main task of this paper.

Industrial upgrading would suffer delay, perhaps for indefinitely long, when the first-move advantage is dominated by the first-move disadvantage. Varieties of economic forces may affect the relative magnitude of the first-move advantage and disadvantage. For instance, there is a vast information economics literature addressing how the risk and uncertainty affect investment decisions including learning behaviors and strategic delay.[1] Another literature emphasizes the importance of early entrance when there exists the increasing return to scales or dynamic comparative advantage.[2] There is also a vast literature examining the role of strategic complementary and coordination in industrial upgrading, the main implication of which is that industrial upgrading cannot take place unless a sufficient large amount of investors are well coordinated to enter simultaneously.[3]

In this paper, we focus on the situation where the first-move advantage mainly comes from the private information about the new technology (and other relevant information), which gives the first mover monopoly power before other competitors learn enough and enter. The first-move disadvantage mainly lies in the fact that the first mover exhibits positive information externality to the followers but only the first mover pays all the exploration cost including the certainty equivalence for the risk it bears, and the followers can freely imitate the successful practice.

The key novel feature of this paper is that we will explicitly examine the role of endowment structure (measured by the capital-labor ratio in the aggregate economy) in shaping the optimal decision of the technology adoption and industrial upgrading, where the market structure may also change endogenously with the adoption of the new technology. To this end, we develop a dynamic general equilibrium model to explain how the first mover's behavior and the aggregate industrial upgrading process are affected by the relative factor prices, which in turn are determined by the endowment structure and the market structure in a general-equilibrium fashion.

To fix ideas, imagine there are two technologies (industries). One is labor intensive and publicly available (old technology) and the other is capital intensive, which is new and initially only privately known. The capital-intensive technology is superior if and only if the rental price of capital is sufficiently small

[1] Chamley (2004) is an excellent monograph on this topic.

[2] See, for example, Krugman (1987).

[3] See, for example, Murphy, Shleifer and Vishny (1989), Rodrik (1997), Krugman (1991), Matsuyama (1991), Andres (2007). Ju, Lin and Wang (2011) study the impact of Marshallian externality on industrial upgrading in a dynamic growth model with both capital and labor.

relative to the wage. Both the endowment structure and the market structure of the consumption goods would ultimately affect the relative cost of capital and labor, hence affect the optimal choice between the two different technologies. We ask under what conditions the capital-intensive technology is adopted and how it takes place in a dynamic economy with endogenous capital accumulation. We first develop a static model to illustrate how the exogenous endowment structure affects the market equilibrium under different market structures, and then we move to the dynamic analysis by incorporating the endogenous changes both in the endowment structure and the market structure.

In the static model, two different market structures are compared. The first one is when the market is always perfectly competitive by assuming that both technologies are publicly and freely available. The second one is when the capital-intensive technology is only freely accessible to one lucky firm, which may operate with a monopoly power. Not surprisingly, in both cases the capital-intensive technology is adopted if and only if the capital endowment is larger than some finite cutoff value. Moreover, this cutoff value is shown to be invariant to the two different market structures. On the other hand, the labor-intensive technology will completely stop operating when the capital endowment is larger than another finite cutoff value, which does depend on the market structure. The most surprising and interesting result is perhaps that the monopoly can still achieve the Pareto efficiency when capital is sufficiently large. This is because the positive income effect on the demand due to the extra monopoly profit and the negative price effect due to the monopoly markup pricing exactly cancel out each other via the general equilibrium feedback. This is different from the standard partial equilibrium result.

Monopoly would cause economic inefficiency only when the capital endowment falls on certain intermediate range, in which circumstance both technologies are operating but the monopolist of the capital-intensive technology produces less than the socially optimal amount. Again, the reason is different from the standard argument in the partial equilibrium framework, but rather that, by doing so, the monopolist can depress the relative rental price of capital to wage so that the monopoly rent is maximized. To put it differently, the old (labor-intensive) technology is "overly used" from the social welfare point of view. It is shown that, when capital endowment falls to a certain interval, the labor-intensive technology would be fully abandoned under the perfect competition market structure, but this technology can still survive when the new technology is monopolized.

In the two-period dynamic model, we also compare the perfect competition (first best) and the monopoly. In the first best case, we show that there could exist six different dynamic patterns of industrial development, depending on the initial capital endowment and the efficiency in the capital good production. In the monopoly case, we examine whether the potential monopolist chooses to implement the new technology immediately or delay the operation. The timing decision affects the market structure dynamically because, the new technology, once operated, will become publicly and freely available to all the firms one period later, hence the market structure will change from monopoly to perfect competition. In other words, the information externality generated by the first-mover monopolist is realized with a one-period lag.

We show that adoption of the new technology (industrial upgrading), if adopted, will be delayed if and only if the initial capital endowment is sufficiently small for two reasons. First, when the initial capital endowment is sufficiently small, the current monopoly rent is small in the first period. Second, the equilibrium inter-temporal interest rate would be small so that the future monopoly profit is not discounted too much. Consequently, adoption (industrial upgrading) will be delayed until the profit is sufficiently large as capital accumulates.

Our paper is different from the standard literature of "creative destruction" in several important ways.[4] First, we emphasize the role of the endowment structure with both capital and labor, which endogenously determines which technology is superior and when to upgrade the industries, while in the creative destruction literature labor is typically assumed to be the only production factor and which technology is superior is presumed rather than endogenously affected by the endowment structure. Second, the natures of the first mover's behavior and the analytical focuses are different. The standard creative destruction literature mainly studies the developed countries, where the first mover obtains monopoly power by conducting very costly R&D to invent brand-new products and the analytical focus is on how individual firms make optimal R&D decisions that determines the speed of the aggregate technological progress. By contrast, our paper focuses on industrial upgrading in developing countries, where the first mover mainly adopts and adapts the existing foreign technologies. The monopoly power is *de facto,* coming from the fact that some firms happen to be luckier than their competitors by having earlier access to the foreign better technology and related information (perhaps after some costly exploration). Our analytical focus is on the economic consequence of such potential monopoly power instead of how a firm optimally acquires the monopoly power. Third, the policy implications are different. The major policy implications of the creative destruction literature center on how to achieve the socially optimal level of R&D by subsidy or designing a better patent system. In this paper, the *de facto* monopoly power bestowed on the first mover typically lasts for a very short period because the exploration activities such as collection of market information and technology imitation are much less costly and their practices are more easily imitated as they are not patentable or legally protected in other forms.[5] Correspondingly, our policy implications are more about how to crack down the *de facto* monopoly power by lowering the information barrier to encourage technology adoption (by subsidizing the training program or encouraging FDI, etc) and facilitating the competition (by deregulation, for example), or alternatively, how to rectify the price signals in the factor markets to facilitate the timely industrial upgrading.

[4] See , for example, Aghion and Howitt (1992), Grossman and Helpman (1991), *etc.*

[5] Even if the foreign better technology is adopted in the form of a legal license from abroad, the violation of the international property rights protection is rampant in many developing countries.

The paper is structured as follows. Section 2 studies the static model, followed by the analysis of the dynamic model in Section 3. Based on these two sections, industrial policies are discussed in Section 4. The last section concludes.

2 Static Model

Consider a static autarky populated by a continuum of identical households with a measure equal to unity. Each household is endowed with K units of capital and L units of labor (time). Define the endowment structure as $k \equiv \frac{K}{L}$. There is only one consumption good, which can be produced with two alternative C-D technologies, called technology 1 and technology 2, respectively. Throughout the paper we also interchangeably call them industry 1 and industry 2, respectively. The corresponding production functions are given by $F^{[1]}(K_1, L_1) = A_1 K_1^{\alpha_1} L_1^{1-\alpha_1}$ and $F^{[2]}(K_2, L_2) = A_2 K_2^{\alpha_2} L_2^{1-\alpha_2}$, where A_i, K_i, L_i and α_i are the total factor productivity, capital, labor, and capital share for technology $i \in \{1, 2\}$. Without loss of generality, assume technology 2 is more capital intensive: $0 < \alpha_1 < \alpha_2 < 1$. Following the pertinent literature, when technology 2 is adopted, it is referred to as industry upgrading. A representative household's utility function is

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \text{ where } \sigma \leq 1.$$

We first analyze the competitive equilibrium when both technologies are freely and publicly available, and then we analyze the case in which technology 2 is accessible only to a monopolist.

2.1 Perfect Competition

The second welfare theorem holds, so the competitive equilibrium can be characterized by solving the following central planner problem:

$$\begin{aligned} G(K, L) \quad = \quad & \max_{\{K_n, L_n\}_{n=1}^{2}} A_1 K_1^{\alpha_1} L_1^{1-\alpha_1} + A_2 K_2^{\alpha_2} L_2^{1-\alpha_2} \\ & s.t. \\ & K_1 + K_2 = K, \\ & L_1 + L_2 = L, \\ & K_n \geq 0, L_n \geq 0, n = 1, 2. \end{aligned}$$

The resource allocation problem is a standard nonlinear programming problem with a strictly concave objective function. As a result, there exists a unique solution characterized by the Kuhn-Tucker condition.

If the solution is interior, then it satisfies two first-order conditions that equate the marginal productivity of labor and capital across the two different technologies. Let $k_n \equiv \frac{K_n}{L_n}$ denote the capital-labor ratio in sector n. We obtain

$$k_1^* = \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_2}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}, \quad (1)$$

$$k_2^* = \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}. \quad (2)$$

Since $\frac{k_1^*}{k_2^*} = \left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right) < 1$, the capital per worker is higher in industry 2 than industry 1 when the two industries coexist.

Using the following factor market clearing conditions

$$\begin{aligned} k_1^* L_1 + k_2^* L_2 &= K, \\ L_1 + L_2 &= L, \end{aligned}$$

together with (1) and (2), we get

$$L_1^* = \frac{k_2^* L - K}{k_2^* - k_1^*};\ L_2^* = \frac{K - k_1^* L}{k_2^* - k_1^*}, \quad (3)$$

$$K_1^* = k_1^* L_1^*;\ K_2^* = k_2^* L_2^*. \quad (4)$$

To satisfy the interior physical constraint $L_1 > 0$ and $L_2 > 0$, we must have

$$k_1^* < \frac{K}{L} < k_2^*.$$

If instead $\frac{K}{L} \leq k_1^*$ or $\frac{K}{L} \geq k_2^*$, then the solution is a corner one. To put it more formally, we have the following proposition.

Proposition 1 *Let k_1^* and k_2^* be defined as in (1) and (2). In the perfect competitive equilibrium,*
(a) If $\frac{K}{L} \leq k_1^*$, only technology 1 operates with the aggregate production function $G(K,L) = A_1 K^{\alpha_1} L^{1-\alpha_1}$.
(b) If $k_1^* < \frac{K}{L} < k_2^*$, then both technologies operate with resource allocated according to (3) and (4). The implied aggregate production function is

$$G(K,L) = aK + bL,$$

where $a \equiv A_1\alpha_1 (k_1^*)^{\alpha_1 - 1}$ and $b \equiv (1-\alpha_1)A_1 (k_1^*)^{\alpha_1}$. (c) If $\frac{K}{L} \geq k_2^*$, only technology 2 operates and the aggregate production function $G(K,L) = A_2 K^{\alpha_2} L^{1-\alpha_2}$.

From the Proposition, the aggregate production function $G(K,L)$ is constant return to scale, continuously differentiable, concave and strictly increasing in both arguments. Intuitively, this proposition can be illustrated by Figure 1.

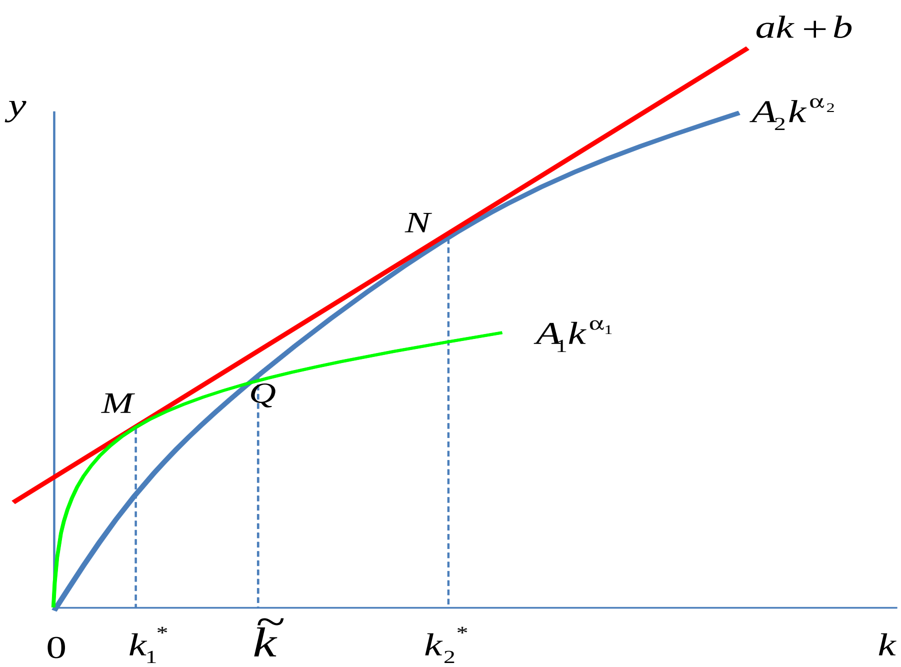

Figure 1. Static Competitive Equilibirum

It plots the output per worker y as a function of the capital-labor ratio k. The two different technologies are represented by the two different concave curves, which cross each other at the origin and point Q, where the corresponding capital-labor ratio is denoted by $\widetilde{k} \equiv \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}$. Clearly, technology 1 is better than technology 2 if and only if $k \leq \widetilde{k}$. The two curves have one unique cotangent straight line $y = ak + b$ and the x-coordinates of the two tangent points M and N exactly correspond to k_1^* and k_2^* given by (1) and (2). The aggregate production function per labor ($\frac{G(K,L)}{L}$) is the convex envelope of the two technology curves. In particular, when $k_1^* < k < k_2^*$, both technologies are used simultaneously and the aggregate production function per labor is linear (denoted by segment MN), in which case the equilibrium rental price of capital is just the slope a and the wage rate is just the intercept b. When $k \leq k_1^*$, only technology 1 is operating so $\frac{G(K,L)}{L} = A_1 (k)^{\alpha_1}$. When $k \geq k_2^*$, only technology 2 is operating, so $\frac{G(K,L)}{L} = A_2 (k)^{\alpha_2}$.

It is worth emphasizing that capital is not subject to the decreasing return to scale when $k_1^* < k < k_2^*$, even though there is no productivity change in either of the two specific technologies (i.e., A_1 and A_2 are fixed) or any non-convexity such as Marshallian externality. It is the resource reallocation during the technology upgrading that sustains the capital return.

2.2 Monopoly in Technology 2

Now suppose technology 2 is a new technology and only one potential entrant has access to it. Suppose this potential entrant is randomly chosen from the whole population. Technology 1 is still publicly and freely accessible. The ownership, hence the dividend, of each firm is equally divided by all the households. The question is whether the potential entrant will operate the second technology, given the endowment K and L. Here this potential entrant can be also interpreted as the "effective" coalition of all the people or firms that have access to technology 2.

Denote the wage rate by W and the gross rental rate of capital by R. Cost minimization implies that the unit costs under these two technologies are given, respectively, by

$$\mu_1(W,R) = \frac{R^{\alpha_1} W^{1-\alpha_1}}{A_1 (\alpha_1)^{\alpha_1} (1-\alpha_1)^{1-\alpha_1}}$$

and

$$\mu_2(W,R) = \frac{R^{\alpha_2} W^{1-\alpha_2}}{A_2 (\alpha_2)^{\alpha_2} (1-\alpha_2)^{1-\alpha_2}}. \tag{5}$$

Normalize $A_1 = 1$ and let $A_2 = A$. Thus $\mu_2(W,R) < \mu_1(W,R)$ if and only if

$$\frac{R}{W} < \psi \equiv \left[A \frac{\alpha_2^{\alpha_2} (1-\alpha_2)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1} (1-\alpha_1)^{(1-\alpha_1)}} \right]^{\frac{1}{\alpha_2-\alpha_1}}. \tag{6}$$

The factor markets are assumed to be perfectly competitive. Given the factor prices, cost minimization (no matter whether a firm has monopoly power or not) always implies that to produce Q units of output with technology 2 will require the following amount of the production factors:

$$L_2^*(Q,W,R) = \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2} \left(\frac{W}{R}\right)^{\alpha_2}}; \; K_2^*(Q,W,R) = \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2-1} \left(\frac{W}{R}\right)^{\alpha_2-1}}. \tag{7}$$

Similarly, to produce Q units of output with technology 1, it requires

$$L_1^*(Q,W,R) = \frac{Q}{\left(\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1} \left(\frac{W}{R}\right)^{\alpha_1}}; \; K_1^*(Q,W,R) = \frac{Q}{\left(\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1-1} \left(\frac{W}{R}\right)^{\alpha_1-1}}.$$

Observe that

$$k_1(Q,W,R) \equiv \frac{K_1^*(Q,W,R)}{L_1^*(Q,W,R)} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R}, \tag{8}$$

$$k_2(Q,W,R) \equiv \frac{K_2^*(Q,W,R)}{L_2^*(Q,W,R)} = \frac{\alpha_2}{1-\alpha_2}\frac{W}{R}. \tag{9}$$

If technology 2 is operated in the equilibrium, then it must be that $\frac{R}{W} \leq \psi$. The equilibrium price of the output, denoted by P, must be no larger than

$\mu_1(W, R)$ because of the free entry of competitive firms accessible to technology 1. What is the necessary and sufficient condition under which only technology 2 is operating (that is, the monopolist of technology 2 serves the whole economy)? The monopolist tries to maximize the profit:

$$\Pi = \max_{P \leq \mu_1(W,R)} [P - \mu_2(W, R)] \frac{Y}{P},$$

where Y is the total consumption expenditure of the economy. For the moment, suppose the monopolist takes Y and factor prices W and R as exogenously given. Then the unit elasticity implies that the monopolist would choose the highest possible price $P^* = \mu_1(W, R)$. Thus

$$\Pi = [1 - \frac{\mu_2(W, R)}{\mu_1(W, R)}]Y. \tag{10}$$

The total expenditure is equal to the total household wealth, which is the sum of profits, labor income and capital income:

$$Y = \Pi + WL + RK. \tag{11}$$

Combining (10) and (11), we obtain

$$Y = \frac{\mu_1(W, R)}{\mu_2(W, R)} (WL + RK).$$

In this general equilibrium, the market clearing conditions for the consumption good, capital and labor jointly imply

$$AK^{\alpha_2} L^{1-\alpha_2} = \frac{Y}{P},$$

or equivalently,

$$AK^{\alpha_2} L^{1-\alpha_2} = \frac{WL + RK}{\mu_2(W, R)}. \tag{12}$$

The right-hand side is the total production cost divided by unit cost, so it is equal to the total output given by the left-hand side. (12) and (5) jointly determine the equilibrium rental wage ratio:

$$\frac{R}{W} = \frac{\alpha_2}{(1 - \alpha_2) k}. \tag{13}$$

Thus the monopolist's operation condition (6) is precisely equivalent to

$$k > k_2^*, \tag{14}$$

where k_2^* is given by (2). Interestingly, recall that in the competitive equilibrium only technology 2 is operated if and only if $k > k_2^*$ (see Proposition 1). So the cutoff values for the capital labor ratio are identical for the perfect competition and monopoly. Moreover, when $k > k_2^*$, the monopoly also achieves the Pareto optimality. This is because the negative effect of the price markup on the consumption demand is exactly cancelled out by the positive income effect of the extra profit earning on the demand through the general equilibrium channel. Nonetheless, the equilibrium prices for the output are different for these two different market structures. Moreover, the monopoly profit is strictly positive, given by $\frac{\Pi}{W} = \frac{\left[\left(\frac{k}{k_2^*}\right)^{\alpha_2-\alpha_1}-1\right]}{(1-\alpha_2)}L$ when measured by the wage rate, while the profit is certainly zero in the perfectly competitive equilibrium.

If the ownership of the monopolist firm is no longer equally distributed among all the households, then the equilibrium is no longer egalitarian in terms of consumption distribution across different households, but the equilibrium is still Pareto optimal because the income distribution does not affect the total wealth of the economy or the aggregate demand for the consumption goods. The competitive equilibrium is, by contrast, always egalitarian independent of the ownership distribution of firm 2 because the dividend is always zero.

One key assumption in the above analysis is that the monopolist takes Y and factor prices W and R as exogenously given. However, the monopolist produces all the output in this closed economy , so it can actually affect the factor prices by choosing different output levels. Hence we will assume from now on that the monopolist is sophisticated enough to take all the general equilibrium effect into account.

For, simplicity, first consider the necessary and sufficient condition under which both technologies are operating (the monopolist of technology 2 only serves a fraction of total demand in the whole economy). The monopolist maximizes the profit by choosing the optimal price P and quantity Q:

$$\Pi = \max_{P \leq \mu_1(W,R); Q \geq 0} \left[P - \mu_2(W,R)\right] Q \tag{15}$$

Combining (7), (8) and factor markets clearing conditions yields

$$\frac{K_1^*}{L_1^*} = \frac{K - \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2-1}\left(\frac{W}{R}\right)^{\alpha_2-1}}}{L - \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(\frac{W}{R}\right)^{\alpha_2}}} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R},$$

which can be rewritten as

$$Q = Q(\frac{R}{W}) \equiv \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\left(\frac{R}{W}\right)^{1-\alpha_2}K - \frac{\alpha_1}{1-\alpha_1}L\left(\frac{R}{W}\right)^{-\alpha_2}\right]. \tag{16}$$

Thus $Q'(\frac{R}{W}) > 0$, implying a monotonic general-equilibrium relationship between the monopolist output Q and the rental wage ratio $\frac{R}{W}$. [[[[wrong: This is because when the rental wage ratio becomes smaller, the monopolist technology becomes more cost effective than the labor-intensive technology, so the substitution effect induces the monopolist to produce more, which means that the competitive firms (with technology 1) will produce less due to the resource constraint (general equilibrium effect). Since the monopolist will charge a markup price and produce less than the social optimal amount for any given demand, the increase in the output produced by the monopolist cannot compensate the decrease in the output produced by the competitive firms, so the net change in the total output goes down.]]]]

The total output is

$$Q_{total} = Q + Q_{1c}, \tag{17}$$

where Y_{1c} denotes the total output produced by the competitive firms with technology 1, so

$$Q_{1c} = \left(\frac{R}{W}\right)^{-\alpha_1} \left[\frac{\alpha_2}{1-\alpha_2} - \left(\frac{R}{W}\right)\frac{K}{L}\right] \frac{(1-\alpha_2)\,\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{(\alpha_2-\alpha_1)} L$$

SO

$$Q = \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\left(\frac{R}{W}\right)\frac{K}{L} - \frac{\alpha_1}{1-\alpha_1}\right] L \left(\frac{R}{W}\right)^{-\alpha_2}$$

Therefore

$$Q_{total} = \widehat{Y}(K, L, \frac{R}{W}) \equiv \left\{ \begin{array}{c} \alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left[\frac{\alpha_2}{1-\alpha_2} - \left(\frac{R}{W}\right)\frac{K}{L}\right]\left(\frac{R}{W}\right)^{-\alpha_1} \\ +A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)\left[\left(\frac{R}{W}\right)\frac{K}{L} - \frac{\alpha_1}{1-\alpha_1}\right]\left(\frac{R}{W}\right)^{-\alpha_2} \end{array} \right\} \frac{(1-\alpha_2)}{(\alpha_2-\alpha_1)} L \tag{18}$$

Now the monopolist solves (15) subject to (16). So the profit (measured by wage) is given by

$$\begin{aligned} \frac{\Pi}{W} &= \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\left(\frac{R}{W}\right)K - \frac{\alpha_1}{1-\alpha_1}L\right] \cdot \\ &\quad \left[\frac{\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]. \end{aligned} \tag{19}$$

Normalizing $W = 1$, (15) is equivalent to

$$\max_{R\le\psi}\left[\frac{R^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\left[RK - \frac{\alpha_1}{1-\alpha_1}L\right],$$

which yields the following first-order condition

$$\frac{(1+\alpha_1-\alpha_2)\,R^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}}k + \frac{(\alpha_2-\alpha_1)\,R^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)}\alpha_1^{1-\alpha_1} - \frac{(1-\alpha_1)^{1-\alpha_1}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}k = 0. \tag{20}$$

It can be verified that the second order condition also holds, so there exists a unique solution R to (20), denoted as $R = \Gamma(k)$, where function $\Gamma(k)$ is continuously differentiable and strictly decreasing.

This condition also implies that $R^* < \psi \Leftrightarrow k > k_1^* \Leftrightarrow Q > 0$. That is, the monopolist technology will operate if and only if the capital is sufficiently abundant ($k > k_1^*$). Again, this cutoff value is the same as in the perfect competitive equilibrium characterized previously.

In addition, according to (20), $k > 0$ and $R > 0$ would require that $R > (1+\alpha_1-\alpha_2)^{\frac{1}{\alpha_2-\alpha_1}}\psi$, which, together with (20), implies that R is a strictly decreasing function of k. Obviously, $\Gamma(k_2^*) < \Gamma(k_1^*) = \psi$. Let $\theta \in ((1+\alpha_1-\alpha_2)^{\frac{1}{\alpha_2-\alpha_1}}, 1)$, then $R = \theta\psi$ if and only if

$$k = \frac{A\alpha_1^{1-\alpha_1}\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}(\alpha_2-\alpha_1)(\theta\psi)^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)^{2-\alpha_1}\left[1-(1+\alpha_1-\alpha_2)\theta^{\alpha_1-\alpha_2}\right]}, \tag{21}$$

which implies that $k'(\theta) < 0$ or equivalently, $\theta'(k) < 0$. Thus (19) is reduced to

$$\Pi(\theta) = \frac{\alpha_1 L}{\alpha_2-\alpha_1}\frac{\left(\theta^{\alpha_1-\alpha_2}-1\right)^2}{\left[1-(1+\alpha_1-\alpha_2)\theta^{\alpha_1-\alpha_2}\right]}.$$

Thus Π is a strictly increasing function of k since $\Pi'(\theta) < 0$ and $\theta'(k) < 0$.

Since the monopolist serves only a fraction of the whole market, we have

$$Q(\frac{R}{W}) < AK^{\alpha_2}L^{1-\alpha_2}, \tag{22}$$

which is equivalent to $\frac{R}{W} < \frac{\alpha_2}{(1-\alpha_2)k}$ due to (16). Recall W is normalized to unity, so (22) holds if and only if

$$k = \frac{A\alpha_1^{1-\alpha_1}\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}(\alpha_2-\alpha_1)R^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)^{2-\alpha_1}\left[1-(1+\alpha_1-\alpha_2)\left(\frac{R}{\psi}\right)^{\alpha_1-\alpha_2}\right]} < \frac{\alpha_2}{(1-\alpha_2)R},$$

from which we obtain $k < k^*$, where we define

$$k^* \equiv k_2^*\left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1}{\alpha_2-\alpha_1}}. \tag{23}$$

Observe that $k^* > k_2^*$.

When $k = k^*$, only technology 2 is operating and the corresponding interest rate

$$R = \left[\frac{(1-\alpha_1)^{2-\alpha_1}\alpha_1^{\alpha_1}}{A\alpha_2^{\alpha_2-1}(1-\alpha_2)^{(1-\alpha_2)}[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2]}\right]^{\frac{1}{\alpha_1-\alpha_2}}.$$

The profit at this threshold value is

$$\Pi = \frac{L(\alpha_2 - \alpha_1)^2}{[1-\alpha_2][\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2]}.$$

When $k \geq k^*$, only technology 2 is operating and the market structure is monopoly, but there are potential entrants with technology 1. Since both technologies satisfy the Inada condition, both production factors must be fully used with positive equilibrium factor prices, which implies that the equilibrium aggregate output must be equal to $AK^{\alpha_2}L^{1-\alpha_2}$. However, the monopolist could choose to sell less than the total output if she wants. The total production cost is still $WL + RK$. So her optimization problem (after normalizing $W = 1$) is

$$\Pi = \max_{R, P \leq \mu_1(1,R); Q \leq AK^{\alpha_2}L^{1-\alpha_2}} P \cdot Q - [L + RK]. \tag{24}$$

Here we still impose the constraint $P \leq \mu_1(1, R)$ because otherwise the potential entrant would have stolen the whole market from the firm that has private access to technology 2.[6] The monopolist also understands that the total sales revenue $P \cdot Q$ must be equal to the aggregate wealth of all the households given by (11) with $W = 1$. Given that the monopolist can fully control the quantity to sell, she has no incentive to charge a price lower than $\mu_1(1, R)$, thus the optimal price $P = \mu_1(1, R)$. After substituting the optimal price into (24), we obtain the following first-order condition with respect to R :

$$R = \frac{\alpha_1}{1-\alpha_1}\left[\frac{K}{Q}\right]^{\frac{1}{\alpha_1 - 1}}. \tag{25}$$

The second-order condition is satisfied for the maximization problem. Substituting (25) back to (24) we obtain

$$\begin{aligned}
\Pi(K,L) &= \max_{Q \leq AK^{\alpha_2}L^{1-\alpha_2}} \frac{Q}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \left\{\frac{\alpha_1}{1-\alpha_1}\left[\frac{K}{Q}\right]^{\frac{1}{\alpha_1-1}}\right\}^{\alpha_1} - \left\{L + \frac{\alpha_1}{1-\alpha_1}\left[\frac{K}{Q}\right]^{\frac{1}{\alpha_1-1}} K\right\} \\
&= \max_{Q \leq AK^{\alpha_2}L^{1-\alpha_2}} Q^{\frac{1}{1-\alpha_1}} K^{\frac{\alpha_1}{\alpha_1-1}} - L \\
&= \left(A^{\frac{1}{1-\alpha_1}} k^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right) L,
\end{aligned} \tag{26}$$

where the third equality obtains by setting $Q^* = AK^{\alpha_2}L^{1-\alpha_2}$. So it is optimal for the monopolist to sell all the output. Thus (25) becomes

$$R = \frac{\alpha_1}{1-\alpha_1}\left[\frac{K}{AK^{\alpha_2}L^{1-\alpha_2}}\right]^{\frac{1}{\alpha_1-1}} = \frac{\alpha_1}{1-\alpha_1} A^{\frac{1}{1-\alpha_1}} k^{\frac{1-\alpha_2}{\alpha_1-1}}. \tag{27}$$

[6] One may think that once all the factors are used for production by the monopolist, potential entrants would be no longer able to affect the pricing decision of the monopolist because all the factors have been used up at that stage. However, in this static model we rule out this possibility by assuming that the pricing decisions by all the firms including the monopolist must be announced with full commitment at the same time as (or before) the wage and rental rate of capital are posted.

For comparison, in the perfect competition case with the same inputs, the rental price is equal to the marginal product of the capital:

$$\frac{R}{W} = \frac{\alpha_2 A k^{\alpha_2 - 1}}{(1-\alpha_2) A k^{\alpha_2}} = \frac{\alpha_2 k^{-1}}{(1-\alpha_2)},$$

so when $k = k^*$, the rental price of capital is smaller under the monopoly than in the perfect competition case.

Thus

$$\Pi(k^* L, L) = \left[\frac{\alpha_2 (1-\alpha_1)}{1-\alpha_2} \left[\frac{(\alpha_1)^{\alpha_1} (1-\alpha_1)}{\alpha_1 \alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2} \right]^{\frac{1}{1-\alpha_1}} - 1 \right] L.$$

We can show that

$$\Pi(k^* L, L) > \frac{L (\alpha_2 - \alpha_1)^2}{[1-\alpha_2] [\alpha_1 \alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2]}.$$

This means that the profit function $\Pi(K, L)$ is not continuous at the point when $K = k^* L$. It jumps up at that point. The reason is that the rental price of capital is depressed under the monopoly. However, the monopolist still achieves social efficiency since the aggregate output and aggregate consumption are the same as in the perfect competition. When $k < k_1^*$, which is given by (1), only technology 1 is operating in the equilibrium and $\frac{R}{W}$ must be strictly larger than ψ.

When $k \in (k_1^*, k^*)$, both technologies are operating. Let Q_i^m and Q_i^c denote the output produced with technology i ($i = 1, 2$) when technology 2 is monopolized and publicly available (so perfect competition for technology 2), respectively.

Lemma 2 *When $k \in (k_1^*, k^*)$, we must have $Q_1^m > Q_1^c$, $Q_2^m < Q_2^c$, $Q_1^m + Q_2^m < Q_1^c + Q_2^c$.*

Proof. Let $\widetilde{k}_1$ and $\widetilde{k}_2$ denote the equilibrium capital-labor ratios for the two technologies in the monopoly. Then by the factor market clearing conditions we obtain the total output for each technology:

$$Q_1^m = \Phi(\widetilde{k}_1, \widetilde{k}_2) \equiv \left(\frac{\widetilde{k}_2 L - K}{\widetilde{k}_2 - \widetilde{k}_1} \right) \left(\widetilde{k}_1 \right)^{\alpha_1}; \; Q_2^m = \Psi(\widetilde{k}_1, \widetilde{k}_2) \equiv A \left(\frac{K - \widetilde{k}_1 L}{\widetilde{k}_2 - \widetilde{k}_1} \right) \left(\widetilde{k}_2 \right)^{\alpha_2}, \tag{28}$$

where $\widetilde{k}_1$ and $\widetilde{k}_2$ are given by (8) and (9), respectively and $\frac{R}{W}$ is endogenously determined by (20). In the competitive equilibrium, we have

$$Q_1^c = \Phi(k_1^*, k_2^*); \; Q_2^c = \Psi(k_1^*, k_2^*),$$

where k_1^* and k_2^* are given by (1) and (2). Since (8)-(9) always hold, independent of the market structure in the goods market, $\frac{R}{W} < \psi$ implies $\widetilde{k}_1 > k_1^*$ and

$\widetilde{k}_2 > k_2^*$. Moreover, $\widetilde{k}_2 > k > \widetilde{k}_1$. Consequently. $Q_1^m > Q_1^c$ and $Q_2^m < Q_2^c$. This is because function $\Phi(\cdot,\cdot)$ strictly increases in both arguments while function $\Psi(\cdot,\cdot)$ strictly decreases in both arguments. The monopoly obviously distorts the resource allocation compared with the competitive equilibrium, so $Q_1^m + Q_2^m < Q_1^c + Q_2^c$. ■

Furthermore, we have

Proposition 3 *When $k \leq k_1^*$, technology 2 is not operating and the market achieves the competitive equilibrium with technology 1; When $k \in (k_1^*, k^*)$, both technologies are operating, the equilibrium $\frac{R}{W}$ (WLOG, $W \equiv 1$) is uniquely determined by (20), the total profit is given by (19), and the total outputs by technologies 1 and 2 are given by (28) ; When $k \geq k^*$, only technology 2 is operating with $\frac{R}{W}$ determined by (13) and the profit being $\frac{\Pi}{W} = \left(A^{\frac{1}{1-\alpha_1}} k^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right) L$, the total output is $AK^{\alpha_2}L^{1-\alpha_2}$, same as the competitive equilibrium output.*

When compared with the social optimal equilibrium (perfect competition), the new technology (technology 2) is adopted under monopoly at the same threshold value for capital endowment, but the total output is smaller than the first best when $k \in (k_1^*, k^*)$, and the old technology 1 is "overused" in the sense that industry 1 should have been abandoned when $k \in (k_2^*, k^*)$ but it still operates in the monopoly.

2.3 A Detour: Costly Acquisition of Technology 2

Now suppose technology 2 is still privately accessible to one firm but it would require a fixed cost to operate it for the first time. After one period, the technology can be freely imitated by all the other firms.

One Formulation:

Denote the entry cost by η (in the units of capital). Technology 2 will be adopted if and only if

$$\Pi(K, L) \geq R\eta.$$

Consider the simplest case when k is sufficiently large so that only technology 2 is operating.

$$\frac{\Pi}{W} = \frac{\left[\left(\frac{K-\eta}{Lk_2^*}\right)^{\alpha_2-\alpha_1} - 1\right]}{(1-\alpha_2)} L \geq \frac{R}{W}\eta,$$

where $\frac{R}{W} = \frac{\alpha_2 L}{(1-\alpha_2)(K-\eta)}$, so

$$\begin{aligned} \frac{\left[\left(\frac{K-\eta}{Lk_2^*}\right)^{\alpha_2-\alpha_1} - 1\right]}{(1-\alpha_2)} L &\geq \frac{\alpha_2 L}{(1-\alpha_2)(K-\eta)}\eta \\ \left(\frac{K-\eta}{Lk_2^*}\right)^{\alpha_2-\alpha_1} - 1 &\geq \frac{\alpha_2}{(K-\eta)}\eta \end{aligned}$$

There exists a unique $K^*(L,\eta)$ such that the above inequality holds if and only if $K \geq K^*(L,\eta)$, where $K_1^*(L,\eta) > 0$ and $K_2^*(L,\eta) > 0$.

Alternative Formulation: Denote the fixed cost by η (in the units of consumption goods, paid by credit). Technology 2 will be adopted if and only if

$$\Pi(K,L) \geq P\eta. \tag{29}$$

When $k \in (k_1^*, k^*)$, the above inequality becomes

$$\Pi(\theta) = \frac{\alpha_1 L}{\alpha_2-\alpha_1} \frac{\left(\theta^{\alpha_1-\alpha_2}-1\right)^2}{\left[1-(1+\alpha_1-\alpha_2)\,\theta^{\alpha_1-\alpha_2}\right]} \geq \frac{(\theta\psi)^{\alpha_1}\eta}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \tag{30}$$

so there must exist a unique $\theta^* \in ((1+\alpha_1-\alpha_2)^{\frac{1}{\alpha_2-\alpha_1}}, 1)$ such that (30) holds if and only if $\theta \in ((1+\alpha_1-\alpha_2)^{\frac{1}{\alpha_2-\alpha_1}}, \theta^*]$, or equivalently,

$$k \geq \frac{A\alpha_1^{1-\alpha_1}\alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}(\alpha_2-\alpha_1)(\theta^*\psi)^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)^{2-\alpha_1}\left[1-(1+\alpha_1-\alpha_2)\,\theta^{*\alpha_1-\alpha_2}\right]}.$$

When $k \geq k^*$, inequality (29) becomes

$$\frac{\left[\left(\frac{\alpha_2}{(1-\alpha_2)\frac{R}{W}k_2^*}\right)^{\alpha_2-\alpha_1}-1\right]}{(1-\alpha_2)}L \geq \frac{\left(\frac{R}{W}\right)^{\alpha_1}\eta}{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},$$

which determines an upper bound for $\frac{R}{W}$ and hence a lower bound for k. Moreover, this lower bound for k decreases with $\frac{L}{\eta}$ and A.

3 Dynamic Models

Now we extend our analysis to a dynamic environment to study the dynamic pattern of technology adoption with possible changes in the market structure. The key insights can be illustrated with a simple two-period model. Again, we first characterize the socially efficient equilibrium, where both technologies are publicly and freely available. The derived perfect competitive dynamic equilibrium serves as an analytical benchmark. Then we explore what happens when the capital-intensive technology is initially privately accessible to only one firm (or an effective coalition of some firms). We compare these two scenarios and discuss the possible welfare-improving role that government can play.

3.1 Perfect Competition

Consider a two-period perfect competition dynamic model.[7] A representative household is endowed with K_0 capital and L labor. Let E_t and $C_t$denote,

[7] Ju, Lin and Wang (2010) analytically characterize the equilibirum in an infinite-horizon dynamic growth model with infinite industries, which differ in their capital intensities. Wang (2011) study how international trade and dynamic trade policies affect the industrial upgrading when there are infinite industries in each country.

respectively, the capital used for production and consumption at period $t \in \{1, 2\}$. Capital good itself cannot be consumed but it can be produced with AK technology with the investment-specific technological progress parameter ξ. Without loss of generality, assume all the capital used for production fully depreciates. Consumption goods are perishable and thus are not saved in the equilibrium. Therefore, the artificial social planner solves the following problem.

$$\max \frac{C_1^{1-\sigma}-1}{1-\sigma}+\beta\frac{C_2^{1-\sigma}-1}{1-\sigma}$$

subject to

$$\begin{aligned}
E_2 &= \xi(K_0 - E_1), \\
C_1 &\leq G(E_1, L), \\
C_2 &\leq G(E_2, L), \\
E_1 &\geq 0, E_2 \geq 0, C_1 \geq 0, C_2 \geq 0, \\
& K_0 \text{ and } L \text{ are given.}
\end{aligned}$$

To make the analysis more interesting and empirically more relevant, we assume $\beta\xi > 1$ so that the capital production is sufficiently productive to support capital accumulation and technology upgrading. Depending on the different combination of technologies over different periods, we have six possible equilibrium patterns of technology adoption. Next, we characterize the necessary and sufficient conditions for all these patterns one by one.

Pattern 1: Only Technology 1 in both periods Establish the Lagrangian:

$$\mathcal{L} = \frac{\left[E_1^{\alpha_1}L^{1-\alpha_1}\right]^{1-\sigma}-1}{1-\sigma}+\beta\frac{\left[E_2^{\alpha_1}L^{1-\alpha_1}\right]^{1-\sigma}-1}{1-\sigma}+\lambda\left[\xi(K_0-E_1)-E_2\right],$$

which yields the following two first-order conditions relative to E_1 and E_2, respectively:

$$\begin{aligned}
\alpha_1\left[E_1^{\alpha_1}L^{1-\alpha_1}\right]^{-\sigma}E_1^{\alpha_1-1}L^{1-\alpha_1} &= \lambda\xi, \\
\beta\alpha_1\left[E_2^{\alpha_1}L^{1-\alpha_1}\right]^{-\sigma}E_2^{\alpha_1-1}L^{1-\alpha_1} &= \lambda.
\end{aligned}$$

We obtain

$$E_1 = \frac{K_0}{1+\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}};\ E_2 = K_0\frac{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{1+\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}.$$

$$\frac{1}{-\alpha_1+1+\alpha_1\sigma}$$

Since $\beta\xi > 1$and $\sigma \in [0, 1]$, to ensure $E_1 \leq k_1^*L$ and $E_2 \leq k_1^*L$, we must have

$$\frac{K_0}{L} \leq \theta_1 k_1^*, \tag{31}$$

where k_1^* is given by (8) and θ_1 is defined as

$$\theta_1 \equiv \frac{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}.$$

In other words, technology 1 alone is adopted in both periods if and only if (31) holds.

Pattern 2: Technology 2 in both periods Following the same method, we have:

$$E_1 = \frac{K_0}{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}};\ E_2 = K_0 \frac{(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}}{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}},$$

To ensure E_1 and E_2 larger than $k_2^* L$, we require

$$\frac{K_0}{L} \geq \theta_6 k_1^*, \tag{32}$$

where k_2^* is given by (9) and θ_6 is defined as

$$\theta_6 \equiv \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2)} \left(1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right).$$

In other words, technology 2 alone is adopted in both periods if and only if (32) holds.

Pattern 3: Technologies 1 and 2 in both periods We can derive from the first order conditions that

$$\begin{aligned} E_1 &= \frac{\left[1 - (\beta\xi)^{\frac{1}{\sigma}}\right] (1-\alpha_1) k_1^* L + \alpha_1 \xi K_0}{[(\beta\xi)^{\frac{1}{\sigma}} + \xi]\alpha_1}, \\ E_2 &= \xi(\frac{\alpha_1 (\beta\xi)^{\frac{1}{\sigma}} K_0 - \left[1 - (\beta\xi)^{\frac{1}{\sigma}}\right] (1-\alpha_1) (k_1^*) L}{[(\beta\xi)^{\frac{1}{\sigma}} + \xi]\alpha_1}). \end{aligned}$$

To ensure $k_1^* L < E_1, E_2 < k_2^* L$, we must have

$$\theta_3 k_1^* < \frac{K_0}{L} < \theta_4 k_1^*,$$

where

$$\theta_3 \equiv \frac{(\beta\xi)^{\frac{1}{\sigma}} + \xi\alpha_1 - (1-\alpha_1)}{\alpha_1 \xi}; \theta_4 \equiv \left[\frac{1}{\xi} + \frac{(\beta\xi)^{-\frac{1}{\sigma}} - (1-\alpha_2)}{\alpha_2}\right] \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2)}.$$

The non-emptiness of K_0 further requires $\theta_3 < \theta_4$, or

$$(\beta\xi) < \left[\frac{(1-\alpha_1)}{(1-\alpha_2)}\right]^{\sigma}. \tag{33}$$

That is, ξ has to be sufficiently small.

Pattern 4: Technology 1 in period 1, Technologies 1 and 2 in period 2 The Euler equation is

$$\begin{aligned} & \left(A_1 (k_1^*)^{\alpha_1} - \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} k_1^*\right) L + \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} \xi (K_0 - E_1) \\ = & \left[\frac{\beta\xi \left[A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}\right]}{\alpha_1 L^{(1-\alpha_1)(1-\sigma)} (k_2^* - k_1^*)}\right]^{\frac{1}{\sigma}} E_1^{\frac{\sigma\alpha_1 - \alpha_1 + 1}{\sigma}}. \end{aligned}$$

To ensure $E_1 \leq k_1^* L$, we need

$$\frac{K_0}{L} \leq \theta_3 k_1^*. \tag{34}$$

On the other hand,

$$\begin{aligned} & \left(A_1 (k_1^*)^{\alpha_1} - \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} k_1^*\right) L + \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} E_2 \\ = & \left[\frac{\beta\xi \left[A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}\right]}{\alpha_1 L^{(1-\alpha_1)(1-\sigma)} (k_2^* - k_1^*)}\right]^{\frac{1}{\sigma}} \left(K_0 - \frac{E_2}{\xi}\right)^{\frac{\sigma\alpha_1 - \alpha_1 + 1}{\sigma}}, \end{aligned}$$

To ensure $k_1^* L < E_2 < k_2^* L$, we get

$$\theta_1 k_1^* < \frac{K_0}{L} < \theta_2 k_1^*, \tag{35}$$

where

$$\theta_2 \equiv [\beta\xi]^{\frac{1}{-\sigma\alpha_1 + \alpha_1 - 1}} \left[\frac{1-\alpha_1}{1-\alpha_2}\right]^{\frac{\sigma}{\sigma\alpha_1 - \alpha_1 + 1}} + \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2) \xi}.$$

The non-emptiness for the set of K_0 requires $\theta_1 < \theta_2$, or equivalently,

$$[\beta\xi]^{\frac{1}{-\sigma\alpha_1 + \alpha_1 - 1}} < \frac{[\beta\xi]^{\frac{1}{\sigma}} - 1}{\alpha_1 \xi} + 1,$$

which must hold because $\beta\xi > 1$. Also we require

$$\theta_1 < \theta_3,$$

or equivalently,

$$\alpha_1 \xi < (\beta\xi)^{\frac{1}{-\alpha_1 + 1 + \alpha_1 \sigma}} \left[(\beta\xi)^{\frac{1}{\sigma}} + \xi\alpha_1 - 1\right],$$

which automatically holds. In summary, the equilibrium demonstrates this industrial pattern if and only if (34) and (35) hold.

Pattern 5: Technology 1 in period 1, Technology 2 in period 2 The Euler equation is

$$\beta\xi\alpha_2\left[\xi(K_0-E_1)\right]^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}=\alpha_1 E_1^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)}.$$

so $E_1 \leq k_1^* L$ implies

$$\beta\xi\alpha_2\left[\xi(K_0-k_1^*L)\right]^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}\geq\alpha_1\left(k_1^*L\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

which is equivalent to

$$\frac{K_0}{L}\leq\theta_5 k_1^*,$$

where

$$\theta_5\equiv\left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}+1.$$

On the other hand

$$\beta\xi\alpha_2 E_2^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}=\alpha_1\left(K_0-\frac{E_2}{\xi}\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)}$$

so $E_2 \geq k_2^* L$ implies

$$\beta\xi\alpha_2\left(k_2^*L\right)^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}\geq\alpha_1\left(K_0-\frac{Lk_2^*}{\xi}\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

which is reduced to

$$\frac{K_0}{L}\geq\theta_2 k_1^*.$$

In summary, we must have

$$\theta_2 k_1^*\leq\frac{K_0}{L}\leq\theta_5 k_1^*,$$

the non-emptiness of which requires $\theta_2 \leq \theta_5$, or equivalently

$$\left[(\beta\xi)^{-1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}}\leq\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)\xi}\left\{\left[(\beta\xi)^{-1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\right]^{-\frac{1}{1-\alpha_2+\sigma\alpha_2}}-1\right\}+1, \tag{36}$$

which holds if and only if (33) does not hold. In other words, Pattern 3 and Pattern 5 are incompatible with each other.

Pattern 6: Technologies 1 and 2 in period 1, Technology 2 in period 2:

$$\beta\xi\alpha_2\left[AE_2^{\alpha_2}L^{1-\alpha_2}\right]^{-\sigma}AE_2^{\alpha_2-1}L^{1-\alpha_2}$$

$$= \left[\begin{array}{c}\left(A_1\left(k_1^*\right)^{\alpha_1}-\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}k_1^*\right)L\\+\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}E_1\end{array}\right]^{-\sigma}\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}.$$

From $k_1^*L<E_1<k_2^*L$, we obtain

$$\theta_6k_1^*>\frac{K_0}{L}>\theta_5k_1^*. \tag{37}$$

To ensure $\theta_6>\theta_5$, we require

$$\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)}\left(1+\xi^{-1}\left(\beta\xi\right)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right)>\frac{\alpha_2}{\alpha_1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}+1,$$

which must be true because $\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}>1$ and

$$\frac{(1-\alpha_1)}{(1-\alpha_2)}\xi^{-1}\left(\beta\xi\right)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}>\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}.$$

On the other hand $E_2\geq k_2^*L$ requires

$$\frac{K_0}{L}\geq\theta_4k_1^*. \tag{38}$$

To ensure $\theta_6>\theta_4$, we require

$$\left(\beta\xi\right)^{\frac{1}{-\alpha_2+1+\sigma\alpha_2}}>\left[\frac{\left(\beta\xi\right)^{-\frac{1}{\sigma}}-1}{\alpha_2}\right]\xi+1,$$

which automatically holds because $\beta\xi>1$. In summary, the equilibrium has this pattern if and only if (37) and (38) hold.

To summarize, we have the following proposition:

Proposition 4 *Suppose $\beta\xi>1$ and $\sigma\in[0,1]$. When $\beta\xi\geq\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$ is satisfied, the dynamic equilibrium follows Pattern 1 when $\frac{K_0}{L}\in(0,\theta_1]$, Pattern 4 when $\frac{K_0}{L}\in(\theta_1,\theta_2)$, Pattern 5 when $\frac{K_0}{L}\in[\theta_2,\theta_5]$, Pattern 6 when $\frac{K_0}{L}\in(\theta_5,\theta_6)$, and Pattern 2 when $\frac{K_0}{L}\in[\theta_6,\infty)$. When $\beta\xi<\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$, then the dynamic equilibrium follows Pattern 1 when $\frac{K_0}{L}\in(0,\theta_1]$, Pattern 4 when $\frac{K_0}{L}\in(\theta_1,\theta_3]$, Pattern 3 when $\frac{K_0}{L}\in(\theta_3,\theta_4)$, Pattern 6 when $\frac{K_0}{L}\in[\theta_4,\theta_6)$, and Pattern 2 when $\frac{K_0}{L}\in[\theta_6,\infty)$.*

This proposition states that the dynamic pattern depends on the initial capital labor ratio and the technological parameter ξ. More specifically, when (33) is violated (that is, ξ is sufficiently large), then the equilibrium dynamic pattern of technology adoption is given by Figure 2.

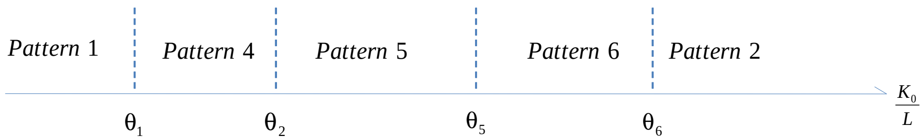

Figure 2: Dynamic Pattern of Technology Adoption when $\beta\xi \geq \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$

When (33) is satisfied (that is, ξ is sufficiently small, but still strictly larger than $\frac{1}{\beta}$), then

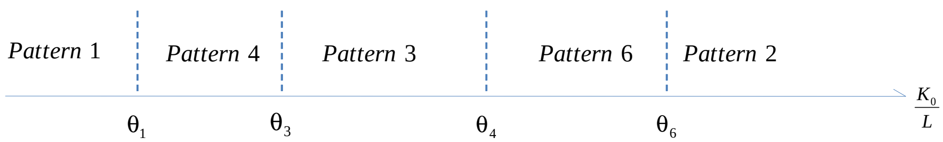

Figure 3: Dynamic Pattern of Technology Adoption when $\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$

3.2 Monopoly with Technology 2

Now consider a dynamic environment identical to the previous one except that technology 2 is privately accessible to only one firm, which decides when to implement this new technology. We call this firm "firm M" thereafter. If technology 2 is implemented in period 1, then we assume that the technology becomes publicly known in period 2 because people can successfully imitate this new technology after it is operated. If the implementation of technology 2 is delayed until period 2, then the monopoly rent can be reaped only in period 2. Since the second welfare theorem is not directly applicable, we need to solve the decentralized optimal decisions of households and firms. A representative household solves

$$\max \frac{C_1^{1-\sigma}-1}{1-\sigma} + \beta \frac{C_2^{1-\sigma}-1}{1-\sigma}$$

subject to

$$\begin{aligned} P_1C_1 + \frac{P_2C_2}{\widetilde{R}} &\leq (W_1L + R_1E_1 + \Pi_1) + \frac{(W_2L + R_2E_2 + \Pi_2)}{\widetilde{R}}, \qquad (39) \\ E_2 &= \xi(K_0 - E_1), \\ E_1 &\geq 0, E_2 \geq 0, C_1 \geq 0, C_2 \geq 0, \\ & K_0 \text{ is given.} \end{aligned}$$

From the household optimization, we obtain

$$\beta \left(\frac{C_2}{C_1} \right)^{-\sigma} = \frac{P_2}{P_1 \widetilde{R}}, \tag{40}$$

which, together with (39), implies

$$C_1 = \frac{(W_1 L + R_1 E_1 + \Pi_1) + \frac{(W_2 L + R_2 E_2 + \Pi_2)}{\widetilde{R}}}{P_1 + \beta P_1 \left[\frac{P_2}{\beta P_1 \widetilde{R}} \right]^{1-\frac{1}{\sigma}}}, \tag{41}$$

$$C_2 = C_1 \left[\frac{P_2}{\beta P_1 \widetilde{R}} \right]^{-\frac{1}{\sigma}}. \tag{42}$$

To ensure $E_1 E_2 > 0$, we must have the interest rate

$$\widetilde{R} = \frac{\xi R_2}{R_1}. \tag{43}$$

Substituting (43) back to (40), we obtain

$$\beta\xi \left(\frac{C_2}{C_1} \right)^{-\sigma} = \frac{P_2/R_2}{P_1/R_1}. \tag{44}$$

All the firms with public technology maximize the total profit by taking the factor prices as given. Firm M also maximizes its present discounted value of total profit. Firm M understands that it can change the relative factor prices by changing the output when technology 2 is operated for the first time. Firm M has three options. Option 1 is to start operating technology 2 in period 1; option 2 is to start operating technology 2 in period 2; option 3 is never to operate technology 2. It is obvious that the profit from option 3 is zero for this firm. In this closed economy, the goods market, labor market and capital market must be all cleared at each period. Next we will characterize the consequences of the first two options for Firm M.

Option 1: Immediate Operation of Technology 2

In period 1, technology 2 is either operated together with technology 1 (Possibility A) or solely operated (Possibility B). Therefore, the market structure is either a mixture of competition and monopoly or a pure monopoly in the first period, but in the second period the market structure is perfect competitive no matter what technologies will be used. Thus $\Pi_2 = 0$ and $\Pi_1 > 0$. In addition, there are three possible cases for the technology operations for period 2: only technology 1, only technology 2, and both technologies.

Suppose firm M uses up all the capital in period 1, then the output in period 2 is zero, which implies that P_2 will be positive infinity (assuming $\sigma \in (0, 1]$) driving up the rental price for capital high enough in period 1 such that it is not optimal for firm M to use so much capital. Thus, there are two possibilities.

Possibility A for Option 1. Both technologies are operated in period 1.

In this case, as suggested by (19), we have

$$
\begin{aligned}
P_1/R_1 &= \mu_1(W_1, R_1)/R_1 = \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1 - 1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
&= \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1 - 1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}},
\end{aligned}
$$

where, like (20), function $\Gamma(\frac{E_1}{L})$ is defined as the implicit solution of $\frac{R_1}{W_1}$ as a function of $\frac{E_1}{L}$ in the following equation

$$
\frac{(1+\alpha_1-\alpha_2)\left(\frac{R_1}{W_1}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}}\frac{E_1}{L} + \frac{(\alpha_2-\alpha_1)\left(\frac{R_1}{W_1}\right)^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)}\alpha_1^{1-\alpha_1} - \frac{(1-\alpha_1)^{1-\alpha_1}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\frac{E_1}{L} =
$$

$$
\frac{(1+\alpha_1-\alpha_2)\left(\frac{R_1}{W_1}\frac{E_1}{L}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}} + \frac{(\alpha_2-\alpha_1)\left(\frac{R_1}{W_1}\frac{E_1}{L}\right)^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)}\alpha_1^{1-\alpha_1} - \frac{(1-\alpha_1)^{1-\alpha_1}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\left(\frac{E_1}{L}\right)^{(\alpha_1-\alpha_2)} =
$$

so $\Gamma'(\frac{E_1}{L}) < 0$ but $\frac{E_1}{L}\Gamma(\frac{E_1}{L})$ as a whole is a strictly increasing function of $\frac{E_1}{L}$.

It remains to determine $\frac{E_1}{L}$. Note that the total output in period 1 is

$$
Y_1(E_1, L) = y_1\left(\frac{E_1}{L}\right)L,
$$

where

$$
y_1\left(\frac{E_1}{L}\right) \equiv \left\{ \begin{array}{c} \alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left[\frac{\alpha_2}{1-\alpha_2} - \Gamma(\frac{E_1}{L})\frac{E_1}{L}\right]\Gamma^{-\alpha_1}(\frac{E_1}{L}) \\ +A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)\left[\Gamma(\frac{E_1}{L})\frac{E_1}{L} - \frac{\alpha_1}{1-\alpha_1}\right]\Gamma^{-\alpha_2}(\frac{E_1}{L}) \end{array} \right\} \frac{(1-\alpha_2)}{(\alpha_2-\alpha_1)}. \tag{45}
$$

In period 2, both technologies are freely available and the market is perfectly competitive.

Possibility **A1-a.** Suppose $\frac{E_2}{L} \geq k_2^*$ so that only technology 2 is operated in period 2.

The market clearing conditions jointly imply that

$$
C_2 = A{E_2}^{\alpha_2}L^{1-\alpha_2}
$$

and

$$
\begin{aligned}
P_2/R_2 &= \mu_2(W_2, R_2)/R_2 = \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}} \\
\frac{W_2}{R_2} &= \frac{(1-\alpha_2)E_2}{\alpha_2 L} = \frac{(1-\alpha_2)\xi(K_0-E_1)}{\alpha_2 L}.
\end{aligned}
$$

In the equilibrium, $C_1 = Y_1$, hence (44) implies

$$\beta\xi\left(\frac{A\left[\xi(\frac{K_0}{L}-\frac{E_1}{L})\right]^{\alpha_2}}{y_1(\frac{E_1}{L})}\right)^{-\sigma} = \frac{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}$$

$$\beta\xi\left(y_1(\frac{E_1}{L})\right)^{\sigma} = \frac{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left(\frac{\xi(K_0-E_1)}{L}\right)^{1-\alpha_2+\alpha_2\sigma}}{A^{1-\sigma}\alpha_2\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}} \quad (46)$$

which uniquely determines $\frac{E_1}{L}$ because the left hand side strictly increases with $\frac{E_1}{L}$ while the right hand side strictly decreases with it. To support such an equilibrium, we require

$$\frac{E_1}{L}\in(k_1^*,k^*) \text{ and } \frac{\xi(K_0-E_1)}{L}\geq k_2^*.$$

which jointly imply that

$$\frac{K_0}{L}>\frac{k_2^*}{\xi}+k_1^*.$$

[[[observe that in possibility B1-a when $\sigma=1$, we have

$$\frac{K_0}{L}\geq\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*>\frac{k_2^*}{\xi}+k_1^*.$$

so it is compatible. However, when the initial endowment can support both **A1-a** and B1-a, firm M will choose B1-a since the latter gives a higher profit.

In addition, in possibility B1-c when $\sigma=1$, we have

$$\frac{\left[(\beta+\alpha_1)\xi-(1-\alpha_1)\right]k_1^*}{\alpha_1\xi}<\frac{K_0}{L}<\frac{(\beta+1)k_2^*}{\beta\xi},$$

and we also require

$$\beta\xi<\frac{(1-\alpha_1)}{(1-\alpha_2)}\left[\frac{\beta+\alpha_2}{\beta+\alpha_1}\right](<\frac{\alpha_2}{\alpha_1}\frac{1-\alpha_1}{1-\alpha_2})$$

which is compatible with **A1-a** only if

$$\begin{aligned}\frac{k_2^*}{\xi}+k_1^* &< \frac{(\beta+1)k_2^*}{\beta\xi}\\ &\Leftrightarrow \beta\xi<\frac{k_2^*}{k_1^*}=\frac{\alpha_2}{\alpha_1}\frac{1-\alpha_1}{1-\alpha_2}.\end{aligned}$$

Thus to summarize, B1-c and **A1-a** are mutually compatible only if

$$\frac{(\beta+1)k_2^*}{\beta\xi}>\frac{K_0}{L}>\max\{\frac{k_2^*}{\xi}+k_1^*,\frac{\left[(\beta+\alpha_1)\xi-(1-\alpha_1)\right]k_1^*}{\alpha_1\xi}\}$$

and

$$\beta\xi < \frac{\alpha_2}{\alpha_1}\frac{1-\alpha_1}{1-\alpha_2}.$$

Note that

$$\begin{aligned}\frac{k_2^*}{\xi} + k_1^* &> \frac{\left[\left(\beta+\alpha_1\right)\xi - \left(1-\alpha_1\right)\right]k_1^*}{\alpha_1\xi} \\ &\Leftrightarrow \frac{1-\alpha_1}{1-\alpha_2} > \beta\xi.\end{aligned}$$

However, firm M will prefer B1-c to **A1-a** when both are feasible.

]]]

The profit is given by

$$\Pi_1 = \pi_1(\frac{E_1}{L})W_1 L,$$

where

$$\begin{aligned}\pi_1(\frac{E_1}{L}) \equiv\ & \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(1-\alpha_1\right)\left(1-\alpha_2\right)}{\alpha_2-\alpha_1}\left[\Gamma(\frac{E_1}{L})\frac{E_1}{L} - \frac{\alpha_1}{1-\alpha_1}\right]\cdot \\ & \left[\frac{\Gamma^{\alpha_1-\alpha_2}(\frac{E_1}{L})}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}} - \frac{1}{A\left(\alpha_2\right)^{\alpha_2}\left(1-\alpha_2\right)^{1-\alpha_2}}\right]. \qquad (47)\end{aligned}$$

Thus $\pi_1'(\frac{E_1}{L}) > 0$.

As a special case, when $\sigma = 1$, then (46) becomes

$$\beta y_1(\frac{E_1}{L}) = \frac{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}\frac{(K_0-E_1)}{L}}{\alpha_2\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}}.$$

.

Possibility **A1-b.** Suppose $\frac{E_2}{L} \in (k_1^*, k_2^*)$ so that both technologies will be operated in period 2.

The market clearing conditions jointly imply that

$$C_2 = aE_2 + bL,$$

where $a \equiv \alpha_1\left(k_1^*\right)^{\alpha_1-1}$ and $b \equiv \left(1-\alpha_1\right)\left(k_1^*\right)^{\alpha_1}$. Observe that

$$\begin{aligned}\frac{P_1}{R_1} &= \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}} = \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}}{\left(\alpha_1\right)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}} \\ \frac{P_2}{R_2} &= \frac{1}{a} = \frac{1}{\alpha_1\left(k_1^*\right)^{\alpha_1-1}}\end{aligned}$$

(44) becomes

$$\beta\xi\left(\frac{a\xi(\frac{K_0}{L}-\frac{E_1}{L})+b}{y_1(\frac{E_1}{L})}\right)^{-\sigma} = \frac{\frac{P_2}{R_2}}{\frac{P_1}{R_1}} = \left(\frac{1-\alpha_1}{\alpha_1}k_1^*\Gamma(\frac{E_1}{L})\right)^{1-\alpha_1}, \qquad (48)$$

which can uniquely determine $\frac{E_1}{L}$.

Therefore, the profit is $\pi_1((\frac{E_1}{L})^*)W_1 L$. To support this equilibrium, we require

$$\xi(\frac{K_0}{L}-\frac{E_1}{L})\in(k_1^*,k_2^*)$$

and

$$\frac{E_1}{L}\in(k_1^*,k^*).$$

which jointly imply that

$$\frac{k_1^*}{\xi}+k_1^*<\frac{K_0}{L}<\frac{k_2^*}{\xi}+k^*.$$

[[[observe that in possibility B1-a when $\sigma=1$, we have

$$\frac{K_0}{L}\geq\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*>\frac{k_2^*}{\xi}+k^*.$$

]]]

Possibility **A1-c.** Suppose $\frac{E_2}{L}\in(0,k_1^*]$ so that only technology 1 will be operated in period 2.

Note that

$$\begin{aligned}\frac{P_1}{R_1} &= \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\\ \frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\\ \frac{W_2}{R_2} &= \frac{(1-\alpha_1)E_2}{\alpha_1 L}\end{aligned}$$

thus (44) becomes

$$\beta\xi\left(\frac{\xi^{\alpha_1}(\frac{K_0}{L}-\frac{E_1}{L})^{\alpha_1}}{y_1(\frac{E_1}{L})}\right)^{-\sigma}=\left(\frac{(1-\alpha_1)}{\alpha_1}\xi(\frac{K_0}{L}-\frac{E_1}{L})\Gamma(\frac{E_1}{L})\right)^{1-\alpha_1}\tag{49}$$

which can uniquely pin down $\frac{E_1}{L}$. We require

$$\begin{aligned}0 &< \xi(\frac{K_0}{L}-\frac{E_1}{L})\leq k_1^*\\ k_1^* &< \frac{E_1}{L}<k^*.\end{aligned}$$

which jointly imply that

$$\frac{k_1^*}{\xi}+k_1^*<\frac{K_0}{L}<\frac{k_1^*}{\xi}+k^*.$$

[[[observe that in possibility B1-a when $\sigma = 1$, we have

$$\frac{K_0}{L} \geq \left(\beta \frac{\alpha_2}{\alpha_1} + 1\right) k^* > \frac{k_1^*}{\xi} + k^*.$$

]]]

As a special case, when $\sigma = 1$, then (49) becomes

$$\beta y_1(\frac{E_1}{L}) = (\frac{K_0}{L} - \frac{E_1}{L}) \left(\frac{(1-\alpha_1)}{\alpha_1} \Gamma(\frac{E_1}{L})\right)^{1-\alpha_1}.$$

Possibility B for Option 1 : Only technology 2 is operated in period 1.

$$P_1/R_1 = \frac{\left(\frac{W_1}{R_1}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} = \frac{\left(\frac{E_1}{L}\right)^{1-\alpha_2}}{\alpha_1 A}$$

because

$$\frac{R_1}{W_1} = \frac{\alpha_1}{1-\alpha_1} A^{\frac{1}{1-\alpha_1}} \left[\frac{E_1}{L}\right]^{\frac{1-\alpha_2}{\alpha_1 - 1}}.$$

The profit is

$$\Pi_{B1} = \left(A^{\frac{1}{1-\alpha_1}} \left(\frac{E_1^*}{L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right) W_1 L.$$

Possibility **B1-a.** Only technology 2 is adopted in the perfect competitive market in period 2.

$$P_2/R_2 = \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}} = \frac{\left[\frac{(1-\alpha_2)}{\alpha_2}\frac{\xi(K_0-E_1)}{L}\right]^{1-\alpha_2}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}$$

Then (44) yields:

$$E_1^* = \frac{\xi K_0}{\left(\beta\xi\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{1-\alpha_2+\alpha_2\sigma}} + \xi} \tag{50}$$

and

$$E_2^* = \frac{\xi\left(\beta\xi\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{1-\alpha_2+\alpha_2\sigma}}}{\left(\beta\xi\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{1-\alpha_2+\alpha_2\sigma}} + \xi} K_0.$$

Thus the profit becomes

$$\Pi_{B1a} = \left[\left(A\left[\frac{\xi K_0/L}{\left(\beta\xi\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{1-\alpha_2+\alpha_2\sigma}} + \xi}\right]^{\alpha_2-\alpha_1}\right)^{\frac{1}{1-\alpha_1}} - 1\right] W_1 L.$$

To justify possibility B for option 1, we must have

$$E_1^* \geq k^* L \text{ and } E_2^* \geq k_2^* L$$

which are reduced to

$$\frac{\xi K_0}{\left(\beta \xi \frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{1-\alpha_2+\alpha_2\sigma}} + \xi} \geq k^* L. \tag{51}$$

In particular, when $\sigma = 1$, then (50) becomes

$$E_1^* = \frac{K_0}{\beta \frac{\alpha_2}{\alpha_1} + 1},$$

(51) becomes

$$\frac{K_0}{\beta \frac{\alpha_2}{\alpha_1} + 1} \geq k^* L. \tag{52}$$

and

$$\Pi_{B1a} = \left[\left(A \left[\frac{K_0/L}{\beta \frac{\alpha_2}{\alpha_1} + 1} \right]^{\alpha_2 - \alpha_1} \right)^{\frac{1}{1-\alpha_1}} - 1 \right] W_1 L.$$

Possibility **B1-b.** Only technology 2 in period 1 and only technology 1 in period 2.

Then (44) becomes

$$L^{(1-\sigma)(\alpha_2-\alpha_1)} \beta A^{\sigma-1} E_1^{\sigma\alpha_2+(1-\alpha_2)} = \xi^{-\alpha_1+\sigma\alpha_1} \left[(K_0 - E_1)\right]^{1-\alpha_1+\sigma\alpha_1}, \tag{53}$$

which uniquely determines E_1. Moreover

$$\frac{\partial E_1^*}{\partial \beta} < 0; \frac{\partial E_1^*}{\partial A} \geq 0; \frac{\partial E_1^*}{\partial L} \leq 0; \frac{\partial E_1^*}{\partial \xi} \leq 0; \frac{\partial E_1^*}{\partial K_0} > 0,$$

where "=" holds only when $\sigma = 1$, in which case we also have

$$E_1^* = \frac{K_0}{1+\beta}; E_2^* = \frac{\beta \xi K_0}{1+\beta}.$$

We require

$$\frac{K_0}{1+\beta} \geq k^* L; \ \frac{\beta \xi K_0}{1+\beta} \leq k_1^* L.$$

which can hold only when

$$k^* \left[1+\beta\right] \leq \frac{K_0}{L} \leq \frac{k_1^* \left[1+\beta\right]}{\beta \xi},$$

which is impossible because $\beta\xi > 1$.

When $\sigma < 1$, we require

$$E_1 \geq k^* L \text{ and } \xi(K_0 - E_1) \leq k_1^* L.$$

$$\beta\xi A^{\sigma-1}\left(\frac{E_1}{L}\right)^{\sigma\alpha_2+(1-\alpha_2)} = [\xi(K_0 - E_1)]^{1-\alpha_1+\sigma\alpha_1}$$

thus we must have

$$\beta\xi A^{\sigma-1}(k^*)^{\sigma\alpha_2+(1-\alpha_2)} \leq [\xi(K_0 - E_1)]^{1-\alpha_1+\sigma\alpha_1} \leq [k_1^*]^{1-\alpha_1+\sigma\alpha_1}.$$

And the related profit is

$$\Pi_{B1b} = \left(A^{\frac{1}{1-\alpha_1}}\left(\frac{E_1^*}{L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right) W_1 L.$$

Possibility **B1-c.** Only technology 2 in period 1 (that is, $\frac{E_1^*}{L} \in (k_1^*, k^*)$). Both technologies are adopted in the perfect competitive market in period 2.(that is, $\frac{\xi(K_0-E_1^*)}{L} \in (k_1^*, k_2^*)$)

(44) becomes.

$$\left(a\xi(\frac{K_0}{L} - \frac{E_1}{L}) + b\right)^{-\sigma} = \frac{A^{1-\sigma}}{\beta\xi (k_1^*)^{\alpha_1-1}\left(\frac{E_1}{L}\right)^{(1-\alpha_2)+\sigma\alpha_2}}, \tag{54}$$

which uniquely determines $\frac{E_1}{L}$. In particular, when $\sigma = 1$, we have

$$\begin{aligned}\frac{E_1}{L} &= \frac{a\xi\frac{K_0}{L} + b}{\beta\xi (k_1^*)^{\alpha_1-1} + a\xi} \\ \frac{E_2}{L} &= \frac{\xi(K_0 - E_1^*)}{L} = \frac{\frac{K_0}{L}\beta\xi - (1-\alpha_1)k_1^*}{\beta + \alpha_1}\end{aligned}$$

Recall $a \equiv \alpha_1 (k_1^*)^{\alpha_1-1}$ and $b \equiv (1-\alpha_1)(k_1^*)^{\alpha_1}$

hence the profit is given by

$$\Pi_{B1c} = \left(A^{\frac{1}{1-\alpha_1}}\left(\frac{a\xi\frac{K_0}{L} + b}{\beta\xi (k_1^*)^{\alpha_1-1} + a\xi}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right) W_1 L.$$

We require

$$\begin{aligned}\frac{[(\beta+\alpha_1)\xi - (1-\alpha_1)]k_1^*}{\alpha_1\xi} &< \frac{K_0}{L} < \frac{[(\beta+\alpha_1)\xi]k^* - (1-\alpha_1)k_1^*}{\alpha_1\xi} \\ \frac{(\beta+1)k_1^*}{\beta\xi} &< \frac{K_0}{L} < \frac{(\beta+1)k_2^*}{\beta\xi}\end{aligned}$$

which jointly imply that

$$\frac{\left[\left(\beta+\alpha_1\right)\xi-\left(1-\alpha_1\right)\right]k_1^*}{\alpha_1\xi}<\frac{K_0}{L}<\frac{\left(\beta+1\right)k_2^*}{\beta\xi}.$$

The above set is not empty iff

$$(\frac{1}{\beta}<)\xi<\frac{\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)}\left[\frac{\alpha_2+\beta}{\beta+\alpha_1}\right]\frac{1}{\beta} \tag{55}$$

Comparing B1-a and B1-c: Suppose (55) is satisfied,

$$\begin{aligned}\frac{K_0}{L} &\geq \left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^* \\ \frac{\left[\left(\beta+\alpha_1\right)\xi-\left(1-\alpha_1\right)\right]k_1^*}{\alpha_1\xi} &< \frac{K_0}{L}<\frac{\left(\beta+1\right)k_2^*}{\beta\xi}\end{aligned}$$

they are compatible if and only if

$$\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*<\frac{\left(\beta+1\right)k_2^*}{\beta\xi},$$

which is impossible because

$$\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)\left[\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1}{\alpha_2-\alpha_1}}<\frac{\left(\beta+1\right)}{\beta\xi}.$$

Thus B1-a and B1-c are mutually exclusive.

Comparing B1-c and A2,

$$\begin{aligned}\frac{\left[\left(\beta+\alpha_1\right)\xi-\left(1-\alpha_1\right)\right]k_1^*}{\alpha_1\xi} &< \frac{K_0}{L}<\frac{\left(\beta+1\right)k_2^*}{\beta\xi}; \\ \frac{K_0}{L} &\geq \frac{\left(1+\beta\right)}{\beta\xi}\left[\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}}A^{-\frac{1}{\alpha_2-\alpha_1}} \\ \beta\xi &< \frac{\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)}\left[\frac{\alpha_2+\beta}{\beta+\alpha_1}\right],\end{aligned}$$

which is impossible because

$$\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}>1.$$

Thus A2 and B1-c are mutually exclusive.

Option 2: Start Technology 2 in Period 2.

The market structure is perfectly competitive in period 1 with only technology 1 in operation. Thus

$$\begin{aligned}
\frac{P_1}{R_1} &= \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{R_1}{W_1} &= \frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1} \\
P_1 &= \frac{(R_1)^{\alpha_1} W_1^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
C_1 &= E_1^{\alpha_1} L^{1-\alpha_1}.
\end{aligned}$$

In period 2, there are two possibilities: Possibility A is that only technology 2 is operated in period 2. Possibility B is that both technologies are operated in period 2.

Possibility A for Option 2: only technology 2 is operated in period 2

$$\begin{aligned}
P_2 &= \frac{(R_2)^{\alpha_1} W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{R_2}{W_2} &= \frac{\alpha_1}{1-\alpha_1} A^{\frac{1}{1-\alpha_1}}\left(\frac{E_2}{L}\right)^{\frac{1-\alpha_2}{\alpha_1-1}} \\
C_2 &= A\left[\xi(K_0-E_1)\right]^{\alpha_2} L^{1-\alpha_2}
\end{aligned}$$

(44) implies

$$\beta\xi\left(\frac{A\left[\xi(K_0-E_1)\right]^{\alpha_2} L^{1-\alpha_2}}{E_1^{\alpha_1} L^{1-\alpha_1}}\right)^{-\sigma} = \left(\frac{L^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}}}{A^{\frac{1}{1-\alpha_1}} E_1 E_2^{\frac{1-\alpha_2}{\alpha_1-1}}}\right)^{1-\alpha_1} \tag{56}$$

which is reduced to

$$E_1^{1-\alpha_1+\sigma\alpha_1}\beta A^{1-\sigma} = L^{(\alpha_2-\alpha_1)(1-\sigma)}\xi^{(\sigma-1)\alpha_2}(K_0-E_1)^{1-\alpha_2+\sigma\alpha_2}. \tag{57}$$

Thus E_1^* can be uniquely determined by (57). In addition, we have

$$\frac{\partial E_1^*}{\partial \beta} < 0; \frac{\partial E_1^*}{\partial A} \leq 0; \frac{\partial E_1^*}{\partial L} \geq 0; \frac{\partial E_1^*}{\partial \xi} \leq 0; \frac{\partial E_1^*}{\partial K_0} > 0,$$

where "=" holds if and only if $\sigma = 1$, in which case

$$\begin{aligned}
E_1^* &= \frac{K_0}{1+\beta}; E_2^* = \frac{\beta\xi K_0}{1+\beta}; \\
\frac{R_1}{W_1} &= \frac{\alpha_1(1+\beta)}{(1-\alpha_1)}\frac{L}{K_0}; \frac{R_2}{W_2} = \frac{\alpha_1}{1-\alpha_1}A^{\frac{1}{1-\alpha_1}}\left(\frac{\beta\xi K_0}{L(1+\beta)}\right)^{\frac{1-\alpha_2}{\alpha_1-1}}; \\
\Pi_2 &= \left(A^{\frac{1}{1-\alpha_1}}\left(\frac{\beta\xi K_0}{(1+\beta)L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right)LW_2 \\
\frac{\Pi_{A2}}{\widetilde{R}} &= \beta\left(1 - \left[A\left(\frac{K_0\beta\xi}{L(1+\beta)}\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1
\end{aligned}$$

To ensure a non-negative profit, we require

$$\frac{K_0}{L} \geq \frac{(1+\beta)}{\beta\xi}A^{-\frac{1}{\alpha_2-\alpha_1}}. \tag{58}$$

To justify that firm M can serve the whole market in period 2 with technology 2, we require

$$E_2^* \geq k^* L,$$

which means

$$\frac{K_0}{L} \geq \frac{(1+\beta)}{\beta\xi}\left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} A^{-\frac{1}{\alpha_2-\alpha_1}}, \tag{59}$$

which automatically ensures (58) because

$$\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2} > 1 \text{ and } \left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1} > 1.$$

For more general $\sigma \in [0, 1)$, we have

$$\frac{\Pi_{A2}}{\widetilde{R}} = \left[\frac{L^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}}}{\xi^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}}E_1^*\left[(K_0-E_1^*)\right]^{\frac{1-\alpha_2}{\alpha_1-1}}}\left(\left(\frac{\beta\xi K_0}{(1+\beta)L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - A^{-\frac{1}{1-\alpha_1}}\right)\right]LW_1.$$

Now we can compare $\frac{\Pi_{A2}}{\widetilde{R}}$ with the profit in possibility B for option1, Π_{B1}. In particular, when $\sigma = 1$, we have

$$\begin{aligned}
\frac{\Pi_{A2}}{\widetilde{R}} &> \Pi_{B1a} \Leftrightarrow \\
\beta + 1 - \left(A\left[\frac{1}{\beta\frac{\alpha_2}{\alpha_1}+1}\right]^{\alpha_2-\alpha_1}\right)^{\frac{1}{1-\alpha_1}}(K_0/L)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} &> \beta\left[A\left(\frac{\beta\xi}{(1+\beta)}\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}(K_0/L)^{-\frac{\alpha_2-\alpha_1}{1-\alpha_1}}
\end{aligned}$$

which holds if and only if

$$x_1 < (K_0/L)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} < x_2 \tag{60}$$

where

$$x_{1,2} \equiv \frac{(1+\beta) \mp \sqrt{\Delta}}{2\left(A\left[\frac{1}{\beta\frac{\alpha_2}{\alpha_1}+1}\right]^{\alpha_2-\alpha_1}\right)^{\frac{1}{1-\alpha_1}}}$$

$$\Delta \equiv (1+\beta)^2 - 4\beta\left(\frac{(1+\beta)}{\beta\xi\left[\beta\frac{\alpha_2}{\alpha_1}+1\right]}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} > 0$$

Therefore, when (60) holds, $\frac{\frac{\Pi_{A2}}{\tilde{R}}}{\Pi_{B1}} > 1$; otherwise $\frac{\frac{\Pi_{A2}}{\tilde{R}}}{\Pi_{B1}} \leq 1$. Recall, we have to require (51) and (59). Therefore, when K_0/L is sufficiently large, it is better to operate technology 2 immediately (in period 1) than to delay to period 2. Recall that under Possibility B1-c, we require

$$\frac{K_0}{L} \geq \vartheta_1 \equiv \left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*. \tag{61}$$

and now under Possibility A2, we require (59):

$$\frac{K_0}{L} \geq \frac{(1+\beta)}{\beta\xi}k^*.$$

Thus they are comparable only when (61) is satisfied. Define

$$\upsilon_{1,2} \equiv A^{\frac{1-\alpha_1}{1-\alpha_2}}\left[\frac{2\left[\beta\frac{\alpha_2}{\alpha_1}+1\right]^{\frac{1-\alpha_2}{1-\alpha_1}}k^*}{(1+\beta)\mp\sqrt{\Delta}}\right]^{\frac{(1-\alpha_1)(\alpha_2-\alpha_1)}{1-\alpha_2}}.$$

Lemma 5 *Suppose $\sigma = 1$. [1] When $A > \upsilon_1$, option B1-a is better than option A2 if $\frac{K_0}{L} \in [\vartheta_1, x_1) \cup (x_2, \infty)$ and the opposite is true if $\frac{K_0}{L} \in (x_1, x_2)$. These two options are indifferent for the firm M when $\frac{K_0}{L} = x_1$ or x_2. [2] When $A \in (\upsilon_2, \upsilon_1)$, option B1-a is better than option A2 if $\frac{K_0}{L} \in (x_2, \infty)$ and the opposite is true if $\frac{K_0}{L} \in (\vartheta_1, x_2)$. These two options are indifferent for the firm M when $\frac{K_0}{L} = x_2$; [3] When $A < \upsilon_2$, option B1-a is better than option A2 whenever $\frac{K_0}{L} \in (\vartheta_1, \infty)$.*

Possibility B for Option 2: both technologies are operated in period 2

$$
\begin{aligned}
P_2 &= \frac{(R_2)^{\alpha_1} W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{R_2}{W_2} &= \Gamma(\frac{E_2}{L})
\end{aligned}
$$

Euler equation (44) becomes

$$
\beta\xi\left(\frac{\widehat{Y}(\xi(K_0-E_1),L,\Gamma(\frac{\xi(K_0-E_1)}{L}))}{E_1^{\alpha_1}L^{1-\alpha_1}}\right)^{-\sigma}=\frac{P_2/R_2}{P_1/R_1}=\left(\frac{W_2}{R_2}\frac{R_1}{W_1}\right)^{1-\alpha_1},
$$

where function $\widehat{Y}(.,.,.)$ is defined in (18). This implies

$$
\beta\xi\left(\frac{\widehat{Y}(\xi(K_0-E_1),L,\Gamma(\frac{\xi(K_0-E_1)}{L}))}{E_1^{\alpha_1}L^{1-\alpha_1}}\right)^{-\sigma}=\left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}\right)^{1-\alpha_1}, \tag{62}
$$

which uniquely determines E_1, and hence everything else. We need to impose that

$$
\frac{\xi(K_0-E_1)}{L}\in(k_1^*,k^*).
$$

thus

$$
\frac{k_1^*}{\xi}<\frac{K_0}{L}<\frac{k^*}{\xi}+\frac{E_1}{L}
$$

The related present discounted profit is

$$
\begin{aligned}
\frac{\Pi}{\widetilde{\widetilde{R}}} &= \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\Gamma(\frac{E_2}{L})\frac{E_2}{L}-\frac{\alpha_1}{1-\alpha_1}\right]\cdot \qquad (63)\\
&\left[\frac{\Gamma(\frac{E_2}{L})^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}-\frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\frac{1}{\xi\Gamma(\frac{E_2}{L})}\frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}LW_1.
\end{aligned}
$$

This implies that, for any given K_0, $\frac{\Pi}{\widetilde{\widetilde{R}}}$ strictly increases when E_2 increases and E_1 decreases, thus

$$
\begin{aligned}
\frac{\Pi}{\widetilde{\widetilde{R}}} &< \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\Gamma(k^*)k^*-\frac{\alpha_1}{1-\alpha_1}\right]\cdot\\
&\left[\frac{\Gamma(k^*)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}-\frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\frac{1}{\xi\Gamma(k^*)}\frac{\alpha_1}{1-\alpha_1}\frac{1}{\frac{K_0}{L}-\frac{k^*}{\xi}}LW_1
\end{aligned}
$$

In particular, when $\sigma = 1$, (62) becomes

$$\beta\xi\widehat{Y}^{-1}(\xi(K_0 - E_1), L, \Gamma(\frac{\xi(K_0 - E_1)}{L})) = \frac{1}{E_1}\left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\right)^{1-\alpha_1}$$

When comparing option B2 and B1a, suppose the initial endowment can support both options, then it can be shown that option B1a will bring a higher discounted present value of profit. The reason is that, not only the current value of profit in period 2 will be larger under option B1a, but also that the discounted rate will be smaller under option B1a. To see the latter, note that in order to transform the period-2 current value of profit into discounted present value at period 1 in the units of period-1 wage, the effective discounting multiplier is $\frac{W_2}{R_2}/\frac{W_1}{R_1}$, which is larger under option B1a because more capital is used in period 2 for given initial capital endowment.

Proposition 6 *Suppose* $\sigma = 1$, $\xi \geq \frac{(1-\alpha_1)}{(1-\alpha_2)}\left[\frac{\alpha_2+\beta}{\beta+\alpha_1}\right]\frac{1}{\beta}$ *and* $\frac{K_0}{L} \in [\vartheta_1, \infty)$. *The dynamic market equilibrium is the following: [1] When* $A > \upsilon_1$, *option B1-a is realized if* $\frac{K_0}{L} \in [\vartheta_1, x_1) \cup (x_2, \infty)$ *and option A2 is realized if* $\frac{K_0}{L} \in (x_1, x_2)$. *There are two equilibria (option B1-a and option A2) when* $\frac{K_0}{L} = x_1$ *or* x_2. *[2] When* $A \in (\upsilon_2, \upsilon_1)$, *option B1-a is realized if* $\frac{K_0}{L} \in (x_2, \infty)$ *and the option A2 is realized if* $\frac{K_0}{L} \in [\vartheta_1, x_2)$. *There are two equilibria (option B1-a and option A2) when* $\frac{K_0}{L} = x_2$; *[3] When* $A < \upsilon_2$, *option B1-a is realized whenever* $\frac{K_0}{L} \in [\vartheta_1, \infty)$.

4 Industrial Policy

The previous positive analysis implies that there is scope for industrial policies in some cases because there exist discrepancies in the outcomes between the first best case (all technologies are publicly and freely available so the market is perfect competitive) and the market equilibrium when the knowledge to best operate the new technology (which is more capital intensive) first becomes accessible to only a subset of the investors who can form coalition as a single monopolist and such knowledge becomes publicly information only after it is operated for a while. The sole source of market inefficiency comes from the fact that the new technology is not publicly available until it is operated for a while by the first mover. The magnitude of this inefficiency, however, endogenously and non-monotonically depends on the endowment structure. More specifically, efficiency will be restored when capital endowment is sufficiently large or sufficiently small. Therefore, welfare-enhancing industrial policies could be in two forms: one is to change the endogenous monopoly market structure, the other is to change the relative factor prices faced by firms. We will discuss these two alternatives one by one.

4.1 Changing the Market Structure

The monopoly in this model is due to the fact that the new technology and how to operate it profitably in the local context is only privately known at the beginning and imitation takes time before it becomes fully publicly available. Who acquires this information advantage is taken as given in this paper. Antitrust law does not necessarily help or even makes things worse because it can delay the implementation of the new technology even further, as the potential investor who has private information to this technology would simply hold the idea without implementing it. Antitrust laws may work only after the first mover already enters.

A more efficient way, theoretically speaking, is for government to help collect and disseminate the information about the new technology and the relevant information how to operate it profitably such as the information on marketing, on searching for the right market and right customer group, *etc.* For example, the government may support the training programs for workers and entrepreneurs, collect the general market information as public service, reduce the regulations and lower the entry barrier for new firms, *etc.*

Of course information collection is costly and it is impossible for the government to collect all the information for all the infinite technologies on the candidate menu. So in addition to the aforementioned industry-neutral general policies, the government may also take some industrial policies that are more industry-specific. Given that the government may not know better than the individual firms when selecting the most promising technology before collecting the relevant information, it is desirable for the government to conduct timely surveys from a widely covered group of potential investors asking about which new industry and what kind of information they are most interested in, and then the government could help collect the specific information or provide the necessary public goods that are most commonly demanded.[8]

To the extent that the government may not be able to collect the full information partly because investors/producers may have heterogeneous characteristics and therefore may need different types of information, it is desirable to reward the individual firms who take the initiatives to explore actively rather than wait passively for the government help. In that sense, the temporary monopoly rent can partly compensate the first mover cost. When the incurred information cost is sufficiently small, the technology upgrading can be achieved by the market without government support, although the monopoly causes some social inefficiency as described in the model. This partly explains why industrial upgrading sometimes takes place without active industrial policies. However, when the incurred cost is too large, partly because the monopoly power is by nature not

[8] Lin and Monge (2010) provide a very concrete six-step policy precedures for the government in the developing countries to identify the right industrial target and then facilitate industrial upgrading. For example, a rule of thumb is to check what are the most prosperous industries in those economies whose GDP per capita is about twice larger. Those industries are on average more likely to be the industries consistent with your latent comparative advantage.

legally protected, appropriate government intervention would be necessary to help overcome this upgrading hurdle.

Another useful way is to encourage foreign direct investment, and thus facilitate the technology diffusion subject to the market incentives. Essentially it not only helps facilitating the technology diffusion and learning, but also increase the number of potential first movers and reduces the likelihood or the duration of the monopoly.

Despite the importance, some of these recommendations of industrial policies and the caveats are not entirely new. According to our model, what seems insufficiently emphasized in the literature, is the set of policies aiming to rectify the relative factor prices so as to indirectly facilitate the adoption of new technology and simultaneously induce the timely abandonment of the obsolete technology, which we now turn to.

4.2 Changing the Relative Factor Prices

Our model suggests that when to implement the new technology and how much the monopolist wants to produce both depend on the relative prices of capital and labor, which is an important novel feature of this paper. Recall in our general equilibrium model, the monopolist may produce less than the social optimal because it fears that the rising rental price of capital relative to the wage would erode its monopoly profit if it increases its output (hence the demand for production factors). Correspondingly, there are several ways to improve the economic efficiency by rectifying the relative factor prices.

One way is to subsidize the capital cost for the monopolist of the new technology to the extent that the monopolist is willing to produce the social optimal amount. The subsidy could be in the form of loan credit or investment credit. Tax holidays could also help the capital accumulate faster and hence facilitates the adoption of the new technology. The rationale is different from the credit constraint argument.

Another way is to impose labor income taxes to discourage the employment in the labor-intensive industry so that the old labor-intensive industry shrinks to the optimal level, and the production scale of the new technology is expanded as the result of the changing market-clearing factor prices.

From a dynamic point of view, it may be desirable to encourage saving and capital accumulation by, for example, levying consumption tax, such that the potential inefficiency could be mitigated or even alleviated even if monopoly exists. Moreover, it also facilitate industrial upgrading.

For small economies, would it help improve economic efficiency by liberalizing the capital account and allowing for international capital flow? Not necessarily. First of all, although the monopolist behavior could no longer affect the price of the international capital market, yet the potential monopolist could still indirectly control the wage rate by changing its own output because the labor demand would shift across the two technologies unless there is a sufficiently large pool of surplus labor, which is less likely in small economies. Moreover, international capital flow weakens the resource constraint but the budget constraint for the households may not change much, therefore the general equilibrium effect is not clear.

5 Conclusion

In this paper, we develop a two-factor dynamic general equilibrium model to examine the industrial upgrading process, where the market structure may change endogenously depending on when the first mover chooses to start operating the new technology, which is initially privately accessible but becomes publicly available after one period lag. We show that endowment structure plays a very important role in determining the magnitude of the monopoly rent, thus affecting whether or not and when the new technology will be implemented. In particular, when the (initial) capital endowment is sufficiently scarce or sufficiently abundant, the equilibrium allocation (not prices) always achieves economic efficiency, no matter whether the new technology is privately or publicly available, hence independent from the market structure. The reason why monopoly may also achieve the first best allocation is because, different from the standard partial equilibrium analysis, the demand curve is not exogenous in the general equilibrium. It turns out that the negative price effect due to the price markup and the positive income effect due to the extra monopoly rent exactly cancel out each other. This is true in both static and dynamic economies.

However, inefficiency arises when the capital endowment falls onto some intermediate range, in which case both technologies are operating in the equilibrium and the potential monopolist will produce less than the social optimal amount. This is not because of the textbook partial-equilibirum argument that the monopolist reduces the supply of the output at given demand curve, but mainly because it wants to indirectly depress the rental price of capital relative to wage by demanding less capital in its production in a general-equilibrium fashion. In addition, in the static equilibrium, the cutoff value for capital above which the new technology starts operating is the same no matter whether this technology is privately or publicly known, but the cutoff value for capital above which the old industry completely exits is larger in monopoly than in the perfect competition case. This static result may suggest that the monopoly of the new technology creates distortions mainly by making the old industry exist "too late" rather than delaying the first adoption of the new technology. In the dynamic model, we show there are cases in which the potential monopolist would choose to delay operating the new technology but the new technology would be operating throughout the time if the new technology is initially publicly available.

Based on the positive analysis, we discuss how the market equilibrium may be improved by adopting industrial policies. One set of industrial policies is to try to alleviate the source that may cause monopoly while at the same time maintaining sufficiently strong incentives for private firms to adopt the new technologies in time and also fully exploit it. The other set of industrial policies is to rectify the relative factors price without trying to change the endogenous market structure.

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