Search for Optimal CEO Compensation: Theory and Empirical Evidence*

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Abstract

We integrate an agency problem into search theory to study executive compensation in the market equilibrium. Assuming that a CEO can choose whether to stay with a firm or quit and search after privately observing an idiosyncratic productivity shock to the firm, we show that it is optimal for the firm to set the pay-to-performance ratio to be less than one even when the CEO is risk neutral. More importantly, our market equilibrium endogenizes CEOs' and firms' outside options to reflect the externalities. In equilibrium, the indirect effects of the risks on the contract via the outside options dominate the direct effects of the risks. As a result, the equilibrium incentive contract exhibits new and important features, which are confirmed by our empirical tests using executive compensation data from 1992 to 2009. First, the equilibrium pay-to-performance sensitivity depends positively on a firm's idiosyncratic risk, and negatively on the systematic risk. This is in contrast to agency models with exogenous outside options, where the two risks always affect the pay-to-performance sensitivity in the same way. This result offers a plausible explanation for why the empirical relationship between the pay-to-performance sensitivity and a firm's total risk is ambiguous. Second, the ratio of a CEO's total compensation to firm value depends positively on idiosyncratic risks and negatively on systematic risks.

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We integrate an agency problem into search theory to study executive compensation in the market equilibrium. Assuming that a CEO can choose whether to stay with a firm or quit and search after privately observing an idiosyncratic productivity shock to the firm, we show that it is optimal for the firm to set the pay-to-performance ratio to be less than one even when the CEO is risk neutral. More importantly, our market equilibrium endogenizes CEOs' and firms' outside options to reflect the externalities. In equilibrium, the indirect effects of the risks on the contract via the outside options dominate the direct effects of the risks. As a result, the equilibrium incentive contract exhibits new and important features, which are confirmed by our empirical tests using executive compensation data from 1992 to 2009. First, the equilibrium pay-to-performance sensitivity depends positively on a firm's idiosyncratic risk, and negatively on the systematic risk. This is in contrast to agency models with exogenous outside options, where the two risks always affect the pay-to-performance sensitivity in the same way. This result offers a plausible explanation for why the empirical relationship between the pay-to-performance sensitivity and a firm's total risk is ambiguous. Second, the ratio of a CEO's total compensation to firm value depends positively on idiosyncratic risks and negatively on systematic risks.

Keywords: executive compensation, principal-agent problem, search, endogenous outside options, dynamic market equilibrium.

JEL classifications: J33, G13.

1. Introduction

Two issues concerning the executive compensation deserve particular attention. The first is how a firm's risk affects the executive's pay-to-performance sensitivity (hereafter PPS), i.e., the ratio of incentive pay to firm performance. Standard agency models predict that the PPS does not change with the firm's risk if the agent is risk neutral and decreases with the firm's risk if the agent is risk averse. Notable examples are Bolton and Dewatripont (2005, pp160-162), Holmstrom (1982), and Murphy (1999, pp27-28). In contrast to this theoretical prediction, the empirical evidence on the effect of the firm's risk on the PPS is ambiguous. For example, Core and Guay (1999) and Oyer and Shaefer (2005) find a positive relationship while Aggarwal and Samwick (1999) document a negative relationship.[1]

The second issue is the large increase of executive compensation along with the increase in firm size in the past three decades. This large increase has generated an intense debate in the public and the academia on whether CEOs are over-compensated. Although the increase in firm value contributed partly to the increase in executive pay, a closer look at the data reveals two notable features (see section 5 for a detailed description of the data). First, incentive pay, as the predominant component of executive pay, has increased more rapidly than the increase in firm value. From 1994 to 2009, median incentive pay increased by 276% in real terms, compared with a 42.5% increase in median firm value, and its share in total pay increased from 41.2% to 79.6%. Second, and related to the first feature, total executive pay outpaced firm value. The ratio between CEO pay (in millions) and firm value (in billions) increased from 1.62 in 1994 to 2.05 in 2009. These features suggest that the key to understanding the increase in executive compensation is to understand what factors determine the PPS.

We believe that two factors are intuitively important for the PPS, both arising from the notion that executive contracts should be designed to maximize firm value in a market economy. One is job mobility of CEOs. When different firms compete for CEOs, each firm has incentive to design contracts to increase the retention probability. Thus, changes in the market conditions can affect the PPS by affecting the severity of competition for CEOs. Another factor is the composition of risks faced by a firm. By switching from one firm to another, a CEO can change the amount of

[1] Prendergast (2002) summarizes additional conflicting empirical evidence on this relationship.

idiosyncratic risks to which he is exposed, but not the aggregate systematic risk since all firms face the same systematic risk. Thus, the PPS should depend on the two types of risks differently.

To incorporate these factors, we integrate an agency model into search theory to determine incentive contracts in a market equilibrium, and then empirically evaluate the model. Search theory endogenizes CEOs' and firms' outside options and it enables us to distinguish idiosyncratic risks from systematic risks. The integrated model captures the intuitive mechanism that competition among firms for CEOs affects incentive contracts in the equilibrium by affecting a CEO's incentive to participate in a firm. To isolate the effect of competition on the incentive contract from the effect of risk aversion, we focus on risk-neutral and effort-averse CEOs.

In our model, there are many firms and CEOs. In each period, a firm's output depends on an aggregate shock, an idiosyncratic shock, and the CEO's effort. The aggregate shock is publicly observed while the idiosyncratic shock, measuring the match quality between the firm and the CEO, is the CEO's private information. The firm offers an incentive contract which can be contingent on its output and the aggregate shock, but not directly on the idiosyncratic shock and the CEO's effort. The CEO decides whether to accept the offer after observing the idiosyncratic shock. If he quits, he can search for a new job. Due to the competition among firms, a CEO's outside option depends on the probability of getting a new job and the compensation at the new job. This link between a CEO's outside option and other firms' contracts implies that a market equilibrium must determine all firms' contracts and agents' outside options simultaneously. We focus on a stationary and symmetric equilibrium where all firms offer the same type of contracts.

As a step to determine the equilibrium, we first analyze an individual firm's optimal contract under arbitrarily fixed outside options for CEOs and firms. The optimal PPS is less than one, in spite of a risk neutral CEO. This result arises because a CEO can choose whether to quit after privately observing the idiosyncratic shock. If the idiosyncratic shock is contractible or the CEO is forbidden to quit, the optimal contract would set the PPS to one, as is well known in agency models with a risk-neutral agent. Such a contract would align the CEO's effort perfectly with the objective of maximizing the joint surplus of the match, and the firm would vary the salary with the idiosyncratic shock to obtain the maximum share of the joint surplus. When the idiosyncratic shock is the CEO's private information, it is not feasible to make the salary payment contingent on such a shock. The CEO will choose to stay to obtain the high payoff when the idiosyncratic shock is high, and will quit to insulate himself from the low payoff when the shock is low. In this setting, it is optimal for the firm to set the PPS below one in order to get part of the high surplus when the idiosyncratic shock is high and compensate for the low payoff when the CEO quits. In fact, the firm chooses the PPS and the salary to obtain the optimal trade-off between the retention probability and the expected profit conditional on retention.

Once the optimal PPS is below one, it can be affected by the aggregate and idiosyncratic risks. When the outside options are arbitrarily fixed, the two risks have the same qualitative effect on the PPS; namely, they reduce the PPS if and only if the sum of the CEO's and the firm's outside options for a period is positive. This effect of the risks on the PPS arises from the new mechanism in our model that a firm makes a trade-off between retention and profit conditional on retention, not from risk aversion as in standard agency models cited above. Specifically, an increase in either the aggregate or the idiosyncratic risk increases a firm's expected profit conditional on retention, which induces the firm to re-optimize. When the sum of the outside options for a period is initially high, the retention probability is low, in which case it is optimal for the firm to increase retention by increasing the salary and reducing the PPS. When the sum of the outside options for a period is initially low, then the retention probability is high, in which case it is optimal for the firm to increase the conditional profit by reducing the salary and increasing the PPS. The critical level of the sum of the outside options for a period that divides these two cases turns out to be zero. It is worth noting that negative outside options for a period are normal in an intertemporal setup where the CEO and the firm are sufficiently patient.

Next, we endogenize the outside options, determine the market equilibrium and explore the new predictions arising from the equilibrium. The outside options reflect two externalities. One is the interactions between different firms' contracts and the other is the dependence of the matching probability on the contracts through competitive entry of vacancies. We find that, for any given contract with a PPS not too large, the sum of a firm's and a CEO's outside options is increased by the idiosyncratic risk and reduced by the aggregate risk. This is intuitive. An increase in the idiosyncratic risk increases dispersion in match value, which induces both the CEO and the firm to search for a new match that might draw a high idiosyncratic productivity. In contrast, an increase in the aggregate risk has the same effect on all matches and reduces the value of searching relative to staying in the existing match. Moreover, we find that endogenous outside options are often the dominating force which determines the response of the equilibrium PPS to the risks. An increase in the sum of the outside options increases the optimal PPS by intensifying the competition for CEOs among the firms. When the PPS is not too large, an increase in the idiosyncratic risk increases the equilibrium PPS by increasing the sum of the outside options, while an increase in the aggregate risk reduces the equilibrium PPS by reducing the sum of the outside options. This difference between the two risks' effects on the optimal PPS is unique to the market equilibrium with search. When the outside options are exogenous, as in the agency literature cited above, the PPS responds to the two risks in the same direction.

Finally, we empirically test two new predictions of our model under the empirically supported assumption that the PPS is not too large. First, the equilibrium PPS depends negatively on the systematic risk and positively on the idiosyncratic risk; second, because the PPS responds to the two risks differently, so does the ratio of a CEO's total compensation to firm value in the equilibrium. This ratio depends negatively on the systematic risk and positively on the idiosyncratic risks. The empirical tests find robust support for these predictions which help addressing the two issues raised at the beginning of this introduction. Specifically, because the equilibrium PPS depends on the two risks in opposite ways, the theory can reconcile with the mixed empirical evidence on the role of total risk on the PPS. Moreover, the increase in firms' idiosyncratic risks between 1994 and 2004 (see section 5) may partly explain why executive compensation outpaced firm value in that period.

Our contribution to the labor search literature (e.g., Mortensen and Pissarides, 1994) is that the current study is the first paper to integrate incentive contracts into a search model to examine executive compensation.[2] To the principal-agent literature (e.g., Bolton and Dewatripont 2005 and references therein), our paper contributes in three dimensions. First, we explicitly model CEOs' quitting decisions and study incentive contracts that induce both optimal effort and optimal retention. Second, we endogenously determine the effects of market conditions on a CEO's outside option. Third, we analyze the optimal contract in a dynamic equilibrium in which firms

[2] In a summary of the new perspectives of search theory, Shi (2008) points out the integration of contract theory with search theory as a promising research agenda.

interact in the CEO job market. This dynamic equilibrium structure contrasts with a typical agency model that analyzes the optimal contract with a single agent-firm pair in a static setting. With the current setup, we show that a firm's specific and systematic risks have opposite effects on the PPS. Our result offers a possible explanation for the mixed evidence on the empirical relationship between a firm's total risk and the PPS.[3]

A more specific comparison is with Oyer (2004), who also recognizes that an agent may choose not to participate in a contract in certain states of the world. However, he assumes that the outside option is exogenous and does not study a market equilibrium. Moreover, he studies broad-based stock option plans for lower-ranked workers and abstracts from the effort-inducing mechanism on the ground that such plans have limited incentive effects on workers.

Our paper is related to Edmans, Gabaix and Landier (2009) in the assumption that the shocks and the agent's effort are multiplicative in a firm's profit function. However, they study a different mechanism (i.e., positively assortative matching) and their objective is to explain the negative relationship between the CEO's effective equity stake and firm size. They do not analyze the effects of risks on the PPS.

The rest of the paper is organized as follows. Section 2 describes the model, formulates individuals' decision problems and defines the market equilibrium. Section 3 examines the optimal contract under fixed outside options. Section 4 discusses the contracting externalities and determines the market equilibrium. Section 5 presents the empirical analyses, and Section 6 concludes the paper. Proofs and tables are relegated to the Appendix.

2. A Search Market with Incentive Contracts

In this section we describe the environment of the model economy, set up individual CEOs' and firms' decision problems and define the market equilibrium.

[3] A few other studies focus on the relationship between a firm's risk and the PPS from different perspectives. For example, Jin (2002) studies a CEO's portfolio diversification effect on the PPS. Shi (2011) differentiates respondable and non-respondable risk. Guo and Ou-Yang (2006) focus on the wealth effects while Bhattacharyya and Lafontaine (1995) on role of risk in franchising.

2.1. The environment of the model economy

Consider an infinite-horizon economy in discrete time. There are many infinitely-lived CEOs whose measure is normalized to one. In each period, a CEO is either employed or unemployed. If a CEO is unemployed, he receives utility B in the period, which includes the utility of extra leisure as well as monetary benefit during unemployment. In addition, the CEO can search for a job. If a CEO is employed, he chooses how much effort to exert at the job, e, and earns income w. Utility in the period is $u(w,e) = w - \frac{c}{2}e^2$, where the constant $c > 0$ reflects a CEO's effort aversion. Note that a CEO is risk neutral in income. This assumption ensures that risk aversion is not a determinant of the PPS as emphasized in the agency literature. Instead, we focus on a new mechanism that centers on the interactions between firms in the market equilibrium.

There are also many firms whose measure is endogenously determined by job creation. In each period, a firm either has or does not have a CEO. A firm without a CEO can incur a recruiting cost H to search for a CEO. If a firm has a CEO, profit in the period before paying the CEO is

$$\pi \equiv \pi(e,x,y) = e\, y^{\alpha}\, x^{1/2}, \quad \alpha > 1/2 \tag{2.1}$$

where x is a shock specific to the firm-CEO pair in the period and y is an aggregate or systematic shock in the period. The idiosyncratic shock x is identically and independently distributed on $[\underline{x}, \bar{x}]$ across matches and over time, where $\bar{x} > \underline{x} > 0$. We assume that the cumulative distribution function, $F_1(x)$, is uniform so that the mean of x is $\mu_x = (\underline{x} + \bar{x})/2$ and the standard deviation is $\sigma_x = (\bar{x} - \underline{x})/(2\sqrt{3})$. The aggregate shock y is identically and independently distributed over time according to the cumulative distribution function $F_2(y)$, with the mean as μ_y and the standard deviation as σ_y^2. Note that the profit function is multiplicative between effort e and the shocks (x,y), which captures the intuitive notion that the marginal productivity of effort is higher when a firm experiences higher shocks.[4] When CEOs are risk neutral, this specification is necessary for a non-trivial analysis. Namely, if the profit function is additive between effort and the shocks, then the shocks do not affect the optimal choice of effort and, with risk neutrality, this implies that the risks generated by the shocks have no effect on the PPS.

[4] For example, in the literature of CEO compensation, Edmans, Gabaix and Landier (2009) promote the multiplicative specification by arguing that a majority of CEO actions are "rolled out" across the entire company and hence have a greater effect in a larger firm. It is useful to note that a generalization of the profit function is $\pi = ey^{\alpha}x^{\beta}$. The analytical results are the same for all $\beta \geq 1/2$, but the algebra is simpler with $\beta = 1/2$.

The idiosyncratic risk can be understood as the match quality between the CEO and the firm in the current period, rather than a permanent characteristic of the firm, the CEO, or the match. For example, a high match quality means that a CEO's talent, experience, education, and personal objective match well in this particular period with the firm's size, nature of the business, strategic direction, organizational culture, and so on. A CEO who is well matched with a firm at one point of time may not be well matched with the firm at another time if one of the above mentioned features of the CEO or the firm has changed.[5] To capture the realistic feature that a CEO might have a better idea than a firm about the match quality, we assume that the realization of the idiosyncratic shock x is a CEO's private information. This assumption on x is central to the results in this paper because a CEO's quitting decision is non-trivial only when x is non-contractible, as we will demonstrate later.

A firm offers a sequence of one-period contracts to the CEO.[6] As in the literature, effort is a CEO's private information and not verifiable. In contrast, the aggregate shock y and profit π are publicly observed. However, knowing π and y is not sufficient for an outsider to disentangle effort e and the idiosyncratic shock x. To simplify the analysis and to facilitate the comparison with well-known models, we assume that the contract in each period has the following linear form:

$$w = w(\psi, \pi) = a + b\,\pi, \quad \text{where } \psi \equiv (a, b).$$

That is, total compensation for a CEO consists of salary, a, and a profit-sharing payment, $b\pi$. The profit-sharing ratio, b, is referred to as the PPS. Note that a and b can be functions of y, but they cannot be functions of the unobservable e and x. For brevity, we refer to ψ as a contract.

To clarify the elements of the economy, we specify the timing of events in each period in Fig. 1. A period consists of four stages. The first stage is exogenous separation in which a

[5]We focus on idiosyncratic heterogeneity occurring ex post rather than ex ante. It is well-known that a large fraction of the wage differential among workers cannot be explained by observable heterogeneity (see Mortensen, 2005). This is also likely to be the case for managers. An excellent CEO in a mining firm may or may not be a good CEO in a software firm. Specifically, Graham, Harvey and Puri (2010) document the evidence that CEOs' personal or behavioral traits such as optimism and managerial risk-aversion are related to corporate financial policies. They also show that certain types of firms appear to attract executives with particular psychological profiles and that CEOs' behavioral traits help explain compensation structure.

[6]In Appendix C, we add a long-term retention reward to the contract which resembles the option grant practice. We show that the qualitative features of the incentive contract remain the same. We do not consider fully dynamic (recursive) contracts because such contracts tend to generate the uncommon result that an agent's utility stochastically approaches the minimum, sometimes $-\infty$, in the long run (see Spear and Srivastava, 1987).

CEO separates exogenously from the firm into unemployment with probability $\delta \in [0, 1)$. This exogenous separation represents the turnover of CEOs caused by reasons other than the ones modeled explicitly here, such as job separation caused by family relocation. The second stage is contract offers and quitting decisions. In this stage, a firm with a CEO offers a contract to the CEO first and then the idiosyncratic shock x is realized, after which the CEO chooses whether to accept the contract or to quit into unemployment. The third stage is effort choice and production, whereby the aggregate shock y is realized, a CEO who stays with the firm chooses effort, profit is generated and the CEO is paid according to the contract.[7] The fourth stage is search and matching. Here, an unmatched CEO receives the benefit B and searches for a match, while a firm without a CEO pays the recruiting cost H to seek a CEO. Entry of vacancies is competitive. After search and matching, the period ends and another period starts.

The matching process is modeled as in Mortensen and Pissarides (1994). Denote v as the number of vacancies and s the number of searching CEOs at the search/matching stage in a period. The total number of new matches is given by the matching function $m(v, s) = vs/(v+s)$.[8] Denote the job market tightness as $\theta = s/v$, the matching probability of a searching CEO as $\lambda \equiv m(v, s)/s$, and the matching probability of a vacancy as $q \equiv m(v, s)/v$. We have:

$$\lambda = 1/(1+\theta) \quad \text{and} \quad q = \theta/(1+\theta) = 1 - \lambda.$$

These expressions reflect the intuitive property that, when there are more searching CEOs per vacancy, the matching probability is lower for a searching CEO and higher for a vacancy. Each CEO or firm takes the tightness and matching probabilities as given, because these characteristics depend only on the aggregate numbers of vacancies and searching CEOs.

2.2. Decisions of individual CEOs and firms

In each period, firms choose contracts first and then CEOs choose whether to quit or stay, followed by CEOs' effort choice. We analyze these decisions recursively in this subsection. In

[7] Whether y is realized before or after a CEO chooses effort matters only slightly for the analysis. If y is realized before the effort choice, y affects the decisions through $\mathbb{E}(y^{2\alpha})$, as shown in sections 3 and 4. If y is realized after the effort choice, y affects the decisions through $[\mathbb{E}(y^{\alpha})]^2$. If $\alpha \neq 1$ and if α is not too small, then the two terms have the common property that they increase in the variance of y. This is the property we will need in the analysis.

[8] The specific matching function has constant returns to scale and is strictly concave in the two arguments, v and s. The intuition for the main results of our paper should hold for more general matching functions, but the algebra becomes more complicated.

these decisions, an individual firm or CEO takes other firms' and CEOs' choices as given. Also, because contracts are assumed to be one-period, individuals take as given future contracts which affect the current period's choices only through the future value functions.

Examine first the optimal choice of effort by a CEO who has chosen to stay with the firm in the current period. Given the contract $\psi = (a, b)$ and the realizations of (x, y), the CEO chooses effort e to maximize utility $u(w, e)$, where $w = a + b\pi(e, x, y)$. Under the specified forms of u and π, the optimal choice of effort is given by the following first-order condition:

$$e^* = e^*(\psi, x, y) = by^\alpha x^{1/2}/c. \tag{2.2}$$

As is expected, optimal effort decreases in the effort-aversion parameter c, increases with the PPS, and is independent of the fixed salary. In addition, because the effort and shocks are multiplicative in the profit function, higher shocks induce higher effort. Given any contract ψ and the induced effort e^*, denote profit, the CEO's income and the CEO's utility, respectively, as follows:

$$\begin{aligned} \pi^* &= \pi^*(\psi, x, y) \equiv \pi(e^*(\psi, x, y), x, y) = by^{2\alpha}x/c, \\ w^* &= w^*(\psi, x, y) \equiv a + b\pi^* = a + b^2y^{2\alpha}x/c, \\ u^* &= u^*(\psi, x, y) \equiv a + \tfrac{1}{2}b^2y^{2\alpha}x/c. \end{aligned} \tag{2.3}$$

Next, we examine a CEO's quitting decision after observing x. If the CEO chooses to quit, he becomes unmatched. The value of this CEO is the same as that of a CEO who is unemployed at the beginning of the period, which is denoted V_S. If the CEO chooses to stay with the firm, utility in the current period is $\mathbb{E}_y(u^*)$, where $\mathbb{E}_y$ denotes the expectation over y. In addition, the CEO will start the next period as being matched, the value of which is denoted $V_{E,+1}$ where the subscripts $+1$ indicate the next period. The CEO accepts the contract if and only if $\mathbb{E}_y(u^*) + \beta V_{E,+1} \geq V_S$, where $\beta \in (0, 1)$ is the discount factor. We write this acceptance condition as

$$\mathbb{E}_y[u^*(\psi, x, y)] \geq \underline{u} \equiv V_S - \beta V_{E,+1}. \tag{2.4}$$

We call $\underline{u}$ the CEO's *effective outside option for the current period*. We substitute u^* from (2.3) and express (2.4) as a cut-off rule on the idiosyncratic shock x. That is, the CEO accepts the contract if and only if the realization of x satisfies $x \geq \rho\bar{x}$, where the cut-off ratio ρ is

$$\rho(\psi, \underline{u}) \equiv \frac{2c[\underline{u} - \mathbb{E}_y(a)]}{\bar{x}\,\mathbb{E}_y(b^2y^{2\alpha})}. \tag{2.5}$$

We keep a and b^2 inside the expectation operator $\mathbb{E}_y$ because these terms can be contingent on y in principle. As is expected, the cut-off is higher and quitting is more likely if the CEO's effective outside option for the period is higher. As well, a more generous salary and a higher PPS both reduce the cut-off and make the CEO less likely to quit, provided $\rho > 0$.

Let us compute the value function of a CEO. If a CEO enters a period as being matched, the value function is V_E. If the CEO separates from the firm, either exogenously or endogenously, the CEO obtains the value V_S. If the CEO is not separated from the firm in the current period, the additional value above V_S that the CEO obtains is $u^* + \beta V_{E,+1} - V_s = u^* - \underline{u}$. Because the CEO works for the firm if and only if he is not separated from the firm exogenously and if the realization of x is no less than $\rho\bar{x}$, a matched CEO's value obeys:

$$V_E = V_S + (1-\delta)\int_{\bar{x}\rho(\psi,\underline{u})}^{\bar{x}} \mathbb{E}_y[u^*(\psi,x,y) - \underline{u}]\, dF_1(x). \tag{2.6}$$

The integral over x reflects the fact that x is not realized when V_E is measured. If a CEO enters a period as being unmatched, he receives utility B and can search. With probability λ, the CEO finds a match at the end of the period, in which case the CEO enters the next period as a matched CEO whose discounted value is $\beta V_{E,+1}$. With probability $1-\lambda$, the CEO fails to find a match in which case the CEO's discounted value is $\beta V_{S,+1}$. Thus, V_S obeys:

$$V_S = B + \lambda\beta V_{E,+1} + (1-\lambda)\beta V_{S,+1}. \tag{2.7}$$

Now we examine a firm's contract offer and value function. Let J_F and J_H denote the value of a firm that enters the period with and without a CEO, respectively. Denote $\underline{J} = J_H - \beta J_{F,+1}$ as a firm's *effective outside option for the period.* We derive J_F similarly to V_E. For a firm that starts the period with a CEO, the firm may lose the CEO through exogenous separation or endogenous quits in the period, in which case the firm's value is J_H. If the CEO stays, the firm obtains net profit in the current period, $\pi^* - w^*$, and the discounted value in the future, $\beta J_{F,+1}$. The additional value above J_H is $\pi^* - w^* + \beta J_{F,+1} - J_H = \pi^* - w^* - \underline{J}$. Thus, J_F obeys:

$$J_F = J_H + (1-\delta)\max_{\psi}\int_{\bar{x}\rho(\psi,\underline{u})}^{\bar{x}} \mathbb{E}_y\left[\pi^*(\psi,x,y) - w^*(\psi,x,y) - \underline{J}\right] dF_1(x). \tag{2.8}$$

We denote the optimal contract for the maximization problem in (2.8) as $\psi^*(\underline{u},\underline{J},y)$ to emphasize its potential dependence on the two sides' outside options $(\underline{u},\underline{J})$ and the aggregate shock. Notice that (2.8) has incorporated the CEO's participation decision through the cut-off rule $\rho(\psi, \underline{u})$ and incentive compatibility of the effort choice through $e^*(\psi, x, y)$ which is embedded in (π^*, w^*).

The value of a firm with a vacant CEO position, J_H, can be computed similarly to V_S. The firm incurs a cost H to recruit in the period. With probability q, the firm will get a match in the period, in which case the firm's value will be $\beta J_{F,+1}$. With probability $1-q$, the firm will fail to get a match in the period, in which case the firm's value will be $\beta J_{H,+1}$. Thus,

$$J_H = -H + \beta \left[q J_{F,+1} + (1-q) J_{H,+1} \right]. \tag{2.9}$$

2.3. Definition of a market equilibrium

Because the outside options depend on the matching probabilities which are functions of the market tightness, we need to determine the number of searching CEOs, s, and the number of firms with vacant CEO positions, v. These numbers are measured immediately before the search process starts (see Fig. 1). Free entry of vacancies determines v. To determine s, we compute the change in the number of searching CEOs between the beginning of the search stages in the current and the next period, $s_{+1} - s$. In the current period, the number of new matches created and, hence, the flow out of the group of searching CEOs is λs, where λ is the matching probability for a searching CEO. Exogenous separation and endogenous quits in the next period generate the flow into the group of searching CEOs. This inflow is $(1 - s + \lambda s)[\delta + (1-\delta)F_1(\bar{x}\rho_{+1})]$ where $1 - s + \lambda s$ measures the number of CEOs in matches at the beginning of the next period and $\delta + (1-\delta)F_1(\bar{x}\rho_{+1})$ measures the probability with which a matched CEO will become unemployed through exogenous separation and endogenous quits in the next period. Thus,

$$s_{+1} - s = (1 - s + \lambda s)[\delta + (1-\delta)F_1(\bar{x}\rho_{+1})] - \lambda s. \tag{2.10}$$

Notice that the number of firms is endogenously determined as $v + 1 - s$, because v number of firms are searching for CEOs and $(1-s)$ number of firms are already matched with CEOs.

We adopt the following definition of an equilibrium. A stationary and symmetric *market equilibrium* consists of an individual firm's contract $\psi^*(\underline{u}, \underline{J}, y)$, an individual CEO's quitting rule $\rho(\psi, \underline{u})$ and effort rule $e^*(\psi, x, y)$, other firms' contracts $\tilde{\psi} = (\tilde{a}, \tilde{b})$, other CEOs' choices $(\tilde{\rho}, \tilde{e}^*)$, value functions (V_E, V_S, J_F, J_H), the implied effective outside options $(\underline{u}, \underline{J})$, and the numbers of searching CEOs and searching firms, (s, v), such that the following requirements are satisfied:

(i) Given any $(\psi, \underline{u})$, a CEO's decision rules $e^*(\psi, x, y)$ and $\rho(\psi, \underline{u})$ satisfy (2.2) and (2.5);

(ii) Given any $(\underline{u}, \underline{J})$, a firm's contract $\psi^*(\underline{u}, \underline{J}, y)$ solves the maximization problem in (2.8);

(iii) The value functions satisfy (2.6), (2.7), (2.8) and (2.9), while the effective outside options are given as $\underline{u} = V_s - \beta V_{E,+1}$ and $\underline{J} = V_H - \beta J_{F,+1}$;

(iv) Symmetry: $\psi = \tilde{\psi}$ and $(\rho, e^*) = (\tilde{\rho}, \tilde{e}^*)$;

(v) The number of searching CEOs, s, obeys the dynamics in (2.10);

(vi) Competitive entry of vacancies: v is such that the net value of creating a vacancy is $J_H = 0$;

(vii) Stationarity: $s_{+1} = s$, $J_{F,+1} = J_F$, $J_{H,+1} = J_H$, $V_{E,+1} = V_E$ and $V_{S,+1} = V_S$.

We focus on a stationary equilibrium because the economy described in subsection 2.1 is stationary. We focus on a symmetric equilibrium because all CEOs and firms conditional on their status (employed or unemployed, with or without a CEO, respectively) are ex ante homogeneous. Note that stationarity and symmetry imply only that the decision rules are time-invariant and symmetric between individuals/firms in the same status. Since these rules are functions of the realizations of the shocks, the outcomes of a firm's or a CEO's choices can still vary from period to period, and different realizations of the shocks make firms or CEOs heterogeneous ex post.

The model presented here differs from a standard static agency model with a single agent-firm pair in three dimensions: (i) The model distinguishes a firm's idiosyncratic risk from the systematic risk, as opposed to lumping them into the firm's total risk; (ii) a CEO can choose to quit after privately observing the idiosyncratic risk; and (iii) there are contracting interactions/externalities among the firms in the market equilibrium which work through endogenous outside options. In the next section we will explore the importance of elements (i) and (ii) by analyzing the optimal contract under any arbitrarily fixed outside options, $(\underline{u}, \underline{J})$. In section 4, we will examine the role of element (iii) and determine the equilibrium.

3. Optimal Contract Under Fixed Outside Options

In this section we determine the optimal contract when the effective outside options $(\underline{u}, \underline{J})$ are fixed. We also analyze how the two risks and the quitting decision affect the optimal contract. Even with fixed $(\underline{u}, \underline{J})$, our model differs from a static agency model not only in the presence of the two risks and the allowance for quitting, but also in the admissible region of the outside options. A static model assumes that $(\underline{u}, \underline{J})$ are positive or zero, which is an inappropriate assumption in an intertemporal setting because a match has future values. In the equilibrium, a firm's outside option for a period, $\underline{J} = J_H - \beta J_{F,+1}$, is negative because $J_H = 0$ by the free-entry condition of vacancies. A CEO's outside option for a period, $\underline{u} = V_S - \beta V_{E,+1}$, can also be negative when the discount factor is sufficiently close to one. Thus, the case with $\underline{u} + \underline{J} < 0$ is the normal case in the equilibrium. To maintain generality in this section, we allow for both $\underline{u} + \underline{J} < 0$ and $\underline{u} + \underline{J} \geq 0$.

3.1. The optimal contract

In the contracting problem in (2.8), a firm chooses $\psi = (a, b)$ by anticipating that the CEO's cut-off rule for quitting responds to the contract as $\rho(\psi, \underline{u})$. To emphasize the importance of the quitting choice, we reformulate the contracting problem by using (b, ρ) as the firm's choices. That is, the firm chooses b and recommends a cut-off ρ on $x/\bar{x}$, leaving the fixed salary a to ensure that the recommended cut-off ρ be consistent with the CEO's optimal quitting rule $\rho(\psi, \underline{u})$. This reformulation also simplifies the analysis of the optimal contract. We provide the reformulation in Appendix A. To facilitate the presentation of the results, denote $\Omega \equiv \frac{\bar{x}}{2c}\mathbb{E}_y(y^{2\alpha})$. Appendix A also rewrites the firm's objective function as $[1 - F_1(\rho\bar{x})]p(b, \rho)$, where $p(b, \rho)$ is the firm's expected surplus over y conditional on retaining the CEO and is defined as

$$p(b, \rho) \equiv \left[b^2\rho + b(1-b)(1+\rho)\right]\Omega - (\underline{u} + \underline{J}). \tag{3.1}$$

Proposition 3.1. *Assume that $(\underline{u}, \underline{J})$ are fixed and satisfy $\frac{\mu_x}{2\bar{x}}(\frac{3\underline{x}}{\bar{x}} - 1)\Omega < \underline{u} + \underline{J} < \Omega$. (i) The optimal choices (b^*, ρ^*) are unique and independent of the realization of the aggregate shock y. (ii) b^* and ρ^* are interior and satisfy the first-order conditions:*

$$\rho^* = 2b^* - 1, \quad \rho^* = \frac{1}{3}\left[1 + \frac{2(\underline{u} + \underline{J})}{b^*\Omega}\right]. \tag{3.2}$$

Expected salary is $\mathbb{E}_y(a^*) = \underline{u} - {b^*}^2\rho^*\Omega$. The unique (admissible) solution to (3.2) is:

$$b^* = \frac{1}{3} + \frac{1}{3}\left[1 + \frac{3}{\Omega}(\underline{u} + \underline{J})\right]^{1/2}, \quad \rho^* = \frac{2}{3}\left[1 + \frac{3}{\Omega}(\underline{u} + \underline{J})\right]^{1/2} - \frac{1}{3}. \tag{3.3}$$

(iii) An increase in either $\underline{u}$ or $\underline{J}$ increases (b^*, ρ^*) and the CEO's incentive pay $b^*\pi^*$. A higher $\underline{u}$ increases the expected salary $\mathbb{E}_y(a^*)$ but a higher $\underline{J}$ reduces the expected salary.

(iv) An increase in either the aggregate or the idiosyncratic risk (σ_y or σ_x) reduces (b^*, ρ^*) when $\underline{u} + \underline{J} > 0$ and increases (b^*, ρ^*) when $\underline{u} + \underline{J} < 0$. Also, an increase in either risk increases the CEO's incentive pay $b^*\pi^*$ at $x = \rho^*\bar{x}$ and reduces the expected salary $\mathbb{E}_y(a^*)$.

The optimal cut-off ratio ρ^* is independent of y because a CEO's quitting decision is made before observing y. The driving force for $b^* < 1$, which will be explained below, is a CEO's option to quit after privately observing x. Because this driving force is independent of the realization of y, so is the optimal b^*. For b^* and ρ^* to lie in the interior of $(0, 1)$, it is necessary and sufficient that $(\underline{u}+\underline{J})$ satisfies the bounds in the proposition. These bounds are satisfied in the equilibrium under certain restrictions on the parameters specified later in Proposition 4.1. In fact, these bounds on $(\underline{u} + \underline{J})$ yield $\frac{1}{2}(1 + \frac{\underline{x}}{\bar{x}}) < b^* < 1$ and $\rho^* \in (\underline{x}/\bar{x}, 1)$.

In contrast to our result $b^* < 1$, standard static agency models predict $b^* = 1$ for a risk neutral agent (e.g., Bolton and Dewatripont, 2005, pp160-162; Holmstrom 1982; and Murphy, 1999, pp27-28). This difference is caused by the assumptions that x is a CEO's private information and a CEO can quit. It is easy to see the role of quitting: if a CEO is forbidden to quit, then it is always optimal for a firm to extract all the surplus and set $b = 1$ to induce effort. As for the role of private information about x, consider the alternative assumption that x is publicly observable and contractible. In this case, a contract takes the form $\psi(\underline{u}, \underline{J}, x, y)$ instead of $\psi(\underline{u}, \underline{J}, y)$, and a CEO can still quit. In fact, it is optimal for the firm to induce the CEO to quit if the realization of x is so low that the joint surplus of the match is negative. When x is high enough to generate a positive joint surplus, the firm will use the contract to squeeze the CEO's expected surplus over y to zero. That is, (2.4) holds with equality for such x, which yields $\mathbb{E}_y(a) = \underline{u} - \frac{x}{2c}\mathbb{E}_y(b^2 y^{2\alpha})$. Then, for such x, the firm's expected surplus is

$$\mathbb{E}_y(\pi^* - w^* - \underline{J}) = \frac{x}{c}\mathbb{E}_y\left\{[\frac{b^2}{2} + b(1-b)]y^{2\alpha}\right\} - (\underline{u} + \underline{J}),$$

after substituting (π^*, w^*) from (2.3) and $\mathbb{E}_y(a)$ from the result just obtained. For each pair (x, y), the derivative of the firm's expected surplus with respect to $b(\underline{u}, \underline{J}, x, y)$ is $\frac{x}{c}[1 - b(\underline{u}, \underline{J}, x, y)]y^{2\alpha}$, which is strictly positive for all $b < 1$. Thus, when x is contractible, the optimal PPS is $b(\underline{u}, \underline{J}, x, y) = 1$ for all x high enough to make it worthwhile for the firm to keep the CEO. With $b^* = 1$, the firm makes the CEO's incentive in the effort choice perfectly aligned with the goal of maximizing the total expected surplus of the match.

Notice that this contract with $b^* = 1$ requires the expected salary over y to vary with the idiosyncratic shock in the form that $\mathbb{E}_y(a) = \underline{u} - \frac{x}{\bar{x}}\Omega$. That is, when x is high, the firm rewards the CEO through the incentive pay by giving $b = 1$ but, at the same time, reduces the salary. In fact, the firm adjusts the CEO's salary conditional on x to the extent that the CEO's value in the match in the period is equal to the effective outside option for the period.

In contrast, when x is only observed by the CEO, contracts cannot be contingent on the realization of x. As a result, the firm cannot squeeze the CEO's expected surplus to zero by adjusting the salary. Rather, the CEO is shielded from the negative surplus in the case $x < \rho^* \bar{x}$ by quitting. For all $x > \rho^* \bar{x}$, the CEO's expected surplus, $\mathbb{E}_y(u^* - \underline{u})$, is strictly positive and increases with the size of the "pie" generated by a higher x. Because the firm cannot condition the salary payment on x, the only way for the firm to get a share of this larger pie is to set the PPS below one.[9]

Once $b^* < 1$, the optimal PPS interacts with the quitting decision and is affected by the two risks, as stated in parts (iii) and (iv) of Proposition 3.1 and explained in the next two subsections.

3.2. The interactions between the optimal PPS and the quitting decision

The optimal choices (b^*, ρ^*) maximize a firm's expected surplus, $[1 - F_1(\rho\bar{x})]p(b, \rho)$, as stated before Proposition 3.1. That is, a firm makes the optimal trade-off between the retention probability, $1 - F_1(\rho\bar{x})$, and the expected surplus conditional on retention, $p(b, \rho)$. This trade-off is described by the first-order conditions in (3.2) which are depicted in Fig. 2. The left panel is the case $\underline{u} + \underline{J} > 0$ and the right panel is the case $\underline{u} + \underline{J} \leq 0$. In both panels, the straight line FOCb depicts the first equation in (3.2) which is the first-order condition of b^*, and the curve FOCρ depicts the second equation in (3.2) which is the first-order condition of ρ^*. The intersection between FOCb and FOCρ depicts the pair (b^*, ρ^*) given by (3.3).

The first-order condition of b^* gives a positive relationship between b^* and ρ^*. To explain this

[9] We do not model the possibility of contract renegotiation when the CEO chooses to quit. Such renegotiation complicates the analysis significantly because a CEO might pretend to quit just to renegotiate the contract even when x is high. Also, we assume that the firm has some inalienable asset of knowledge necessary for the operation, which makes it not optimal for the firm to sell the operation to the CEO. This assumption is implicit in the contract that all payments between the firm and the CEO must occur after production is carried out.

relationship, note that for any given ρ, the optimal choice of b maximizes the firm's conditional surplus $p(b, \rho)$; i.e., the marginal effect of b on the conditional surplus is zero. A higher PPS affects the firm's conditional surplus in two ways. One is that a higher PPS enables the firm to cut the salary and still induce the CEO to stay when $x = \rho\bar{x}$. This effect works through the term $b^2\rho$ in the firm's conditional surplus. The other effect of a higher PPS on the conditional surplus works through the amount of profit that the firm keeps, $b(1-b)(1+\rho)$. The overall effect of a higher PPS on the firm's conditional surplus, given by $(1+\rho-2b)\Omega$, increases with ρ. That is, the PPS and the cut-off are complementary with each other in the firm's conditional surplus. In particular, when ρ is higher, an increase in the PPS generates a larger increase in the CEO's incentive pay when $x = \rho\bar{x}$, which enables the firm to cut the expected salary by a larger amount and still induce the CEO to stay. Because of this complementarity, the PPS needs to increase with ρ in order to keep the marginal effect of b on the conditional surplus at zero.

The first-order condition of ρ^* gives an ambiguous relationship between the PPS and the cut-off. The relationship is positive if $\underline{u} + \underline{J} < 0$ and negative if $\underline{u} + \underline{J} > 0$. To explain this relationship, note that the cut-off affects both the retention probability and the firm's conditional surplus. A higher cut-off reduces the retention probability. Because the value of x conditional on a higher cut-off is higher in the first-order stochastic dominance, a higher cut-off also increases the firm's conditional surplus. When the cut-off is optimally set, the marginal effect on the firm's expected surplus is zero. The implied relationship between b^* and ρ^* is positive if and only if the PPS and the cut-off are complementary with each other in the firm's expected surplus, $[1 - F(\rho\bar{x})]p(b, \rho)$. Because the retention probability depends only on ρ, the firm's expected surplus has complementarity between the two choices if and only if the conditional surplus plays a relatively more important role than the retention probability in the firm's decision. This is the case when the retention probability is already high, in which case a further increase in the probability has only a small effect on the firm's expected surplus. In turn, for the retention probability to be sufficiently high, the sum of the two sides' effective outside options must be sufficiently low. The critical level of the sum of the outside options turns out to be zero. This explains why the relationship between b^* and ρ^* arising from the first-order condition of the cut-off is positive when $\underline{u} + \underline{J} < 0$ and negative when $\underline{u} + \underline{J} > 0$.

3.3. The effects of the outside options and the two risks

Part (iii) of Proposition 3.1 describes the effect of the outside options on the optimal contract. The outside options affect the optimal (b^*, ρ^*) only as the sum $(\underline{u} + \underline{J})$. An increase in $(\underline{u} + \underline{J})$ shifts up the curve FOCρ in Fig. 2 and leaves the curve FOCb intact, thus increasing the optimal PPS and cut-off. These effects are intuitive. When the CEO's outside option is higher, the CEO is less likely to stay, and so the cut-off on the idiosyncratic shock above which the CEO stays is higher. When the firm's outside option is higher, it is worthwhile for the firm to keep a match only if the firm's profit is increased. This requires the value of x conditional on $x \geq \rho^* \bar{x}$ to be higher and, hence, requires the optimal cut-off to be higher. When the optimal cut-off is higher, the optimal PPS is also higher because the two choices are complementary with each other in the firm's conditional surplus, $p(b, \rho)$, as explained in the last subsection.

Part (iv) of Proposition 3.1 states the effect of the risks on the optimal contract. We model an increase in a risk as an increase in the variance of the associated shock while the mean is fixed. A higher mean or variance of the aggregate shock increases $\mathbb{E}(y^{2\alpha})$, while a higher idiosyncratic risk increases $\bar{x} = \mu_x + \sigma_x \sqrt{3}$. Hence, the two risks affect (b^*, ρ^*) exclusively by increasing Ω (see 3.2). When Ω increases, the line FOCb in Fig. 2 does not change, while the curve FOCρ shifts down in the left panel and up in the right panel. Thus, an increase in either risk reduces (b^*, ρ^*) when $\underline{u} + \underline{J} > 0$ and increases (b^*, ρ^*) when $\underline{u} + \underline{J} < 0$.

The key to understanding these effects of the risks is to understand that, for any given contract, an increase in either risk increases the firm's conditional surplus defined in (3.1). Consider first an increase in the aggregate risk. Because the CEO's optimal effort is an increasing function of the aggregate shock and the profit function is multiplicative in effort and y, the combined effect of the aggregate shock and the induced effort on profit is $\mathbb{E}(y^{2\alpha})$. When the variance of the aggregate shock increases, $\mathbb{E}(y^{2\alpha})$ increases, and so does profit. Because the size of the "pie" increases, the firm's conditional surplus increases under any given contract. Consider next an increase in the idiosyncratic shock. For any given contract, a higher variance of the idiosyncratic shock increases the firm's conditional surplus by increasing the average value of x conditional on the CEO's acceptance of the match. Specifically, because the CEO stays only if $x \geq \rho^* \bar{x}$, the firm's profit behaves like a call option written on the idiosyncratic shock, with a strike price being set to the reservation value $\rho^*\bar{x}$. A higher variance of the idiosyncratic shock widens the interval of possible realizations of x on both sides of the mean. However, the widening of the interval of x on the left-side of the mean has no impact on the firm's profit since the CEO quits if the realization is below $\rho^*\bar{x}$. In contrast, the widening of the interval of x on the right-side of the mean increases the firm's profit and conditional surplus.

When the firm's conditional surplus increases with either risk, the firm must re-optimize in order to restore the optimal trade-off between the retention probability and the conditional surplus. The required adjustments in (b^*, ρ^*) depend on $\underline{u} + \underline{J}$. If $\underline{u} + \underline{J} > 0$, the cut-off is relatively high and the firm is more concerned about the term $(1 - \rho)$ associated with the retention probability than the conditional expected surplus (see the previous subsection). In this case, it is optimal for the firm to increase the retention probability by inducing the cut-off ρ^* to fall. If $\underline{u}+\underline{J} < 0$, however, the cut-off is low and the firm is more concerned about the conditional expected surplus than the term $(1 - \rho)$ associated with the retention probability. In this case, it is optimal for the firm to increase the conditional surplus and reduce the retention probability by inducing ρ^* to rise. In both cases, the PPS adjusts in the same direction as ρ^* because the two choices are complementary with each other in the firm's conditional surplus. Moreover, in order to induce effort, the firm uses part of the increase in profit to increase the CEO's incentive pay. Thus, even when b^* and ρ^* fall in the case $\underline{u} + \underline{J} > 0$, the CEO's incentive pay at $x = \rho^*\bar{x}$ still increases with Ω. Because total expected pay to the CEO is fixed at $\underline{u}$, the increase in the incentive pay must be accompanied by a fall in the expected salary.

Note that the optimal contract here responds to changes in the risks because the firm makes the trade-off between the retention probability and conditional surplus. This mechanism contrasts with risk considerations in standard static agency models with a risk-averse agent (Bolton and Dewatripont, 2005, pp160-162; and Murphy, 1999, pp27-28). There, it is optimal for a firm to reduce the PPS in order to limit the agent's exposure to increased risk. Because the agent dislikes risk, regardless of whether the risk is aggregate or idiosyncratic, the optimal PPS decreases with both risks.[10] We deliberately model a risk-neutral agent. Moreover, standard models presume

[10]Standard models typically assume that the profit function is additive in the agent's effort and the shocks, rather than the multiplicative form used in our model. However, even if standard models are modified to have a multiplicative profit function, risk aversion will remain to be the reason why the PPS is below one there, and so

that the outside options are non-negative. Because this presumption is likely to be invalid in an intertemporal setup with sufficiently patient players, our analysis in this section uncovers the result that the risks can increase the PPS when the outside options are fixed at negative levels.

4. Contracting Externalities and Properties of the Market Equilibrium

In this section we determine the effective outside options and the market equilibrium. By determining the outside options, we show how individual firms' choices generate contracting externalities. Moreover, the analysis reveals that the two risks affect the optimal contract differently in the equilibrium, in contrast to the result with fixed outside options. These properties of the equilibrium form a list of hypotheses that we will test empirically in Section 5.

Contracting externalities arise because the outside options $(\underline{u}, \underline{J})$ and the matching rates (λ, q) are endogenous in the equilibrium but are taken as given by individual firms. To understand these externalities, suppose that all firms choose a contract $\psi = (a, b)$ and that, as in an optimal contract, the cut-off on $x/\bar{x}$ is $\rho = 2b - 1 > 0$ and the expected salary is $\mathbb{E}_y(a) = \underline{u} - b^2 \rho \Omega$. We find how the outside options $(\underline{u}, \underline{J})$ depend on b. Given the contract ψ and the induced cut-off ρ, we can derive the following equations (see Appendix B):

$$\underline{u} = B - qL\Omega b^2(1-b)^2, \quad \underline{J} = -\beta J_F = -2L\Omega b(1-b)^2, \tag{4.1}$$

$$q = H/[2L\Omega b(1-b)^2], \quad \text{where } L \equiv \beta(1-\delta)\bar{x}/(\sigma_x\sqrt{3}). \tag{4.2}$$

These equations reveal two externalities. The first externality is that individual firms ignore the inter-dependence between their choices. This externality appears in (4.1): for any given matching probability q, the outside options $\underline{u}$ and $\underline{J}$ increase in b, provided $\rho = 2b - 1 > 0$. To explain this externality, recall that when a firm chooses a higher PPS, the firm also increases the recommended cut-off ρ and reduces the expected salary. For any $b > 1/2$, the loss to a CEO from the reduction in the expected salary outweighs the gain from the increased PPS. Thus, an employed CEO's expected surplus in the period falls. For any given q, competition among firms intensifies in order to retain a CEO, which reduces the value of each firm with a matched CEO. The value of a searching firm relative to the value of a matched firm increases.

the optimal PPS will likely to be decreasing with the two risks.

The second externality is that individual firms and CEOs ignore the equilibrium relationship between the matching probability and the contract. To see this relationship, note that free-entry of vacancies requires $q = H/(\beta J_F)$. Because J_F is a decreasing function of b (see 4.1), then q increases in b in the equilibrium. That is, when the value of a firm with a CEO falls with the PPS, fewer vacancies are created in order for each vacancy to break even. This equilibrium response of q to b reduces a searching CEO's matching probability $\lambda = 1 - q$ and, hence, reduces a CEO's outside option $\underline{u}$ (see 4.1). This is an externality because individual CEOs and firms take the matching probability as given. Moreover, for a CEO's outside option, this externality through the matching probability dominates the first externality. To see this, we substitute q from (4.2) into (4.1) to obtain $\underline{u} = B - bH/2$. Since this is a decreasing function of b, the overall effect of the two externalities is that if all firms increase the PPS, a CEO's outside option falls.

Because of these externalities, the properties of the PPS in the equilibrium differ from those under fixed outside options. Before stating these properties, let us denote the ratio of a CEO's total expected pay to firm value as $R_{pay/size}$, where firm value is J_F and total expected pay is $(1-\delta)\int_{\rho^*\bar{x}}^{\bar{x}} \mathbb{E}_y(a^* + b^*\pi^*)dF_1(x)$. The following proposition is proven in Appendix B:

Proposition 4.1. *A unique market equilibrium exists in a non-empty region of the parameters $(H, \mu_x/\sigma_x, B)$ specified in Appendix B. Moreover, the equilibrium PPS, the cut-off ρ^* and the matching probability q are all interior. Assuming $\beta > 1/[2(1-\delta)]$, the equilibrium has the following properties: (i) Equilibrium PPS decreases with μ_y and σ_y. (ii) Equilibrium PPS increases with the idiosyncratic risk σ_x if and only if $b^* < b_2 \in (2/3, 1)$ where b_2 is defined by (B.4). (iii) The pay/size ratio $R_{pay/size}$ is $\frac{2b^*-1}{1-b^*} + (1-b)L$, which decreases with μ_y and σ_y. It increases with the idiosyncratic risk σ_x if and only if $b^* < b_3 \in (\mu_x/\bar{x}, 2/3)$ where b_3 is defined by (B.5).*

Let us describe verbally the restrictions on the parameters $(H, \mu_x/\sigma_x, B)$ for an interior equilibrium to exist (see B.3 in Appendix B for the precise restrictions).[11] First, the hiring cost H should not be very high. If H is very high, the vacancy-filling probability must be one in order for a firm to create a vacancy. In this case, a CEO's job-finding probability is zero. Second, the ratio μ_x/σ_x should not be too high. If μ_x/σ_x is very high, the idiosyncratic risk is low, in which case

[11] Because these restrictions guarantee that b^* and ρ^* are interior, they ensure that the sum $(\underline{u} + \underline{J})$ generated by the equilibrium satisfies the bounds imposed in Proposition 3.1.

a CEO never quits. Third, the benefit to a searching CEO should be bounded below and above. If this benefit is very low, there is no gain for a CEO to quit one job and search for another. If this benefit is very high, quitting happens very often. In this case, a firm needs to set the PPS to be very high which is not profitable. In addition, we impose the assumption $\beta > 1/[2(1-\delta)]$ to simplify the algebra. This assumption is easily satisfied when the exogenous separation rate δ is small. For example, when $\delta = 0$, the assumption requires only that $\beta > 1/2$.

Parts (i) and (ii) of Proposition 4.1 show that the effects of the two risks on the PPS in equilibrium have two main differences from those in Proposition 3.1 under fixed outside options. First, an increase in μ_y or σ_y always reduces the PPS in equilibrium, while it does so with fixed outside options if and only if the sum of outside options is positive. Second, the two risks can affect the equilibrium PPS in opposite directions. In the equilibrium, an increase in σ_x increases the PPS when the PPS is not too large, while σ_y always reduces the equilibrium PPS. With fixed outside options, in contrast, the two risks always affect the PPS in the same way. These differences demonstrate in a concrete way the importance of the market equilibrium. Also, the finding that the two risks can affect the PPS in opposite directions contrasts with the agency literature with a single agent-firm pair, where the two risks both reduce the PPS.

To explain why the market equilibrium differs from the partial equilibrium in these aspects, we examine how the sum of outside options varies with the risks. To this aim, we combine the two externalities discussed above and derive the following expression (see Appendix B):

$$\underline{u} + \underline{J} = B - \frac{H}{2}b - 2L\Omega b(1-b)^2, \tag{4.3}$$

where L is defined in (4.2). For any given b, the sum of the outside options depends on σ_y entirely through $\Omega = \frac{\bar{x}}{2c}\mathbb{E}_y(y^{2\alpha})$, and on σ_x entirely through $\bar{x}^2/\sigma_x$ which appears in the term $L\Omega$ in (4.3). An increase in σ_y or μ_y increases Ω. An increase in σ_x reduces $\bar{x}^2/\sigma_x$, even after taking into account the relationship, $\bar{x} = \mu_x + \sigma_x\sqrt{3}$. Thus, for any given b, the sum of the outside options decreases in σ_y and increases in σ_x.

It is intuitive that the two risks affect the outside options in opposite directions. As is well known in search theory, the return to search is a convex function of the underlying match value because it is truncated at the value of search. That is, a match with a high value is accepted, but a match with a low value is rejected in which case a searcher retains the value of search. Convexity implies that the return to search increases when match value is more dispersed. An increase in σ_x increases dispersion in match value and, hence, increases the return to search relative to staying in a match for both the CEO and the firm. In contrast, an increase in σ_y affects all matches in the same way and reduces the return to search relative to staying in the existing match.

Because of this difference in the effect on the outside options, the two risks can affect the equilibrium PPS in opposite directions. Recall from Proposition 3.1 that an increase in the sum of the outside options $(\underline{u} + \underline{J})$ increases the PPS. When the aggregate risk increases, the ratio $(\underline{u} + \underline{J})/\Omega$ falls and, by (3.3), the equilibrium PPS falls. When the idiosyncratic risk increases, the sum of the outside options increases, which increases the PPS. However, for any given sum of the outside options, an increase in the idiosyncratic risk also has the direct effect of increasing the PPS if and only if the sum of the outside options is negative. The overall effect of a higher σ_x on the equilibrium PPS is positive if and only if the sum of the outside options is not too large. Because $\underline{u} + \underline{J} > 0$ if and only if $b^* > 2/3$ (see 3.3), σ_x increases the PPS if and only if $b^* < b_2$ for some $b_2 > 2/3$, as in Proposition 4.1. This property reveals that the response of the sum of the outside options to the risks is often the dominating force that determines the response of the equilibrium PPS to the risks. By fixing the outside options, a partial equilibrium model is likely to reach incorrect conclusions about how the risks affect the PPS.

The equilibrium effects of the risks σ_y and σ_x on the PPS lead to the behavior of the pay/size ratio described in part (iii) of Proposition 4.1. When σ_y increases, the equilibrium PPS decreases and the retention probability increases (see 3.2). Both responses increase a firm's expected value. Total expected pay to a CEO may also increase, but it increases by a smaller proportion because the CEO's profit-sharing ratio is reduced. As a result, the pay/size ratio falls with σ_y. In contrast, an increase in σ_x can increase the pay/size ratio by increasing the PPS and the incentive pay. Because an increase in σ_x increases the PPS only when the PPS is relatively small, it is not surprising that it increases the pay/size ratio only when the PPS is relatively small.

5. Empirical Analysis

Before beginning the empirical analyses, let us recapitulate. We have constructed a theoretical model to integrate an agency problem in executive compensation with search theory and analyzed the market equilibrium with many firms and CEOs. Each firm offers an incentive contract to the CEO that achieves the optimal trade-off between the probability of retaining the CEO and expected profit conditional on retention. This trade-off implies that the optimal PPS is below one, despite that a CEO is risk neutral. More importantly, the search process endogenously determines CEOs' and firms' outside options which reflect the externalities in the market equilibrium. The externalities induce novel effects of the risks on incentive contracts. While an increase in the idiosyncratic risk increases search by widening dispersion in match value and induces search, an increase in the aggregate risk reduces search by compressing dispersion in match value. As a result, we have found that the idiosyncratic risk increases the PPS when the PPS is not too large, but the aggregate risk always reduces the PPS. Also, the two risks affect the pay-size ratio differently. Because these differences between the two risks' effects arise from the externalities, they are unique to the market equilibrium with search. In particular, if the outside options are fixed at positive levels, as in the agency literature, then the two risks both reduce the PPS.

The objective of our empirical analyses is three-fold: (i) to evaluate our model's predictions on the PPS; (ii) to clarify the existing mixed evidence on the relationship between firms' risks and the PPS; and (iii) to provide new evidence on the relative growth between executive pay and firm size. Because the PPS in the data is not very large, the condition $b^* < b_3$ in Proposition 4.1 is satisfied. In this case, Proposition 4.1 leads to the following two testable predictions:

> *Prediction 1:* The PPS, b, decreases with the aggregate state and a firm's systematic risk, and increases with the firm's idiosyncratic risk.
>
> *Prediction 2:* The relative growth of total pay to firm size decreases with the aggregate state and a firm's systematic risk, and increases with the firm's idiosyncratic risk.

5.1. Data and variable definitions

Executive compensation data are retrieved from the ExecuComp for the period of 1992 to 2009. Firm characteristics and returns are obtained from the COMPUSTAT and CRSP. Firms are classified into 48 Fama-French industries. As is standard in the empirical compensation literature (e.g., Coles, Daniel and Naveen 2006), we exclude financial and utility firms. Our final sample consists of 13,051 firm-years for 1,905 firms and 3,226 executives.

For the intended empirical tests, we first identify the empirical measures for the PPS and $R_{pay/size}$. A typical compensation package includes salary, bonus, and restricted stock and option grants (see Murphy 1999). Since most incentive payments are related to firms' equities, we therefore focus on the PPS related to stock and option grants. Following Jensen and Murphy (1990), we define b as the change in the value of CEO pay with respect to a change of \$1000 in shareholders' wealth. This measure is widely used in the existing literature (e.g., Aggarwal and Samwick 1999). For our empirical analyses, we calculate two versions of b. The first is calculated based on the current year stock and option grants, which are straightforward to obtain since ExecuComp data provide detailed information. We call it the new equity incentive. The second version of b is computed from the accumulated stock and option grants up to the current year. We call it the total equity incentive. However, ExecuComp data offer no details on past option grants prior to 2005. As an alternative, we use Core and Guay's (2002) one-year approximation method to compute the total equity incentive.

Total compensation is taken as the flow compensation (TDC1) which consists of salary, bonus, other annual (short-term) compensation, total value of restricted stock granted, total value of stock options granted, long-term incentive payouts and other miscellaneous compensation. $R_{pay/size}$ is calculated as the ratio between annual total compensation and firm size, where firm size is proxied by either the firm's total assets or its market capitalization.

We then formulate three major explanatory variables: the aggregate state, a firm's systematic risk and idiosyncratic risk. In our model, the aggregate shock includes all changes common to the industry that affect the marginal contribution of a CEO's effort to firm profit. Aggregate shocks to total factor productivity, such as the one measured by the Solow residual, are part but not the only part of this aggregate shock. For example, industry-wide changes in the inputs of physical capital, energy and regular (non-CEO) labor can all change the marginal contribution of a CEO's effort to firm profit. For this reason, we use four broad measures as the proxy for the aggregate state: industry sales, GDP, the commercial paper spread and the credit spread. Intuitively, a high industry sales growth represents a healthy business environment for the industry while a high GDP growth indicates a good economy. The use of the commercial paper spread is based on Bernanke and Blinder (1992) who suggest that a high commercial paper spread at the beginning of the year signals a bad economy since it tends to rise sharply during credit crunches. Therefore, in the regression analysis, we use the negative lagged commercial paper spread as a proxy for aggregate shock. The use of the negative credit spread is based on Gilchrist, Yankov and Zakrajsek (2009), and Gomes and Schmid (2010).[12] The annual industry sales growth is computed from COMPUSTAT while the annual GDP growth is retrieved from the websites of the Bureau of Economic Analysis. The commercial paper spread is the difference between the annualized three-month commercial paper rate and the T-bill rate (see Friedman and Kuttner 1993; and Korajczyk and Levy 2003) and the credit spread is the difference between the yield of Baa bond and Aaa bond. Both spreads are obtained from the Federal Reserve Board.

Following Core and Guay (1999), a firm's risk is proxied by the volatility of its stock returns.[13] A firm's total risk is the volatility of stock returns over the 60 months prior to the fiscal year. Its beta is obtained from the market model using the same set of monthly return data. A firm's systematic risk is equal to the firm's beta multiplied by the stock market risk, while the firm's idiosyncratic risk is the square-root of total return variance minus the systematic return variance.

We take two additional steps to bring the model closer to the data. First, to control for heterogeneity that exists in the data but not in our model, we include other control variables such as the executive's age and tenure, firm size, and firm growth (e.g., Milbourn 2003). Tenure is defined as the number of years a person has been an executive with a firm. A firm's growth is

[12] We thank the editor for suggesting the use of credit spread as a proxy.

[13] We also consider the volatility of dollar returns as an alternative measure proposed by Aggarwal and Samwick (1999). They use this measure to ensure that risks are expressed in dollars since a firm's profit in their model is the sum of the executive's effort and the noise term. However, in our model, a firm's profit is the product of the aggregate variable, the firm's specific shock variable, and the executive's effort. If the executive's effort has the same measure as the profit, it is not necessary to measure the aggregate variable and the match-specific shock variable in dollars. Therefore, stock return volatilities are proper measures for our tests. Moreover, the correlations among the firm's total dollar risk, its systematic risk and specific risks are higher than 0.92. Such high correlations create multicollinearity problems.

proxied by its sales growth while firm size is proxied by its asset value or market capitalization. Second, it is well known that there are outliers in executive compensation. To reduce the effect of outliers on the empirical results, we winsorize the data of executive compensation and firm characteristics at the 1% and 99% levels, a standard procedure used in the compensation literature (e.g., Garvey and Milbourn 2006; and Coles, Daniel and Naveen 2006).

Table 1 provides summary statistics for the winsorized compensations and characteristics of the executives, characteristics of firms, and un-winsorized macroeconomic proxies. All monetary variables are also inflation adjusted, using the 2005 dollar as the base. Therefore, the industry sales growth and GDP growth are in real terms. Panel A of Table 1 shows that the average annual salary for a CEO is about $726,000, which is slightly higher than the average annual bonus of $636,000. However, the median annual salary of $671,000 is twice as high as the median bonus of $325,000, indicating that bonuses are more skewed toward the high end. Similar patterns are observed for equity related pay and total pay. In particular, the average equity related pay and total compensation are $3,4880,00 and $4,916,000, respectively, which are about twice of the corresponding median values but about one-seventh of the corresponding maximum values. It is worth noting that the average equity related pay is more than four times of the average annual salary while the average total pay is more than six times, indicating that the main income for an executive is from equity-related compensation. The average new equity incentives granted for a fiscal year is $1.99 with respect to the $1000 change in shareholders' wealth, compared to the average accumulated total equity incentives $23.16. An average executive is almost 55 years old and stays with a firm for about eight years. The youngest executive is 39 years old while the oldest is 75. The longest tenure is 37 years, in contrast to the shortest job duration of six months.

Summary statistics of firms' characteristics suggest that firms in the sample are skewed toward large sizes. In particular, the average market capitalization is $6,880 million, almost five times as large as the corresponding median value, $1,479 million; average asset value is $5,213 million, almost four times as large as the median asset value, $1,346 million. The average firm's total risk represented by the return volatility is 40%, which is slightly higher than the median 35%; average firm's systematic risk is 15%, about one-third of the average total risk.

During the sample period of 1992 to 2009, the growth rates of industry sales range from -43%

to 61%, with a mean and median of 4%. The average GDP growth rate is 3%, with a standard deviation of 2%, indicating significant fluctuations. The commercial paper spread and credit spread are even more volatile, with standard deviations of 17 and 39 basis points, in contrast to corresponding means of 27.72 and 94 basis points.

Table 2 presents the correlations among explanatory variables. As expected, there is a positive correlation between the industry sales growth and the GDP growth and between the commercial paper spread and the credit spread. Also, the two spreads are negatively correlated with the two growth series. For example, the credit spread has a correlation of -0.355 with the industry sales growth, -0.861 with the GDP growth. Lastly, the low correlations among most explanatory variables reassure the absence of potential multicollinearity problems.

5.2. Test of Prediction 1: Effects of idiosyncratic and systematic risks on the PPS

To empirically evaluate the opposite effects of the idiosyncratic and systematic risks on the PPS, we run the following regression:

$$\begin{aligned} b \;&= a_1 + a_2\text{macro proxy} + a_3\text{idiosyncratic risk} + a_4\text{Firm-systematic risk} + a_5\text{Age} \\ &+ a_6\text{Tenure} + a_7 \log(\text{Firm size}) + a_8\text{Firm growth} + a_9\text{Industry dummy} + \varepsilon, \end{aligned} \tag{5.1}$$

where 'macro proxy' includes the industry sales growth, the GDP growth, or the negative lagged commercial paper spread (hereafter NCP spread), and the negative credit spread (NCredit Spread hereafter).[14] The executive's age and tenure, firm size, and firm growth are included as control variables. Equation (5.1) is run as both OLS and median regressions. Since the results of the two versions are similar and since median regressions are more reliable for skewed compensation data, we only report the median regression results. By the same token, results are similar when firm size is proxied by asset value or market capitalization. For brevity, we only report the cases with asset value measuring the firm size. Panel A of Table 3 presents the results for new equity incentives, while Panel B for total equity incentives. The main findings are as follows.

First, the four macro proxies generate very similar R^2, suggesting that they are equivalent proxies for the U.S. macro condition during the period of 1992 to 2009.

[14]To simplify the language, the phrase "commercial paper spread" and "credit spread" from this point on refer to the "negative lagged commercial paper spread" and "the negative credit spread", respectively.

Second, aggregate factors have a negative effect on b, regardless of whether new equity incentives or total equity incentives are used in the regression. All coefficients are significant at the 1% level. The impact of aggregate shock on the PPS is economically significant. For example, an increase of one standard deviation in GDP growth (i.e., 2%) will reduce the total equity incentive pay by $\$1,386,917$ ($= 58.978 \times 2\% \times \$6,880$ million$\times 17.09\%/1000$) while a decrease of one standard deviation in the commercial spread (i.e., 17 basis points) will reduce the total equity incentive by $\$1,219,296$ ($= 0.061 \times 17 \times \$6,880$ million$\times 17.09\% * 1000$).[15]

Third, consistent with the model's predictions, b depends positively on the idiosyncratic risk and negatively on firm-systematic risk in all regressions. All coefficients for the idiosyncratic risk are significant at the 1% level while most of the coefficients for the firm-systematic risk are significant at the 1% level. Given that b is determined in each period in our model, tests on the PPS of new equity grants are more direct. Thus, we use results in Panel A to discuss the impact of a firm's risks on the PPS. Suppose the GDP growth is used as the macro proxy, a rise of one standard deviation in firms' idiosyncratic risk (i.e., 19%) increases new equity incentives by $\$468,471$ ($= 2.097 \times 19\% \times \$6,680$ million $\times 17.09\%/1000$) while a rise of one standard deviation in firms' systematic risk (i.e., 10%) decreases new equity incentives by $\$70,430$ ($= 0.599 * 10\% * \$6,880$ million $\times 17.09\%/1000$). The above analyses demonstrate that the impacts of firms' idiosyncratic risk and systematic risk on the PPS are economically significant.

Fourth, to contrast our predictions with those of a standard agency model, we re-run regressions (5.1) by removing the "idiosyncratic risk" and "systematic risk" while adding the firm's "total risk." For brevity, we only report the coefficient and t-value for the firm's "total risk," as well as the corresponding R^2. The relationship between b and a firm's total risk is positive and significant at the 1% level for all regressions. This finding is consistent with the results in Core and Guay (1999) but inconsistent with the prediction of a standard agency model.

To summarize, our model predictions are generally borne out by the empirical results. In particular, the PPS represented by b is affected negatively by the firm's systematic risks and positively by the firm's idiosyncratic risks.

[15] $\$6,880$ million is the average market value of equity, and 17.09% is the average stock return in our sample period. Therefore, $\$6,880$ million$\times 17.09\%$ is the average change in the shareholder's wealth during a year.

5.3. Test of Prediction 2 on the ratio between total compensation and firm size

To gain better understanding of the time trend in annual compensation and firm size, we report the median annual compensation and firm size in Table 4. There is an upward trend in annual compensation, which is confirmed in Fig. 3. Since the samples of 1992 and 1993 are relatively small and are biased towards large firms, we use the 1994 sample as the base case for the following discussion. The median equity-related pay and total compensation increased from $\$811,000$ and $\$1,969,000$ in 1994 to $\$3,049,000$ and $\$3,830,000$ in 2009 respectively. The corresponding percentage increases are 276% and 95%. This suggests that the increase in total pay is mostly due to the increase in equity related pay.

As already revealed in Fig. 3, Table 4 shows a positive growth in the median firm size during the sample period. The asset value and the market capitalization increased from 1994 to 2009 by 47.3% and 42.5% respectively, much lower increase rates compared with equity related pay and total pay, as revealed by the time series median ratios between total pay and firm size in Table 4. The median ratio exhibits a positive trend (see Fig. 4), increasing from 0.150% in 1994 to 0.203% to 2009 when firm size is measured by asset value. A similar observation can be made when market capitalization is used to measure firm size. Given the important influence of firm-systematic and idiosyncratic risks on compensation, firm size and $R_{pay/size}$, we also present medians for firms' risks in Table 4. The median idiosyncratic risk is much higher than the median firm-systematic risk (see Fig. 5). Also, the median idiosyncratic risk increased from 1994 to 2004 and declined in the recent years. This observation is consistent with evidence documented in asset pricing literature.[16] Since our theory shows a positive influence of aggregate shock on compensation and firm size, we also plot the two aggregate proxies in Fig 6.

To summarize, the statistics in Table 4 exhibit two important features: (1) equity related compensation and total compensation have increased; (2) the increase in total compensation has outpaced the increase in firm size.

[16] In the asset pricing literature, Campell, Lettau, Malkiel, and Xu (2001) first document a noticeable increasing trend in firm-specific risk for the period of 1962 to 1997. However, Brandt, Brav, Graham and Kumar (2010) show that, during recent years, idiosyncratic volatility has fallen substantially, reversing any time trend documented by Campell, Lettau, Malkiel and Xu (2001). They also find that the late 1990s surge and 2000s reversal in idiosyncratic volatility is most evident in firms with low stock prices and limited institutional ownership. Another recent paper by Bekaert, Hodrick, and Zhang (2010) examines aggregate idiosyncratic volatility in 23 developed equity markets and finds no evidence of upward trends.

To investigate which factors are important in determining the growth of executive compensation relative to firm size, we run the following regression based on *Prediction 2*:

$$\begin{aligned} R_{pay/size} &= a_1 + a_2\text{macro proxy} + a_3\text{idiosyncratic risk} + a_4\text{Firm-systematic risk} \\ &\quad + a_5\text{Age} + a_6\text{Tenure} + a_7\text{Firm growth} + a_8\text{Industry dummy} + \varepsilon. \end{aligned} \tag{5.2}$$

Table 5 reports the median regression results.

First, the negative impact of aggregate shock is confirmed with all four proxies. Except for the coefficients associated with the industry sales growth, all coefficients are significant at the 1% level. Our regression results also indicate that the ratio $R_{pay/size}$ is affected positively by the firm's idiosyncratic risk and negatively by the firm's systematic risk, confirming *Prediction 2*.

To determine the economical significance and the order of importance among the aggregate state, the idiosyncratic risk and the systematic risk, we use the regression coefficients from the second column in Panel A where the macro proxy is the negative commercial paper spread and firm size is proxied by firm's asset value. Specifically, an increase of one standard deviation in commercial paper spread (viz. 17 basis points) decreases the ratio between annual total pay and assets by 0.238×10^{-3} ($= 0.014 \times 17 \times 10^{-3}$). The economic impact of the idiosyncratic and systematic risk on the ratio is computed in a similar way: an increase of one standard deviation in idiosyncratic risk (viz. 19%) results in an increase of 1.41×10^{-3} ($= 7.434 \times 19\% \times 10^{-3}$) in the ratio while an increase of one standard deviation in systematic risk (viz. 10%) results in a reduction 0.4017×10^{-3} ($= 4.017 \times 10\% \times 10^{-3}$) in the pay/size ratio. Compared to the sample average ratio of 3.53×10^{-3}, we can conclude that the positive impact of the idiosyncratic risk (1.41×10^{-3}) on the pay-size ratio is the highest among the three factors and it is economically significant. In other words, our empirical evidence suggests that the change in $R_{pay/size}$ is mainly due to the change in firms' idiosyncratic risks.

6. Conclusion

This paper addresses two issues regarding executive compensation: (1) how does the pay-to-performance sensitivity depend differently on the systematic and idiosyncratic risks? (2) how does the pay-size ratio depend on these risks? To address these issues, we integrate an agency problem with search theory and analyzed the market equilibrium with many firms and CEOs.

Our model differs from a standard static agency model with a single agent-firm pair in three dimensions. First, it distinguishes a firm's idiosyncratic risk from the systematic risk, rather than lumping them into the firm's total risk; second, a CEO can choose to quit after privately observing the idiosyncratic risk; third, there are contracting interactions/externalities among the firms in the market equilibrium which work through endogenous outside options. In our setup, each firm offers an incentive contract to the CEO that achieves the optimal trade-off between the probability of retaining the CEO and expected profit conditional on retention. This trade-off generates an optimal pay-to-performance sensitivity which is less than one for a risk-neutral CEO. More importantly, the search process endogenously determines CEOs' and firms' outside options which reflect the externalities in the market equilibrium. The externalities induce novel effects of the risks on incentive contracts, which are confirmed by our empirical tests using executive compensation data from 1992 to 2009. First, the equilibrium pay-to-performance sensitivity depends positively on a firm's idiosyncratic risk, and negatively on the systematic risk. This is in contrast to agency models with exogenous outside options, where the two risks always affect the pay-to-performance sensitivity in the same way. This result offers a plausible explanation for why the empirical relationship between the pay-to-performance sensitivity and a firm's total risk is ambiguous. Second, the ratio of a CEO's total compensation to firm value depends positively on idiosyncratic risks and negatively on systematic risks.

A natural extension of the current study is to investigate whether the model predictions hold for the international executive compensation practice. For example, Oxelheim, Wihlborg and Zhang (2010) show that macroeconomic influences on Swedish CEOs' compensation are substantial. It will be relevant to investigate how macro factors together with firms' systematic and idiosyncratic risks affect the European executive compensation. Moreover, although European executives receive less compensation than their American counterparts, their compensation has also been rising in recent years (see Oxelheim and Wihlborg 2008). It will be informative to check how European compensation has been evolving relative to firm size and whether the macro factors and firm risk factors significantly affect the behavior of the relative growth of compensation to firm size.

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Appendix

A. Proof of Proposition 3.1

We start by reformulating the contracting problem with (b, ρ) as a firm's choices, instead of (b, a). To do so, we invert (2.5) to obtain: $\mathbb{E}_y(a) = \underline{u} - \frac{\rho\bar{x}}{2c}\mathbb{E}_y(b^2 y^{2\alpha})$. Substituting this expression and using (2.3) to compute $\mathbb{E}_y(\pi^* - w^*)$, we rewrite (2.8) as

$$J_F = J_H + (1-\delta)\max_{(b,\rho)} \int_{\rho\bar{x}}^{\bar{x}} \left[\frac{\rho\bar{x}}{2c}\mathbb{E}_y(b^2 y^{2\alpha}) + \frac{x}{c}\mathbb{E}_y[b(1-b)y^{2\alpha}] - (\underline{u} + \underline{J})\right] dF_1(x)\,. \tag{A.1}$$

The maximization problem above is the reformulated contracting problem.

Parts (i) and (ii) of Proposition 3.1: Consider the maximization problem in (A.1). For any given $(\underline{u}, \underline{J})$, the objective function in (A.1) is continuous in the choices (b, ρ). Because the set of feasible choices is $(b, \rho) \in [0, 1] \times [\underline{x}/\bar{x}, 1]$, which is compact, then the Theorem of the Maximum (see Stokey et al., 1989, p62) implies that the maximum is attained by a feasible choice. In this proof, let us denote the optimal choices as $(b^*(y), \rho^*)$ by suppressing their dependence on $(\underline{u} + \underline{J})$. Note that ρ is independent of y. Also note that the objective function is twice continuously differentiable in (b, ρ). With differentiability, we can verify that the objective function is strictly concave in b and ρ separately. However, the objective function is not necessarily concave in (b, ρ) jointly. To circumvent this problem, we use a two-step procedure to prove that the optimal choices are unique. First, for any fixed ρ, we prove that the optimal choice of b is unique. Second, taking into account the dependence of the optimal choice of b on ρ obtained in the first step, we prove that the resulting objective function is strictly concave in ρ and, hence, the optimal choice ρ^* is unique. In this procedure we also prove that $b^*(y)$ is independent of y and that the interior optimal choices satisfy (3.2).

Take the first step. For any given $\rho \in [\underline{x}/\bar{x}, 1]$, denote the optimal choice of b as $\hat{b}(\rho, y)$. Clearly, $\hat{b}(\rho^*, y) = b^*(y)$. Because the objective function in (A.1) is strictly concave in $b(y)$ for any given ρ, the optimal choice, $\hat{b}(\rho, y)$, is unique. If $\hat{b}(\rho, y)$ is at either corner of $[0, 1]$, then clearly it is independent of y. So, consider interior $\hat{b}(\rho, y)$. Because the objective function is continuously differentiable in $b(y)$, then the interior $\hat{b}(\rho, y)$ is characterized by the first-order condition, which is $\hat{b}(\rho, y) = (\rho + 1)/2$. Because ρ is independent of y, $\hat{b}(\rho, y) = \hat{b}(\rho)$ is independent of y. Then, $b^*(y) = \hat{b}(\rho^*)$ is independent of y. This procedure has also established that the first equation in

(3.2) holds whenever b^* is interior.

Take the second step. Substituting $b(y) = \hat{b}(\rho)$, we write the objective function in (A.1) as $f(\rho) = [1 - F_1(\rho\bar{x})]p(\hat{b}(\rho), \rho)$, where $p(b, \rho)$ is defined by (3.1). Using the uniform distribution F_1, we can directly verify $f''(\rho) < 0$. Thus, the optimal choice ρ^* is unique and independent of y. This implies that b^* is also unique, because $b^* = \hat{b}(\rho^*)$ and because $\hat{b}(\rho)$ is unique for any given ρ. If ρ^* is interior, then $f'(\rho^*) = 0$, which can be written as $b^* - 2\rho^* + \frac{\underline{u}+\underline{J}}{b^*\Omega} = 0$. Substituting $b^* = (\rho^* + 1)/2$ into the first term of this condition, we get the second equation in (3.2).

Now we prove that, under the restrictions on $(\underline{u}+\underline{J})$ specified in Proposition 3.1, the equations in (3.2) have a unique solution which is interior. Substituting ρ^* from the second equation into the first equation in (3.2), we get

$$3b^2 - 2b - (\underline{u} + \underline{J})/\Omega = 0.$$

This quadratic equation has a real solution if and only if $\underline{u} + \underline{J} \geq -\Omega/3$. Maintain this condition. Then, the above quadratic equations has two real solutions generically. Because the quadratic expression is minimized at $b = 1/3$, the smaller solution is less than or equal to $1/3$ and, hence, less than $1/2$, in which case the first equation in (3.2) implies $\rho^* < 0 < \underline{x}/\bar{x}$. Thus, the smaller solution is not admissible. The larger solution for b^* to the quadratic equation above is given by the first equation in (3.3), and the implied solution for ρ^* is given by the second equation. This solution satisfies $b^* < 1$ if and only if $\underline{u} + \underline{J} < \Omega$. This condition also guarantees $\rho^* < 1$, because $\rho^* = 2b^* - 1$ by the first equation in (3.2). Moreover, $\rho^* > \underline{x}/\bar{x}$ if and only if $b^* > \frac{1}{2}(\frac{\underline{x}}{\bar{x}} + 1) = \frac{\mu_x}{\bar{x}}$, which is equivalent to $\underline{u} + \underline{J} > \frac{\Omega\mu_x}{2\bar{x}}(\frac{3\underline{x}}{\bar{x}} - 1)$. Note that this lower bound on $(\underline{u} + \underline{J})$ is greater than $-\Omega/4$ and, hence, greater than $-\Omega/3$ that was imposed earlier for b^* to be a real number. Thus, the unique solution for the pair (b^*, ρ^*) to (3.2) is interior if and only if $\frac{\Omega\mu_x}{2\bar{x}}(\frac{3\underline{x}}{\bar{x}} - 1) < \underline{u} + \underline{J} < \Omega$. Finally, (2.5) yields $\mathbb{E}_y(a^*) = \underline{u} - (b^*)^2\rho^*\Omega$.

Parts (iii) and (iv) of Proposition 3.1: Consider the solution for (b^*, ρ^*) given by (3.3). Clearly, $\underline{u}$ and $\underline{J}$ affect (b^*, ρ^*) only through the sum $(\underline{u} + \underline{J})$. Also, (3.3) shows that an increase in $(\underline{u} + \underline{J})$ increases b^* and ρ^*. Because the CEO's incentive pay is equal to $(b^*)^2\frac{\underline{x}}{\bar{x}}\Omega$, it increases with $\underline{u}$ and $\underline{J}$. Expected salary is equal to $\mathbb{E}_y(a^*) = \underline{u} - (b^*)^2\rho^*\Omega$. By increasing (b^*, ρ^*), an increase in $\underline{J}$ reduces $\mathbb{E}_y(a^*)$. Using (3.3) we can calculate:

$$\frac{d(\mathbb{E}_y(a^*))}{d\underline{u}} = \frac{1}{3}\left\{2 - \left[1 + \frac{3}{\Omega}(\underline{u} + \underline{J})\right]^{1/2}\right\}.$$

This is positive if and only if $\underline{u} + \underline{J} < \Omega$, which is maintained in Proposition 3.1.

A higher σ_y is reflected by a higher value of $\mathbb{E}_y(y^{2\alpha})$ and a higher σ_x by a higher value of $\bar{x}$. Both risks increase Ω. It is clear from (3.3) that the two risks affect (b^*, ρ^*) only through Ω. Also, (3.3) shows that an increase in Ω increases (b^*, ρ^*) if and only if $\underline{u} + \underline{J} < 0$. The incentive pay at $x = \rho^* \bar{x}$ is equal to $(b^*)^2 \rho^* \Omega$. We can compute:

$$\frac{d}{d\Omega}[(b^*)^2 \rho^* \Omega] = \frac{b^*}{9} \left\{ 1 - \frac{3}{\Omega}(\underline{u} + \underline{J}) + \left[1 + \frac{3}{\Omega}(\underline{u} + \underline{J}) \right]^{1/2} \right\}.$$

This is clearly positive if $\underline{u} + \underline{J} \leq \Omega/3$. Consider the case where $\underline{u} + \underline{J} > \Omega/3$. In this case, the above derivative is positive if and only if $1 + \frac{3}{\Omega}(\underline{u} + \underline{J}) > \left[\frac{3}{\Omega}(\underline{u} + \underline{J}) - 1\right]^2$. This condition is equivalent to $\underline{u} + \underline{J} < \Omega$, which is maintained. Thus, an increase in Ω increases $(b^*)^2 \rho^* \Omega$. Because $\mathbb{E}_y(a^*) = \underline{u} - (b^*)^2 \rho^* \Omega$, the expected salary decreases in Ω. **QED**

B. Proof of Proposition 4.1

According to the definition in subsection 2.3, we determine a market equilibrium by solving for the contract $\psi = (a, b)$, the induced choices by the CEO (ρ, e^*), the value functions (V_E, V_S, J_F, J_H), and the measures (v, s). The effort level e^* is given by (2.2) as a function of ψ. In Proposition 3.1, we have already solved (b, ρ) and $\mathbb{E}_y(a)$ as functions of $(\underline{u}, \underline{J})$. We will derive (4.1) - (4.2) below, which give $(\underline{u}, \underline{J})$ and q as functions of b. We will also solve $(\underline{u}, \underline{J})$ and b jointly from (3.3) and (4.1) - (4.2). Once this is done, we can determine other equilibrium objects easily. Specifically, we can recover J_F and q from (4.1) and (4.2), compute a searching CEO's matching probability as $\lambda = 1 - q$, compute the value function J_H as $J_H = 0$, and compute the value functions (V_E, V_S) from (2.6) and (2.7). Moreover, we can compute s from (2.10) by setting $s_{+1} = s$, solve $\theta = \lambda^{-1} - 1$, and recover $v = s/\theta$. In the procedure below, we suppress the superscript $*$ on optimal choices and the tilde on other firms' choices.

To derive (4.1) - (4.2), we impose symmetry between firms' choices and stationarity, as required by the equilibrium. Substituting the properties of the optimal contract, $\rho = 2b - 1 > 0$ and $\mathbb{E}_y(a) = \underline{u} - b^2 \rho \Omega$, we can solve V_E from (2.6), V_S from (2.7), and J_F from (A.1). With stationarity, the outside options are $\underline{u} = V_S - \beta V_E$ and $\underline{J} = -\beta J_F$, where we have used the free-entry condition $J_H = 0$. Substituting the solutions of (V_E, V_S) into the expression for $\underline{u}$ and using $\lambda = 1 - q$, we get $\underline{u}$ as in (4.1). Inverting (3.3) to get $\underline{u} + \underline{J} = b(3b - 2)\Omega$ and substituting it into the solution just obtained for J_F, we get J_F and $\underline{J}$ as in (4.1). Setting $J_H = 0$ in (2.9) yields $q = H/(\beta J_F)$, which produces (4.2) after substituting J_F from (4.1).

To solve $(\underline{u}, \underline{J})$ and b jointly from (3.3) and (4.1) - (4.2), we substitute q from (4.2) into (4.1) to get $\underline{u} = B - \frac{H}{2}b$. Adding this result to the expression for $\underline{J}$ in (4.1), we get (4.3). Substituting (4.3) into the expression for b in (3.3), we find that the equilibrium PPS solves $G(b) = 0$, where

$$G(b) \equiv 4Lb(1-b)^2 + 2b(3b-2) + \frac{Hb - 2B}{\Omega}. \tag{B.1}$$

The constant L is defined in (4.2) as $L = \beta(1-\delta)\bar{x}/(\sigma_x\sqrt{3})$. Let us determine the admissible interval for b in the equilibrium. From Proposition 3.1 we know that b must satisfy $1 > b > \frac{1}{2}(1 + \frac{\underline{x}}{\bar{x}}) = \frac{\mu_x}{\bar{x}}$. Note that $\frac{\mu_x}{\bar{x}} > \frac{1}{2}$ because $\underline{x} > 0$. In addition, the equilibrium must satisfy $q \in (0,1)$. From (4.2), we know that q lies in $(0,1)$ if and only if $b(1-b)^2 > H/(2L\Omega)$. For all $b \in (\mu_x/\bar{x}, 1)$, the function $b(1-b)^2$ is strictly decreasing and, hence, achieves the maximum at the left corner $\mu_x/\bar{x}$. Also, the function is equal to zero at $b = 1$. Thus, a necessary condition for $q \in (0,1)$ is $H < 2L\Omega\frac{\mu_x}{\bar{x}}(1 - \frac{\mu_x}{\bar{x}})^2$. Under this condition, $q \in (0,1)$ if and only if $b \in (\mu_x/\bar{x}, b_1)$, where $b_1 \in (\mu_x/\bar{x}, 1)$ is defined by

$$b_1(1-b_1)^2 = H/(2L\Omega). \tag{B.2}$$

Note that b_1 is independent of the parameter B. The admissible interval for b is $(\mu_x/\bar{x}, b_1)$.

Now we establish that a unique solution for equilibrium b exists. Compute:

$$G'(b) = 4(3b-1)[1 - L(1-b)] + \tfrac{H}{\Omega}, \qquad G''(b) = 8L(3b-2) + 12.$$

Assume that $G''(1/2) \geq 0$, $G(\mu_x/\bar{x}) < 0$ and $G(b_1) > 0$, which we will support with explicit restrictions on the parameters. Because $G''(b)$ is strictly increasing in b, the assumption $G''(1/2) \geq 0$ implies that for all $b > 1/2$, $G''(b) > 0$ and hence $G'(b)$ is strictly increasing. It is evident that $G'(1) > 0$. If $G'(1/2) \geq 0$, then $G'(b) > 0$ for all $b > 1/2$. If $G'(1/2) < 0$, then there exists $b_0 \in (1/2, 1)$ such that $G'(b) < 0$ for $b \in (1/2, b_0)$ and $G'(b) > 0$ for $b \in (b_0, 1)$. In both cases, the assumptions $G(\mu_x/\bar{x}) < 0$ and $G(b_1) > 0$ ensure that a unique solution exists in the admissible interval $(\mu_x/\bar{x}, b_1)$. Furthermore, the solution has the property $G'(b) > 0$.

To summarize the above proof, we find that there is a unique and admissible solution for equilibrium b if the following conditions hold: $H < 2L\Omega\frac{\mu_x}{\bar{x}}(1 - \frac{\mu_x}{\bar{x}})^2$, $G''(1/2) \geq 0$, $G(\mu_x/\bar{x}) < 0$

and $G(b_1) > 0$. Let us express these conditions more explicitly as follows:

$$\begin{array}{l} H < H_1 \equiv 2\beta(1-\delta)\Omega\frac{\mu_x\sigma_x\sqrt{3}}{\bar{x}^2}, \\ \frac{\mu_x}{\sigma_x\sqrt{3}} \leq \frac{3}{\beta(1-\delta)} - 1, \\ B > B_1 \equiv \frac{\mu_x}{\bar{x}}\left\{\frac{\Omega}{\bar{x}}\left[\mu_x - 2\sigma_x\sqrt{3}\left(1-\beta(1-\delta)\right)\right] + \frac{H}{2}\right\}, \\ B < B_2 \equiv H\left(1+\frac{b_1}{2}\right) + \Omega b_1(3b_1-2). \end{array} \tag{B.3}$$

There is a non-empty region of $(H, \mu_x/\sigma_x)$ that satisfies the first two conditions, and this region is independent of the parameter B. Given $(H, \mu_x/\sigma_x)$ in this region, the interval (B_1, B_2) that satisfies the last two conditions is non-empty by construction. Thus, there is a non-empty region of the parameters $(H, \mu_x/\sigma_x, B)$ that satisfies all conditions above.

For comparative statics, let us establish the results $L > \frac{\bar{x}}{2\sigma_x\sqrt{3}} > 1$, $B_1 > H/4$ and $b^* < 2B/H$ under restriction $\beta \geq \frac{1}{2(1-\delta)}$. Because $\underline{x} > 0$, we have $\mu_x > \sigma_x\sqrt{3}$, $\bar{x} > 2\sigma_x\sqrt{3}$ and $2\mu_x > \bar{x}$. The restriction on β implies $L > \frac{\bar{x}}{2\sigma_x\sqrt{3}} > 1$ and $B_1 > \mu_x H/(2\bar{x}) > H/4$. To prove $b^* < 2B/H$, note that it is equivalent to $G(2B/H) > 0$ which, in turn, is equivalent to

$$L > \left(1 - \frac{3B}{H}\right) / \left(1 - \frac{2B}{H}\right)^2.$$

Because $B > B_1 > H/4$, then $1 - 6B/H < 0$, and so the right-hand side of the above inequality is an increasing function of (B/H). A sufficient condition for the inequality to hold is that it holds at $B = H/4$, which is equivalent to $L > 1$ that we just established.

We now establish results (i) and (ii) stated in Proposition 4.1. The aggregate shock affects b^* exclusively through $\mathbb{E}_y(y^{2\alpha})$ which appears in Ω in the function $G(b)$. A higher μ_y or σ_y leads to a higher $\mathbb{E}_y(y^{2\alpha})$ and, hence, a higher Ω. Compute:

$$\frac{db^*}{d\Omega} = \frac{-\partial G/\partial\Omega}{G'(b^*)} = -\frac{2B - Hb^*}{\Omega^2 G'(b^*)} < 0.$$

The inequality follows from the result $b^* < 2B/H$, which we just established, and the fact $G'(b^*) > 0$. To examine the effect of σ_x on b^*, note that $\bar{x} = \mu_x + \sigma_x\sqrt{3}$ and $\frac{d}{d\sigma_x}(\frac{\bar{x}}{\sigma_x\sqrt{3}}) = -\frac{\mu_x}{\sigma_x^2\sqrt{3}}$. Using $G(b^*) = 0$, we can derive

$$\frac{db^*}{d\sigma_x} = \frac{2\sqrt{3}b^*}{\bar{x}G'(b^*)}\left[2L(1-b^*)^2\left(\frac{\mu_x}{\sigma_x\sqrt{3}} - 1\right) - 3b^* + 2\right].$$

Thus, $db^*/d\sigma_x > 0$ if and only if $2L(1-b^*)^2\left(\frac{\mu_x}{\sigma_x\sqrt{3}} - 1\right) - 3b^* + 2 > 0$. The left-hand side of this inequality is a strictly decreasing function of b^*. It is positive at $b^* = 2/3$ and negative at $b^* = 1$.

Thus, there exists $b_2 \in (2/3, 1)$ such that

$$2L(1-b_2)^2\left(\frac{\mu_x}{\sigma_x\sqrt{3}}-1\right)-3b_2+2=0. \tag{B.4}$$

Moreover, $db^*/d\sigma_x > 0$ if $b^* < b_2$ and $db^*/d\sigma_x < 0$ if $b^* > b_2$.

Finally, we turn to part (iii) of Proposition 4.1. Recall $J_F^* = \frac{2}{\beta}L\Omega b^*(1-b^*)^2$ from (4.1) and $\mathbb{E}_y(a^*) = \underline{u} - {b^*}^2\rho^*\Omega$ from Proposition 3.1. Also, since $b^*\pi^* = {b^*}^2 y^{2\alpha}x/c$, we have $\mathbb{E}_y(b^*\pi^*) = \frac{2x}{\bar{x}}{b^*}^2\Omega$. Substituting $\mathbb{E}_y(a^*)$ and $\mathbb{E}_y(b^*\pi^*)$, and integrating over x, we get:

$$\begin{aligned}&(1-\delta)\int_{\rho^*\bar{x}}^{\bar{x}}\mathbb{E}_y(a^*+b^*\pi^*)dF_1(x) = (1-\delta)\int_{\rho^*\bar{x}}^{\bar{x}}\left[\underline{u}+{b^*}^2(\tfrac{2x}{\bar{x}}-\rho^*)\Omega\right]dF_1(x)\\ &= \tfrac{\bar{x}(1-\delta)(1-b^*)}{\sigma_x\sqrt{3}}\left[\underline{u}+{b^*}^2\Omega\right] = \tfrac{2}{\beta}L\Omega b^*(1-b^*)(2b^*-1)+(1-b^*)LJ_F^* = J_F^*\left[\tfrac{2b^*-1}{1-b^*}+(1-b^*)L\right].\end{aligned}$$

The ratio between the expected total pay and firm value, denote as $R_{pay/size}$, is computed as:

$$R_{pay/size} = \tfrac{\mathbb{E}(a^*+b^*\pi^*)}{J_F^*} = \tfrac{2b^*-1}{1-b^*}+(1-b^*)L.$$

We obtain $\frac{\partial R_{pay/size}}{\partial b} = \frac{1}{(1-b^*)^2} - L$. Since $(1-b^*)^{-2} > 4$ for $b^* > \frac{1}{2}$ and $L < 3$ due to $\frac{\mu_x}{\sigma_x\sqrt{3}} \leq \frac{3}{\beta(1-\delta)} - 1$, we have $\frac{\partial R_{pay/size}}{\partial b} > 1$. It is easy to establish the following results:

$$\tfrac{\partial R_{pay/size}}{\partial \mu_y} = \tfrac{\partial R_{pay/size}}{\partial b}\tfrac{\partial b^*}{\partial \mu_y} < 0 \qquad \text{and} \qquad \tfrac{\partial R_{pay/size}}{\partial \sigma_y} = \tfrac{\partial R_{pay/size}}{\partial b}\tfrac{\partial b^*}{\partial \sigma_y} < 0.$$

To examine the effect of σ_x on $R_{pay/size}$, we obtain

$$\begin{aligned}&\tfrac{\partial R_{pay/size}}{\partial \sigma_x} = \tfrac{\partial R_{pay/size}}{\partial b}\tfrac{\partial b}{\partial \sigma_x} - (1-b^*)L\tfrac{\mu_x}{\bar{x}\sigma_x}\\ &= \tfrac{1}{\bar{x}\sigma_x G'(b^*)}\left[2\sqrt{3}b^*(2-3b^*)\left(\tfrac{1}{(1-b^*)^2}-L\right)+4L\underline{x}b^*\left[1-L(1-b^*)^2\right]-(1-b^*)L\mu_x G'(b^*)\right].\end{aligned}$$

Tedious algebra shows that $\frac{\partial R_{pay/size}}{\partial \sigma_x} > 0$ for $b^* \in (\frac{\mu_x}{\bar{x}\sigma_x}, b_3)$ where $b_3 \in (\frac{\mu_x}{\bar{x}\sigma_x}, \frac{2}{3})$ is determined by

$$2\sqrt{3}b_3(2-3b_3)\left(\frac{1}{(1-b_3)^2}-L\right)+4L\underline{x}b_3\left[1-L(1-b_3)^2\right]-(1-b_3)L\mu_x G'(b_3)=0. \tag{B.5}$$

This completes the proof of part (iii) in Proposition 4.1. QED

C. Equilibrium Long-Term Incentive Contract

Now we extend the contract studied in Section 4 to include a retention reward payment. This extension is motivated by the existing option grant practice which exhibits the following features. First, a firm normally grants options to its CEO on a yearly basis, which are intended to increase retention and incentive. Second, an option grant has a vesting period which normally lasts for five years and a CEO can only exercise a fraction of his accumulated options that have been vested. If the CEO is fired, he may get part of the retention account as settlement but if the CEO voluntarily quits, he has to forgo the remaining un-vested options. Based on these features, we introduce a retention reward mechanism as follows. If the CEO is newly matched with a firm, he starts with a zero balance in the retention account. The firm will put i amount into the account in the current period. The CEO can receive a fraction ϕ of the retention account balance if he works for the firm next period. This mechanism is carried out as long as the CEO continues to work for the firm. Denote a contract with this reward mechanism as $\psi = (a, b, i)$, where a and b have the same meanings as in the baseline model. Denote the balance of the retention account at the beginning of a period as k. The balance in the next period is:

$$k_{+1} = (1 + r) \left[i + (1 - \phi) k \right], \tag{C.1}$$

where $(1 - \phi) k$ is the amount in the retention account immediately after paying the CEO in the current period. This amount is augmented by the firm's injection into the account, i. The retention account earns a risk-free interest at the rate r.

At the beginning of the current period, if a matched CEO is separated from the firm exogenously, he can negotiate with the firm to settle the retention account. This settlement can be understood as the severance pay if the CEO is fired by the firm. Negotiation is costly to the two sides. Let us model the negotiation cost as a reduction in the retention account by $n(k)$. We use the Nash bargaining rule to determine the split of the remaining amount, $k - n(k)$, between the firm and the CEO, where the CEO's bargaining power is $\eta \in (0, 1)$. That is, the CEO receives $\eta \left[k - n(k) \right]$ and the firm receives $(1 - \eta) \left[k - n(k) \right]$. However, if a CEO who has survived the exogenous job separation shock decides to quit, he has to forgo the money in the retention account. This treatment is similar to the foregone unvested options in practice. In this case, the firm will claim the entire balance of the retention account.

The value function of a searching CEO is still V_S. Note that in the event that a CEO just separated from a job exogenously and settled an account k with the firm, V_S is measured after the settlement is already paid. If a searching CEO gets a match, he will start the next period with zero balance in the retention account, and so the future value function in this case will be

$V_E(k_{+1}=0)$. Modifying (2.7), we have:

$$V_S = B + \beta \left[\lambda V_E(k_{+1}=0) + (1-\lambda) V_{S,+1} \right]. \tag{C.2}$$

The value function of a CEO who enters a period as being matched is now denoted as $V_E(k)$. Similar to the baseline model, we can compute an employed CEO's expected utility over y in the current period as $\mathbb{E}_y(u) = a + \frac{b^2}{2c} x \mathbb{E}_y(y^{2\alpha})$. A CEO accepts a contract if and only if $a + \frac{b^2}{2c} x \mathbb{E}_y(y^{2\alpha}) + \beta V_E(k_{+1}) > V_S$. This acceptance condition can be written as $x \geq \rho(k)\bar{x}$, where the cut-off ratio $\rho(k)$ is:

$$\rho(k) = \frac{1}{b^2 \Omega} \left[V_S - \beta V_E(k_{+1}) - a \right]. \tag{C.3}$$

If a CEO separates from the job exogenously, he obtains the settlement income $\eta[k-n(k)]$. Incorporating this income, we modify (2.6) as:

$$\begin{aligned} V_E(k) = \; & \delta\{\eta\left[k-n(k)\right] + V_S\} \\ & +(1-\delta)\left[\int_{\rho\bar{x}}^{\bar{x}} \left(a + b^2 \tfrac{x}{\bar{x}} \Omega + \beta V_E(k_{+1})\right) dF_1(x) + F_1(\rho\bar{x}) V_S\right]. \end{aligned} \tag{C.4}$$

Substituting salary a in terms of ρ from (C.3) and integrating over x, we obtain:

$$V_E(k) = V_S + \delta\eta\left[k-n(k)\right] + (1-\delta)[1-F_1(\rho\bar{x})] \frac{b^2}{2}(1-\rho)\Omega. \tag{C.5}$$

Denote the value function of a firm with a CEO as $J_F(k)$ and the value function of a firm without a CEO as J_H. If a searching firm just settled a retention account with a separating CEO, J_H is measured after the settlement receipts are counted. If a searching firm gets a match, the firm with start the next period with zero balance in the retention account, and so the future value function in this case will be $J_F(k_{+1}=0)$. Thus,

$$J_H = -H + \beta \left[q J_F(k_{+1}=0) + (1-q) J_{H,+1} \right]. \tag{C.6}$$

For a firm with a matched CEO, the value function is:

$$J_F(k) = \max_{(a,b,i)} \left\{ \begin{array}{l} \delta\left[(1-\eta)\left(k-n(k)\right) + J_H\right] + (1-\delta)\left(k+J_H\right) F_1\left(\rho\bar{x}\right) + \\ (1-\delta) \displaystyle\int_{\rho(\psi)\bar{x}}^{\bar{x}} \left[b(1-b)\tfrac{x}{\bar{x}} 2\Omega - a - i + \beta J_F(k_{+1})\right] dF_1\left(x\right) \end{array} \right\} \tag{C.7}$$

s.t.

$$a = V_S - \beta V_E(k_{+1}) - b^2 \rho \Omega \qquad \text{and} \qquad i = \frac{k_{+1}}{1+r} - k(1-\phi).$$

As in the baseline model, $J_F(k)$ is computed before x and y are realized and, hence, is independent of x and y. The Bellman equation (C.7) modifies the one in the baseline model in three ways. First, the firm receives $(1-\eta)(k-n(k))$ from the retention account when the firm's CEO separates exogenously and receives k when the CEO quits. Second, the firm injects i in the retention account when the CEO stays in the period, besides paying salary and the incentive amount. As a result, the firm's profit in the current period is $\pi - w = 2b(1-b)\Omega\frac{x}{\bar{x}} - a - i$, which appears inside the integral in (C.7). Third, because the future balance in the retention account k_{+1} will depend on the current injection i, the firm incorporates this dependence as a constraint.

Since there is a one-to-one mapping between i and k_{+1}, we can restate the contract as $\psi = (a, b, k_{+1})$. By doing so, we can easily formulate the dynamic contracting problem by taking the retention account balance in the current period k as the state variable. In other words, the firm is to choose the contract for the current period, $\psi = (a,\, b,\, k_{+1})$ while taking J_H, $J_{H,+1}$ and $J_F(k_{+1})$ as given. Also, the firm anticipates that the CEO's effort e^* and acceptance rule $\rho(k)$ will depend on the contract. Solving the dynamic maximization problem in (C.7) leads to the following optimal contract:

$$\begin{aligned}
&k_{+1} \text{ is solved from } n'(k_{+1}) = \tfrac{1}{\delta}\left(1 - \tfrac{1}{\beta(1+r)}\right), \\
&b = \tfrac{1}{3} + \tfrac{1}{3}\left[1 + \tfrac{3}{\Omega}\left(\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \tfrac{k_{+1}}{1+r}\right)\right]^{1/2}, \\
&a = \underline{u}(k_{+1}) - \phi k - \Omega b^2(2b-1), \\
&\rho(k) = 2b - 1,
\end{aligned} \tag{C.8}$$

with $\underline{u}(k_{+1}) = V_S - \beta V_E(k_{+1})$ and $\underline{J}(k_{+1}) = J_H - \beta J_F(k_{+1})$, interpreted as the effective outside options for a CEO and a firm, respectively. The above optimal contract indicates that, when $\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \frac{k_{+1}}{1+r} > 0$, the PPS decreases with σ_y and σ_x. The effects of σ_y and σ_x on the PPS are opposite when $\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \frac{k_{+1}}{1+r} < 0$. However, we will show that σ_y and σ_x have different effects on the PPS in a market equilibrium when V_S, J_H, $V_E(k_{+1})$, and $J_F(k_{+1})$ are endogenously determined.

For the market equilibrium, we can modify the definition in Section 2.3 to incorporate the retention account. The equilibrium values of (a, b, k_{+1}, ρ), $(V_E, V_S, J_F,\ J_H)$, and (q, λ) can be solved from the following equations:

$$\underline{u}(k_{+1}) = V_S - \beta V_E(k_{+1}), \tag{C.9}$$

$$\underline{J}(k_{+1}) = J_H - \beta J_F(k_{+1}), \tag{C.10}$$

$$n'(k_{+1}) = \tfrac{1}{\delta}\left(1 - \tfrac{1}{\beta(1+r)}\right), \tag{C.11}$$

$$a = \underline{u}(k_{+1}) - \phi k - \Omega b^2(2b-1), \tag{C.12}$$

$$b = \tfrac{1}{3} + \tfrac{1}{3}\left[1 + \tfrac{3}{\Omega}\left(\underline{u}(k_{+1}) + \underline{J}(k_{+1}) + \tfrac{k_{+1}}{1+r}\right)\right]^{1/2}, \tag{C.13}$$

$$\rho(k) = 2b - 1, \tag{C.14}$$

$$J_F(k) = k\,(1-\eta\delta) - \delta\,(1-\eta)\,n(k) + (1-\delta)\,b(1-b)^2\tfrac{2\bar{x}\Omega}{\sigma_x\sqrt{3}}, \tag{C.15}$$

$$J_H = -H + \beta\left[qJ_F(0) + (1-q)J_H\right], \tag{C.16}$$

$$V_S = B + \beta\left[\lambda V_E(0) + (1-\lambda)V_S\right], \tag{C.17}$$

$$V_E(k) = V_S + \eta\delta\left[k - n(k)\right] + (1-\delta)\,b^2(1-b)^2\tfrac{\bar{x}\Omega}{\sigma_x\sqrt{3}}, \tag{C.18}$$

$$s_{+1} = s + (1 - s + \lambda s)\left[\delta + (1-\delta)F_1(\bar{x}\rho_{+1})\right] - \lambda s, \tag{C.19}$$

$$\beta q J_F(k) = H, \tag{C.20}$$

$$q = 1 - \lambda. \tag{C.21}$$

First, we find the expressions for V_S based on V_E. Working with $(C.17)$ and $(C.18)$ yields:

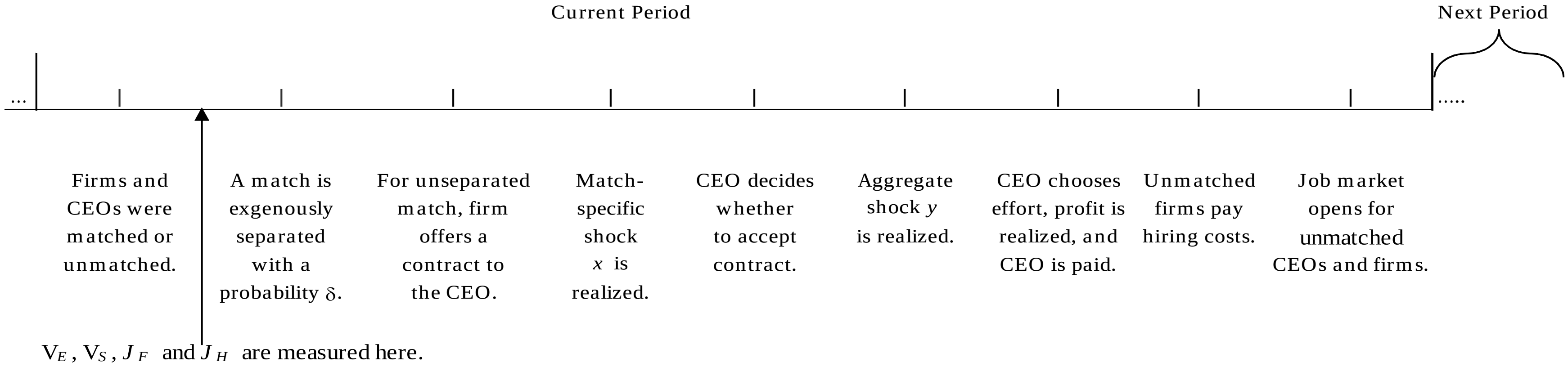
$$V_S = \frac{1}{1-\beta}\left(B - \frac{b}{2}H + A\beta\,(1-\delta)\,b^2(1-b)^2\frac{\bar{x}\Omega}{\sqrt{3}\sigma_x}\right),$$

which is the same as in the baseline case. We then substitute the expression for V_S, $V_E(k_{+1})$, and $J_F(k_{+1})$ in to $(C.13)$. Using $(C.20)$ and $(C.21)$, we can rewrite $(C.13)$ as follows:

$$g(b) = G(b) + 2\left(\beta k - \frac{k}{1+r} - \beta\delta n(k)\right) = 0,$$

where $G(b)$ is defined in (B.1) in Appendix B. Since $g(b)$ depends on b in the same way as $G(b)$ does, we conclude that the dependence of the equilibrium PPS on μ_y, σ_y, and σ_x remains the same as in Proposition 4.1. In other words, the introduction of the long-term incentive reward changes the level of b but not the dependence of b on the risks.

Fig. 1: timing of events in each period



V_E is the value function of an employed CEO.

V_S is the value function of a CEO searching for a job.

J_F is the value function of a firm with a matched CEO.

J_H is the value function of a hiring firm with a vacant CEO position.

Fig. 2: determination of optimal *b* and ρ

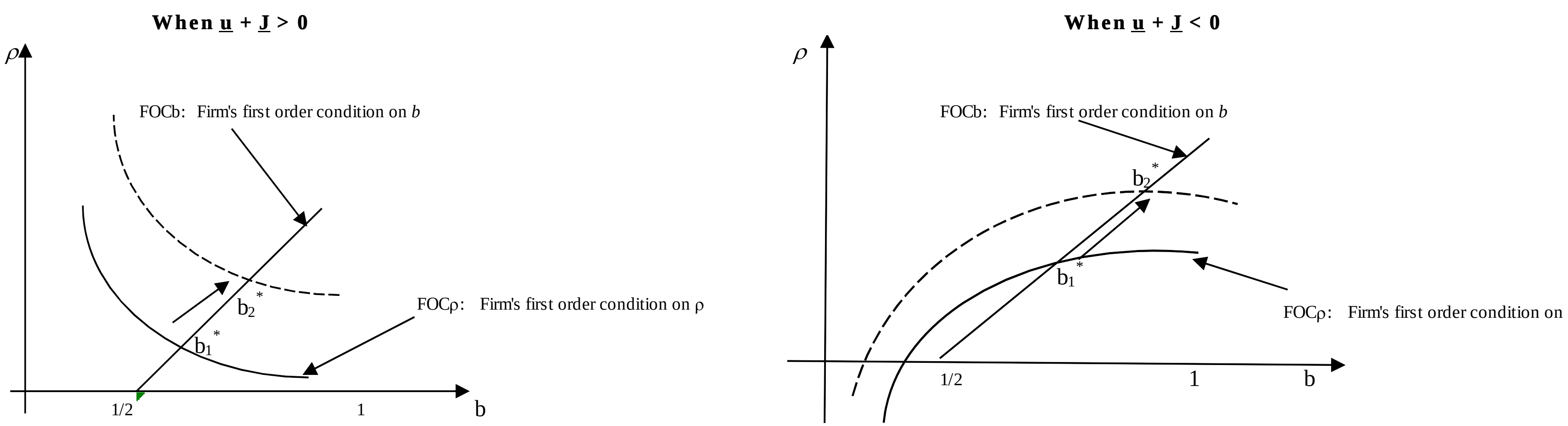


Fig. 3: time trend for median annual pay during 1992–2009

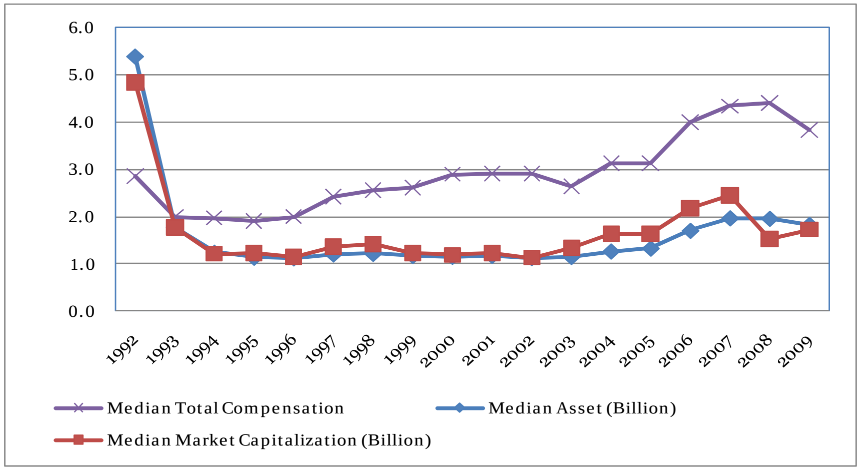


Fig. 4: time trend for median ratio between pay and size during 1992–2009

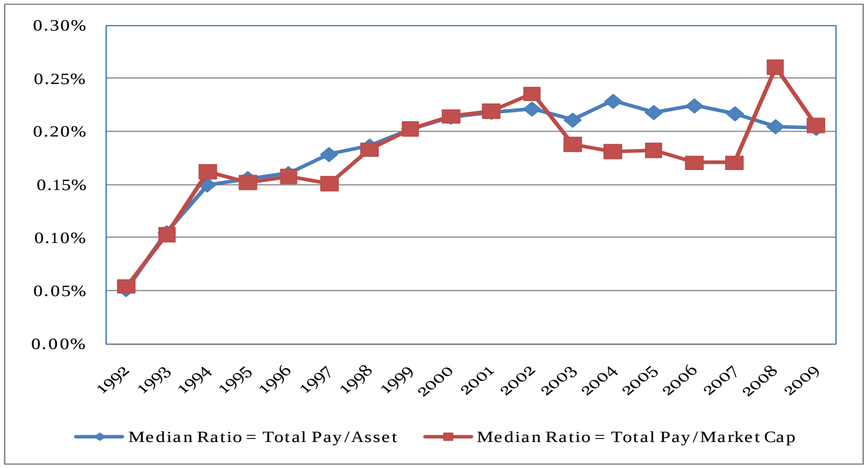


Fig. 5: time trend for median firm risks during 1992-2009

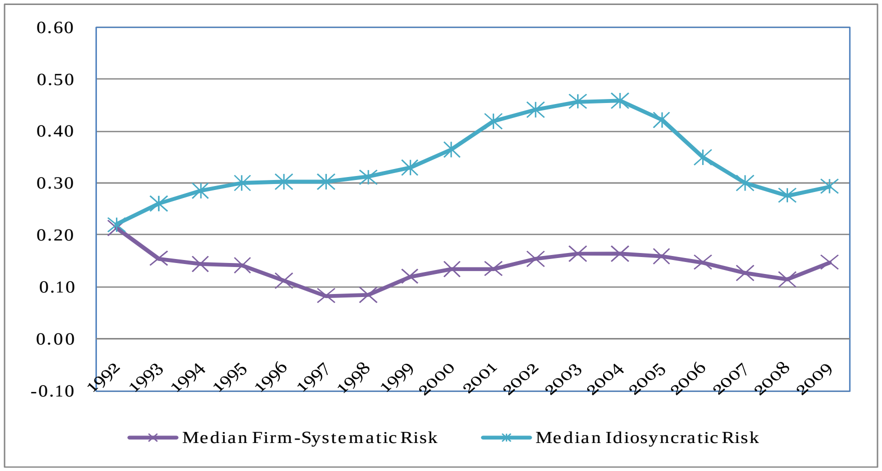


Fig. 6: time trend for macro proxies during 1992-2009

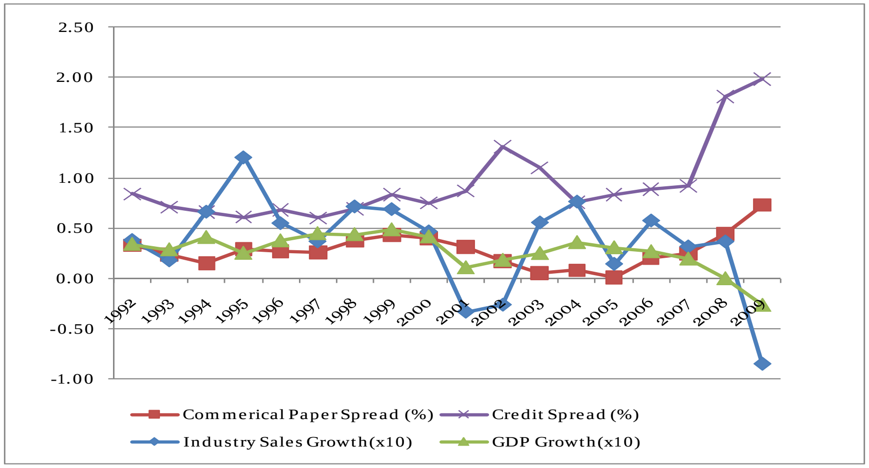


Table 1: summary statistics

This table reports the summary statistics on the executive compensation and characteristics, the firm characteristics, and macroeconomic variables for the period of 1992 to 2009 with a sample size of 13,051 firm-years. The executive compensation and characteristics data are retrieved from ExecuComp. New equity incentive is the pay-to-performance sensitivity of a CEO based on the stock and option grant for the fiscal year with respect to the $1,000 change in shareholders' wealth. Total equity incentive is the sensitivity for a CEO based on the cumulative stock and option grants with respect to the $1,000 change in shareholder's wealth. Firm characteristics data are from COMPUSTAT and CRSP. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. The commercial paper spread is defined as the difference between the annualized rate on three-month commercial paper and the three-month T-bill rate while the credit spread is the difference between the yield of Baa bond and Aaa bond.

Variables	Mean	Std Dev	Min.	25% Percentile	Median	75% Percentile	Max.	Skewness	Kurtosis
	Panel A: Executive Characteristics and Compensation								
Salary (Thousand)	$726	$338	$38	$471	$671	$922	$1,911	0.92	4.02
Bonus (Thousand)	$636	$933	$0	$0	$325	$819	$5,322	2.72	11.91
Equity Incentive Pay (Thousand)	$3,488	$5,177	$0	$506	$1,545	$4,130	$29,178	2.83	12.03
Total Compensation (Thousand)	$4,916	$5,969	$251	$1,410	$2,794	$5,791	$34,620	2.76	11.81
New Equity Incentive (Per $1,000 Change in Shareholders' Wealth)	$1.99	$2.96	$0.00	$0.28	$0.99	$2.39	$18.18	3.13	14.76
Total Equity Incentive (Per $1,000 Change in Shareholders' Wealth)	$23.16	$47.18	$0.09	$2.53	$6.67	$19.01	$328.91	3.86	19.84
Executive Tenure	7.84	7.12	0.50	2.75	5.67	10.51	37.02	1.72	6.24
Executive Age	55.31	7.04	39.00	50.00	55.00	60.00	75.00	0.04	2.88
	Panel B: Firm Characteristics								
Total Firm Return Volatility (Annualized)	43%	20%	16%	29%	38%	53%	115%	1.29	4.60
Idiosyncratic Volatility (Annualized)	40%	19%	14%	26%	35%	49%	107%	1.23	4.48
Systematic Firm Return Volatility (Annualized)	15%	10%	1%	9%	13%	19%	54%	1.44	5.64
Market Capitalization (Million)	$6,880	$17,105	$40	$543	$1,479	$4,771	$119,311	4.69	27.33
Assets (Million)	$5,213	$11,205	$63	$513	$1,346	$4,196	$78,174	4.26	24.12
Sales Growth	11%	24%	-49%	0%	8%	19%	110%	1.27	7.29
	Panel C: Macroeconomic Variables								
Industry Sales Growth	4%	11%	-43%	-2%	4%	9%	61%	0.28	6.34
Commercial Paper Spread (Basis Points)	27.72	17	1	17	27	38	73	0.78	4.12
GDP Growth	3%	2%	-3%	2%	3%	4%	5%	-1.47	5.11
Credit Spread (Basis Points)	94	39	60	69	83	92	198	1.74	4.90

Table 2: correlation

This table reports the correlations among explanatory variables and control variables for the period of 1992 to 2009 with a sample size of 13,051 firm-years. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance.

	Industry Sales Growth	GDP Growth	Lagged CP Spread	Credit Spread	Assets	Market Capitalization	Sales Growth	Tenure	Age	Total Firm Return Volatility	Specific Firm Return Volatility	Systematic Firm Return Volatility
Industry Sales Growth	1.000											
GDP Growth	0.421	1.000										
Lagged CP Spread	-0.203	-0.428	1.000									
Credit Spread	-0.355	-0.861	0.457	1.000								
Assets	-0.015	0.019	-0.041	0.039	1.000							
Market Capitalization	0.013	0.009	0.008	-0.005	0.805	1.000						
Sales Growth	0.273	-0.108	0.205	-0.193	-0.021	0.025	1.000					
Tenure	0.026	-0.003	0.044	-0.043	-0.078	-0.049	0.061	1.000				
Age	0.009	-0.012	0.044	-0.048	0.092	0.066	-0.041	0.408	1.000			
Total Firm Return Volatility	0.010	-0.270	0.002	-0.005	-0.253	-0.212	0.115	0.027	-0.188	1.000		
Idiosyncratic Volatility	0.013	-0.265	0.019	-0.016	-0.276	-0.233	0.125	0.029	-0.190	0.987	1.000	
Systematic Firm Return Volatility	-0.009	-0.180	-0.076	0.055	-0.057	-0.042	0.024	0.012	-0.109	0.653	0.531	1.000

Table 3: test of prediction 1 - effects of macroeconomic variable and firm risks on Pay-to-Performance Sensitivity (PPS)

This table reports the results for median regression (4.1): $PPS\ b = a_1 + a_2$ (macro proxy) + a_3 idiosyncratic risk + a_4 Firm-systematic risk + a_5 Age + a_6 Tenure + a_7 log(Firm size) + a_8 Firm growth + ε. The sample size is 13,051 firm-years for the period of 1992 to 2009. The dependent variables in Panels A and B are, respectively, the new equity incentive calculated with stock and option grants for the fiscal year and the total equity incentive calculated with the cumulative stock and option grants, with respect to the $1,000 change in shareholders' wealth. Industry sales growth and GDP growth are the respective growths in the fiscal year. NCP spread is the negative lagged commercial paper spread while NCredit spread is the negative credit spread. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. Firm size and firm growth are proxied by the firm's asset value and its sales growth, respectively. We also run regression (4.1) by replacing "specific " and "systematic" risks with "total risk." The coefficient and *t*-value for "total risk" are reported at the bottom of the table. For all regressions, we control for industry-fixed effects. Standard errors are calculated by bootstrapping with 500 replications. *, **, and *** indicate significance levels at 10%, 5%, and 1%, respectively.

	Prediction								
	This Model	Panel A: New Equity Incentive				Panel B: Total Equity Incentive			
Industry Sales Growth	-	-0.907 ***				-4.086 ***			
		(7.951)				(4.986)			
NCP Spread (basis points)	-		-0.011 ***				-0.061 ***		
			(14.407)				(14.858)		
GDP Growth	-			-9.162 ***				-58.978 ***	
				(14.363)				(14.480)	
NCredit Spread	-				-0.499 ***				-2.776 ***
					(15.404)				(15.052)
Firm-Specific Risk (annualized)	+	2.044 ***	2.504 ***	2.097 ***	2.058 ***	5.784 ***	8.012 ***	6.096 ***	5.907 ***
		-13.596	-17.246	-15.263	-15.253	-7.134	-9.876	-8.274	-8.129
Firm-Systematic Risk (annualized)	-	-0.45 **	-0.456 **	-0.599 ***	-0.598 ***	-2.692 **	-2.851 **	-3.805 ***	-3.256 ***
		(2.139)	(2.212)	(2.980)	(2.941)	(2.166)	(2.350)	(3.229)	(2.803)
Age		-0.008 ***	-0.006 ***	-0.006 ***	-0.005 ***	-0.078 ***	-0.057 ***	-0.061 ***	-0.058 ***
		(3.535)	(2.654)	(2.768)	(2.672)	(5.573)	(4.407)	(4.205)	(3.907)
Tenure		-0.008 ***	-0.009 ***	-0.008 ***	-0.008 ***	1.074 ***	1.057 ***	1.064 ***	1.054 ***
		(4.012)	(4.493)	(3.830)	(3.799)	-31.022	-29.094	-30.471	-28.483
log(Firm Size)		-0.213 ***	-0.194 ***	-0.216 ***	-0.217 ***	-1.949 ***	-1.873 ***	-2.003 ***	-2.013 ***
		(20.339)	(18.151)	(20.468)	(21.110)	(28.429)	(28.154)	(29.560)	(28.746)
Firm Growth		-0.108	-0.102 *	-0.064	-0.047	0.727	0.73 *	1.361 ***	1.352 ***
		(1.572)	(1.762)	(1.036)	(0.761)	-1.59	-1.781	-2.925	-2.906
Pseudo R^2		0.094	0.100	0.099	0.101	0.109	0.112	0.112	0.112
	Traditional Model								
Firm Total Risk	-	1.637 ***	2.087 ***	1.692 ***	1.681 ***	4.083 ***	6.206 ***	4.176 ***	4.185 ***
		-13.522	-17.445	-14.015	-14.595	-7.507	-10.874	-7.498	-7.18
Pseudo R^2		0.093	0.098	0.097	0.099	0.109	0.111	0.112	0.112

Table 4: median statistics for annual pay, firm Size, ratio between annual pay and firm size and firm risks during 1992–2009

This table reports the median statistics for annual compensation, firm size, ratio between annual pay and firm size, and firm risks. Firm size is either proxied by the firm's asset value or its market capitalization. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. The sample size is 13,051 firm-years for the period of 1992 to 2009.

Year	Total	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Sample Size	13,051	124	411	577	640	703	753	790	811	829	882	957	989	1,011	990	576	649	684	675
Annual Pay (millions)																			
Equity Incentive Pay		1.022	0.754	0.811	0.731	0.879	1.158	1.256	1.425	1.458	1.670	1.562	1.372	1.673	1.834	2.869	3.501	3.447	3.049
Total Compensation		2.850	1.996	1.969	1.909	1.997	2.422	2.549	2.606	2.892	2.909	2.913	2.638	3.117	3.129	3.998	4.335	4.396	3.830
Firm Size (billions)																			
Size 1 = Asset		5.382	1.779	1.241	1.137	1.122	1.205	1.216	1.173	1.154	1.175	1.128	1.151	1.267	1.329	1.703	1.966	1.956	1.828
Size 2 = Market Capitalization		4.832	1.769	1.211	1.233	1.148	1.370	1.423	1.232	1.190	1.234	1.132	1.335	1.628	1.626	2.169	2.444	1.517	1.726
$R_{pay/size}$ = Total Pay/Size																			
Total Pay / Size 1		0.051%	0.105%	0.150%	0.156%	0.160%	0.179%	0.187%	0.203%	0.214%	0.219%	0.222%	0.211%	0.229%	0.218%	0.225%	0.217%	0.205%	0.203%
Total Pay / Size 2		0.054%	0.103%	0.162%	0.152%	0.158%	0.151%	0.183%	0.202%	0.214%	0.219%	0.235%	0.188%	0.181%	0.183%	0.171%	0.171%	0.261%	0.205%
Firm Risks																			
Firm-Systematic Risk		0.213	0.155	0.144	0.142	0.112	0.083	0.084	0.120	0.134	0.135	0.154	0.164	0.163	0.159	0.148	0.127	0.115	0.148
Idiosyncratic Risk		0.220	0.261	0.285	0.301	0.302	0.302	0.311	0.329	0.365	0.418	0.441	0.457	0.459	0.422	0.349	0.300	0.276	0.293
Macroeconomic Variables																			
Industry Sales Growth		3.8%	1.8%	6.6%	12.0%	5.5%	3.7%	7.1%	6.9%	4.6%	-3.3%	-2.6%	5.5%	7.6%	1.4%	5.7%	3.1%	3.6%	-8.5%
Commerical Paper Spread (basis points)		33	24	15	29	27	26	38	43	40	31	17	5	8	1	20	25	44	73
GDP Growth		3.4%	2.9%	4.1%	2.5%	3.7%	4.5%	4.4%	4.8%	4.1%	1.1%	1.8%	2.5%	3.6%	3.1%	2.7%	1.9%	0.0%	-2.6%
Credit Spread (basis points)		84	71	66	61	68	60	69	83	75	87	131	110	76	83	89	92	181	198

Table 5: test of prediction 2 – effects of macroeconomic variable and firm risks on ratio between total compensation and firm size

This table reports median regression results for (4.2): $R_{pay/size}$ x 10^3 = a_1 + a_2 (macro proxy) + a_3 idiosyncratic risk + a_4 Firm-systematic risk + a_5 Age + a_6 Tenure + a_7 Firm growth + ε. The sample size is 13,051 firm-years for the period of 1992 to 2009. The dependent variable in Panel A is the ratio between an executive's total compensation and the firm's asset value while the dependent variable in Panel B is the ratio between an executive's total compensation and the firm's market capitalization. Industry sales growth and GDP growth are the respective growths in the fiscal year. NCP spread is the negative lagged commercial paper spread while NCredit spread is the negative credit spread. Total firm return volatility is the stock return volatility over the 60 months prior to the fiscal year. Systematic firm return volatility is equal to a firm's beta multiplied by the stock market risk while specific firm return volatility is the square root of the difference between the total return variance and the systematic return variance. Firm growth is proxied by the firm's sales growth. We also run regression (4.2) by replacing "specific " and "systematic" risks with "total risk." The coefficient and *t*-value for "total risk" are reported at the bottom of the table. We control for industry-fixed effects. Standard errors are calculated by bootstrapping with 500 replications. *, **, and *** indicate significance levels at 10%, 5%, and 1%, respectively.

	Prediction	Panel A:				Panel B:			
	This Model	$R_{pay/size}$ = Annual Total Pay/Asset Value				$R_{pay/size}$ = Annual Total Pay/Market Cap			
Industry Sales Growth	-	-0.278 **				-0.11			
		(1.943)				(0.625)			
NCP Spread (basis points)	-		-0.014 ***				-0.016 ***		
			(13.122)				(13.480)		
GDP Growth	-			-4.066 ***				-7.195 ***	
				(4.645)				(7.401)	
NCredit Spread	-				-0.202 ***				-0.491 ***
					(5.186)				(9.129)
Firm-Specific Risk (annualized)	+	7.033 ***	7.434 ***	7.077 ***	7.028 ***	6.362 ***	6.817 ***	6.392 ***	6.305 ***
		(38.961)	(36.390)	(38.223)	(38.742)	(36.627)	(35.640)	(35.566)	(35.702)
Firm-Systematic Risk (annualized)	-	-4.188 ***	-4.017 ***	-4.262 ***	-4.198 ***	-4.539 ***	-4.354 ***	-4.615 ***	-4.601 ***
		(13.922)	(12.831)	(13.781)	(13.689)	(17.518)	(16.439)	(16.869)	(17.577)
Age		-0.016 ***	-0.013 ***	-0.016 ***	-0.016 ***	-0.004	-0.003	-0.001	-0.001
		(5.735)	(4.519)	(5.577)	(5.670)	(1.338)	(1.064)	(0.442)	(0.322)
Tenure		0.013 ***	0.011 ***	0.013 ***	0.013 ***	0.009 ***	0.01 ***	0.009 ***	0.009 ***
		(4.807)	(3.930)	(4.775)	(4.778)	(3.037)	(3.351)	(3.013)	(3.203)
Firm Growth		0.333 ***	0.528 ***	0.388 ***	0.392 ***	-0.953 ***	-0.897 ***	-0.844 ***	-0.835 ***
		(3.459)	(4.623)	(3.903)	(4.037)	(10.856)	(9.167)	(9.578)	(9.571)
Pseudo R^2		0.131	0.136	0.131	0.131	0.104	0.112	0.106	0.108
Firm Total Risk		5.2 ***	5.619 ***	5.196 ***	5.178 ***	4.551 ***	5.008 ***	4.514 ***	4.463 ***
		(31.675)	(31.001)	(30.521)	(30.990)	(28.293)	(29.857)	(27.981)	(26.273)
Pseudo R^2		0.113	0.117	0.113	0.113	0.083	0.090	0.083	0.085