Information Acquisition in a War of Attrition*

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Abstract

This paper analyzes information acquisition in a war of attrition with the stochastic arrival of a public signal that reveals the state of nature and ends the game. Players can acquire information about the state of nature any time during the game. We study how the incentive to acquire information interacts with verifiability of the acquired information. When information is verifiable, players have only an incentive to free ride on the opponent's information acquisition, so there is an inefficient delay in information acquisition. When information is unverifiable, there is an additional incentive to catch up on information acquisition to prevent the opponent from extracting information rents, which causes duplication in information acquisition. We show that the interplay of these incentives has interesting policy implications and, in particular, conflicts are resolved faster when information is unverifiable.

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1 Introduction

Information plays a central role in many economic situations. While its importance has long been recognized, the fact that information often has to be acquired, rather than exogenously given, has not been fully incorporated into economic analysis yet. Several papers, however, show that costly information acquisition may have a significant impact on economic outcome,

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such as in bargaining (Dang, 2008; Shavell, 1994), in auctions (Morath and Münster, 2010; Dang, 2007; Persico, 2000; Matthews, 1984), in war of attrition (Morath, 2010), in principal-agent models (Crémer and Khalil, 1992; Crémer et al. 1998a, 1998b), and in repeated games (Liu 2010).[1]

When information has to be acquired, one central question is its efficiency: can information be acquired efficiently? There are two fundamental incentives that work against efficiency. First, information reduces uncertainty for everyone and, therefore, has a public good property. This implies that players have an incentive to free ride on others' information acquisition (the free-riding incentive), which would lead to underinvestment in information acquisition. On the other hand, if acquired information remains private then a player with the information can extract informational rents (the information-rent incentive). This may induce players to overinvest in information acquisition. In much of the literature about information acquisition, one of these two incentives operates. The purpose of this paper is to understand how these two incentives interact when both are operative.

Our context is a game of war-of-attrition with two players. Different from the standard war of attrition in which the game ends only when one of the two players concedes, our game can be terminated by the arrival of a public signal that determines the outcome according to a state of nature.[2] For example, in a competition over which technology standard an industry should adopt, a public signal may be exogenously generated that unambiguously shows one standard to be superior in quality than the other and thus ends the conflict with that standard being the winner. In a legal conflict, the trial will eventually generate a public signal to determine whether the defendant is guilty or not. The arrival of such a public signal can be modeled as a stochastic or deterministic deadline. We focus on the stochastic case, but the deterministic case can be easily accommodated. This setup allows us to derive some policy implication on whether improving the availability of the public signal or decreasing the information acquisition cost would help or hurt social welfare. In the literature, models of war of attrition with a deadline have been adopted to study litigation with a trial date (Ordover and Rubinstein, 1986), competition for technology standard in an emerging industry, labor-management disputes, or recruitment decision making, etc. (Damiano, Li and Suen, 2010).

The state of nature, which will be revealed by the public signal, is an important piece of information to both sides of the conflict, so both sides have an incentive to acquire this information to guide their decision on whether or not to concede. Such information is likely

[1]Several of these papers will be reviewed in more detail later in the paper.

[2]War of attrition has been widely used to study many forms of conflicts, including bargaining (Ordover and Rubinstein, 1986; Chatterjee and Samuelson, 1987), oligopolistic competition with the option to exit (Fudenberg and Tirole, 1986; Ghemawat and Nalebuff, 1985; Kreps and Wilson, 1982), patent races (Fudenberg et al., 1983), and public good provision (Bliss and Nalebuff, 1984).

to be costly to acquire because otherwise both sides would have been fully informed and the conflict would end at the very beginning. For example, in a competition over an industry's technology standard, both sides can invest in understanding which standard is superior and in a legal conflict both sides can choose to acquire information about the merit of the case.

We first consider a benchmark, where the players are not allowed to acquire information. In the main model of the paper, at any point in time, a player can choose to acquire information or not. We study two versions of this model. The two cases differ in whether acquired information is verifiable or not. When information is verifiable, the acquired information can be disclosed credibly, so it is essentially public. This implies that only the free-riding incentive is operative. When information is not verifiable, if a player remains the only one processing the information, then this player can extract informational rents, which induces the other player to catch up on acquiring information to protect himself from being cheated. This may induce players to overinvest in information acquisition.

Under verifiable information, once a player acquires information, the game ends immediately because information will unravel, so both players save the discounting cost of dragging on the conflict. Therefore, information acquisition has a public good property, and as a result each player wants to free ride on the opponent's information acquisition. There exist asymmetric equilibria even under a symmetric setting, where one player fully free rides on the other player to get information. In the unique symmetric equilibrium, players slowly acquire information and essentially play a war of attrition regarding the information acquisition. This creates inefficiency in terms of *a delay in information acquisition.*

Under unverifiable information, after a player acquires information, information does not unravel and a player who finds himself to be weak would pretend to be a strong one. As a result, the other player would also acquire information soon after.[3] This creates another source of inefficiency in the form of *a duplication in information acquisition.* However, this duplication reduces the delay in information acquisition and speeds up the ending of the conflict, which saves both players the discounting cost. As a result, the players' payoffs in the symmetric equilibrium are the same as in the verifiable case even though the equilibrium behaviors are different.

Another stakeholder in this war of attrition is the general public or the "society", who may prefer the conflict to be resolved according to the state of nature and who may also prefer the conflict to be resolved as early as possible. For example, in the competition for technology standard, the "society" would hope for the high quality standard to win and also that the standard gets implemented as early as possible. Similarly, in a legal conflict,

[3]Here, acquiring information first does not give a player an information advantage because the other player would acquire information soon after.

the "society" would prefer the defendant to lose when he or she is truly guilty and that the verdict be carried out as early as possible. Taking this into account to measure the total social welfare, unverifiable information becomes more socially desirable because it speeds up the conflict resolution.[4]

Often this external stakeholder can do something to influence the parameters of the game. For example, the government can invest in research to improve the availability of the public signal, can improve the legal system to make the trial take place more speedily, can change the cost of gathering information by interfering on the salary of attorneys, and so on. Therefore it is important to investigate the impact of a change in the parameters. We find that when the public signal is more likely to arrive, it would crowd out the incentive of the players to acquire information on their own, which is stronger than the direct benefit of speeding up conflict resolution from the public signal. The policy implication is that improving the system that generates the public signal actually hurts the social welfare. When players are less patient, they also have more incentive to acquire information and to resolve the conflict quickly, which results in a higher social welfare. The impact of a higher information acquisition cost, however, depends on whether the information is verifiable or not. Under verifiable information, a higher information cost hurts welfare because it *directly* reduces the incentive to acquire information. Under unverifiable information, there is an additional effect from a higher information cost, it encourages a solely informed opponent to cheat, and thus reduces the benefit of free-riding on the opponent and thus *indirectly* increases the incentive to acquire information. Depending on the other parameters, this indirect effect can be so strong as to reverse the impact on social welfare.[5]

The insight that there is the free-riding incentive in acquiring information has been raised by several papers. Grossman and Stiglitz (1980) point out that price cannot fully reveal the information of the informed players because in that case no one will acquire costly information. Persico (2004) shows that information is a public good and the free-riding incentive causes information to be underprovided. Li (2001) identifies the free-riding problem in acquiring costly evidence as a rationale for group conservatism. In Persico (2004) and Li (2001), only the free-riding incentive is operative and the informed players cannot extract information rent.

The existence of information rent gives incentive for acquiring information has been

[4]In our model, the resolution is always in agreement with the state of nature because we assumed that the stake in the game is so high for the two players so that an uninformed player will not concede.

[5]As in the previous literature of war of attrition (Riley, 1980; Nalebuff and Riley, 1983; Hendricks et al. 1988), we find a continuum of equilibria. To fix idea, we will do some of the comparisons of payoffs, outcomes and efficiency in terms of a particular equilibrium, which would be the unique symmetric equilibrium if the setting is symmetric.

recognized in many settings. Dang (2008) studies information acquisition in a setting of a seller-buyer bargaining and shows that information acquisition is socially harmful because it creates an endogenous lemon problem. Seller is motivated to get information to gain speculative profit. Shavell (1994) contains both the free-riding and the information rent incentive. It studies information acquisition between a buyer and a seller under voluntary disclosure and mandatory disclosure. Since mandatory disclosure will make the information a public good, it will eliminate the incentive to acquire information. Voluntary disclosure on the other hand allows a player to extract information rent and thus encourages information acquisition. Similar insights are derived when studying information acquisition and disclosure prior to oligopolistic competition by Jansen (2008). [6]

There can be other incentives in the information acquisitions that are not present in our models. Morath and Münster (2010) look at information acquisition prior to an all-pay-auction and show that the willingness to get information is smaller if the opponent acquires information. There, the information that can be learned are independent between the two players, so there is no free-riding incentive. The smaller incentive to get information given that the opponent gets information comes from the value of committing to be more aggressive in the bidding. Morath (2010) studies information acquisition in a war of attrition applied to the provision of public good. He assumes that information acquisition can only happen prior to the war of attrition and at a deadline the winner will be randomly drawn so there is a strategic incentive for a player not to acquire information so as to pre-commit to not concede when the deadline is sufficiently near.

Aside from the papers mentioned above, there is a literature on the deadline effect in war of attrition and bargaining games. However, the interpretation of the deadline in this literature tends to be a moment where the surplus in the relationship dissipates if a decision is not made in time. Several papers study the delay in reaching the agreement because of the presence of this type of deadline. Ma and Manove (1993) find strategic delay in bargaining when there may be exogenous and random delay in offer transmission. Spier (1992) explains the phenomenon of U-shaped settlement in litigation, that is, more settlements happen at the beginning and towards the deadline. Fershtman and Seidmann (1993) show that when players are sufficiently patient they wait until the deadline to reach an agreement if by rejecting an offer one commits to not accepting poorer offers. Ponsati (1994) show that delay in a war of attrition with a deadline can be explained by private information of the payoff loss of conceding.

[6] Whether a verifiable piece of information is subjective to mandatory disclosure or not is not an issue in our model as a player is known to be informed once he acquires information. Unverifiable information by definition cannot be disclosed credibly.

The remainder of this paper organizes as follows. Section 2 presents and analyzes a model with information acquisition not allowed, which serves as a benchmark. Then Section 3 presents the model with information acquisition. Section 4 and Section 5 analyze the case where acquired information is verifiable and the case where it is not, respectively. Section 6 highlights the role of the verifiability of the information by comparing equilibrium outcomes, payoffs, efficiency as well as comparative statics across the two models. In Section 7, we check the robustness of the results by looking at a model where the deadline is prefixed and deterministic[7] and where there is a fighting cost. Section 8 concludes.

2 War of Attrition with Arrival of Public Signals

2.1 The Model

Our underlying model is the standard war of attrition. There are two players, player 1 and player 2. Each player chooses the time to concede, $t_i \in \mathcal{R}_+$. If player j concedes faster ($t_i > t_j$), then player i receives utility $e^{-rt_j}h$, while player j receives utility $e^{-rt_j}l$, where $h > l > 0$. In other words, the loser (who concedes earlier) receives l, the winner receives h, and and the common discount rate is $r > 0$. For simplicity, we assume that if $t_i = t_j$ then both players obtain $e^{-rt_i}l$.[8] Let d denote the reward to the winner, that is, $d \equiv h - l$.

Different from the standard game, our game may conclude exogenously. There is an underlying state of nature, $\omega \in \{1, 2\}$. The state ω is initially unknown, but a public signal that reveals the state arrives according to a Poisson rate $\lambda > 0$. Upon arrival of the signal, the game ends and player i receives utility h if $\omega = i$ and utility l if $\omega = j \neq i$. It is commonly known that $\omega = 1$ with probability $p_1 \in [0.5, 1)$ and $\omega = 2$ with probability $p_2 = 1 - p_1$.

Within each unit in time $[t, t + dt)$, the timing of the game is as follows: a public signal arrives with probability $1 - e^{-\lambda dt}$ and, if so, the game ends. If a signal does not arrive, then players simultaneously decide whether to concede or not.

Although we study a continuous-time model, the model can be interpreted as the limit of the following discrete-time models as the time interval Δ tends to zero: In each period $t = \Delta, 2\Delta, ...,$ a public signal arrives with probability $1 - e^{-\lambda\Delta}$. If a signal does not arrive, then players simultaneously decide whether to concede or not. We use this discrete-time version of the model to clarify some of our results.

[7]A deterministic deadline can have another interpretation. The decision has to be made at a certain point in time, at which a third party is brought in to decide on the outcome. With that interpretation, the state of nature is simply this third party's preference.

[8]This is without loss of generality, as the event that both players concede simultaneously occurs with probability 0 in all the cases we consider.

2.2 Characterization

Let a distribution function $G_i : \mathcal{R}_+ \to [0,1]$ represent player i's strategy where $G_i(t)$ is the cumulative probability that player i concedes at or before time t. By a standard argument, G_i has no atom in its interior. In addition, unless it is a degenerate equilibrium (a player concedes immediately), the supports of G_1 and G_2 are common and an interval starting from 0. Denote by g_i the density of G_i over the interior of its support.

As in the standard war of attrition, at each t in the interior of the support, players must be indifferent between conceding and waiting an instant more. Therefore, for both $i = 1, 2$,

$$rl = \frac{g_j(t)}{1 - G_j(t)} d + \lambda p_i d.$$

The left-hand side is player i's marginal cost of waiting an instant more, while the right-hand side is the corresponding marginal benefit.[9] Player i receives l if he concedes. His marginal cost of staying an instant more is to collect the payoff an instant later. If player i wait an instant more, he obtains an additional payoff d in the following two contingencies: (1) Player j concedes before $t + dt$, whose arrival rate is $\frac{g_j(t)}{1-G_j(t)}$. Or, (2) a public signal arrives and the state is revealed to be favorable to player i (that is, $\omega = i$). The arrival rate of the signal is λ and the probability that $\omega = 1$ is p_i.

If $rl < \lambda p_i d$, then the marginal benefit is always larger than the marginal cost. Therefore, player i never concedes. If $rl \geq \lambda p_i d$, then the equation has a closed-form solution:

$$G_j(t) = 1 - (1 - G_j(0)) \exp\left(-\left(\frac{rl - \lambda p_i d}{d}\right) t\right),$$

where $G_j(0) \in [0, 1]$ is unknown.

There are three cases to consider, except for borderline cases. If $rl < \lambda p_2 d$, then no player is willing to concede. In this case, it is the unique equilibrium that both players wait for a public signal forever. If $\lambda p_2 d < rl < \lambda p_1 d$, then player 1 never concedes and, given player

[9] The corresponding indifference condition in the discrete-time model is

$$l = \frac{G_j(t) - G_j(t-\Delta)}{1 - G_j(t-\Delta)} h + \frac{1 - G_j(t)}{1 - G_j(t-\Delta)} e^{-r\Delta} \left((1 - e^{-\lambda\Delta})(l + p_i d) + e^{-\lambda\Delta} l\right)$$

The left-hand-side is player i's payoff by conceding, while the right-hand-side is his payoff by waiting for one more period. If player i does not concede, then player j concedes with probability $\frac{G_j(t)-G_j(t-\Delta)}{1-G_j(t-\Delta)}$ in this period. With the complementary probability, the game moves to the next period. In the next period, a public signal arrives with probability $1 - e^{-\lambda\Delta}$, in which case player i receives h with probability p_i and l with probability $1 - p_i$. If a signal does not arrive, for small enough Δ, player i is indifferent between conceding and waiting one more period again, and thus player i's expected payoff is l. It is straightforward to show that this discrete-time equation converges to the continuous-time condition as Δ tends to zero.

1's strategy, player 2 strictly prefer conceding immediately. It is the unique equilibrium in which player 2 concedes immediately.

If $rl > \lambda p_1 d$, then both G_1 and G_2 are well-defined. One restriction for the two unknowns, $G_1(0)$ and $G_2(0)$, is that at least one of them is equal to 0. This is because if player i acquires information with a positive probability at time 0 then player j strictly prefers waiting an instant more to conceding immediately. There is no further restriction on $G_1(0)$ and $G_2(0)$, and thus $G_i(0)$ can take any value in $[0, 1]$, as long as $G_j(0) = 0$.

In the later case, as is familiar in the war of attrition, there are degenerate equilibria: one player concedes never or only after a sufficiently long time, and the opponent concedes immediately. Those equilibria are essentially irrelevant, because their equilibrium outcomes coincide with either the equilibrium with $G_1(0) = 1$ or the one with $G_2(0) = 1$. For clarify of the exposition, we ignore all such equilibria throughout the paper.

The following proposition summarizes all the findings.

Proposition 1 *(1) If $rl < \lambda p_2 d$, then there is a unique equilibrium in which both players wait for a public signal forever. Player i's expected payoff is $\frac{\lambda}{r+\lambda}(l + p_i d)$.*

(2) If $\lambda p_2 d = rl \leq \lambda p_1 d$, then there is a continuum of equilibria. If $rl < \lambda p_1 d$, there is a continuum of equilibria: for any $\alpha \in [0, 1]$, there is an equilibrium in which player 2 concedes with probability α at date 0 and never concedes with the complementary probability and player 1 never concedes. Player 2's expected payoff is always $l = \frac{\lambda}{r+\lambda}(l + p_2 d)$, while player 1 can obtain any utility between $[l, h]$. If $rl = \lambda p_1 d$, the roles of the players can be switched, and thus both players can obtain any expected payoff in $[l, h]$, provided that the opponent obtains l.

(3) If $\lambda p_2 d < rl \leq \lambda p_1 d$, then it is the unique equilibrium outcome that player 2 concedes immediately. Player 2 obtains l, while player 1 obtains h.

(4) If $rl > \lambda p_1 d$, there is a continuum of equilibria. For any $G_1(0), G_2(0) \in [0, 1]$ such that $G_1(0) = 0$ or $G_2(0) = 0$, it is an equilibrium in which player i concedes according to a distribution function $G_j(t) = 1 - (1 - G_j(0)) \exp\left(-\left(\frac{rl - \lambda p_i d}{d}\right) t\right)$ for both $i = 1, 2$. Each player can obtain any expected payoff in $[l, h]$, provided that the opponent receives l.

Intuitively, if a public signal arrives sufficiently fast (λ is high), players are sufficiently patient (r is small), or the winning reward is sufficiently large (d is large), players are unwilling to concede early and, therefore, wait forever. In the opposite case, public signals are essentially irrelevant, and the game is almost identical to the standard war of attrition. In the asymmetric case ($p_1 > p_2$), there are intermediate cases: a public signal arrives fast enough, so that player 1 is willing to wait, but not too fast, so that player 2 does not want to bear fighting costs. In those cases, player 2 concedes immediately, and player 1 obtains the highest possible payoff.

3 The Model with Information Acquisition

In the game studied in the previous section, players have an incentive to learn about the state ω, in order to save unnecessary delay. From this section, we allow players to acquire information about ω. Information acquisition is costly: each player must incur a cost $c >$ in order to be informed. If a player incurs the cost, then he becomes fully informed about ω.

Within each unit of time $[t, t + dt)$, the timing of the game is as follows:

1. Signal stage: a public signal arrives with probability $1 - e^{-\lambda dt}$. If it arrives, then the game concludes.

2. Information acquisition stage: If a public does not arrive, players simultaneously decide whether to acquire information or not. A player's information acquisition is observable by the opponent.

3. Disclosure stage: If acquired information is verifiable, then the player who acquired information can disclose his information. Otherwise, this stage is skipped.

4. Concession stage: Players simultaneously decide whether to concede or not.

We focus on the case where both the public signal and information acquisition are relevant. Formally, we make the following two assumptions.

Assumption 1

$$\lambda p_i d > rl, i = 1, 2.$$

This assumption states that a public signal arrives fast enough (λ is high), the reward of winning is large enough (d is large), or players are patient enough (r is small), so that both players are unwilling to forgo the opportunity to win the game. Under this assumption, if no player acquires information, by Proposition 1, both players wait forever and the game concludes only by arrival of a public signal.

Assumption 2

$$c < \frac{rl}{\lambda}.$$

To understand this assumption, suppose that no player would acquire information and both players would wait forever. If a player believes that the state is favorable to him with probability p, then his expected payoff would be $\frac{\lambda}{r+\lambda}(l + pd)$. Now suppose the player acquires information and the game concludes immediately with payoffs according to the true state. In this case, the player's expected payoff is $-c+l+pd$. The assumption states that the latter payoff is strictly larger than the former, as long as a player does not strictly prefer conceding immediately to acquiring information, that is, $-c+l+pd > \frac{\lambda}{r+\lambda}(l+pd)$ for any p such that $-c+l+pd \geq l$. The cost of information acquisition is not too large, and thus players have a non-trivial incentive to acquire information.

The two assumptions together imply that $c < \frac{r}{\lambda+r}(l+p_i d)$ for both $i=1,2$.[10] This in turn implies that $-c+l+p_i d > l$ for both $i=1,2$.

In the following, for notational simplicity, we refer to a player who has acquired information and found that the state is favorable (unfavorable) to him as the "strong" ("weak") type.

4 Information Acquisition with Verifiable Information

This section considers the case where players can verify acquired information.

We begin with three immediate results. First, the game ends immediately once at least one player acquires information. This is because it is a dominant strategy for the strong type (who knows that the state is favorable to him) to disclose acquired information. If an informed player does not disclose information, then the opponent would know that the state is favorable to him and, therefore, would never concede. The weak informed player would concede immediately then.[11] Second, the game endogenously ends only when at least one player acquires information. If no player acquires information, by Assumption 1, both players wait forever. Third, by Assumption 1, 2 and the previous result, if player j never acquires information, then player i strictly prefers acquiring information to waiting.

The above results imply that the only strategic problem is who acquires information. When information is verifiable, players cannot collect any information rents. Therefore, only the free-riding incentive is operative. Players want the opponent to acquire information. Therefore, the game is essentially a war of attrition. The difference from the standard war of attrition is now it is not about who concedes first, but who acquires information first.

To formally describe equilibrium, let a distribution function $F_i : \mathcal{R}_+ \to [0,1]$ represent player i's information acquisition strategy where $F_i(t)$ is the cumulative probability that player i acquires information by time t. By a standard argument, F_i has no atom in its interior. Denote by f_i the density of F_i over the interior of its support. As familiar, if t is in

[10] This is because $\frac{r}{\lambda+r}(l+p_2 d) > \frac{r}{\lambda+r}(l+\frac{rl}{\lambda}) = \frac{rl}{\lambda} > c$.

[11] The logic of this result is familiar in the persuasion game literature. See Grossman (1981) and Milgrom (1981) for seminar contributions.

the interior of the support of F_i, player i must be indifferent between acquiring information and waiting an instant more. Therefore,

$$r\left(-c+l+p_i d\right)=\left(\lambda+\frac{f_j(t)}{1-F_j(t)}\right)c.$$

The left-hand side is player i's marginal cost of delaying information acquisition an instant, while the right-hand side is the corresponding marginal benefit.[12] The marginal cost is his collecting the payoff by acquiring information, $-c+l+p_i d$, an instant later. On the other hand, during an instant, a public signal may arrive, whose arrival rate is λ, or the opponent may acquires information, whose arrival rate is $\frac{f_j(t)}{1-F_j(t)}$. In both cases, player i saves the information acquisition cost c.

Solving the first-order ordinary differential equation,

$$F_j(t)=1-(1-F_j(0))\exp\left(-\left(r\frac{-c+l+p_i d}{c}-\lambda\right)t\right),$$

where $F_j\in[0,1]$ is unknown. Under Assumption 2, the function is always well-defined. One restriction for the two unknowns $F_1(0)$ and $F_2(0)$ is that at least one of them must be equal to zero, that is, $F_1(0)F_2(0)=0$. Similarly to the standard argument, this is because if a player acquires information with a positive probability at date 0, then the other player strictly prefers waiting an instant to acquiring information immediately. As in Section 2, there is no further restriction for the two unknowns. Therefore, $F_1(0)$ and $F_2(0)$ can take any values from $[0,1]$ as long as $F_1(0)F_2(0)=0$.

The following proposition summarizes the findings and completely characterize equilibria with verifiable information. Given the characterization above, the proof is straightforward and thus omitted.

Proposition 2 *When acquired information is verifiable, there is a continuum of equilibria: for each $F_1(0),F_2(0)\in[0,1]$ such that $F_1(0)F_2(0)=0$, there is an equilibrium in which player i acquires information according to the distribution function $F_i(t)=1-(1-F_i(0))\exp\left(-\left(r\frac{-c+l+p_j d}{c}-\lambda\right)t\right)$ for both $i=1,2$ and $j=-i$.*[13] *The set of players' expected*

[12] The discrete-time analog to this equation is

$$-c+l+p_i d=\frac{F_j(t)-F_j(t-\Delta)}{1-F_j(t-\Delta)}(l+p_i d)+\frac{1-F_j(t)}{1-F_j(t-\Delta)}e^{-r\Delta}\left((1-e^{-\lambda\Delta})(l+p_i d)+e^{-\lambda\Delta}(-c+l+p_i d)\right).$$

[13] As usual, there are degenerate equilibria, whose outcomes coincide with either the one with $F_1(0)=1$ or the one with $F_2(0)=1$.

payoffs is given by

$$\{(v_1, v_2) : v_i \in [-c + l + p_i d, l + p_i d], \text{ and } (v_1 - (-c + l + p_1 d))(v_2 - (-c + l + p_2 d)) = 0\},$$

that is, player i can achieve any payoff in $[-c + l + p_i d, l + p_i d]$, as long as the opponent receives $-c + l + p_j d$.

5 Information Acquisition with Unverifiable Information

This section studies the case where acquired information is not verifiable. The difference from the previous section is that there is no disclosure stage (or acquired information cannot be verified).

We first characterize the outcome of the game in which one player is informed about the state and the other is uninformed. This is the subgame right after one player acquires information. We use the outcome of this game to characterize equilibrium in the original game. The analysis of this subgame also has its own merit, as it reveals how much informational rents an informed player enjoys.

5.1 Subgame in which one player is informed

Although players begin with symmetric information in the original game, this subgame is one with incomplete information. This game is quite similar to that of Ordover and Rubinstein (1986). The difference lies in the uninformed player's strategy set. In Ordover and Rubinstein (1986), the uninformed player can choose only whether to concede or not, while in our game the uninformed player can acquire information and become informed. We show that this difference makes the equilibrium dynamics of our game significantly different from that of Ordover and Rubinstein (1986).

For expositional clarity, consider the discrete-time version of the model. We begin with two results regarding the equilibrium behavior of the weak informed player (who knows that the state is unfavorable to him) in the first period. First, the weak informed player must concede with a positive probability. Otherwise, the uninformed player would either acquire information or wait for a public signal, and then, due to discounting, the weak informed player would get strictly less than l. Second, the weak informed player does not concede with probability 1. If he concedes with probability 1, then in the next period the uninformed player would concede without acquiring information for sure. But then the weak informed player strictly prefers waiting to conceding in the first period, which is a contradiction.

The fact that the weak informed player stays with a positive probability but still obtains l implies the following two results regarding the uninformed player's behavior in the second period. First, the uninformed player must concede without acquiring information with a certain probability. Otherwise, the weak informed player would get only l in the second period, whether the uninformed player acquires information or waits forever. But this is a contradiction because he could get the same payoff earlier. Second, the probability that the uninformed player concedes must be small enough. Otherwise, the weak informed player would strictly prefer waiting to conceding in the first period.

In equilibrium, the weak informed player randomizes between conceding and waiting in the first period. The uninformed player randomizes between acquiring information and conceding in the second period. They do so with just enough probabilities so that both the weak informed player and the uninformed player are indifferent between their two actions. The game ends after the second period.

Formally, let α be the probability that the weak informed player concedes in the first period. Also, let β be the probability that the uninformed player acquires information. Then the following two conditions must be met:

1. Weak player i's indifference:

$$l = e^{-r\Delta}\left((1-e^{-\lambda\Delta})l + e^{-\lambda\Delta}\left(\beta l + (1-\beta)h\right)\right).$$

   If weak player i does not concede, then in the next period a public signal arrives with probability $1-e^{-\lambda\Delta}$, in which case the weak player i receives l. Conditional on the event that a signal does not arrive, uninformed player j acquires information with probability β and concedes with the complementary probability. Weak player i receives l and h in each event. Solving this equation,

$$\beta = 1 - \frac{(1-e^{-r\Delta})l}{e^{-(r+\lambda)\Delta}d},$$

   which is well-defined as long as Δ is sufficiently small.

2. Uninformed player j's indifference:

$$-c + l + \frac{p_j(1-\alpha)}{p_i + p_j(1-\alpha)}d = l.$$

   The left-hand side is player j's expected payoff by acquiring information in the second period. Conditional on the event that player i did not concede in the first period, by

Bayes' rule, player j's belief over the state is $\frac{p_j(1-\alpha)}{p_i+p_j(1-\alpha)}$. The right-hand side is his payoff by conceding. Solving the equation,

$$\alpha = 1 - \frac{p_i c}{p_j(d-c)}.$$

This probability is also well-defined because, by Assumptions 1 and 2,

$$c < \frac{r}{\lambda + r}(l + p_j d) < \frac{r}{\lambda + r}\left(\frac{\lambda p_j d}{r} + p_j d\right) = p_j d.$$

The probability α is independent of Δ, while β approaches zero as Δ tends to zero. Therefore, in the continuous-time model, in equilibrium the weak informed player immediately concedes with probability α. If the informed player does not concede, then an instant later the uninformed player acquires information with probability 1. One may wonder why the weak informed player does not prefer conceding immediately to waiting, given that the uninformed player acquires information with probability 1 in the second period and thus the weak informed player cannot obtain more than l. This is because in continuous time the cost of waiting an instant is also negligible. In compensating the weak informed player's cost of waiting an instant, it is enough for the uninformed player to concede with negligible probability. For this to be true, the uninformed player must remain indifferent between acquiring information and conceding, even though in equilibrium he acquires information with probability 1.

Proposition 3 *(Subgame outcome) In the subgame in which player i is informed about the state and player j is uninformed, there is a unique equilibrium. In equilibrium, the weak player i concedes with probability $\frac{p_j d - c}{p_j(d-c)}$. If the informed player does not concede, then the uninformed player acquires information and ends the game. The strong player i obtains h, and the weak player i obtains l. Player j's expected payoff is $l + \frac{p_j d - c}{d-c} d$.*

Proof. A precise proof is in the Appendix. ■

Let us conclude this subsection by calculating the amount of informational rents and the value of the uninformed player's information acquisition opportunity.

Informational rents If player i is informed and his opponent is not, then his expected payoff is $l + p_i d$. If the opponent is informed but player i is not informed, then his expected payoff is $l + \frac{p_i d - c}{d-c} d$. Therefore, the additional payoff player i collects by being informed, relative to the opponent's being informed, amounts to

$$(l + p_i d) - \left(l + \frac{p_i d - c}{d - c} d\right) = \frac{(1 - p_i)cd}{d - c}. \tag{1}$$

Value of the opportunity to acquire information Suppose the uninformed player cannot acquire information. Then the game is essentially the stationary version of Ordover and Rubinstein (1986)'s model. In this game, the weak informed player is indifferent between conceding and waiting from the first period on, while the uninformed player strictly prefers waiting to conceding in the first period and is indifferent between conceding and waiting from the second period on. In equilibrium, the weak informed player concedes only in the first period, while the uninformed player gradually concedes from the second period.

Let α be the probability that weak informed player i concedes in the first period. Also, let a distribution function $G_j : \mathcal{R}_+ \to [0, 1]$ represent uninformed player j's concession strategy. Then the following two conditions must be satisfied:

1. Weak player i's indifference: In the limit as Δ tends to zero,

$$rl = \frac{g_j(t)}{1 - G_j(t)} d.$$

   If weak player i does not concede, uninformed player j may concede, whose arrival rate is $\frac{g_j(t)}{1-G_j(t)}$. Different from the previous cases, arrival of a public signal does not contribute to the marginal benefit, because weak player i surely loses once a signal arrives. Solving this equation,

$$G_j(t) = 1 - \exp\left(-\frac{rl}{d} t\right).$$

   Notice that $G_j(0) = 0$, because if $G_j(0) > 0$ then weak player i would strictly prefer waiting an instant to conceding immediately.

2. Uninformed player j's indifference:

$$rl = \lambda \frac{p_j(1 - \alpha)}{p_i + p_j(1 - \alpha)} d.$$

   The left-hand side is uninformed player j's marginal cost of waiting an instant, while the right-hand side is the corresponding marginal benefit. The latter only comes from the possibility of arrival of a public signal, because informed player i never concedes, whether he is strong or weak. Solving the equation,

$$\alpha = \frac{p_j \lambda d - rl}{p_j(\lambda d - rl)}.$$

In this game, uninformed player j obtains

$$l + p_j \alpha d = l + \frac{p_j \lambda d - rl}{\lambda d - rl} d.$$

The value of the uninformed player's information acquisition opportunity is the difference between his payoff in Proposition 3 and this payoff, which amounts to

$$\left(l + \frac{p_j d - c}{d - c} d \right) - \left(l + \frac{p_j \lambda d - rl}{\lambda d - rl} d \right) = \frac{(1 - p_j) d (rl - \lambda c)}{(d - c)(\lambda d - rl)} d.$$

Under Assumption 2, this value is always positive. This result proves that the option to acquire information indeed help the uninformed player.

Weak informed player i again obtains l, while strong informed player i's expected payoff is

$$\int_0^\infty e^{-rt} hd \left(1 - e^{-\lambda t}(1 - G_j(t))\right) = \frac{\lambda + \frac{rl}{d}}{r + \lambda + \frac{rl}{d}} h,$$

which is strictly smaller than the corresponding payoff, h, in Proposition 3. This result is somewhat surprising, because when information acquisition is allowed, the opponent (player j) is in a stronger position and indeed obtains a higher payoff. The driving force for this result is that the role of the uninformed player's information acquisition is mainly to reduce unnecessary delay and this helps the informed player as well as the uninformed player.

5.2 The (original) game in which both players are uninformed

Now we consider the original game in which both players are uninformed about the state.

As with verifiable information, the game ends endogenously only when at least one player acquires information. This is because of Assumption 2 and Proposition 3: if player i never acquires information, then player j strictly prefer acquiring information to waiting, because his expected payoff by acquiring information is $-c + l + p_i d$ by Proposition 3 and it is strictly higher than his expected payoff by not acquiring information by Assumption 2.

Also, players prefer the opponent to acquire information first. Although an informed player receives informational rents, the cost of information acquisition outweighs information rents, because

$$c - \frac{(1-p_i)cd}{d-c} = \frac{p_i d - c}{d-c} c > 0.$$

The two results imply that the game is again a war of attrition regarding who acquires information first. The difference from the verifiable case is player's payoffs after one player's information acquisition. An informed player obtains the same payoff as in the verifiable case, but the opponent receives a strictly lower payoff than in the verifiable case. The latter is because in equilibrium an uninformed player also has to acquire information with a positive probability.

For direct comparison, let us use the same notations for players' information acquisition strategies as in the verifiable case. As usual, if t is in the interior of the support of F, then player i must be indifferent between acquiring information and delaying it an instant. Therefore,

$$r\left(-c + l + p_i d\right) = \lambda c + \frac{f_j(t)}{1 - F_j(t)} \frac{p_i d - c}{d - c} c.$$

The left-hand side is player i's marginal cost of acquiring information an instant later, while the right-hand side is the corresponding marginal benefit. If player i does not acquire information right now, then a public signal may arrive, who arrival rate is λ, or the opponent may acquire information, whose arrival rate is $\frac{f_j(t)}{1-F_j(t)}$. In the latter case, player i saves the cost of information acquisition c but loses informational rents, $\frac{(1-p_i)d}{d-c}c$.

The solution to this first-order ordinary differential equation is given by

$$F_j(t) = 1 - (1 - F_j(0)) \exp\left(-\frac{d-c}{p_i d - c}\left(r\frac{-c + l + p_i d}{c} - \lambda\right) t\right).$$

Again, as usual, $F_1(0)$ and $F_2(0)$ can take any values from $[0,1]$ as long as at least one of them is equal to zero. Therefore, there is a continuum of equilibria.

Proposition 4 *When acquired information is not verifiable, there is a continuum of equilibria: for each* $F_1(0), F_2(0) \in [0,1]$ *such that* $F_1(0)F_2(0) = 0$*, there is an equilibrium in which player* i *acquires information according to the distribution function* $F_i(t) = 1 - (1 - F_i(0)) \exp\left(-\frac{d-c}{p_j d - c}\left(r\frac{-c+l+p_j d}{c} - \lambda\right) t\right)$ *for both* $i = 1, 2$ *and* $j = -i$*. The set of players' payoffs is given by*

$$\left\{(v_1, v_2) : v_i \in \left[-c + l + p_i d, l + \frac{p_j d - c}{d - c} d\right], \text{ and } (v_1 - (-c + l + p_1 d))(v_2 - (-c + l + p_2 d)) = 0\right\},$$

that is, player i *can achieve any payoff in* $\left[-c + l + p_i d, l + \frac{p_j d - c}{d-c} d\right]$*, as long as the opponent receives* $-c + l + p_j d$*.*

6 The Role of Information Verifiability

6.1 Comparison of payoffs and welfare

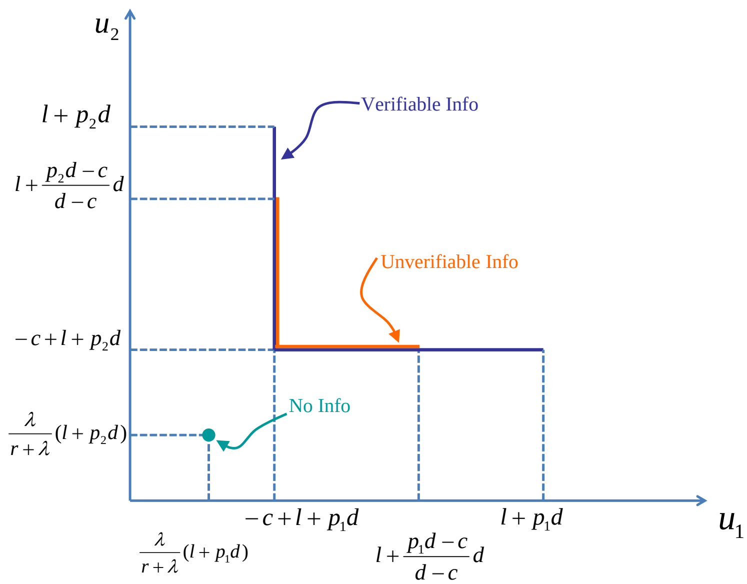

Figure 1: The sets of equilibrium payoffs

Figure 1 presents the set of payoff across three models. Not surprisingly, information acquisition increases players' payoffs.

There are some payoff vectors that are attainable only in the verifiable case. Those payoffs are unattainable in the unverifiable case because of the informed player's informational rents: in the unverifiable case, the game turns into an incomplete information once a player acquires information. The informed player must receive informational rents, but it can be created only by destroying the uninformed player's payoff. This limits the extent to which a player can benefit from the other's information acquisition.

In both the verifiable and the unverifiable setting, there are multiple equilibria due to the freedom in the initial probability of information acquisition. We call the equilibrium where the game ends the slowest the "slowest equilibrium". That is, it is the equilibrium where $F_1(0) = F_2(0) = 0$. In the slowest equilibria, players' payoffs reaches the lower bounds of their payoffs and are the same no matter information is verifiable or not. There are however several important differences.

First, the lower bound is exogenously given in the verifiable case, while it is endogenously determined in the unverifiable case. In the former case, acquired information is essentially public, and the lower bound simply derives from here. The lower bound holds even if a player does not play an equilibrium strategy. In the latter case, there is no a priori reason why the game ends immediately after a player's information acquisition. It is only in equilibrium that the game ends immediately after because the other player acquires information immediately after.

Second, although the equilibrium strategies share some qualitative properties, they are quantitatively different. In particular, the game ends faster with unverifiable information. This comparison is valid not only for the slowest equilibria, but also for all equilibria that share the same initial probability $F_1(0)$ and $F_2(0)$ across the two settings. See Figure 2 for a graphical representation of speed of game ending for the slowest equilibria.

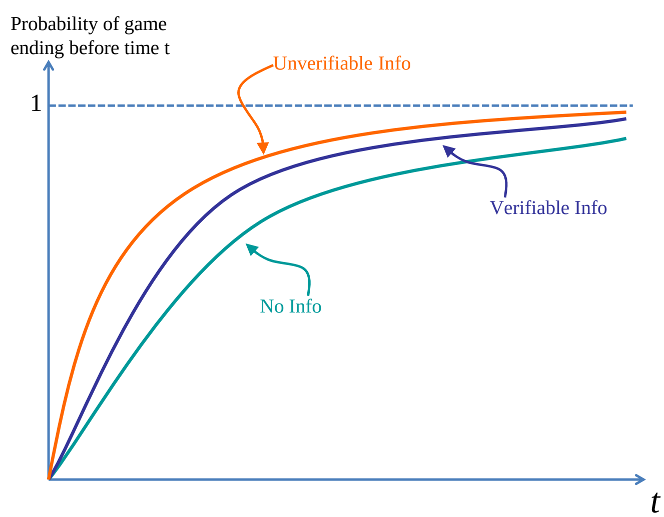

Figure 2: The Cumulative Probability of the Game Ending for the Slowest Equilibria.

Proposition 5 *Let $S_V(t)$ and $S_U(t)$ be the cumulative probabilities that the game ends by time t in the verifiable case and in the unverifiable case, respectively for a given pair of $F_1(0)$ and $F_2(0)$. Then $S_V(t)$ first order stochastically dominates $S_U(t)$.*

Proof. From the characterization in Section 4,

$$\begin{aligned} S_V(t) &= 1 - e^{-\lambda t}(1 - F_1(t))(1 - F_2(t)) \\ &= 1 - e^{-\lambda t} \exp\left(-\left(r\frac{-c+l+p_1 d}{c} - \lambda\right)t\right) \exp\left(-\left(r\frac{-c+l+p_2 d}{c} - \lambda\right)t\right). \end{aligned}$$

Similarly,

$$S_U(t) = 1 - e^{-\lambda t} \exp\left(-\frac{d-c}{p_1 d - c}(r\frac{-c+l+p_1 d}{c} - \lambda)t\right) \exp\left(-\frac{d-c}{p_2 d - c}(r\frac{-c+l+p_2 d}{c} - \lambda)t\right).$$

Since $S_V(t) < S_U(t)$ for any $t > 0$, $S_V(t)$ first order stochastically dominates $S_U(t)$. ■

The social welfare should include more than just the two players in this war of attrition. For example, in a war of technology standard, the general public should prefer the higher quality standard to be adopted and also that the adoption happens earlier, so that all the production and consumption can happen earlier. In a legal conflict, the general public cares for the legal outcome to be just and fair. Therefore, these outsiders' utility should also be considered in the comparison of social welfare between the verifiable case and the unverifiable case. As in Proposition 5, all our comparison from now on is done by comparing equilibria that share the same $F_1(0)$ and $F_2(0)$ across the verifiable and the unverifiable setting.

Case 1. If the social planner only cares about the two players' payoffs, then there is no welfare difference between the two cases. For all the non-degenerate equilibria, players randomize between acquiring and not acquiring information under both verifiable and unverifiable information. The two cases have the same set of degenerate equilibria.

Case 2. If the social planner, in addition to Case 1, cares about whether the right person wins the war of attrition, i.e., whether the outcome is consistent with the state of nature, then again there is no welfare difference between the two cases. This is because in both cases, players do not concede until someone has acquired information or the public signal has arrived, so the outcome is always always determined by the true state of nature.

Case 3. If the social planner, in addition to Case 2, cares about the speed of the resolution of the conflict, then unverifiable information is better. Game ends faster under unverifiable information as shown by Proposition 5. Under unverifiable information, the incentive to protect oneself from being exploited by an informed opponent give one more incentive to acquire information, and thus speeds up the ending of the game.

There is however a source of inefficiency brought by unverifiability of information. Under verifiable information, game ends as soon as one side acquires information, so there is no duplication of information acquisition. Under unverifiable information, however, after one side acquires information, the other side will also acquire information with probability close to one in the next period, so there is almost complete duplication of information acquisition. The increase in the information duplication and the decrease in the delay of information acquisition cancels each other out when comparing the payoff of the two players.

6.2 Comparatives Statics

The comparative statics with respect to some parameters do not depend on whether the information is verifiable or not.

Proposition 6 *When λ decreases, or r increases, the incentive to acquire information increases under both verifiable and unverifiable information in the sense that given the same $F_i(0)$, $F_i(t)$ increases.*

Proof. Under verifiable information,

$$\frac{\partial}{\partial \lambda} F_i(t) = (1 - F_i(0)) \left(r \frac{-c + l + p_j d}{c} - \lambda \right) \exp \left(- \left(r \frac{-c + l + p_j d}{c} - \lambda \right) t \right) (-t) < 0$$
$$\frac{\partial}{\partial r} F_i(t) = (1 - F_i(0)) \left(r \frac{-c + l + p_j d}{c} - \lambda \right) \exp \left(- \left(r \frac{-c + l + p_j d}{c} - \lambda \right) t \right) t > 0$$

Under unverifiable information,

$$\frac{\partial}{\partial \lambda} F_i(t) = (1 - F_i(0)) \frac{d - c}{p_j d - c} \left(r \frac{-c + l + p_j d}{c} - \lambda \right) \exp \left(- \left(r \frac{-c + l + p_j d}{c} - \lambda \right) t \right) (-t) < 0$$
$$\frac{\partial}{\partial r} F_i(t) = (1 - F_i(0)) \frac{d - c}{p_j d - c} \left(r \frac{-c + l + p_j d}{c} - \lambda \right) \exp \left(- \left(r \frac{-c + l + p_j d}{c} - \lambda \right) t \right) t > 0$$

■

A lower λ means it is less likely to get "free" information from the public signal, so the players have more incentive to acquire information on their own. When r increases, the cost of discounting is higher, so the players have more incentive to end the game by acquiring information. The following corollary immediately follows.

Corollary 1 *When λ decreases or when r increases, the social welfare as defined in Case 3 increases.*

The arrival of the public signal speeds up the resolution of the conflict by itself, but at the same time, as shown above, it also crowds out the incentive for the players to acquire information on their own. The second effect is stronger, so the combined effect is that the more likely the signal arrives, the slower the game ends, and as a result, the lower is the social welfare. When the discount rate increases, the players have more incentive to acquire information and the game ends faster.

The comparative statics with respect to some parameters depend on whether the information is verifiable or not.

Proposition 7 *When c decreases, the incentive to acquire information increases under verifiable information. However, this may not be true under unverifiable information. When d*

increases, the incentive to acquire information increases under verifiable information. However, this may not be true under unverifiable information.

Proof. Under verifiable information,

$$\frac{\partial}{\partial c}F_i(t) = (1 - F_i(0))\left(r\frac{-c+l+p_jd}{c} - \lambda\right)\exp\left(-\left(r\frac{-c+l+p_jd}{c} - \lambda\right)t\right)(-\frac{l+p_jd}{c^2}) < 0$$
$$\frac{\partial}{\partial d}F_i(t) = (1 - F_i(0))\left(r\frac{-c+l+p_jd}{c} - \lambda\right)\exp\left(-\left(r\frac{-c+l+p_jd}{c} - \lambda\right)t\right)\frac{rp_j}{c} > 0$$

Under unverifiable information,

$$\frac{\partial}{\partial c}F_i(t) = (1 - F_i(0))\left(r\frac{-c+l+p_jd}{c} - \lambda\right)\exp\left(-\left(r\frac{-c+l+p_jd}{c} - \lambda\right)t\right)(-\frac{l+p_jd}{c^2} + \frac{p_id}{(p_jd-c)^2})$$
$$\frac{\partial}{\partial d}F_i(t) = (1 - F_i(0))\left(r\frac{-c+l+p_jd}{c} - \lambda\right)\exp\left(-\left(r\frac{-c+l+p_jd}{c} - \lambda\right)t\right)(\frac{rp_j}{c} - \frac{p_id}{(p_j-c)^2})$$

■

Under verifiable information, when the cost of acquiring information c decreases, the marginal cost of waiting an instant more before acquiring information increases because now the amount being discounted is higher, while the marginal benefit of waiting an instant more to free-ride (either on the public signal or on the opponent's information acquisition) decreases because the saving in the information cost is lower. Both effects gives more incentive to acquire information right away. Under unverifiable information, there is an additional effect: when c decreases, a solely informed player would cheat less to make the uninformed player indifferent between getting information or not immediately after, so the benefit of free-riding on the other to acquire information increases.[14] This is a countervailing effect that makes the total effect dependant on the other parameters' values.

Under verifiable information, when the reward for winning d increases, the expected payoff increases so the cost of discounting increases. This gives a player more incentive to end the game early. Under unverifiable information, however, there is an additional effect: when d increases, an informed opponent would cheat less to make the uninformed player indifferent between getting information or not immediately after, so the benefit of free-riding on the other to acquire information increases.[15] This is a countervailing effect that makes the total effect dependant on the other parameters' values. The following corollaries are immediate.

[14] If j is uninformed, the probability of weak i to concede (not cheat) is $\alpha = 1 - \frac{p_ic}{p_j(d-c)}$ which increases when c decreases.

[15] If j is uninformed, the probability of weak i to concede (not cheat) is $\alpha = 1 - \frac{p_ic}{p_j(d-c)}$ which increases when d increases.

Corollary 2 *A lower c increases social welfare as defined in Case 3 under verifiable information. However, this may not be true under unverifiable information. A higher d increases social welfare as defined in Case 3 under verifiable information. However, this may not be true under unverifiable information.*

7 Deterministic Deadline and Fighting Cost

In this section, we show that the qualitative results do not change if we instead have a pre-fixed deadline instead of stochastic deadlines and a fighting cost instead of discounting.

7.1 Without information acquisition

We normalize the deadline to be at time $t = 1$. Denote the per unit time fighting cost by k. The payoff is as follows. For $i = 1, 2$,

$$U_i(t_i, t_j, \omega) = \begin{cases} h - kt_i, & \text{if } (t_i > t_j) \text{ or } (t_i = t_j = 1 \text{ and } \omega = i), \\ l - kt_i, & \text{if } (t_i < t_j) \text{ or } (t_i = t_j = 1 \text{ and } \omega = j), \text{ or } (t_i = t_j < 1). \end{cases}$$

Assume $0 < k < d$. The rest of the assumptions are the same as the model in Section 2. We have the following proposition that is parallel to proposition 1.

Proposition 8

1. *If $k < p_2 d$, then there is a unique equilibrium where both players wait until the deadline.*

2. *If $p_2 d = k \leq p_1 d$, then there is a continuum of equilibrium where player 1 never concedes and player 2 randomizes at $t = 0$. Player 2's payoff is l and player 1's payoff can be anywhere between $[l, h]$. If $k = p_1 d$ as well, then both players can obtain any payoff between $[l, h]$.*

3. *If $p_2 d < k \leq p_1 d$, then the unique equilibrium outcome is that player 2 concedes immediately. Player 2 obtains l while player 1 obtains h.*

4. *If $2p_1 d > k > p_1 d > p_2 d$, then there exist degenerate equilibria, where player 2 concedes immediately at $t = 0$.*

5. *If $k \geq 2p_1 d$ and $p_1 > p_2$, there exist two types of degenerate equilibria. One has player 1 conceding immediately at $t = 0$, and the other has player 2 conceding immediately at $t = 0$.*

6. *If $k > p_1 d$ and $p_1 = p_2 = p$, then there is a continuum of equilibria (in addition to degenerate equilibria). For any $G_1(0), G_2(0) \in [0,1]$ such that $G_1(0)G_2(0) = 0$, it is an equilibrium in which player i concedes according to a distribution function $G_i(t) = 1 - (1 - G_i(0)) \exp\left(-\left(\frac{k}{d}\right)t\right)$ for $t \in [0, \bar{t}]$ for both $i = 1, 2$, where $\bar{t} = 1 - \frac{pd}{k}$. Each player can obtain any expected payoff in $[l, h]$, provided that the opponent receives l.*

Note that under parameter case $k > p_1 d > p_2 d$, there is no equilibrium where both players use mixed strategy. This is because if so, time $\bar{t}_1 \equiv 1 - \frac{p_1 d}{k}$ will be reached with positive probability. After $\bar{t}_1$, player 1 will never concede. Then player 2 will concede with probability one at $\bar{t}_1$. This implies that for $t < \bar{t}_1$ but very close to $\bar{t}_1$, player 1 will stay with probability one, which is a contradiction.

7.2 Information Acquisition with Verifiable Information

From this section, we allow players to acquire information. At any point of time before the deadline, players can incur a cost $c > 0$ and find the true state of the nature.

Parallel to Assumption 1 and Assumption 2, we make the following two assumptions:

Assumption 1'

$$k < p_i d, i = 1, 2.$$

Assumption 2'

$$c < k.$$

The first assumption says that the cost of fighting is small enough such that both players will fight until the deadline if they cannot acquire information. The second assumption says that the information acquiring cost is low enough relative to the fighting cost so it is not the case that both players do not acquire information.[16]

We first examine the case where information acquisition is observable and verifiable. Because the information is verifiable, the game ends as soon as someone acquires information.

There exists a continuum of equilibria where both players randomizes on information acquisition.

Let a distribution function $F_i : [0,1] \to [0,1]$ represent player i's information acquisition strategy where $F_i(t)$ is the cumulative probability that each player acquires information by time t. Denote by f_i the density of F_i.

[16] In an alternative setup with deterministic deadline and positive discounting instead of fighting cost, the two corresponding assumptions are the following. Assumption 1" is $r < \ln(1 + \frac{p_i d}{l}), i = 1, 2$. Assumption 2" is $c < (1 - e^r)l$, where r is the discount rate.

If t is in the support of F_i, then

$$k = \frac{f(t)}{1 - F(t)} c.$$

The left-hand side is the marginal cost of delaying information acquisition an instant, while the right-hand side is the corresponding marginal benefit. When information acquisition is verifiable and observable, the game ends immediately once a player obtains information. Therefore, the benefit of a player's delaying information acquisition comes from the possibility that the other player pays the cost c instead. Therefore, the marginal benefit is equal to the conditional rate at which the other player acquires information times the information acquisition cost. Solving this ordinary differential equation,

$$F_i(t) = 1 - (1 - F_i(0))e^{-\frac{k}{c}t}.$$

Let $\bar{t}$ be defined by the following equality, i.e., both players are indifferent between acquiring information and waiting until the end of the game at $\bar{t}$. Therefore,

$$l + p_i d - k(1 - \bar{t}) = -c + l + p_i d \quad \Rightarrow \bar{t} = 1 - \frac{c}{k}$$

The value $\bar{t}$ is well defined because we assumed $c < k$.

There also exist two degenerate equilibria where players both use pure strategies and one player acquires information at the beginning of the game.[17] These equilibria are outcome equivalent to some of the equilibria in the continuum of equilibrium described above.

Proposition 9 *When acquired information is verifiable there is a continuum of mixed strategy equilibria: for any $F_1(0), F_2(0) \in [0, 1]$ such that $F_1(0)F_2(0) = 0$, there is a equilibrium where each player acquires information according to a distribution function $F_i(t) = 1 - (1 - F_i(0))e^{-\frac{k}{c}t}$ over time $[0, \bar{t}]$ where $\bar{t} = 1 - \frac{c}{k}$, and they both do not acquire information and do not concede after $\bar{t}$. The set of equilibrium payoff is*

$$\{(v_1, v_2) : v_i \in [-c + l + p_i d, l + p_i d], \text{ and } (v_1 - (-c + l + p_1 d))(v_2 - (-c + l + p_2 d)) = 0\}.$$

[17] The strategy profile of such an equilibrium is the following: any player concedes immediately if the other is disclosed to be strong, otherwise one player acquires information immediately at any time $t < \bar{t}$ and never acquires information and never concedes for $t \geq \bar{t}$, and the other player never acquires information and never concedes.

7.3 Information Acquisition with unverifiable information

This section studies the case where the acquired information is not verifiable.

Let's first consider a subgame where one player is informed, but the other is not. The analysis is the same as in Section 5. The weak informed player concedes with probability less than one in the first period, and then the uninformed player randomizes between conceding and acquiring information in the second period. Formally, let α be the probability that the weak informed player concedes in the first period. Also, let β be the probability that the uninformed player acquires information. Weak player i's indifference implies:

$$l = -k\Delta + \beta l + (1-\beta)h \quad \rightarrow \beta = 1 - \frac{k\Delta}{d}$$

Uninformed player j's indifference:

$$-c + l + \frac{p_j(1-\alpha)}{p_i + p_j(1-\alpha)} d = l.$$

Therefore, α is independent of Δ, while β approaches zero as Δ tends to zero. Therefore, the subgame outcome is exactly the same as in Proposition 3.

Now we consider the original game in which both players are uninformed about the state. There exists a continuum of mixed strategy equilibria. Let a distribution function $F_i : [0,1] \rightarrow [0,1]$ represent player i's information acquisition strategy where $F_i(t)$ is the cumulative probability that each player acquires information by time t. Denote by f_i the density of F_i. As usual, if t is in the interior of the support of F, then player i must be indifferent between acquiring information and delaying it an instant. Therefore,

$$k = \frac{f_j(t)}{1 - F_j(t)} \frac{p_i d - c}{d - c} c.$$

The left-hand side is player i's marginal cost of acquiring information an instant later, while the right-hand side is the corresponding marginal benefit. If player i does not acquire information right now, then a public signal may arrive, who arrival rate is λ, or the opponent may acquire information, whose arrival rate is $\frac{f_j(t)}{1-F_j(t)}$. In the latter case, player i saves the cost of information acquisition c but loses Informational rents, $\frac{(1-p_i)d}{d-c}c$. The solution to this first-order ordinary differential equation is given by

$$F_j(t) = 1 - (1 - F_j(0)) \exp\left(-\frac{d-c}{p_i d - c}\frac{k}{c}t\right).$$

Again, as usual, $F_1(0)$ and $F_2(0)$ can take any values from $[0,1]$ as long as at least one of them is equal to zero. Therefore, there is a continuum of equilibria.

Let $\bar{t}$ be defined by the following equality, i.e., both players are indifferent between acquiring information and waiting until the end of the game at $\bar{t}$. Therefore,

$$l + p_i d - k(1-\bar{t}) = -c + l + p_i d \quad \Rightarrow \bar{t} = 1 - \frac{c}{k}$$

There also exist two degenerate equilibria where one player strictly prefers to acquire information at the beginning of the game.[18] These equilibria are outcome equivalent to some of the equilibria in the continuum of equilibria described above.

Proposition 10 *When acquired information is unverifiable, there is a continuum of mixed strategy equilibria: for any $F_1(0), F_2(0) \in [0,1]$ such that $F_1(0)F_2(0) = 0$, there is an equilibrium where each player acquires information according to a distribution function $F_i(t) = 1-(1-F_i(0))e^{-\frac{k}{c}\frac{d-c}{p_i d-c}t}$ over time $[0,\bar{t}]$ where $\bar{t} = 1-\frac{c}{k}$, and they do not acquire information and do not concede after $\bar{t}$. The set of equilibrium payoff is*

$$\left\{(v_1, v_2) : v_i \in [-c + l + p_i d, l + \frac{p_i d - c}{d-c} d], \text{ and } (v_1 - (-c + l + p_1 d))(v_2 - (-c + l + p_2 d)) = 0\right\}.$$

Comparing Proposition 2 and 3, we see that when restricting comparison to equilibria that share the same initial probabilities, game ends faster under unverifiable information. To conclude, the results in this alternative model with deterministic deadline and fighting cost are not qualitatively different from the main model with stochastic deadline and discounting.

8 Conclusion

This paper analyzes information acquisition in a war of attrition with the stochastic arrival of a public signal that reveals the state of nature and ends the game. Players can choose to acquire information about the state of nature any time during the game to resolve the conflict before the public signal does it. When information is verifiable, players have the incentive to free ride on each other to acquire information so there is an inefficient delay in information acquisition. When information is unverifiable, there is an additional incentive to catch up on information acquisition to prevent the opponent from extracting information

[18] The strategy profile of such an equilibrium is the following: if any player has acquired information while the other one has not, then the part of the players' strategy following this history is described as in the subgame; if both players has acquired information, then the weak informed one concedes immediately; otherwise one player acquires information immediately at any time $t < \bar{t}$ and never acquires information and never concedes for $t \geq \bar{t}$, and the other player never acquires information and never concedes.

rent, which causes duplication in information acquisition. We show the game ends faster when the information is unverifiable, and better availability of the public signal can hurt social welfare by crowding out the incentive of the players to acquire information on their own.

Appendix

Proof of Proposition 3

(1) Weak player i obtains only l.

Suppose not. It can happen only when player j concedes without acquiring information with a certain probability. Therefore, player j's expected payoff must be equal to l. But, since weak player i would not quit immediately, player j's belief over the true state would not change, which implies that player j can secure $-c + l + p_j d$ by acquiring information. By Assumptions 1 and 2, the latter payoff is strictly larger than the former, and thus this is a contradiction.

(2) At the beginning of the subgame, weak player i randomizes between conceding and waiting.

The argument given before the proposition applies.

(3) Let α be the probability that weak player i concedes at the beginning of the subgame. An instant later player j must be indifferent between acquiring information and conceding without acquiring information. Therefore,

$$\alpha = \frac{p_j d - c}{p_j(d - c)}.$$

Suppose player j strictly prefers acquiring information to conceding. Then weak player i obtains only l, whether player j acquires information or wait for a public signal. But then an instant before (at the beginning of the subgame), weak player i strictly prefers conceding to waiting, which contradicts (2). Now suppose player j strictly prefers conceding to acquiring information. For this and (2) to be simultaneously true, both weak player i and player j must be indifferent between conceding and waiting for a public signal. This implies that, again, weak player i obtains only l, which creates the same contradiction as the previous case.

(4) An instant after the beginning of the subgame, player j either acquires information or concedes. That is, player j does not simply wait for a public signal.

Suppose player j strictly prefers waiting to conceding. For (2) to be true, weak player i must strictly prefer waiting to conceding as well (otherwise, weak player i must prefer conceding earlier). The latter implies that player j's belief does not change at the next instant and thus he again strictly prefers waiting to conceding or acquiring information. In addition, by the same argument as above, weak player i does not concede. This process will continue until a public signal arrives. But then weak player i's expected payoff is strictly smaller than l, which contradicts (4).

Now suppose player j is indifferent between waiting, conceding, and acquiring information and he waits with a positive probability. Similarly to the previous case, weak player i must strictly prefer waiting to conceding. He also must strictly prefer waiting to conceding at the next instant as well, because otherwise at the following instant player j's belief would decrease, player j would concede immediately, and then weak player i would obtain strictly more than l, which would contradict (2). This process will continue until either player j acquires information, player j concedes, or a public signal arrives. Suppose player j waits for a signal forever. Since weak player i never concedes, player j's expected payoff is

$$\frac{\lambda}{r+\lambda}\left(l+\frac{p_j(1-\alpha)}{p_i+p_j(1-\alpha)}d\right)=\frac{\lambda}{r+\lambda}\left(l+c\right).$$

Under Assumptions 1 and 2, this payoff is strictly smaller than l, which is a contradiction to the fact that player j is indifferent between waiting and conceding.

(5) Player j acquires information with probability 1, that is, player j concedes with negligible probability.

Otherwise, then weak player i would strictly prefer waiting an instant to conceding immediately, which would contradict (2). As shown in the main content, in discrete time, player j concedes with a positive probability, but the probability approaches zero as Δ tends to zero.

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