Profitability of pay-what-you-like pricing

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ABSTRACT: This paper considers a pricing strategy where consumers choose their own price including zero. Our analysis shows that in this pay-what-you-like pricing strategy in spite of the option to free-ride, all consumers do not free-ride. Even more surprising, we show that, under low marginal cost and for a low demand elasticity range, when consumers are willing to share their surplus with the seller, pay-what-you-like pricing can achieve higher profits for the seller when compared to uniform pricing. When price setting is costly and price discrimination is not feasible, the pay-what-you-like option can serve as an alternative method to practice a form of price discrimination that mimics first-degree, but without incurring the cost associated with screening consumer types.

Keywords: pay-what-you-like pricing; open contract; voluntary payment; social preferences; price discrimination;

JEL codes: C70, D03, D21, L10

1 Introduction

Pay-what-you-like pricing (PWYL) is an unorthodox form of pricing that lets consumers voluntarily choose their own price including zero. The seller uses an open contract design to explore whether PWYL pricing can be used in a market to receive higher profits than a traditional form of pricing based on profit maximization. This form of pricing has existed for quite some time.[1] Nonetheless, it has gained more attention recently when the British band *Radiohead* offered their album *In Rainbows* to consumers on a pay-what-you-like basis in 2007. Between October 9-29, 2007, 1.2 million people worldwide visited their website and among those who downloaded the album, 38 percent worldwide and 40 percent in the U.S.,

[1] Historically PWYL pricing has existed for certain types of services. For example, in most Indian villages, the village priest accepts whatever the host pays for many ceremonies he performs such as naming a newborn, performing a marriage, or other religious services. This tradition still continues. Doctors in rural India are still paid based on how much a patient can afford. In the United States, it is common place to observe church parishioners practicing PWYL pricing when providing contributions (donations) to the church in support of the services/programs offered by the church. Some donations are made out of guilt. Gruber (2003) shows that religious attendance and religious givings are substitutes.

willingly paid. Free-riders were as prevalent in the U.S. as in the rest of the world. However, the average paying consumer in the US paid considerably more, \$8.05 compared to \$4.64 by his international counterpart.[2] On the other hand, based on three recent field experiments in Germany, where three different products were offered using PWYL pricing, Kim, Natter and Spann (2009) report that, although there was a wide distribution of payments made by consumers, surprisingly no instance of free-riding is reported as *all* consumers paid a positive price.

A recent empirical paper by Regner and Barria (2009) analyzes payment patterns of consumers of the online music label Magnatune where consumers can pay what they wish in a specified price range of \$5-\$18. They find that on average customers paid \$8.20 far more than the suggested minimum price of \$5, and even higher than the recommended price of \$8. The authors conclude open contracts could serve as a viable alternate pricing system because such open contracts encourage customers to make voluntary payments. Although the free-rider problem cannot be eliminated completely due to widespread files sharing, it can be minimized via a minimum price requirement but at the expense of limiting the extent of market participation.

Unlike the music industry where the marginal cost of production is negligible, this form of pricing has been used by religious organizations (Gruber, 2004) and restaurants both on a long and short run basis where the marginal cost could be significant. For example, in spite of a rather exclusive and expensive cuisine, an upscale restaurant, *Just Around the Corner*, in London has been operating using this pricing strategy for the last 22 years. Also since 1979, a small (maximum capacity of about 10 people) and exclusive Japanese restaurant, *Mon Cheri*, located in an expensive area of Fukuoka City, Japan, has consistently maintained PWYL pricing. An intimate environment and enjoyable dinning experience together with personal interactions could be the reason for this sustained use of PWYL pricing. On the other hand, the *New Yorker* magazine reported in 2005 that a popular New York City restaurant, *Babu,*

[2] See http://www.inrainbows.com; and http://comscore.com/press/release.asp?press=1883; and the Wall Street Journal, October 3, 2007, p.C14. The band also included a \$2.00 transaction fee.

located in the Village, used a menu without prices. After finishing their meals, consumers were asked to pay what they liked.[3] The owner found that most consumers did pay, and some even paid considerably more than what the owner had expected.[4] There were also a few cases of free riders. Eventually, after running this short run experiment, the owner did switch to a menu with listed prices.

The success in offering products with negligible to considerable marginal costs using PWYL pricing encouraged other firms to experiment with this form of pricing. Today, this pricing strategy is practiced by many firms who supply experience goods and services, for example, by musicians, coffee houses, restaurants, theatres, and magazines; the publisher of *Paste* magazine has adopted this strategy where subscribers can pay what they like for a year's subscription to the magazine.

From the practical experiments and the field studies there does not seem to be a clear theoretical basis for why and when this form of pricing may work? This paper, using a stylized game theoretical model, provides theoretical support for the recent empirical findings. In particular, our results offer explanations for what encourages consumers to pay when they have the option to free ride? What determines the number of free riders? What demand and cost conditions can make this form of pricing a viable and at the same time a more profitable strategy when compared to a uniform pricing strategy? Although intuitively a product with a low marginal cost appears to be a likely candidate for this pricing, is zero marginal cost sufficient for this pricing to be more profitable than other forms of pricing? Is this pricing more suited as a short run experiment to find consumers' willingness to pay (e.g., *Babu* restaurant) when price setting is very costly or can it be sustained as a profitable pricing strategy in the long run for a known market demand (the two restaurants mentioned above)? What role does customer loyalty or repeated interactions play?

[3] See Rebecca Mead, the *New Yorker*, March 21, 2005, for other details. Recently, many cafes and restaurant in the US have also adopted this practice.

[4] Lynn (1990) argues that some customers in a restaurant pay more to avoid the impression of looking cheap.

Our basic framework relies on the growing literature related to social preferences in experimental and behavioral economics. Fair minded consumers are modeled to maximize net utility, where the utility function is comprised of two parts: (1) consumers wish to maximize consumer surplus defined as the difference between consumers' private values for the good and the amount paid; and (2) consumers wish to minimize transaction utility. Transaction utility incorporates the effects of fairness or guilt typically ignored in standard models of utility maximization. PWYL pricing becomes a viable option when the importance of the transaction utility grows.[5] A consumer experiences guilt when the voluntary payment made for the good is below an accepted reference (fair) price. We show that the number of free riders is dependent on the "guilt" parameter (Proposition 1), which measures the relative importance of the transaction utility within net utility. The actual payment depends on how much a consumer is willing to share her additional transaction surplus—the difference between her voluntary payment and the reference price – with the seller. Given the utility framework, PWYL pricing is found to generate higher profits than uniform pricing if the elasticity of demand is between 1 and 2 when evaluated at the profit-maximizing uniform price (Proposition 2). Further a low marginal cost, even zero marginal cost, is not a sufficient condition for PWYL pricing to be more profitable than uniform pricing.

Another relevant factor, at least in the short run, affecting the choice of using PWYL pricing is the cost of price setting. The cost of price setting can be particularly high especially for new products for which information about a consumer's willingness to pay may be unknown. In such situations this pricing strategy can serve as a low cost mechanism to gather information about the patterns of customers' payments in the short run. Sellers can use the observed payment amounts to learn or estimate demand and then use this information to implement other forms of pricing.

Next, we consider the situation where consumers like to support a seller because of very

[5] Regner and Barria (2009) argue that voluntary payments can be due to reciprocity, "warm glow"—act of kindness, or experiencing a large enough guilt from not paying a "fair" price. Repeated interaction is another plausible theoretical explanation.

satisfying experiences in the past (e.g., a good dining experience in a particular restaurant or to support a local community (Duncan and Olshavsky, 1982)). Generally in a smaller community, local firms are frequented by many loyal customers (repeated interactions). When the proportion of loyal customers is large enough, PWYL may be more profitable than uniform pricing. The repeated interactions between consumers and sellers provide an incentive for firms to accept lower prices today in return for more transactions in future periods. Therefore, PWYL pricing may become a viable alternative to traditional forms of pricing in a smaller community where consumers are more likely to become loyal than a market serving mainly tourists.

It is worth stating that one may find a striking similarity between the PWYL offer and the private provision of a public good. In both cases no one can be excluded from consuming the good. In spite of this similarity, the two situations are very different. First, a public good will not be provided by the market unless there is some mechanism to pay for its provision. Certainly, this is not the case with the goods that are being offered using the PWYL option. Firms have an incentive to offer the good under both the PWYL option if they choose or use some other form of pricing. Second, for a known demand, the choice of PWYL is endogenous. A seller compares expected profits with respect to several pricing strategy, including PWYL, and selects the strategy that maximizes profits. No such profit incentive exists in the provision of a public good. Third, when consumers are only self-interested the contribution level to the public good is less than the social optimum. But, whenever PWYL pricing is more profitable than uniform pricing it is also Pareto-improving.

Finally, the PWYL option allows firms to practice a modified version of first-degree price discrimination, when it is typically considered infeasible. PWYL pricing cannot exclude consumers from the market because there is no set price thus making the potential level of market participation the largest. Next, as will be shown in the model section, the voluntary consumer payment is a monotonic transformation of a patient's willingness to pay. Therefore, consumers reveal their willingness to pay voluntarily (or willingness to pay can be inferred from observed payments) and the seller does not have to incur any major costs associated with implementing this screening mechanism.

The rest of the paper is organized as follows. Section 2 presents a model of PWYL pricing where consumers have fairness/guilt considerations. Consumers are assumed to derive disutility from paying a price lower than some reference price. In this section, we derive the necessary conditions such that PWYL pricing yields larger profits than uniform pricing. Additionally, we extend the model to include costly price setting. In Section 3, we present a competing model for PWYL pricing based on repeated interactions between consumers and the firm. Although the assumptions upon consumer behavior differ from those presented in the static model, similar results are obtained. Section 4 contains discussions and summarizes the results.

2 Model

In this section, we present a stylized game theoretical model describing how a "fairness" consideration (the feeling of guilt) in a consumer's utility function can induce voluntary payments even in the presence of a free-ride option (Proposition 1). Proposition 2 states the necessary conditions for PWYL pricing to generate *higher* expected profits than uniform pricing. Proposition 3 states the *sufficient* condition for PWYL to be more profitable than uniform pricing when marginal cost is zero, showing that zero marginal cost in itself is not sufficient. Proposition 4 extends the model by incorporating the costs of price setting. Proposition 5 states the results when consumers are loyal and have repeated interactions with the seller.

Relying on the behavioral economics literature which suggests that consumers do care for the welfare of others and are fair-minded (see, for example, Thaler, 1992; Kahneman et al. (1986); Fehr and Scmidt (2003); Camerer 2003; and recent field study by Kim et al. (2009) and so far the only empirical analysis by Regener and Barria (2009)). Our model incorporates fairness by taking into account the guilt consumers experience when making voluntary payments that are less than some reference or market price.

Let there be N consumers in a market for a highly differentiated good. Following Thaler (2004) if consumers are assumed to be motivated by two desires: (1) to increase consumption utility; and (2) transactional utility from being "fair"— then the utility function can be specified as follows.[6] Consumer i's utility function is

$$U_i = v_i - p_i - \theta(\min[p_u, v_i] - p_i)^2, \tag{1}$$

where v_i is consumer i's private value for the product, p_i is the price paid by consumer i, p_u is some reference or suggested price, which for theoretical tractability and for comparing profits, is assumed to be equal to the profit-maximizing uniform price.[7] The "loss" parameter θ captures the degree of guilt associated with paying a price below the uniform price. Consumers' values are assumed to be distributed uniformly and have a support $v_i \in [0, \overline{v}]$. Further, let $G(v) = \frac{v}{\overline{v}}$ be the cumulative density function over consumer values.

In our formulation, the first term $v_i - p_i$ in (1) is the consumption utility that captures a consumer's surplus. The second part $\theta(p_u - p_i)^2$ represents the transaction utility that internalizes the loss from guilt due to not being fair and also highlights the importance of some reference (fair) price. The net utility is the difference between the positive consumption utility and the disutility from not paying a fair price. In our specification of consumer's utility, the loss parameter θ is identical for all consumers (we relax this assumption in later section) and has a support $\theta \in [0, \infty)$. A consumer is said to be self-interested when $\theta = 0$. A

[6] In a recent paper based on a field experiment, Just and Wansnik (forthcoming) use a similar approach to analyze how variations in flat-fee charges affect consumers' behavior at an all-you-can-eat pizza lunch buffet.

[7] We use this specification of the utility function because it provides closed form solutions that allows us to state results which have an intuitive appeal. Similar resutls can be obtained with a more general utility function. But as often is the case with a general specification, we could not get clear insights without some additional restrictions on functional forms. Also without a speicific functional form it is difficult to make profits comparison.

consumer experiences guilt when $\theta > 0$, and she pays an amount less than the uniform price (or the private value), $p_i < \min[v_i, p_u]$. The cost of guilt is proportional to the additional amount of surplus received from paying below the uniform price. In the case of consumers who are previously excluded from the market, this cost is proportional to their gross surplus as they would have received a surplus of zero under the prevailing uniform price. Note, a consumer will never pay an amount greater than the uniform price (or their private value) because when $v_i \geq p_u$, then a consumer can always purchase the product without any guilt at the uniform or the asked price. If $v_i < p_u$, then the consumer's contribution cannot be greater than their willingness to pay, v_i.

The loss parameter, θ, may be generated from a variety of social preferences or norms, individual code of conduct. Similarly, consumers may construct reference prices from a variety of sources (i.e. advertisements, the price of close substitutes, or a private perception of "fair" based on seller's cost). We argue that the upper bound on the reference price is equal to the reservation price—the most a consumer is willing to pay given their available alternatives. The reference price is not necessarily equal to the market price, but rather it is the minimum between a consumer's willingness to pay and the market price. For example, a consumer may value *Radiohead*'s newest music CD at \$20, but the same CD may be purchased at a market price of \$10 guilt free. On the other hand, a different consumer believes the CD is worth \$8, but will not purchase the CD at the market price of \$10. The first consumer has a reference price of \$10 and the second consumer has a reference price of \$8 for the same product. The utility function captures this definition of the reference price by allowing the loss function to vary with respect to a consumer's willingness to pay, v_i. The reference price is equal to the market price when a consumer's willingness to pay exceeds the market price ($v_i \geq p_u$) else the reference price is equal to a consumer's willingness to pay when the equilibrium uniform price prevents the consumer from participating in the market when $v_i < p_u$. Additionally, the utility function indicates there is no loss from guilt when the consumer pays the market price.[8]

2.1 Structure of the Game

The game is played in two stages. First, the firm chooses a pricing strategy: uniform pricing or PWYL pricing. Second, consumers observe the pricing strategy and choose to participate. Under uniform pricing, all consumers with private values greater than the uniform price, $v_i \geq p_u$, enter into the market and pay the uniform price. Under PWYL pricing all consumers enter into the market and choose a price that maximizes their utility.

We assume consumers make payment decisions independent of other consumers. They neither observe the private values of other consumers nor the firm's cost of production. The reason for this structure is that, in reality, consumers do not formally or informally cooperate with other consumers when making a purchase. Consumers simply observe the pricing options available and make choices that maximize their utility. This assumption departs from the literature on utility reciprocity established by Rabin (1993) and Dufwenberg and Kirchsteiger (2004).

We seek a subgame-perfect Nash equilibrium in pure strategies under asymmetric information: the firm's payoff is taken over the expectation of consumers' strategies. The equilibrium of the game is found using backward induction. The firm solves the consumer's problem under both pricing strategies and then chooses the pricing strategy that yields the highest expected profits.

2.2 Consumer Behavior

Consumer behavior under uniform pricing is defined over a discrete strategy space. Consumers choose either to participate or remain out of the market. Consumers participate

[8] The quadratic loss function would imply that consumers will experience guilt even when paying above the reference price, but voluntary payments in this range are not along the equilibrium path. If a consumer makes a payment greater than the reference price, then they would be failing to maximize utility as such a payment would reduce the magnitute of the consumption utility (consumer surplus).

when $v_i \geq p_u$ and pay $p_i = p_u$. Regardless of the value of θ, consumers do not experience guilt under uniform pricing as the reference price is equal to the uniform price. It is important to highlight the presence of consumers who have positive value for the good $v_i > 0$, but are locked out of the market because $v_i < p_u$. If the firm could price discriminate, then profits would increase by including consumers whose values are greater than the marginal cost, $v_i > c$.

Consumer behavior under PWYL pricing is defined in a continuous strategy space. All consumers participate in the market, but each consumer pays a different price, p_i. The optimal contribution amount maximizes utility conditional on guilt. A consumer's marginal utility with respect to price, p_i, is given by the following equation.

$$\frac{\partial U}{\partial p_i} = -1 - 2\theta \left(p_i - \min\left[p_u, v_i\right]\right) = 0. \tag{2}$$

The optimal price, p_i^*, which maximizes utility, is

$$p_i^* = \min\left[p_u, v_i\right] - \frac{1}{2\theta}, \tag{3}$$

where p_i^* is strictly increasing with respect to the guilt parameter, θ. In the presence of guilt, a consumer is never willing to provide a voluntary contribution in excess of the market price or their own willingness to pay.

From Equation (3) we state the following proposition.

Proposition 1. In the presence of guilt, pay-what-you-like pricing results in free riding behavior by all (or $p_i = 0 \ \forall \ i$) when $\theta \leq \frac{1}{2p_u} = \overline{\theta}$. Consumers with private values $v_i \in \left(0, \frac{1}{2\theta}\right)$ always free-ride.

Remark. The cutoff value $\overline{\theta}$ represents the *minimum* level of guilt (the lower bound on the guilt parameter) required in order to get positive contributions under PWYL pricing. In the presence of guilt, there still exists an incentive to free-ride. The participation constraint requires $U_i \geq 0$. Given the functional form of the utility function, free-riders have positive utility when their private values are represented by $v_i \in \left(0, \frac{1}{\theta}\right)$. Among these consumers, those with the private value $v_i < \frac{1}{2\theta}$ will free-ride and consumers with $v_i \geq \frac{1}{2\theta}$ are better off making a small contribution in accordance with Equation (3), but all consumers within this bound do participate as the minimum utility gained from free riding is zero. A firm can effectively stop free riding by requiring a minimum contribution equal to $\frac{1}{2\theta}$, for example the \$5 minimum set by Magnatune. The minium requirement would exclude these consumers from the market.[9]

2.3 Firm Behavior

The firm chooses between two pricing strategies: uniform pricing or PWYL pricing. Under uniform pricing, the firm chooses a price, p_u, that maximizes profit. The expected profit function, $E\left[\pi_u\right]$, is

$$E\left[\pi_u\right] = \max_{p_u} N\left[1 - G\left(p_u\right)\right]\left(p_u - c\right) - F, \tag{4}$$

where c is the marginal cost of production and F is the fixed cost. The expressions for equilibrium price (p_u), quantity (q), and profits under uniform (π_u) pricing are well known and are given below.

$$p_u = \frac{\overline{v} + c}{2} \ \ \forall\ i,\ \ q = N\left(\frac{\overline{v} - c}{2\overline{v}}\right),\ \text{and}\ \pi_u = \frac{N}{4\overline{v}}\left(\overline{v} - c\right)^2 - F. \tag{5}$$

Note, the firm's strategy space is continuous as it chooses a single price on the support $p_u \in \mathcal{R}_+$. In the case of PWYL pricing, the firm's decision is reduced to a discrete choice of entry or exit. The roles of the firm and the consumer are reversed under PWYL pricing.

[9] On the other hand, a price floor or minimum contribution requirement, as noted by Regner and Barria (2009), cannot rule out the free-rider problem completely in the electronic music market due to widespread availability in P2P network.

The expected profit function under PWYL pricing has the following form

$$E\left[\pi_{pwyl}\right] = \begin{cases} E\left[R\right] - cN - F & p_u > \frac{1}{2\theta} \\ -cN - F & p_u \leq \frac{1}{2\theta} \end{cases}, \tag{6}$$

where revenue, $R = \sum_{i=1}^{N} p_i$, remains uncertain and total costs are pre-determined as no consumer can be turned away. Having previously defined consumer behavior under PWYL pricing. The expected total revenue function is:

$$E\left[R\right] = N\left[\underbrace{\left[1 - G\left(p_u\right)\right]\left(p_u - \frac{1}{2\theta}\right)}_{v_i \geq p_u} + \underbrace{\left[G\left(p_u\right) - G\left(\frac{1}{2\theta}\right)\right]\int_{\frac{1}{2\theta}}^{p_u}\left(v_i - \frac{1}{2\theta}\right)\frac{G'(v_i)}{G(p_u) - G\left(\frac{1}{2\theta}\right)}dv_i}_{v_i < p_u}.\right] \tag{7}$$

The expected revenue function consists of two parts. The first term measures the revenue obtained from consumers who would have otherwise participated in the market under uniform pricing. Note that these consumers have different private values for the good, but each makes the same contribution. Furthermore, the amount paid cannot exceed the uniform price implying that PWYL pricing can only increase welfare for these consumers. The uniform price creates a lower bound on the utility function as no consumer can be made worse off under PWYL pricing compared to uniform pricing.

The second term in (7) accounts for the revenue obtained from those consumers who would not have been served under the uniform price. Only consumers with private values in excess of $\frac{1}{2\theta}$ will contribute as the rest have an incentive to free-ride. Payments made by these individuals can be viewed as an indirect form of first degree price discrimination. Each consumer pays an amount which is a monotonic transformation of their true private values. Therefore, PWYL pricing allows a firm to practice first degree price discrimination without the knowledge of consumers' true willingness to pay. However, such pricing could become more costly as no consumer can be turned away. The expected revenue function simplifies to

$$E\left[R\right] = N\left[\frac{(4\overline{v}\theta - 2p_u\theta - 1)(2p_u\theta - 1)}{8\overline{v}\theta^2}\right], \tag{8}$$

where revenues are increasing both with respect to the uniform price, $\frac{\partial E[R]}{\partial p_u} = N\left(1 - \frac{p_u}{\overline{v}}\right) \geq 0$, and guilt, $\frac{\partial E[R]}{\partial \theta} = \frac{N}{2\theta}\left(1 - \frac{1}{2\overline{v}\theta}\right) \geq 0$.

2.4 Profits Comparison

In this section, we derive conditions when PWYL pricing is more profitable than uniform pricing in terms of three parameters namely the guilt parameter θ, valuation $\overline{v}$, and the marginal cost of production c. $\pi_{pwyl} > \pi_u$, when the following inequality is satisfied.

$$N\left[\frac{(4\overline{v}\theta - 2p_u\theta - 1)(2p_u\theta - 1)}{8\overline{v}\theta^2} - c\right] - F > \frac{N}{4\overline{v}}\left(\overline{v} - c\right)^2 - F. \tag{9}$$

In order for this inequality to hold, the lower and the upper bounds on the guilt parameter $\underline{\theta}$ can be founds as:

$$\frac{1}{2\overline{v} - \sqrt{2c^2 + 4c\overline{v} + 6\overline{v}^2 - 8\overline{v}p_u + 4p_u^2}} < \underline{\theta} < \infty. \tag{10}$$

Derivation is included in appendix. Substituting for the uniform price, $p_u = \frac{\overline{v}+c}{2}$, the bound on $\underline{\theta}$ simplifies to the following inequality

$$\frac{1}{2\overline{v} - \sqrt{3c^2 + 2c\overline{v} + 3\overline{v}^2}} < \underline{\theta} < \infty. \tag{11}$$

In essence, the bound on $\underline{\theta}$ indicates that the level of guilt must increase with the marginal cost for PWYL pricing to be more profitable than uniform pricing. For this reason, PWYL pricing is less likely to be observed for products with a high marginal cost of production. The simple intuition is that as the marginal cost increases, the marginal profit loss from free-riders must also increase. Therefore the number of free-riders must decrease in order to compensate for this profit loss. Consumers with private values $v_i \in \left(0, \frac{1}{2\theta}\right)$ always free-ride. The costs that free-riders impose on the firm are equal to $N\,\Pr\left(v_i < \frac{1}{2\theta}\right)c$. Therefore, an increase in the guilt parameter ($\underline{\theta}$) is required to reduce the number of free-riders to offset the increase in the marginal cost, or a minimum payment requirement can mitigate the profit loss from free riders. But, as the uniform price increases, the required level of guilt needed for PWYL pricing to be more profitable decreases. The uniform price represents the opportunity cost to a consumer for participating in the pay-what-you-like option. A higher uniform price has two effects on consumers: (1) the opportunity cost of purchasing the product "guilt free" increases; and (2) at a higher uniform price more consumers are excluded from the market. These two effects encourage sellers to use the PWYL option to attract consumers.

In Proposition 2, we state the *necessary* conditions for PWYL pricing to be more profitable than uniform pricing.

Proposition 2. If profits under pay-what-you-like pricing were to exceed the profits under uniform pricing then: (i) the marginal cost $c < \frac{\overline{v}}{3}$ *or; (ii) the price elasticity of demand at the profit-maximizing uniform price equilibrium must be less than 2 .*

Proof.

From the bounds on $\underline{\theta}$ (Equation. 11), it is easy to verify that as the marginal cost approaches $\frac{\overline{v}}{3}$, the minimum level of guilt for PWYL pricing to be more profitable approaches infinity $\left(c = \frac{\overline{v}}{3} \Longrightarrow \theta = \infty\right)$. The upper bound on θ will be satisfied only when $c < \frac{\overline{v}}{3}$. The second part of the proposition follows from the Lerner's Index which at the equilibrium profit-maximizing uniform price can be written as $\frac{1}{\eta} = \frac{p_u - c}{p_u}$, where η (defined as a positive number) is the price elasticity of demand. Noting that $p_u = \frac{\overline{v}+c}{2}$ and $c < \frac{\overline{v}}{3}$, it follows that the price elasticity of demand $\eta < 2$.

The rationale behind this result is simple and intuitively appealing. Under a uniform price, the maximum level of profits occurs when the marginal cost of production is zero. At this price, the elasticity of demand equals its lower limit of one. Further, the linear demand case yields the special result that both the consumer surplus and the deadweight loss are equal to each other and each is one-half of the maximum profit. This significant amount of deadweight loss cannot be captured as profits using commonly observed pricing strategies, especially in the presence of increasing consumer heterogeneity. However, the behavioral factors incorporated within PWYL pricing may create an opportunity to eliminate the deadweight loss by sharing it between the consumers and the seller, thus making it profitable for the seller. At the same time the elimination of the deadweight loss increases the consumer surplus for existing consumers and creates surplus for those who were previously excluded from the market. These new entrants gain surplus by paying a price lower than their willingness to pay. The seller gains additional profits from payments made in excess of the marginal cost. Compared to uniform pricing, the seller's profit will be higher under PWYL if the profit earned from the elimination of the deadweight loss is large enough to off-set the revenue lost from existing consumers and the additional cost in production (see Fig. 1).

Proposition 3 below states the sufficient condition for PWYL to be more profitable than uniform pricing when $c = 0$.

Proposition 3. $\pi_{pwyl} > \pi_u$ when the surplus-sharing index under PWYL pricing $\frac{p_u - p_i}{p_u} \leq 0.268$.

Proof. The necessary condition is satisfied when $c = 0$. At zero marginal cost the optimal uniform price is $\frac{\overline{v}}{2} = p_u$. The lower bound on $\underline{\theta}$ reduce to $\underline{\theta} \geqslant \frac{1}{2\overline{v} - \overline{v}\sqrt{3}}$, or $\overline{v}\underline{\theta} \geqslant 3.73$. For consumers who were buying at the uniform price p_u, $p_i = p_u - \frac{1}{2\theta} \Rightarrow \frac{1}{2\theta} = p_u - p_i$. Since the actual value of the guilt parameter θ must exceed its lower bound $\underline{\theta}$, we have $\overline{v}\theta > 3.73$. Substituting for $\frac{\overline{v}}{2} = p_u$ and $\theta = \frac{1}{p_u - p_i}$ into the inequality $\overline{v}\underline{\theta} \geqslant 3.73 \Rightarrow \frac{2p_u}{p_u - p_i} \geqslant 3.73$, we get the stated result.

Although the numerical value of the surplus-sharing index is specific to the utility function specified, it provides several insights. First, low marginal cost and even a zero marginal cost does not guarantee that PWYL is profitable. Second, it strongly suggests that the amount of surplus shared between consumers and the seller is the most important consideration for the use of this form of pricing. These two insights are also in conformity with the empirical findings. As guilt increases, consumers become more generous and the likelihood that Proposition 3 is met increases.

The results of this section can be explained by Figure 1. Profit under uniform pricing is the sum of Area I and Area II. Profit under pay-what-you-like pricing is Area II + Area III - Area IV. Therefore, the difference in profits is Area III - Area I - Area IV. If the marginal profit gained from the new entrants (Area III) is greater than the revenue lost from the existing consumers (Area I) and the associated cost due to expanding the market (Area IV), then PWYL pricing is more profitable than uniform pricing. It is worth noting that (i) as guilt increases (an increase in θ), Areas I and IV decrease but Area II increases allowing the firm to capture more of the deadweight loss; (ii) as marginal cost approaches zero, $c \to 0$, PWYL pricing offers the Pareto efficient outcome and there is no deadweight loss in equilibrium. This happens without incurring any cost for expanding the market (Area IV disappears).

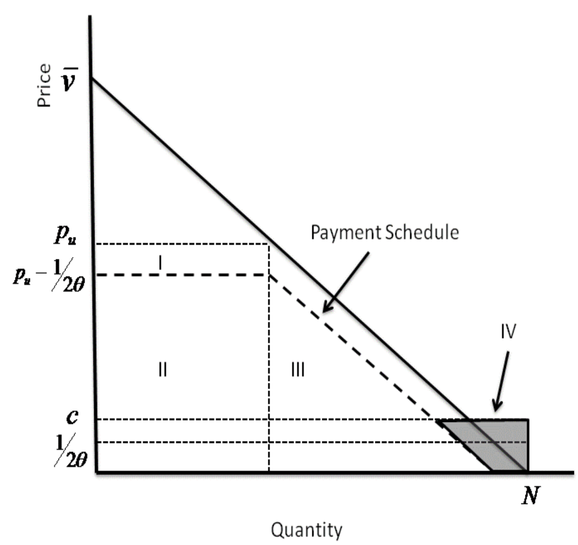

Figure 1: If Area III - Area I - Area IV >0, then $\pi_{pwyl} > \pi_u$.

2.5 Costly Price Setting and other extensions

Thus far we have assumed that setting a uniform price or the use of any other pricing strategy is costless. However, price setting requires demand estimation which in turn requires market research. According to IBISWORLD, the US market research industry generated over \$14 billion of revenue in 2009.[10] The cost of pricing is especially relevant for experience goods or launching a new product. For example, aspiring musicians may have little or no information about their fans' willingness to pay or the quality of their own product. Suppose consumer values are uniformly distributed $v \rightarrow U(0, a)$, but a may take on only two values $a \in \{1, 2\}$ where $\Pr(a = 1) = .5$ and $\Pr(a = 2) = .5$. In this case, the firm is uncertain about both the consumers' valuations for the good and whether to serve high-demand consumers $(a = 2)$ or low-demand $(a = 1)$ consumers. Since it may be quite costly to determine the optimal price given the uncertainty about consumer values and the value of a, firms may choose the less costly option of using PWYL pricing to reduce uncertainty. Thus a potential source of savings associated with PWYL pricing is the reduction in the costs related to setting a price. Now consumers, and not the firm, bare the cost of pricing. In what follows, we provide three extensions to the guilt model where costly price setting, demand learning, and random guilt are introduced into the model.

2.5.1 Costly price setting

The PWYL option offers firms a source of cost savings. We demonstrate this cost savings by allowing the fixed cost to vary between the two pricing options. Let the fixed cost associated with PWYL pricing be $\overline{F} = F - \gamma$ where γ is the cost of pricing in equation (9). As in the previous section, we solve for the minimum level of guilt such that $\pi_{pwyl} > \pi_u$ as a function of the marginal cost, consumer valuation $\overline{v}$, the market size N, and the cost of pricing γ. Incorporating the cost of pricing reduces the minimum level of guilt for profitability

[10] http://www.ibisworld.com/industry/default.aspx?indid=1442

and increases the likelihood of the use of pay-what-you-like pricing.

$$\underline{\theta} > \frac{1}{2\overline{v} - \sqrt{3c^2 + 2c\overline{v} + 3\overline{v}^2 - \frac{8\overline{v}\gamma}{N}}} \tag{12}$$

Proposition 4. When price setting is costly then there exists an incentive to use pay-what-you like pricing when the market is small (low N) and/or pricing cost are large (high γ). The upper bound on marginal cost is $c < \frac{2}{3}\sqrt{\frac{\overline{v}(\overline{v}N+6\gamma)}{N}} - \frac{\overline{v}}{3} = \overline{c}$ for $\theta < \infty$.[11]

Proof. See appendix.

This result provides insight into why some of the firms mentioned earlier use PWYL pricing. The exclusive restaurant in Japan is quite small (low N) having a niche market. On the other hand, in the case of *Radiohead,* both consumer heterogeneity and the market size are quite large (global). Yet, PWYL pricing was adopted because both the costs of market research and the cost of screening consumers are significant considering that music is a highly variable experience good (high γ).

These same cost savings could also affect entry. On the margin, a firm only enters into a market if the expected profits are positive. One can argue that firms who would not have entered the market because a uniform price would have resulted in negative profits ($\pi_u < 0$) may enter under PWYL pricing as the cost saving without market research may be significant resulting in positive profits ($\pi_{pwyl} > 0$).

These results and arguments are similar to those made for class pricing, which also attempts to reduce costs associated with pricing. Wernerfelt (2008) motivates the use of "class pricing" as a method to reduce the cost associated with selecting individual prices for similar product (e.g., one tuition fee for a variety of university classes). Class pricing is preferred in markets with a small number of buyers, a large number of product versions, and a small variance in costs. Class pricing and PWYL pricing share some of the same characteristics in reducing the cost associated with pricing within small markets (low N),

[11]Note that $\frac{\partial\overline{c}}{\partial\gamma} = 2\sqrt{\frac{\overline{v}}{N(N\overline{v}+6\gamma)}} > 0$ and $\frac{\partial\overline{c}}{\partial N} = -\frac{\gamma}{N}\sqrt{\frac{\overline{v}}{N(N\overline{v}+6\gamma)}} < 0$.

but differ with respect to the ex ante difference in demand. PWYL pricing is preferred when there is a great deal of consumer heterogeneity where as class pricing is preferred when there is little ex ante difference in demand between different versions.

2.5.2 Learning

In some instances, PWYL pricing could be viewed as an experiment for "learning demand." A firm could learn the demand for the product by observing the sequence of consumer payments. Recall, each payment is a monotonic function of the consumer's true value as stated in equation (3) but these voluntary contributions may be observed with error.

$$p_i^* = \begin{cases} p_u - \frac{1}{2\theta} + e_{it} & v_i \geq p_u \\ v_i - \frac{1}{2\theta} + e_{it} & v_i < p_u \end{cases}. \tag{13}$$

The error shocks, e_{it}, capture unobserved random perceptions of the reference price p_u. In the short run the firm can infer consumers' true values or use a Bayesian method to update beliefs on consumer's willingness to pay by using observed contribution levels. For this reason some firms may abandon the PWYL option in the long run as they become more established and have learned the demand for their product. This may have been the case with the restaurant *Babu* in the Village that switched to a uniform price after initially practicing PWYL pricing.[12]

2.5.3 Random Guilt

So far, we have assumed that every consumer has the same level of guilt or sense of fairness, θ. Dubner and Levitt (2004) show consumers have varying beliefs about guilt and/or fairness. The article describes payment differences between employees who purchase bagels based on the honor system. In this case, high paid executives are less likely to pay than lower paid employees within the same firm. We can infer that highly paid executives

[12] The guilt model presented here can easly be extended to include variations in the reference price. We exclude this discussion for brevity as the results qualitatively will not change significantly.

have lower values of θ than lowly paid employees. For this reason, we relax the assumption of identical guilt among consumers. Instead assume $\Pr(\theta = 0) = 1 - \phi$ and $\Pr\left(\theta = \widehat{\theta}\right) = \phi$. Adding uncertainty to the guilt parameter unambiguously decreases profit under pay-what-you-like pricing, but does not affect profit under uniform pricing. Expected profit under pay-what-you-like pricing is

$$E\left[\pi(\theta)_{pwyl}\right] = N\left[\frac{\phi\left(4\overline{v}\widehat{\theta} - 2p_u\widehat{\theta} - 1\right)\left(2p_u\widehat{\theta} - 1\right)}{8\overline{v}\widehat{\theta}^2} - c\right] - \overline{F}, \tag{14}$$

where the firm still serves the whole market, but revenues fall as the percentage of guilt free consumers increases. The minimum level guilt required for pay-what-you like pricing to be profitable is $\widehat{\theta} > \frac{1}{2\overline{v} - \sqrt{\frac{2N(\overline{v}+c)^2 - 8\overline{v}\gamma + N\phi(\overline{v}-c)^2}{N\phi}}}$ where $\frac{\partial\widehat{\theta}}{\partial\phi} < 0$ for $\phi \in (0, 1]$. The minimum level of guilt must rise to off-set both the cost associated with expanding the market and the presence of two free-rider types: (1) *strategic free-riders* who have $v_i < \frac{1}{2\theta}$; and (2) *random free-riders* caused by ϕ who are located over the full distribution of private values, v_i.

3 Model of Loyal Consumers

Instead of the earlier model based on guilt, we now present a model that considers repeated interactions between the consumers and the firm. Reputation and repeated interactions over long periods can be another theoretical explanation for generous payments under PWYL pricing. In this model payments are made as a result of a dynamic game played between the firm and its loyal consumers. Loyal (repeat) customers have an incentive to support PWYL pricing as this pricing mechanism offers a higher level of life time consumer surplus than paying uniform prices in every period. Although the underlying assumptions for the guilt model and the dynamic model are very different, both models predict similar payment behavior on the part of consumers. We derive the necessary conditions under which PWYL pricing gives lower expected profits than uniform pricing.

Let there be N infinitely lived consumers with a single period utility function given as: $U_i = v_i - p_i$. Suppose the proportion of repeat (loyal) buyers is $\lambda \in [0,1]$ and each repeat buyer has a discount factor $\beta \in [0,1)$. A consumer maximizes their lifetime utility by selecting a contribution sequence. If the firm chooses a uniform price, then each consumers has a lifetime utility

$$V_u(v_i, \beta) = \frac{v_i - p_u}{1-\beta}, \tag{15}$$

where a consumer chooses to participate if $v_i > p_u$. The consumer's value function under pay-what-you-like pricing for repeat buyers is

$$V(v_i, \beta) = \tfrac{v_i - p_i}{1-\beta} \quad \text{s.t. } E[\pi_{pwyl}] \geq E[\pi_u]. \tag{16}$$

The above expression simplifies to $V(v_i, 0) = v_i - p_i$ for non-repeat consumers ($\beta = 0$). Therefore, a non-repeat consumer always has an incentive to free-ride under the PWYL option.

Consider the following game. In the first stage, the firm chooses a type of pricing strategy: uniform or PWYL. In the second stage, consumers choose to participate and, in the case of PWYL pricing, choose how much to pay. The game is repeated subject to the firm earning positive profits. If the firm's profit under PWYL pricing are less than profit under uniform pricing, then the firm will punish consumers by choosing the uniform price in all future periods. Given the setup of the game, consumers will choose a payment schedule such that the firm is indifferent between uniform and PWYL pricing. Consumers capture the entire surplus in this game, but there exists an infinite number of possible payment schedules, which satisfy the constraint $E[\pi_{pwyl}] \geq E[\pi_u]$ in equilibrium.[13] Further, the payment schedule must

[13] For example, two potential equilibria result in either dividing the net surplus (after payment) among all consumers evenly or giving all the net surplus to one consumer. At that point, no consumer can gain additional surplus without triggering the firm to use uniform price, thereby making everyone else worse off.

satisfy the following consumer participation constraint.

$$\tfrac{v_i - p_i}{1-\beta} \geq v_i + \min\left[\frac{\beta\left(v_i - p_u\right)}{1-\beta}, 0\right]. \tag{17}$$

Consumers will choose not to free ride under PWYL pricing if their life time utility when committing to a positive price exceeds lifetime utility where consumers free ride in the first period and pay the uniform price in all subsequent periods. The participation constraint can be simplified to

$$\overline{p}_i = \beta \min\left[v_i, p_u\right] \geq p_i \tag{18}$$

where the maximum contribution amount, $\overline{p}_i$, leaves the consumer indifferent between free-riding and committing to a positive price.

The maximum amount a consumer is willing to pay is strictly less than their private value, but is proportional to $\min\left[v_i, p_u\right]$. As the discount factor decreases consumers become less patient and the incentive to free-ride increases. Since the marginal benefit of future consumption decreases with the discount factor, consumers are less willing to cooperate with the firm over the long-run. If we consider the discount factor to represent the probability of a repeat visit in the future, then contributions are increasing as consumers become more loyal. This theoretical result is supported by empirical evidence in the tipping literature. Conlin, Lynn, and O'Donoghue (2003) find that repeat consumers, on average, provide larger tips compared to one-time consumers.

Now we can obtain a necessary condition which when satisfied makes pay-what-you-like pricing always less profitable than uniform pricing. First, we find the expected maximum level of revenue that can be generated in the absence of free riding. From equation (18), if consumers have private values $v_i > p_u$, then the maximum contribution is $\overline{p}_i = \beta p_u$, otherwise $\overline{p}_i = \beta v_i$. The expected revenue function is given in (19) where the first term in the brackets is the average payment made by consumers with private values $v_i > p_u$ and the second term is the average payment by all other consumers.

$$\max \; E[R] = \lambda N\left[\left[1-G(p_u)\right]\beta p_u + G(p_u)\beta\int_0^{p_u} v_i \frac{g(v_i)}{G(p_u)}dv_i\right] \tag{19}$$
$$= \lambda\beta N\left[p_u\left(1-\tfrac{p_u}{2\overline{v}}\right)\right].$$

Next, we compute expected profits under both pricing strategies, then solve for the value of the discount factor such that uniform pricing strictly dominates pay-what-you like pricing, $\pi_{pwyl} < \pi_u$. Proposition 5 states the results.

Proposition 5. Given the uniform price $p_u = \frac{\overline{v}+c}{2}$, *profits under uniform pricing are higher than profits under pay-what-you-like pricing when: (i)* $\beta < \frac{2(\overline{v}+c)}{(3\overline{v}-c)\lambda}$; or *(ii)* $\frac{\overline{v}}{3} < c$, $\forall\, \beta$; or *(iii)* $\lambda < \frac{2}{3}, \forall\beta$ and $\forall c$.

Proof. See appendix.

The magnitude of marginal cost again becomes critically important for the adoption of PWYL pricing. A sufficiently low marginal cost and a high proportion of loyal consumers can make PWYL pricing a viable profitable alternative to uniform pricing. The intuition is supported by what is observed in real life. For example, PWYL pricing is not practiced by firms serving mainly tourists because the proportion of repeat buyers is very low. On the other hand, PWYL pricing may turn out to be a profitable strategy in small communities where consumers have a preference for locally owned businesses (high λ and/or high β).

Finally, incorporating pricing cost relaxes the restriction on the discount factor. We include pricing cost as a component of fixed cost, $\overline{F} = F - \gamma$, where $\overline{F}$ is the fixed cost under PWYL pricing and γ is the cost of pricing. After modifying the profit functions with respect to their fixed cost, we solve for the level of the discount factor such that uniform pricing dominates PWYL pricing. The upper bound on the discount factor is represent by

$$\beta < \tfrac{2(\overline{v}+c)^2-\gamma/N}{(3\overline{v}-c)(\overline{v}+c)\lambda} \tag{20}$$

where γ/N is the average savings per consumers (for the derivation of equation (20) see the appendix). Again, the likelihood of PWYL pricing increases as the market size decreases (low N) and/or pricing cost increase (high γ). The savings resulting from forgoing the cost of pricing allows the firm to use PWYL pricing while accepting lower payments. Recall, consumer contributions are a monotonic function of the discount factor. A firm could accept lower payments from consumers (due to lower β) when the loss in revenue is off-set by a corresponding increase in average savings from forgoing the cost of pricing (higher γ/N).

4 Discussion and Conclusions

Our analysis provides a theoretical economic framework that captures both the seller's and the consumers' behaviors under the pay-what-you-like option. We offer two competing models to show why a firm lets consumers choose their own price and why consumers would choose to pay given the option to free ride. The first model introduces guilt into the traditional utility maximization problem. Consumers derive disutility by providing a payment lower than some predetermined reference price. A firm can exploit the presence of guilt to expand the market by offering PWYL pricing. Further, it can generate higher profits compared to uniform pricing when marginal cost is small relative to consumers' valuation, the demand elasticity is low (between 1 and 2), and the surplus-sharing index is low—consumers keep low surplus for themselves. Our theoretical results support the empirical findings of previous studies (Kim et al., 2009; Regner and Barria, 2009; Just and Wansink, 2011) by demonstrating that in equilibrium not all consumers free-ride.

In the absence of any guilt, PWYL pricing can still lead to a sustained Pareto improving equilibrium when consumers are loyal and there are repeated interactions between the consumers and the firm. The second model analyzes PWYL pricing in a dynamic setting. In this case, when the discount factor is high, the proportion of loyal consumers is greater than 2/3, and the marginal cost is low, PWYL pricing can be sustained as an equilibrium. Under these conditions, consumers can maximize their lifetime utility by providing positive payments instead of facing a uniform price in every period and they will choose a payment schedule such that the firm is indifferent between PWYL pricing and uniform pricing. Further, the net-surplus gained by using PWYL pricing can be distributed among all the consumers resulting in a Pareto improvement.

In both models we explain why this pricing strategy can be used as a long run or a short run option; and why this unorthodox form of pricing is rarely observed. First, this pricing strategy requires a very low marginal cost relative to consumers' valuations. Second, the elasticity of demand, evaluated at the profit maximizing uniform price, must be within a narrow range (between one and two). Lastly, the market must contain a captive set of consumers who are driven by either loyalty or guilt. For this reason, PWYL pricing is most likely observed for experience goods within small markets. If the good or the seller of the good is not sufficiently differentiated, then pay-what-you-like pricing is not suited and the firm should compete in prices with other firms. Yet, if the product is sufficiently differentiated or consumer heterogeneity is high, then PWYL pricing facilitates a voluntary segmentation based on consumers' self selection thus facilitating an indirect form of first-degree price discrimination but without incurring the costs that such a practice would generally require.

We extend both models to explicitly include the cost of pricing and show that PWYL pricing can provide a significant source of savings when the cost of setting a price is high due to market research or uncertainty. In smaller markets, the savings in setting price can motivate the use of PWYL pricing.

Finally, the theoretical models presented in this paper have implications on future empirical research. Specifically, both models find similar restrictions on marginal cost and derive consumer payments as monotonic transformations of a consumer's private value for the good. Yet, the underlying assumptions on consumer behavior are quite different. Future empirical work in this area would require a method that can separately identify the reasons for consumer payments, guilt or loyalty. In particular, the relative weights consumers place on these two factors while making payments decisions.

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Appendix. Proofs

DERIVATION OF EQUATION (11)

In the guilt model, $\pi_{pwyl} > \pi_u$, when the following inequality is satisfied.

$$N\left[\frac{\left(2\overline{v}-p_u-\frac{1}{2\theta}\right)\left(p_u-\frac{1}{2\theta}\right)}{2\overline{v}}-c\right]-F>\frac{N}{4\overline{v}}\left(\overline{v}-c\right)^2-F.$$

This equation simplifies to

$$\frac{\left(2\overline{v}-p_u-\frac{1}{2\theta}\right)\left(p_u-\frac{1}{2\theta}\right)}{2\overline{v}}-c>\frac{\left(\overline{v}-c\right)^2}{4\overline{v}},$$

as the fixed cost and the market size are the same in both settings. We next solve the inequality with respect to the guilt parameter, θ. The two roots are found for θ :

$$\begin{aligned}\theta_1 &= \frac{1}{2\overline{v}-\sqrt{2c^2+4c\overline{v}+6\overline{v}^2-8\overline{v}p_u+4p_u^2}}\\ \theta_2 &= \frac{1}{2\overline{v}+\sqrt{2c^2+4c\overline{v}+6\overline{v}^2-8\overline{v}p_u+4p_u^2}},\end{aligned}$$

where $\frac{\partial\theta_1}{\partial c}>0$ and $\frac{\partial\theta_2}{\partial c}<0$. Note, the difference between profits $\pi_{pwyl}-\pi_u$ is decreasing with respect to marginal cost, $\frac{\partial\left(\pi_{pwyl}-\pi_u\right)}{\partial c}<0$ and increasing with respect to the guilt parameter, $\frac{\partial\pi_{pwyl}}{\partial\theta}>0$. Therefore, the second root of the guilt parameter, θ_2, is internally inconsistent as it implies that $\frac{\partial\left(\pi_{pwyl}-\pi_u\right)}{\partial c}>0$. We conclude θ_1 is the unique solution.

PROOF TO PROPOSITION 3

The proof to proposition 3 is similar to the derivation of equation (11). We modify equation (9) to reflect the cost of pricing

$$
\begin{aligned}
N\left[\frac{\left(2\overline{v}-p_u-\frac{1}{2\theta}\right)\left(p_u-\frac{1}{2\theta}\right)}{2\overline{v}}-c\right]-\overline{F} &> \frac{N}{4\overline{v}}\left(\overline{v}-c\right)^2-F \\
N\left[\frac{\left(2\overline{v}-p_u-\frac{1}{2\theta}\right)\left(p_u-\frac{1}{2\theta}\right)}{2\overline{v}}-c\right]+\gamma &> \frac{N}{4\overline{v}}\left(\overline{v}-c\right)^2,
\end{aligned}
$$

where fixed cost under PWYL pricing is given as $\overline{F}=F-\gamma$, F is equal to fixed cost and γ is the cost of pricing setting, which is saved under PWYL pricing. We next set the uniform price $p_u=\frac{\overline{v}+c}{2}$ and solve the inequality for the guilt parameter, θ. The two roots are found for θ:

$$
\begin{aligned}
\theta_1 &= \frac{1}{2\overline{v}-\sqrt{3c^2+2c\overline{v}+3\overline{v}^2-\frac{8\overline{v}\gamma}{N}}} \\
\theta_2 &= \frac{1}{2\overline{v}+\sqrt{3c^2+2c\overline{v}+3\overline{v}^2-\frac{8\overline{v}\gamma}{N}}},
\end{aligned}
$$

where $\frac{\partial\theta_1}{\partial c}>0$ and $\frac{\partial\theta_2}{\partial c}<0$. As in the derivation of equation (11), the root θ_1 is selected as the solution and the lower bound on the guilt parameter is given by (Eq. 12). The bound on marginal cost is found by solving the denominator of θ_1 for zero. In this case, the upper bound on marginal cost for PWYL to be more profitable is

$$
c<\frac{2}{3}\sqrt{\frac{\overline{v}\left(\overline{v}N+6\gamma\right)}{N}}-\frac{\overline{v}}{3}.
$$

which simplifies to $c<\frac{\overline{v}}{3}$ when the cost of pricing is zero, $\gamma=0$.

Proof to Proposition 4

We compare consumer's lifetime utility under the uniform price and the PWYL option. A consumer commits to paying a positive price when the incentive compatibility constraint is satisfied. In this case, the consumer must be better off committing to a positive price than free riding in the first period and paying a uniform price in each subsequent period. The firm punishes free-riders when PWYL profits fall short of uniform profits. Therefore, if a consumer's private value $v_i > p_u$, then the maximum she is willing to pay is

$$\begin{aligned}
\underbrace{\tfrac{v_i - p_i}{1-\beta}}_{\text{paying utility}} &\geq \underbrace{v_i + \tfrac{\beta(v_i - p_u)}{1-\beta}}_{\text{free-rider utility}} \\
v_i - p_i &\geq (1-\beta)\, v_i + \beta\,(v_i - p_u) \\
\beta p_u &\geq p_i.
\end{aligned}$$

or if $v_i \leq p_u$, then the most she will pay is

$$\begin{aligned}
\underbrace{(v_i - p_i)\,/\,(1-\beta)}_{\text{paying utility}} &\geq \underbrace{v_i}_{\text{free-rider utility}} \\
v_i - p_i &\geq (1-\beta)\, v_i \\
\beta v_i &\geq p_i
\end{aligned}$$

Next, we find the maximum possible revenue in the absence of free-riding. The expected revenue function is

$$\begin{aligned}
\max\ E[R] &= \lambda N \left[[1 - G(p_u)]\, \beta p_u + G(p_u)\, \beta \int_0^{p_u} v_i \frac{g(v_i)}{G(p_u)} dv_i \right] \\
&= \lambda \beta N \left[p_u \left(1 - \tfrac{p_u}{2\overline{v}}\right) \right].
\end{aligned}$$

where $p_u = \frac{\overline{v}+c}{2}$ is the uniform price. Therefore, PWYL expected profits take the form

$$\begin{aligned}
E[\pi_{pwyl}] &= \lambda \beta N \left[p_u \left(1 - \tfrac{p_u}{2\overline{v}}\right) \right] - cN - F \\
&= \lambda \beta N \left[\left(\frac{\overline{v}+c}{2} \right) \left(\frac{3\overline{v}-c}{4\overline{v}} \right) \right] - cN - F,
\end{aligned}$$

and we compare these profits to the case of uniform pricing, $\pi_u = \frac{N}{4\overline{v}} (\overline{v} - c)^2 - F$. The expression $E[\pi_{pwyl}] < \pi_u$ simplifies to $\beta < \frac{2(\overline{v}+c)}{\lambda(3\overline{v}-c)}$. Note, as marginal cost $c \rightarrow \frac{\overline{v}}{3}$ the bound on the discount factor approaches infinity regardless of the value on λ. Further, if the proportion of loyal buyers $\lambda < \frac{2}{3}$ then, for all values of the marginal cost and the discount factor, the inequality will be satisfied. For example, let the marginal cost $c = 0$, then the bound simplifies to $\beta < \frac{2}{3\lambda}$. If $\lambda < \frac{2}{3}$, then the bound is always greater than 1, but the discount factor cannot be exceed 1. Therefore, for all relevant values of the marginal cost and the discount factor satisfy the bound.

Derivation of equation (20)

We incorporate pricing cost into the repeat consumer game via the firm's profit function. We use the notation of $\overline{F} = F - \gamma$, where $\overline{F}$ is fixed cost under PWYL pricing and γ represents the cost of pricing. The firm's expected profit function under PWYL pricing is then

$$E\left[\pi_{pwyl}\right] = \lambda\beta N\left[\left(\frac{\overline{v}+c}{2}\right)\left(\frac{3\overline{v}-c}{4\overline{v}}\right)\right] - cN - \overline{F},$$

and we compare these profits to the case of uniform pricing, $\pi_u = \frac{N}{4\overline{v}}\left(\overline{v}-c\right)^2 - F$. The expression $E\left[\pi_{pwyl}\right] < \pi_u$ simplifies to $\beta < \frac{2(\overline{v}+c)-\frac{\gamma}{N}}{\lambda(3\overline{v}-c)}$. Note, as marginal cost $c \to \frac{\overline{v}}{3}$ the bound on the discount factor approaches infinity regardless of the value on λ. Further, if the proportion of loyal buyers $\lambda < \frac{2-\frac{\gamma}{N}}{3}$ then for all values of marginal cost and the discount factor the inequality will be satisfied. Let marginal cost $c = 0$, then the bound simplifies to $\beta < \frac{2-\frac{\gamma}{N}}{3\lambda}$.

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