How I Learn to Love Being Dynamically Inconsistent

(work in progress)

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November 21, 2012

Changing Tastes $\Longrightarrow$ Dynamic Inconsistency

being dynamically inconsistent is mighty inconvenient

- you can't implement your favorite plan
- have to settle with what your future selves are willing to do (Strotz 1955)

in the hyperbolic-discounting literature . . .

- you procrastinate (Akerlof 1991)
- sometimes even preproperate (O'Donoghue and Rabin 1999)
- have to give up flexibility (Laibson 1997)
- get screwed by your health club (DellaVigna and Malmendier 2004)

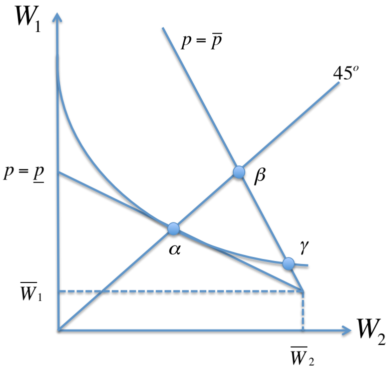
this paper:

- being dynamically inconsistent is a blessing

An Example

- you are risk averse, seeking to buy insurance
- 2 states: bad (1) or good (2)
  - wealth: $\bar{W}_2 > \bar{W}_1$
- probability of good state (p):
  - either $\overline{p}$ or $\underline{p}$; $\overline{p} > \underline{p}$
  - your private information
- insurer risk neutral, has prior belief on p
- you propose a take-it-or-leave-it contract to the insurer
- what's the best you can do?
- ask Maskin and Tirole (1993)

The Rothschild-Stiglitz-Wilson Allocation



RSW allocation $= \alpha$ for low type, γ for high type

Maskin and Tirole (1993):

- under certain prior beliefs, RSW $=$ unique equil contract
- high type cannot get full insurance

Modify the Timeline

1. you propose a contract / mechanism
2. insurer accepts / reject
3. you and the insurer play the mechanism
    - insurance premium and loss compensation determined by the play
4. state realizes, payments made

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you have an urge of early consumption

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for example:

$$V_1 = (1-p)u(W_1)+pu(W_2), \qquad V_2 = Be+(1-p)u(W_1)+pu(W_2)$$

e = early consumption
$B = 0$ corresponds to no change in taste

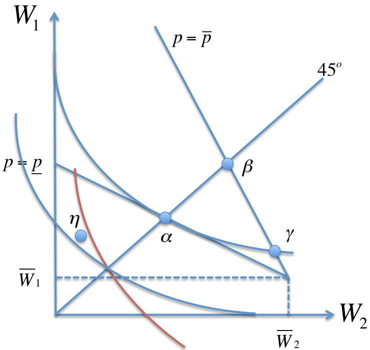
What a Blessing!

propose the following (dynamic) mechanism:

- if insurer accepts this contract . . .
- . . . you have discretion to pick β or α
  - $\alpha =$ low type's best full-insurance outcome, subject to insurer breaking even
  - $\beta =$ high type's best full-insurance outcome, subject to insurer breaking even
- if (and only if) you pick β, insurer comes ask you again in Stage 3.5 . . .
- . . . offer you an option of a huge early consumption in exchange for a huge premium and a meager loss compensation

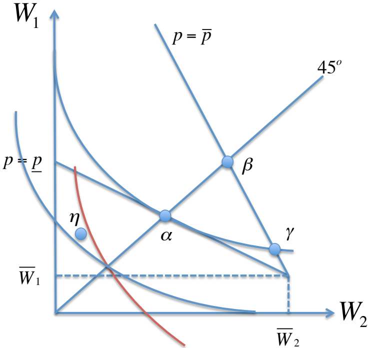
The Stage-3.5 Option

offered only if you chose β in stage 3



- $\eta =$ "huge premium and a meager loss compensation"
- especially undesirable for high type
- $\therefore \; \exists$ early consumption e such that your stage-3.5 self takes the option if and only if you have low type

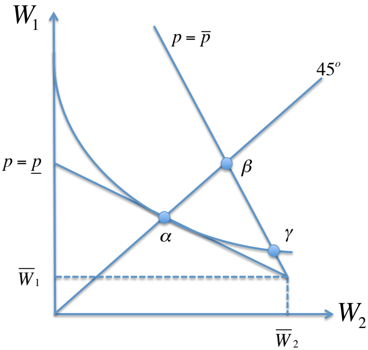
What will you choose in stage 3, α or β?



your stage-3 self does not care about early consumption

- but anticipates that a choice of β will be short-changed into η if and only if you have low type ...
- ... which is worse than α
- you choose β if and only if you have high type

Would the insurer accept the contract?



yes, high type picks β, low type α, insurer breaks even either case

this paper's Theorem 1:

- this is the unique equilibrium outcome for any prior
- low type fares as well as before
- high type does much better

What's going on?

- low type imposes negative externality on high type
- high type can't buy full insurance because
  - no cheap way to convince insurer that he has a high type
- a future self that disagrees with you
  - share your private information
  - but doesn't collude "well" with you
  - a perfect person to testify on your behalf
- you gain credibility exactly because you're dynamically inconsistent

The Model

two parties: an informed principal and an uninformed agent

principal's time-variant vNM utility

- $V_t^i(x_1, x_2, y) = v^i(f_t(x_1, x_2), y)$
- $i = 1, \ldots, n$; principal's type
- $t = 1, 2$; point of time
- $y \in \mathbb{R}$; observable and verifiable action (of the principal)
- $x_1, x_2 \in \mathbb{R}$; two different ways to make monetary payment
- v^i, f_1, f_2 continuously differentiable
- f_1, f_2 strictly increasing

The Model (continue)

agent's (time-invariant) vNM utility

- $U^i(x_1, x_2, y)$
- depends on principal's type as well
- strictly increasing in i (higher $i =$ "better" type)
- continuously differentiable
- strictly decreasing in x_1, x_2

agent's prior beliefs

- $\Pi^i > 0$, $i = 1, \ldots, n$

Agent's Reservation Utilities

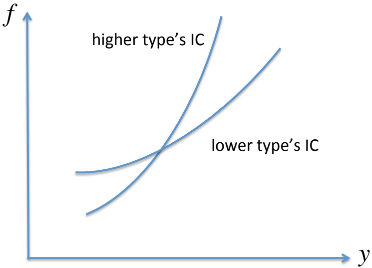
$$U_0^1 \geq U_0^2 \geq \cdots \geq U_0^n$$

example: exclusive licensing agreement

- principal = an inventor; agent = a producer
- if the agent rejects an exclusive licensing agreement . . .
  . . . a competing producer will get the license
- the better is the invention (higher i) . . .
  . . . the more formidable that competitor becomes

The Sorting Assumption

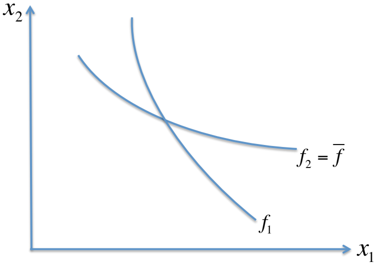
adapted from Maskin and Tirole (1993)



The Sorting Assumption:

1. $x_1, x_2, y \in \mathbb{R}$
2. $-v_y^1/v_f^1 > -v_y^2/v_f^2 > \ldots > -v_y^n/v_f^n > 0$
3. for any number $\bar{u}$ there exists a (finite) solution to the program $\max V_1^i(x_1, x_2, y)$ subject to $U^i(x_1, x_2, y) \geq \bar{u}$

The Assumption of Changing Tastes



The Assumption of Changing Tastes: For any number $\bar{f}$, there exists (x_1, x_2) such that $f_2(x_1, x_2) = \bar{f}$, and there does not exist a (finite) solution to the program min $f_1(x_1, x_2)$ subject to $f_2(x_1, x_2) = \bar{f}$.

e.g., $f_1 = x_1 + x_2$, $f_2 = bx_1 + x_2$; with $1 \neq b > 0$

Examples

insurance

- principal = the insured; agent = the insurer
- x_1, x_2 = (the negative of) insurance premium and early consumption
- y = (the negative of) loss compensation

managerial compensation

- principal = the manager; agent = the boss
- y = managerial output (e.g., cost reduction)

weapon procurement

- principal = government; agent = weapon manufacturer
- government's taste changes when another party takes office
- private information: CIA's intelligence
  (shared by any administration, Democrat or Republican)
  (may affect agent's production costs)

The Contract Proposal Game

- time-1 principal proposes a mechanism $m \in M$
  - a finite message space for time-1 principal; $S_1 \ni s_1$
  - a finite message space for time-2 principal; $S_2 \ni s_2$
  - an outcome $\mu = (x_1, x_2, y)$ for each pair (s_1, s_2)
- agent accepts / rejects
- rejecting $\Longrightarrow$ reservation utilities $\{U_0^i\}_{i=1}^n$
- accepting $\Longrightarrow$
  - time-1 and time-2 principals play m (necessarily sequentially)
  - outcome realized (depending on the play)
  - principal and agent get payoffs (depending on true type i)
- solution concept: perfect Bayesian equilibrium

Ex Post Efficient Allocation

outcome: $\mu = (x_1, x_2, y) \in \mathbb{R}^3$

allocation: $\mu^\bullet = \{\mu^i\}_{i=1}^n$

an allocation $\mu^\bullet = \{\mu^i\}_{i=1}^n$ is ex post efficient (EPE) iff for every i, μ^i maximizes V_1^i subject to $U^i(\mu) \geq U_0^i$

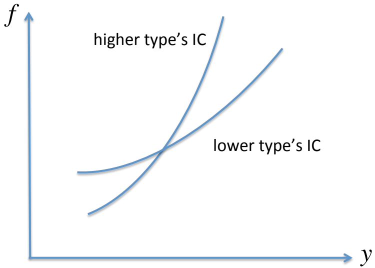
EPE payoffs are unique, although EPE allocation may not be

Theorem 1: The equilibrium payoffs in the contract proposal game are unique, and equal to the EPE payoffs.

Sketch of Proof

- expost efficient allocation: $\{(x_1^i, x_2^i, y^i)\}_{i=1}^n$
- pick $\bar{y} < \min_i y^i$
- Sorting Assumption $\Longrightarrow \forall\, i > 1,\ \exists\, \bar{f}^i$ s.t.

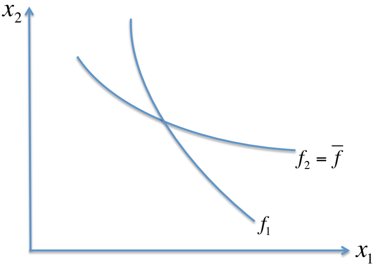
$$\begin{aligned} v^i(f_2(x_1^i, x_2^i), y^i) &> v^i(\bar{f}^i, \bar{y}), \\ v^{i-1}(f_2(x_1^i, x_2^i), y^i) &< v^{i-1}(\bar{f}^i, \bar{y}) \end{aligned}$$



Sketch of Proof

- Changing-Tastes Assumption $\Longrightarrow \forall\, i > 1,\ \exists\, (\bar{x}_1^i, \bar{x}_2^i)$ s.t.

$$\begin{aligned} f_2(\bar{x}_1^i, \bar{x}_2^i) &= \bar{f}^i \\ v^j(f_1(\bar{x}_1^i, \bar{x}_2^i), \bar{y}) &< v^j(f_1(x_1^j, x_2^j), y^j), \quad \forall\, j < i \end{aligned}$$



Sketch of Proof

offer contract m

- give time-1 principal discretion to choose among $\{\mu^1, \dots, \mu^n\}$
- if choice is μ^1, implement μ^1
- if choice is μ^i, $i > 1$, give time-2 principal discretion to choose between μ^i and $(\bar{x}_1^i, \bar{x}_2^i, \bar{y})$

time-2 principal choose $(\bar{x}_1^i, \bar{x}_2^i, \bar{y})$ over μ^i iff principal has type $j < i$

time-1 type-i will not choose μ^j, $j < i$

Discussions

$$V_t^i : (x_1, x_2, y) \longmapsto \mathbb{R}$$

in applications, may be derived from something more fundamental

- time preferences of time-t principal
- how much time-t principal cares about her other self's happiness

the framework presumes

- neither myopia
- not self-centeredness

Discussions

Q: What if it is the uninformed party (UP) making proposal?

A: informed party (IP) driven to her reservation utilities

dynamic inconsistency a curse for higher types
$\because$ they receive no information rent

but this result is not realistic

- in reality, one informed party (IP), with at least some bargaining power
- many uninformed parties (UP), competing in Bertrand manner

. . . , but this result is not realistic

consider modified Rubinstein bargaining game:

- alternating chances to make offers
  - 1 round for IP, T rounds for UP's, . . .
  - $T \gg 1 \implies$ IP has small bargaining power
- for any T, if length of a period short enough, number of UP's big enough . . .
- . . . equilibrium utilities of IP arbitrarily close to $V_1^\bullet(\hat{\mu}^\bullet)$
- dynamic inconsistency a blessing again

intuition:

- Bertrand competition makes each UP a weak bargainer

Conclusion

- dynamic inconsistency can be a blessing
- provided you know how to capitalize on it