Tea Parties

by

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Abstract

In models of tax compliance, the existence of a budget or capacity constraint on tax enforcement creates a complementarity between taxpayers' incentives to comply. All else equal, the higher the level of non-compliance in the population, the lower the likelihood any individual will be caught underreporting. A constraint on audit capacity may therefore lead to multiple equilibria to the income reporting game amongst taxpayers ("tax riots"). In spite of this, our main result shows that this multiplicity must disappear when the number of possible income types in the population is allowed to become arbitrarily large. Our result holds regardless of whether or not the government can commit to its audit policy.

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1 Introduction

Models of legal compliance and enforcement typically exhibit multiple equilibria when there is an ex-post budget or capacity constraint on enforcement activity. The intuition is straightforward. When crime is low, the probability of punishment is high, making it optimal to commit little crime. When crime is high, criminals are protected, as they collectively tie down limited enforcement resources, making crime attractive (Shrag and Scotchmer (1997), Fender (1998), Bond and Hagerty (2010)).

An analogous situation arises in tax compliance and enforcement, where it is typically assumed that the tax agency is constrained by a limited audit budget. Similarly, in optimal taxation, the mechanism designer may be constrained by an ex-post limit on audit capacity. Parallel reasoning with the economics of crime has led researchers to believe that multiple equilibria are also endemic to the area of tax compliance (Holger (1996)). This view has received support from two main sources. The seminal paper of Graetz, Reinganum and Wilde (1986) models tax compliance as an incomplete information inspection game between the IRS and the taxpayers ('interactive model'). This paper briefly discuss a two-type example in which multiple equilibria arise when the budget of the IRS is neither too low nor too high. More recently, in the context of optimal taxation and auditing, Basetto and Phelan (2008) have provided a two-type example to prove that optimal tax and audit mechanisms may possess bad equilibria ('riot' equilibria).[1] In these equilibria taxpayers severely underreport incomes, because other taxpayers are expected to do so as well. Following the literature on full implementation, Basetto and Phelan construct indirect mechanisms that implement the optimal outcome uniquely. Variants of these two models have been used to explain fiscal anarchy in the UK (Besley, Preston and Ridge (1997)), and to estimate the tax evasion social multiplier (Galbiati and Zanella (2008)).

In contrast with this literature, our paper proves two uniqueness results. For the interactive model, we show that when the set of possible income types is allowed to become arbitrarily fine, and there is no masspoint at the highest income level in the population, then there is a unique equilibrium that satisfies a multi-sender version of the D1 refinement (Cho and Sobel (1990)). In the context of optimal taxation, or equivalently for the case where the IRS can commit to audit policy, we show that under these conditions there exists a *direct mechanism* which has a unique equilibrium to the

[1] Probably the most famous of all tax revolts is the Boston Tea Party. For a comprehensive review of tax rebellions, see Burg (2004).

income reporting stage. There is then no need to resort to more complicated indirect mechanisms. We conclude that a budget constraint *alone* is insufficient to generate multiplicity of equilibria.

These results share a common intuition. With a fixed budget or capacity level, when the fraction of taxpayers reporting the lowest income increases, the IRS will optimally shift its audit resources towards auditing that category more intensely. As a consequence, the audit probability for other income reports decreases. This feedback effect makes it attractive for taxpayers to underreport to a lesser degree, thereby eliminating the possibility of multiple equilibria. In keeping with the crime analogy, one might say that as severe crimes become more prevalent in the population, it becomes more attractive to commit lesser crimes instead.

Our results should not be taken to imply that a riot equilibrium never exists. Indeed, in the interactive model, this equilibrium always occurs when the budget or audit capacity are sufficiently low. Instead, what we show is that as the audit budget rises, the degree of underreporting decreases smoothly in the population. Therefore riot equilibria never coexist with other equilibria.

From an economic viewpoint, our paper contains two significant contributions. First, we establish necessary and sufficient conditions for multiple equilibria to arise in the interactive model. These conditions are twofold: there must exist a lower bound to the set of possible income reports (that in equilibrium is binding for the lowest income type), and there must be a masspoint at the upper end of the distribution of income in the population. Without a lower bound on income reports, in a D1 equilibrium the lowest possible income type would have an incentive to separate from higher income types by lowering its income report. The equilibrium is then necessarily separating, and hence unique (Reinganum and Wilde (1986)). In other words, pooling at the lowest possible signal is necessary for congestion to be possible. Additionally, if there is a masspoint at the upper end of the income distribution, multiplicity of equilibrium occurs at the budget level where the highest income type starts to separate from the riot pool.

Our second contribution is to establish a tight connection between equilibria of the optimal tax mechanism with an ex-post constraint on audit capacity, and equilibria of a version of the interactive model, in which the IRS faces the same capacity constraint on audit activity, and maximizes gross revenue from taxes and fines. More precisely, we show that this interactive model always has a D1 equilibrium in which taxpayers report income truthfully. This implies that the optimal mechanism can be decentralized by giving the IRS discretionary authority over audit policy. Importantly, suppose that in implementation the government is limited to symmetric mechanisms in which the audit policy is ex-post optimal (credible mechanisms). We establish that multiple equilibria *necessarily* exist in any symmetric credible implementing mechanism if, and only if, the decentralized version has a "critical" equilibrium at the audit capacity necessary to implement the optimal outcome.[2] This allows us to conclude that whenever there is a continuum of possible income types, there exists a direct mechanism that uniquely implements the optimal outcome.

Our paper also contains some methodological contributions. First, we provide a rigorous formulation of the Bayesian game between the IRS and a continuum of taxpayers, in which taxpayers' incomes are independently distributed, yet the IRS knows the aggregate income distribution. This allows us to uncover assumptions necessary to justify previous analyses of this problem, and drop any symmetry assumptions on players' strategies. Secondly, in order for our game to be a monotonic signalling game, it is necessary to assume that taxpayers incur a nuisance cost from being audited.[3] This assumption introduces some complications, as the resulting signalling game does not satisfy the single crossing property. We show how to handle this problem.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 demonstrates how congestion can arise in the interactive model. Section 4 derives the D1 equilibria of this model. Section 5 studies the optimal taxation problem. Section 6 concludes. All proofs are relegated to two appendices.

2 The Model

We consider a sequential move game of tax compliance. A population of taxpayers, whose true individual incomes are private information, first report their income to the IRS. After observing the reported distribution of income in the population, the IRS then determines the likelihood with which to audit each income report. The IRS has a limited budget to spend on auditing. An audit reveals a taxpayer's true income, and any under-reporting of income discovered results in the IRS levying a penalty that is proportional to the amount of evaded income.

More specifically, we consider the following Bayesian game. There is a continuum of taxpayers, of total mass normalized to 1, and indexed by $i \in [0, 1]$. At the beginning of the game, each taxpayer's income $x(i)$ is randomly and independently selected from a commonly known income

[2] Riot equilibria are always critical if the type space is an interval, or if the type space is discrete, for generic distributions of income.

[3] As emphasized by Cho and Sobel (1990), maintaining monotonicity is important because it provides a rigorous foundation for the D1 refinement : D1 is then equivalent to strategic stability in the sense of Kohlberg and Mertens (1986).

distribution H, whose support is the interval $[0, \bar{x}]$. We assume that each realized income profile $x(\cdot)$ is a measurable function. The strong law of large numbers then guarantees that the distribution of incomes in the population equals H.[4]

Each taxpayer is then privately informed about her income. After observing this report, taxpayers simultaneously report their income to the IRS. Let $y(i) \in [0, \bar{x}]$ denote the reported income level of taxpayer i. When i is not audited, her tax assessment equals $\tau y(i)$, i.e. there is proportional taxation. When i is audited, the IRS ascertains the true income level of $x(i)$ of i, and levies a tax equaling $\tau x(i)$. In addition, the IRS imposes a penalty proportional to evaded income. Thus evasion results in a fine equalling $f(x(i) - y(i))$, for some $f > 0$. Finally, an IRS audit causes i to incur a monetary loss of $F > 0$. This cost reflects time or effort spent complying with the audit, as well as a loss in utility due to the audit process.[5]

Taxpayers are risk neutral. Thus the expected payoff to taxpayer i when she reports an income y that is audited with probability π, and when her true income is x, equals:

$$u_i(y, \pi | x) = \begin{cases} x - \tau y - \pi\left((\tau + f)(x - y) + F\right) & \text{if } y \leq x \\ x - \tau y - \pi F & \text{if } y > x \end{cases}.$$

A pure strategy for taxpayers is a measurable function $g(\cdot, \cdot) : [0, 1] \times [0, \bar{x}] \to [0, \bar{x}]$. Thus if taxpayer i's income equals x, she reports $y(i) = g(i, x)$. Note that while the measurability assumption on g restricts the joint profile of taxpayer reports, it does not constrain any individual's choice of strategies. The measurability of g ensures that for any realized profile of incomes $x(\cdot)$, the associated profile of income reports $y(i) = g(i, x(i))$ is measurable in i.

After observing the reported income profile $y(\cdot)$, the IRS determines the audit probabilities $\pi_i(y(\cdot))$ facing each taxpayer. Note that these probabilities are allowed to depend on the *entire profile* of income reports $y(\cdot)$, rather than the income report $y(i)$ of taxpayer i alone. For every taxpayer the IRS audits, she incurs a cost $c > 0$. The total audit budget of the IRS is fixed at a

[4] It is well known that with a continuum of independent random variables, thorny measure theoretic problems arise in guaranteeing that the sample paths $x(\cdot)$ are measurable, and that the strong law of large numbers applies (Judd (1985), Sun (2006)). Recently, Sun and Zhang (2009) have shown that there exists an extension of the Lebesgue σ-algebra and Lebesgue measure m on $[0, 1]$, such that for almost all draws of nature, the profile $x(\cdot)$ is measurable with respect to this σ-algebra, and such that the measure of the set $\{i : x(i) \leq x\}$ equals $H(x)$ for all x. All of our integrals are with respect to this measure. However, little economic significance is lost by interpreting them as integrals with respect to the ordinary Lebesgue measure instead.

[5] In the words of Adam Smith (1776), these costs are caused by "the frequent visits and the odious examination" of taxpayers by the tax agency, which "exposes them to much unnecessary trouble, vexation, and oppression."

maximum level of $B > 0$. Thus for every profile of income reports, we must have

$$c \int_0^1 \pi_i \left(y(\cdot) \right) \mathrm{d}i \leq B. \tag{1}$$

The IRS maximizes expected tax revenue, net of audit costs. Given a reporting strategy profile g and an income report y by taxpayer i, let $\mu_i(x|y)$ denote the IRS's belief that taxpayer i's true income equals x.[6] Then if the IRS audits an income report of y by taxpayer i with probability π, her expected net revenue from this taxpayer equals

$$R_i(y|\pi) = \tau y + \pi[(\tau + f) \int_0^{\bar{x}} \max\left(x - y, 0\right) d\mu_i(x|y) - c].$$

Facing a profile of income reports $y(\cdot)$ the IRS selects her audit probabilities $\pi_i(y(\cdot))$ so as to maximize

$$\int_0^1 R_i(y(i)|\pi_i(y(\cdot)))\mathrm{d}i$$

subject to the budget constraint (1).

We consider the sequential equilibria of this model, appropriately generalized to a continuum of players, and a continuum of actions. To simplify notation, we shall restrict ourselves to sequential equilibria in which the audit probability of taxpayer i depends on the reports of taxpayers other than i only in as far as this is reflected in the *distribution* of reported incomes G. This distribution is defined from the profile $y(\cdot)$ by $G(z) = m(\{i : y(i) \leq z\})$, where m denotes the Lebesgue measure. More specifically, we let $\pi_i \left(y|G\right)$ denote the probability with which an income report y by taxpayer i is audited, when the distribution of reported incomes in the population equals G.

From the viewpoint of individual taxpayers, the aggregate income profile $x(\cdot)$ is random. The strong law of large numbers implies that for any measurable reporting strategy g, almost all realized income profiles will induce the same distribution of reported incomes G. Since audit probabilities facing a taxpayer depends on other reports only in so far as this is reflected in G, in determining a taxpayer's payoff from an income report, there will therefore be no need to take expectations over the realizations of incomes of other taxpayers.[7]

The set of sequential equilibria of a game is not affected by the specification of players' strategies at information sets that can only be reached through simultaneous deviations. More precisely, to

[6] The consistency requirement of sequential equilibrium and independence imply that this belief can depend on i's report only.

[7] More generally, we could let audit probabilities depend on the entire profile of reported incomes. All of our results hold as stated, provided we interpret $\pi_i(y)$ as taxpayer i's *expected* audit probability when reporting income y, the expectation taken w.r.t. the realization of incomes of all taxpayers other than i.

identify sequential equilibria, it suffices to specify players' strategies only on information sets that can be reached following deviations by at most one player. Because a single taxpayer constitutes a set of measure zero, it therefore suffices to specify the IRS's audit strategy only for the equilibrium reported income distribution G. As a consequence, in our search for equilibria we shall henceforth drop the dependence of π on G.

Finally, since this is a multi-sender signalling game, the set of sequential equilibria includes equilibria that are not strategically stable. For this reason, we shall further restrict equilibria to satisfy a multi-sender version of criterion D1 (Cho and Sobel, 1990).[8]

3 The Possibility of Congestion

First, consider a version of our audit model in which there is only a single taxpayer. Note that in order for the IRS to have a meaningful audit decision, its budget constraint is necessarily not binding. The audit model then becomes a fairly standard signalling game, which has a unique sequential equilibrium satisfying criterion D1.[9]

To see why, consider the special case where there are two income types, $x_H > x_L > 0$. Uniqueness then results from a combination of two features. First, the higher the likelihood with which the high income type selects the report of the low income type, the higher the likelihood with which the IRS will audit this report, and hence the less attractive this report becomes to both types. Single crossing then gives the low income type an incentive to distinguish itself by lowering its income report. Thus the only possible pooled report occurs at the lowest signal level, i.e. a zero income report, and the high income type selects the best higher income report that reveals its type. More precisely, if x_L is sufficiently high the low income type can fully separate; below this threshold the low income type reports $y = 0$, and the equilibrium is semi-pooling.

The results of our paper imply that uniqueness of D1 equilibrium extends to the situation where there are multiple taxpayers, and the IRS has a limited audit budget, provided x_L is sufficiently high. Indeed, in any D1 equilibrium low income taxpayer types can then separate themselves from high income taxpayer types (see also Reinganum and Wilde (1986b)). Thus the existence of a lower

[8] We apply the D1 criterion to histories involving an (observable) deviation by a single taxpayer. This is because deviations by multiple taxpayers play no role in supporting equilibrium, and because our assumptions imply that the IRS's response following such a deviation is limited to the deviating taxpayer only.

[9] Some care must be taken, for the presence of audit costs borne by taxpayers leads to a violation of the required single-crossing condition necessary for separation to occur (see Section 4 for details).

bound to the set of possible income reports, that in equilibrium forms a binding constraint on the report of the lowest income type, is a necessary condition for the existence of multiple equilibria to the tax game. However, we do not take the *theoretical* possibility that a unique fully separating equilibrium may exist very seriously. Indeed, as a practical matter, there will exist multiple possible income types, in which case the lower bound on the distribution of income in the population should be near the lowest possible income report of zero.[10]

With multiple taxpayers, and in the presence of a binding lower bound to the set of possible income reports, the possibility of multiple equilibria arises. To see why, note that just like in the single taxpayer model, the larger the number of high income taxpayers that report minimal income, the more IRS will wish to audit such a report. But now the presence of an IRS budget constraint requires that the probability of such an audit must decrease, thereby making the minimal income report more attractive to all income types. In this way, a budget constraint introduces a strategic complementarity between taxpayers' income reports. Analogizing with the economics of crime, this complementarity has led the profession to believe that the game of tax compliance is riddled with multiple equilibria.

The following simple example briefly discussed in Graetz, Reinganum and Wilde (1986a) lends credence to this belief. Consider again the two type version of our model, in which $x_H > x_L = 0$. For simplicity, let us also assume that the only possible income reports are x_H and x_L, and that $F = 0$.[11] Let ρ denote the fraction of low income types in the population.

To describe the equilibria of this game, let ω denote the equilibrium fraction of the population that cheats in their income report. Let $\widetilde{\omega}$ denote the fraction of cheaters that renders the IRS is indifferent between auditing and not auditing 0, i.e.

$$\frac{\widetilde{\omega}}{\widetilde{\omega} + (1-\rho)} (\tau + f)\, x_H = c$$

Also let $\widetilde{\pi}$ be the audit probability that makes the high type indifferent between reporting x_H and 0, i.e. $\widetilde{\pi} = \frac{\tau}{\tau+f}$.

[10] Allowing taxpayers to report negative incomes will not restore the possibility of full separation in our model. This is because negative income reports yield neither a lower tax bill nor a larger fine for underreporting than a zero income report would. As a consequence, there is no possibility for separation in the region of negative income reports.

[11] Limiting income reports to belong to the set of possible income types will facilitate comparison with the literature on optimal tax design. In particular, our example will imply the existence of non-truthfull equilibria to the reporting stage of the optimal mechanism, analogous to the tax-riot equilibria in Bassetto and Phelan (2008).

Proposition 1 *Suppose that* $(1-\rho)(\tau+f)\,x_H > c$.

If $B < B_{\min} = \widetilde{\pi}(\widetilde{\omega}+\rho)c$ *there is a unique equilibrium. In this equilibrium all taxpayers report* $y = 0$, *and the IRS exhausts its audit budget, i.e.* $\omega = 1-\rho$ *and* $\pi = \frac{B}{c}$.

If $B > B_{\max} = \widetilde{\pi}c$, *there is a unique equilibrium. In this equilibrium the fraction of cheaters equals* $\widetilde{\omega}$, *the IRS audits with probability* $\widetilde{\pi}$, *and the audit budget is not exhausted.*

If $B \in [B_{\min}, B_{\max}]$, *both of these outcomes are equilibrium outcomes.*[12]

The condition $(1-\rho)(\tau+f)\,x_H > c$ states that if every taxpayer reported zero income, the IRS would wish to audit. According to proposition 1, there is an intermediate range of budgets for which there exists both a semi-separating equilibrium in which the fraction of cheaters equals $\widetilde{\omega} < 1-\rho$, and a pooling equilibrium in which all high income taxpayers cheat.[13] In the next section, we prove that if the number of possible income types is allowed to become arbitrarily large, then the congestion effect must disappear. In particular, with a continuous distribution of possible incomes, congestion is necessarily absent.

4 Uniqueness of Equilibrium

Before discussing the structure of D1 equilibria, we must overcome a serious technical difficulty: in our model the single crossing condition required for separation does not hold over the entire parameter space. More specifically, when either the audit probability is sufficiently high, or the reported income level exceeds the true income level, indifference curves may cross twice, or even coincide over a segment of income reports (see Lemmas 2 and 3 in Appendix A).

When $F = 0$, over-reporting income is a strictly dominated strategy for any taxpayer, so her equilibrium utility is bounded below by the utility of reporting income truthfully. This lower bound on utility guarantees that the equilibrium audit probability cannot be too high. Thus, for the case $F = 0$, it is straightforward to establish that single-crossing holds over the relevant part of the parameter space. However, when $F > 0$ over-reporting is no longer a strictly dominated strategy, and a natural lower bound on equilibrium utility no longer exists. Nevertheless, we are able to establish the following:

[12] There also exists a third equilibrium in which the fraction of cheaters is such that the IRS just exhausts its budget when auditing the low income report with probability $\widetilde{\pi}$.

[13] Proposition 5 implies that this multiplicity of equilibria persists when taxpayers can report any income in the range $[0, x_H]$.

Proposition 2 *For any $F \geq 0$, and in any D1 equilibrium with $\pi_i(0) > 0$, the equilibrium indifference curves of different types of taxpayer i are downward sloping and cross exactly once. Furthermore, at the point of intersection, the indifference curve of lower income types are steeper.*

We now turn to the characterization of D1 equilibria. We concentrate on the more realistic case where the audit cost is not so high that if every taxpayer reports $y = 0$, the IRS would not wish to audit:

Assumption 1 [14]

$$(\tau + f) \int_0^{\bar{x}} x dH(x) > c.$$

First, we consider a 'riot' equilibrium, in which all types of every taxpayer pool by reporting zero income.[15]

Proposition 3 *In any sequential equilibrium in which all types of every taxpayer report $y(i) = 0$, we have $B \leq \bar{B}$, where*

$$\bar{B} = \frac{\tau \left(\bar{x} - \int_0^{\bar{x}} x dH(x) \right)}{(\tau + f)\bar{x} + F} c.$$

Furthermore, $\int_0^1 \pi_i(0) di = \frac{B}{c}$, and $\pi_i(0) \leq \frac{\bar{B}}{c}$ for all $i \in [0, 1]$.

Assumption (1) implies that in a riot equilibrium the IRS wishes to audit the income report $y = 0$. Thus the IRS must exhaust its budget, and so $\int_0^1 \pi_i(0) \mathrm{d}i = \frac{B}{c}$. Note that audit probabilities are not uniquely determined. However, if $\pi_i(0) > \frac{\bar{B}}{c}$, then the audit probability for taxpayer i is so high that type $\bar{x}$ would deviate to reporting some income $y > 0$.

Next, we consider D1 equilibria in which a positive measure of taxpayers report a positive income. Using Proposition 2, we show these equilibria are separating, except at the lowest signal level $y = 0$, where there necessarily is some pooling. The equilibria are illustrated in Figure 1.

Proposition 4 *Suppose $H(\cdot)$ has no masspoints*[16]. *Then in any D1 equilibrium in which some type $x < \bar{x}$ of some taxpayer j reports $y > 0$, there exists $\widetilde{x} \in (0, \bar{x})$ such that for all $i \in [0, 1]$ we have $y_i(x) = 0$ for $x \in [0, \widetilde{x}]$ and $y_i(x) = x - \lambda$ for $x \in (\widetilde{x}, \bar{x}]$, where*

$$\lambda = \frac{\int_0^{\widetilde{x}} x dH(x)}{H(\widetilde{x})}. \tag{2}$$

[14] Here, and in the sequel, all integrals against $\mathrm{d}H$ should be interpreted as Riemann-Stieltjes integrals.

[15] If Assumption 1 does not hold, then it can be shown that any D1 equilibrium is a riot equilibrium. If $(\tau + f) \int_0^{\bar{x}} x \mathrm{d}H(x) < c$, then $\pi_i(0) = 0$, whereas if $(\tau + f) \int_0^{\bar{x}} x \mathrm{d}H(x) = c$, then $\pi_i(0) \in [0, \frac{1}{c} \min\{B, \bar{B}\}]$. In this non-generic case, multiplicity arises because the IRS is indifferent between auditing and not auditing $y = 0$.

[16] Our proof in the Appendix shows how to modify (2) and (3) to allow for masspoints at points $x < \bar{x}$.

Letting $\alpha = \frac{F}{\tau+f}$, $\widetilde{x}$ *is the solution to*

$$\frac{B}{c} = \frac{\tau}{\tau+f}\left\{\left(1 - \frac{\lambda+\alpha}{\widetilde{x}+\alpha}e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}\right)H(\widetilde{x}) + \int_{\widetilde{x}}^{\bar{x}}\left(1 - e^{-\frac{\bar{x}-x}{\lambda+\alpha}}\right)dH(x)\right\} \tag{3}$$

whenever $\lambda \geq \frac{c}{\tau+f}$, *and the solution to* $\lambda = \frac{c}{\tau+f}$, *otherwise.*

For every i *the IRS's audit probability is given by:*[17]

$$\pi_i(y) = \begin{cases} \frac{\tau}{\tau+f}\left(1 - \frac{\lambda+\alpha}{\widetilde{x}+\alpha}e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}\right) & \text{if } y = 0 \\ 1 & \text{if } 0 < y \leq \widetilde{x} - \lambda \\ \frac{\tau}{\tau+f}\left(1 - e^{-\frac{\bar{x}-(y+\lambda)}{\lambda+\alpha}}\right) & \text{if } \widetilde{x} - \lambda < y \leq \bar{x} - \lambda \\ 0 & \text{if } y \geq \bar{x} - \lambda. \end{cases}$$

Proposition 4 says that there are two different types of D1 equilibria in which not all taxpayer types pool at a zero income report: those in which the IRS exhausts its budget, and those in which the IRS leaves some part of its audit budget unused. Since equilibrium requires that the IRS be indifferent between auditing all income reports in the interval $\{0\} \cup [\widetilde{x} - \lambda, \bar{x} - \lambda]$, these equilibria are distinguished by the IRS's expected revenue from auditing the income $y = 0$, i.e. by $(\tau + f)\lambda$. If $(\tau + f)\lambda > c$, then the IRS's budget constraint is binding, as expressed by (3). If $(\tau + f)\lambda = c$, then the IRS may leave some of its budget unused. Proposition 4 also shows that equilibrium audit probabilities and equilibrium reporting strategies are independent of i. This prediction is interesting, because in reality the IRS audit policy is blind to the identity of taxpayers.[18] Proposition 4 does not say that equilibrium is unique, as conceivably there could be multiple values of $\widetilde{x}$ solving (3), caused by congestion in tax enforcement. The same economic force could also lead to the coexistence of equilibria in which the IRS does and does not exhaust its budget, or the coexistence of equilibria with incomplete pooling and riot equilibria. Our next results rules out these possibilities.

Theorem 1 *Suppose Assumption 1 holds, and* $H(\cdot)$ *has no masspoint at* $\bar{x}$. *Then there is a unique D1 equilibrium.*

[17] Because the IRS is indifferent between auditing 0 and $\widetilde{x}$, it is consistent with equilibrium to have $\pi_i(\widetilde{x}) \in [\underline{\pi}(\widetilde{x}), 1]$, where $\underline{\pi}(\widetilde{x}) = \lim_{y \downarrow \underline{\pi}(\widetilde{x})} \pi_i(y)$. For $\pi_i(\widetilde{x}) > \underline{\pi}(\widetilde{x})$, it is still uniquely optimal strategy for type $\widetilde{x}$ to report 0. At $\pi_i(\widetilde{x}) = \underline{\pi}(\widetilde{x})$, type $\widetilde{x}$ can randomize in any way between the reports 0 and $\widetilde{x} - \lambda$. This multiplicity in equilibrium is inconsequential, since $\widetilde{x}$ is not a masspoint.

[18] The IRS sorts individual returns into audit classes, based upon income and type of income. All taxpayers in the same audit class face the same audit probability. This audit probability is based upon a complex and top secret computer algorithm known as the "Discriminant Index Function." (Clotfelter (1983), Harcourt (2007)).

In spite of this, multiplicity is possible if the income distribution H has a masspoint at $\bar{x}$. This multiplicity occurs at $B = \bar{B}$, the highest budget level for which a riot equilibrium exists.

Proposition 5 *Suppose that $H(\cdot)$ has a masspoint of size $\rho(\bar{x})$ at $\bar{x}$. Then there is a unique D1 equilibrium for all $B \neq \bar{B}$. Furthermore, at $B = \bar{B}$ there exists a continuum of D1 equilibria. In these equilibria, all types $x < \bar{x}$ select $y = 0$ with probability one. Type $\bar{x}$ selects $y = 0$ with probability η, and $y = \bar{x} - \lambda$ with probability $1 - \eta$, where*

$$\lambda = \frac{E(x) - (1-\eta)\rho(\bar{x})\bar{x}}{1 - (1-\eta)\rho(\bar{x})}.$$

In these equilibria, $\pi_i(\cdot)$ is independent of i. The IRS audits $y = 0$ with probability

$$\pi_i(0) = \frac{\tau(\bar{x} - \lambda)}{(\tau + f)\bar{x} + F},$$

audits all reports $y \in (0, \bar{x} - \lambda)$ with probability one, and does not audit reports $y \geq \bar{x} - \lambda$.

The reason for the multiplicity can be understood as follows. As type $\bar{x}$ lowers the probability with which it selects to report $y = 0$, this frees up audit resources. As a consequence, the IRS can raise the probability with which it audits the income report $y = 0$. At the same time, the decreased revenue from auditing $y = 0$ makes it optimal for the IRS to audit a larger range of low income reports. Thus, while reporting $y = 0$ becomes less attractive for type $\bar{x}$, so does reporting $y = \bar{x} - \lambda$. In equilibrium, these two effects exactly balance each other out.

5 Optimal Income Taxation

In this section, we study the problem of optimal income taxation. In particular, we show that the existence of bad equilibria to the reporting stage of the optimal tax mechanism is intimately connected to the existence of congestion equilibria in the interactive model we studied in the previous two sections. To allow this comparison, we assume that in implementation the mechanism designer is restricted to selecting tax and fine schedules that are proportional to income and evaded income, respectively. We show that this restriction, by itself, does not rule out the possibility of tax riots.

To formulate the mechanism design problem, we assume that each individual's utility w_i is additively separable in the private good m and the public good R, i.e. that $w_i(m, R) = m + P(R)$, where P is strictly increasing and concave in R. The public good is to be financed by the tax proceeds from collecting taxes and fines, net of audit costs. A direct mechanism is then a collection

$\{\tau, f, R, K, \pi_i(\cdot|G)\}$, where τ is the marginal tax rate, f is the marginal fine rate, and K is the audit capacity. Note that a mechanism must specify audit probabilities $\pi_i(\cdot|G)$ as a function of the entire reported income distribution G.

Suppose the mechanism designer is concerned only with implementation, and hence does not worry about the existence of multiple equilibria to the reporting stage of the game. By the revelation principle, there is then no loss of generality in limiting attention to truthtelling equilibria of direct mechanisms. In our context, this has two implications. First, in formulating the design problem, there is no need to specify audit probabilities $\pi_i(\cdot|G)$ for reported income distributions that are inconsistent with truthtelling. Second, the reported income distribution G in the truthtelling equilibrium will coincide with the true income distribution H. Hence in formulating the design problem, we may drop the dependence of π_i on H.

We assume that the mechanism designer is interested in maximizing social welfare. The designer's objective is thus to solve the following problem:

$$Max_{\ \{\tau,f,R,K,\pi_i(\cdot)\}} \int_0^1 u_i(x(i), \pi_i(x(i))|x\,(i))\mathrm{d}i + P(R)$$

subject to:

$$R \leq \tau \int_0^1 x\,(i)\,\mathrm{d}i - cK$$

$$\int_0^1 \pi_i(x(i))\mathrm{d}i \leq K$$

$$\tau + f \leq 1,\ \pi_i(\cdot) \in [0,1]$$

and the incentive compatibility constraints

$$x\,(i) \in \arg\max_m \{u_i(m, \pi_i(m)|x(i)).$$

Let the optimal solution to this problem be denoted by $\{\tau^*, f^*, R^*, K^*, \pi_i^*(\cdot)\}$.

Consider now the alternative in which the designer first commits to a tax rate τ^*, a fine rate f^*, and an audit capacity K^*, and then delegates the authority over audit policy to an independent IRS, whose goal is to maximize net proceeds from taxes and fines subject to the capacity constraint K^*. The net proceeds from tax collection and enforcement are then used to finance the provision of public good. We assume that the IRS cannot commit, so she must select her audit policy after observing the reported income profile y. Let G denote the corresponding distribution of reported incomes. In the sequential game between taxpayers and the IRS, the IRS thus solves:

$$\max_{\pi_i(\cdot|G)} \left\{ \tau \int_0^1 y(i)\mathrm{d}i + (\tau + f) \int_0^1 \max\{x(i) - y(i), 0\}\pi_i(y(i)|G)\mathrm{d}i \right\} \tag{4}$$

$$\text{s.t.} \int_0^1 \pi_i(y(i)|G)\mathrm{d}i \leq K^*$$

We then have:

Proposition 6 *There exists a D1 equilibrium of the sequential game (4) in which taxpayers report their incomes truthfully, and in which the IRS selects* $\pi_i(\cdot|H) = \pi_i^*(\cdot)$.

If one is only concerned about implementation, and not full implementation, then Proposition 6 implies that the designer can delegate authority for tax collection and enforcement to the IRS, and still implement the optimal scheme $\{\tau^*, f^*, R^*, K^*, \pi_i^*(\cdot)\}$.

In a thought provoking recent paper, Basetto and Phelan (2008) demonstrate that with audit capacity limited to $K^* = \int_0^1 \pi_i^*(x(i))\mathrm{d}i$, the reporting stage of the mechanism necessarily has equilibria which do not involve truthtelling by taxpayers. Their paper focuses on the existence of a riot equilibrium, in which every type of every taxpayer reports zero income. The intuition is that faced with massive underreporting by taxpayers, there may be insufficient audit capacity available to deter this underreporting.

Our next result explores the possibility of unique implementation, when the set of implementing mechanisms is unrestricted. For this result, we allow a more general set of possible income types X. We continue to assume that $0 = \min X$ and $\bar{x} = \max X$. We define a mechanism to be symmetric if $\pi_i(\cdot|G)$ is independent of i for all reported income distributions G.

Theorem 2 *There always exists an asymmetric mechanism that uniquely implements the optimal outcome. Furthermore, if* X *contains* x' *such that* $0 < x' < \bar{x}\frac{\tau+f}{\tau}K^*$*, then there exists a symmetric mechanism that uniquely implements the optimal outcome.*

Theorem 2 shows that if asymmetric audit treatment is allowed, the mechanism can be designed such that the reporting stage has truthtelling as the unique equilibrium, even if X contains only two elements, as in Basetto and Phelan (2008). Furthermore, if the set of possible income types is sufficiently fine, as is necessarily the case when $X = [0, \bar{x}]$, then we can design a symmetric mechanism that has truthtelling as the unique equilibrium.[19] The intuition behind the theorem is that for any G different from H, one can design the audit stage such that a positive measure of individuals will have an incentive to report in such a way as to cause the reported distribution to differ from G.

[19] This result also implies that asymmetric treatment can be avoided if the indirect mechanism allows for an income report set Y larger than X. Such mechanisms are indirect, but still very simple.

One might object to theorem 2 on the grounds that the designer should not be able to discriminate in its audit treatment of taxpayers, and that the mechanism designer has too much freedom in specifying audit probabilities when faced with a tax riot. For this reason, let us define a mechanism to be *credible* if $\pi_i(\cdot|G)$ must be an optimal response to G, i.e. must solve (4), and beliefs satisfy criterion D1. Let us also define:

Definition 1 *A symmetric D1 equilibrium of the delegation game is called critical if*

(i) For any equilibrium report $y > 0$ the IRS strictly prefers auditing 0 to y

(ii) For any off equilibrium report $y > 0$, if there exists a D1 belief s.t. the IRS is indifferent between auditing 0 and y, then $\pi(y) = 0$.

Part (ii) in Definition 1 holds if $X = [0, \bar{x}]$, and if X is discrete, for generic distributions of income. Part (i) implies that in a critical equilibrium the IRS receives positive equilibrium revenue from auditing. Furthermore, there exist at most two equilibrium reports, $y = 0$ and possibly also some $y^* > 0$. Furthermore, it must be that $\pi(0) = K^*$, and $\pi(y^*) = 0$. For generic discrete X, or for $X = [0, \bar{x}]$, a riot equilibrium is always critical. However, there often exist critical equilibria even when no riot equilibrium is present.

We may now state:

Proposition 7 *There exists a non truthful equilibrium to the reporting stage of every symmetric credible direct mechanism with $\pi_i(\cdot|H) = \pi_i^*(\cdot)$ if, and only if, the delegation game with audit capacity K^* has a critical equilibrium.*

The proof of Proposition 7 shows that if the delegation game has a critical equilibrium, then it is necessarily an equilibrium to the reporting stage of any symmetric credible direct mechanism that implements the optimum. Conversely, if the delegation game has no critical equilibria, then a symmetric credible direct mechanism can be designed so that there are no non truthful equilibria to the reporting stage of the mechanism.

To show that in general the reporting stage of any implementing symmetric credible direct mechanism may have bad equilibria, consider again a two type example, with $X = \{0, x_H\}$. Let K denote the audit capacity available to the IRS, and ρ denote the fraction of low income types in the population. Also let ω be the proportion of cheaters in the population. Our next proposition describes the sequential equilibria of the delegation game.

Proposition 8 *Let* $\widetilde{\pi} = \frac{\tau}{\tau+f}$. *Then the set of sequential equilibria of the delegation game is as follows:*

(i) For $K < \rho\widetilde{\pi}$, *the riot equilibrium in which* $\omega = 1 - \rho$ *is the unique sequential equilibrium.*

(ii) For $\rho\widetilde{\pi} \leq K \leq \widetilde{\pi}$ *there are three sequential equilibria: the truthtelling equilibrium in which* $\omega = 0$, *a mixed equilibrium in which* $\omega = K\widetilde{\pi}^{-1} - \rho$, *and a riot equilibrium in which* $\omega = 1 - \rho$.

(iii) For $K > \widetilde{\pi}$ *the truthtelling equilibrium in which* $\omega = 0$ *is the unique sequential equilibrium.*

In all equilibria, the IRS fully utilizes its audit capacity.

According to Proposition 8 we have $K^* = \rho\widetilde{\pi}$, since this is the smallest audit capacity for which truthtelling is an equilibrium. Because there also exists a riot equilibrium at $K = K^*$, it follows from Proposition 7 that any implementing symmetric credible direct mechanism necessarily has a riot equilibrium to the reporting stage of the mechanism.

However, the existence of bad equilibria to the reporting stage of the mechanism disappears when the set of possible income types becomes a continuum. More precisely, we have:

Proposition 9 *Suppose the support of the income distribution* H *equals* $[0, \bar{x}]$. *Then there exists a symmetric credible direct mechanism that uniquely implements the optimal outcome* $\{\tau^*, f^*, R^*, K^*, \pi_i^*(\cdot)\}$.

We prove Proposition 9 by showing that the delegation game with $\tau = \tau^*$ and $f = f^*$ has no critical equilibrium at $K = K^*$.

6 Conclusion

We have shown that a budget or capacity constraint on audit activity alone is insufficient to generate multiple equilibria in the game between the IRS and taxpayers. In order for congestion to arise, it is necessary that there exists a lower bound to the signal available to the senders, for otherwise full separation will occur in any D1 equilibrium. When such a bound is present, multiplicity will arise if and only if the income distribution has a masspoint at the highest possible income level. Such a masspoint is necessarily present when there are a finite number of possible income types. However, as this number increases, and the size of the masspoint decreases, the degree of multiplicity falls. In the limit, when there is a continuum of possible income types, and the distribution of incomes is continuous, there is a unique equilibrium for every budget level.

We have also shown that there is a tight connection between the equilibria of an interactive model in which the IRS is constrained in its audit capacity and maximizes gross revenue, and the equilibria of optimal tax and enforcement mechanisms. In particular, we established that symmetric credible mechanisms can be designed to have a unique equilibrium if and only if the decentralized version does not have a critical equilibrium. With a continuum of possible income levels, unique implementation is then possible in our model, regardless of whether or not there exists a masspoint at the upper endpoint of the income distribution.

Our results have important implications for other areas of economics. In accounting, we can envision models in which auditors have multiple clients, and are constrained in audit capacity. If cheating is a binary variable, our results suggest that congestion in auditing will occur, and multiple equilibria will be present. However, when cheating is a continuous variable (such as reporting the value of an asset), audit capacity may uniquely determine the degree of cheating. In the economics of crime and punishment, limited enforcement capacity will generate multiple equilibria in an interactive model if crime is a binary choice variable (as in Brock and Durlauf (2001)). However, if criminal activity is a continuous variable, multiplicity might disappear. We pursue these extensions elsewhere.

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Appendix A: Proof of Proposition 2

The proof of Proposition 2 contains seven lemmas. Lemma 1 describes the structure of indifference curves for any taxpayer i with income x. Lemma 2 and Lemma 3 establish conditions under which the indifference curves of income types x and x' of taxpayer i either never cross, cross once, cross twice, or coincide over a range. Lemma 4 shows that in any D1 equilibrium, taxpayer i must report 0 when his income is 0. Lemma 5 shows that in any D1 equilibrium there are some types $x > 0$ of taxpayer i that report 0 with positive probability. Lemma 5 implies that the IRS's expected revenue from auditing an income report of 0 by taxpayer i is strictly positive. Using Lemma 4 and Lemma 5, we argue in Lemma 6 that the IRS's auditing probability at 0 can not be too high. This rules out double crossing of equilibrium indifference curves. Lemma 7 rules out the possibility that taxpayer i reports an income higher then his true income. This implies that equilibrium indifference curves of different income types of taxpayer i cannot coincide over a range. Finally, we show that Proposition 2 holds.

Let $I_i(y|x, \bar{u})$ denote the indifference curve of taxpayer i with income x at the level $\bar{u}$, i.e.

$$I_i(y|x,\bar{u}) = \begin{cases} \frac{x-\tau y-\bar{u}}{(\tau+f)(x-y)+F} & \text{if } y \leq x \\ \frac{x-\tau y-\bar{u}}{F} & \text{if } y > x \end{cases}$$

Lemma 1 *$I_i(y|x, \bar{u})$ is strictly decreasing and linear with slope $-\frac{\tau}{F}$, for all $y \geq x$. For $y \leq x$,*

(i) If $I_i(0|x, \bar{u}) > \frac{\tau}{\tau+f}$, then $I_i(y|x, \bar{u})$ is strictly increasing and strictly convex;

(ii) If $I_i(0|x, \bar{u}) = \frac{\tau}{\tau+f}$, then $I_i(y|x, \bar{u})$ is constant;

(iii) If $I_i(0|x, \bar{u}) < \frac{\tau}{\tau+f}$ then $I_i(y|x, \bar{u})$ is strictly decreasing and strictly concave.

Proof : Note that regardless of $\bar{u}$, we have $\frac{dI_i}{dy} = -\frac{\tau}{F}$ for all $y > x$. Using $\bar{u} = x - I_i(0|x, \bar{u})\left((\tau + f)x + F\right)$, for $y \leq x$ we may compute

$$I_i(y|x,\bar{u}) = \frac{-\tau y + I_i(0|x,\bar{u})\left((\tau+f)x+F\right)}{(\tau+f)(x-y)+F},$$

$$\frac{dI_i(y|x,\bar{u})}{dy} = \left((\tau+f)I_i(0|x,\bar{u}) - \tau\right)\left((\tau+f)x+F\right)\left((\tau+f)(x-y)+F\right)^{-2},$$

and

$$\frac{d^2I_i(y|x,\bar{u})}{dy^2} = 2\left(\tau+f\right)\left((\tau+f)I_i(0|x,\bar{u}) - \tau\right)\left((\tau+f)x+F\right)\left((\tau+f)(x-y)+F\right)^{-3}$$

from which the desired result follows. *Q.E.D.*

Figure 2 illustrates the shape of taxpayer i's indifference curves.

[Insert Figure 2 here]

Lemma 2 *Let $x' < x$, and suppose that $I_i(0|x',\bar{u}') < \frac{\tau}{\tau+f}$ and $I_i(0|x,\bar{u}) < \frac{\tau}{\tau+f}$. We have:*

(i) if $I_i(y|x',\bar{u}') = I_i(y|x,\bar{u})$ for some $y < x$, then $\frac{dI_i}{dy}(y|x',\bar{u}') < \frac{dI_i}{dy}(y|x,\bar{u}) < 0$. Hence $I_i(y'|x',\bar{u}') > I_i(y'|x,\bar{u})$ for all $y' < y$, and $I_i(y'|x',\bar{u}') < I_i(y'|x,\bar{u})$ for all $y' > y$;

(ii) if $I_i(y|x',\bar{u}') = I_i(y|x,\bar{u})$ for some $y \geq x$, then $I_i(y'|x',\bar{u}') > I_i(y'|x,\bar{u})$ for all $y' < x$ and $I_i(y'|x',\bar{u}') = I_i(y'|x,\bar{u})$ for all $y' \geq x$.

Proof : (i) Let $\bar{\pi} = I_i(y|x,\bar{u})$. It follows from the implicit function theorem that

$$\frac{dI_i}{dy}(y|x,\bar{u}) = -\frac{\frac{\partial u_i}{\partial y}(y,\bar{\pi}|x)}{\frac{\partial u_i}{\partial \pi}(y,\bar{\pi}|x)} = -\frac{\tau - \bar{\pi}(\tau+f)}{(\tau+f)(x-y)+F} \tag{5}$$

Thus if $y < x'$, we have $\frac{dI_i}{dy}(y|x',\bar{u}') < \frac{dI_i}{dy}(y|x,\bar{u}) < 0$. Meanwhile, if $y > x'$, then $\frac{dI_i}{dy}(y|x',\bar{u}') = -\frac{\tau}{F} < \frac{dI_i}{dy}(y|x,\bar{u}) < 0$. A standard argument then establishes the conclusion for $y' \in [0,x]$. Lemma 1 establishes that $I_i(y'|x',\bar{u}')$ and $I_i(y'|x,\bar{u})$ are parallel for $y' \geq x$, completing the argument.

(ii) if $I_i(y|x',\bar{u}') = I_i(y|x,\bar{u})$ for some $y \geq x$, then by Lemma 1 we have $I_i(y'|x',\bar{u}') = I_i(y'|x,\bar{u})$ for all $y' \geq x$. By (i), the indifference curves cannot intersect at any point $y < x$. Since $\frac{dI_i}{dy}(x|x',\bar{u}') < \frac{dI_i}{dy}(x|x,\bar{u}) < 0$, we conclude $I_i(y'|x',\bar{u}') > I_i(y'|x,\bar{u})$ for all $y' < x$. *Q.E.D.*

[Insert Figure 3 here]

Figure 3 illustrates the possible configurations of the indifference curve of types x and x' when the audit probability is not too high, i.e. $I_i(0|x',\bar{u}') < \frac{\tau}{\tau+f}$ and $I_i(0|x,\bar{u}) < \frac{\tau}{\tau+f}$. The indifference curves of types x' and x then either never cross, cross once, in which case they cross in the interval $[0,x)$ and the indifference curve of the lower income type is the steeper one, or they coincide over the entire interval $[x,\infty)$, in which case they do not cross on $[0,x)$. We now turn to the case where audit probabilities are high, i.e. $I_i(0|x',\bar{u}') > \frac{\tau}{\tau+f}$ and $I_i(0|x,\bar{u}) > \frac{\tau}{\tau+f}$.

Lemma 3 *Let $x' < x$, and suppose that $I_i(0|x',\bar{u}') > \frac{\tau}{\tau+f}$ and $I_i(0|x,\bar{u}) > \frac{\tau}{\tau+f}$. We have:*

(i) if $I_i(y|x',\bar{u}') = I_i(y|x,\bar{u})$ for some $y < x'$, then $\frac{dI_i}{dy}(y|x',\bar{u}') > \frac{dI_i}{dy}(y|x,\bar{u}) > 0$. Hence $I_i(y'|x',\bar{u}') < I_i(y'|x,\bar{u})$ for all $y' < y$, and $I_i(y'|x',\bar{u}') > I_i(y'|x,\bar{u})$ for all $y' \in (y,x']$;

(ii) if $I_i(0|x',\bar{u}') = I_i(0|x,\bar{u})$ then $I_i(x|x',\bar{u}') < I_i(x|x,\bar{u})$.

Proof : (i) Suppose that $I_i(y|x',\bar{u}') = I_i(y|x,\bar{u}) = \bar{\pi}$ for some $y < x'$. It follows from Lemma 1 that $I_i(z|x,\bar{u})$ is increasing in z on $[0,x]$, and hence that $\bar{\pi} > \frac{\tau}{\tau+f}$. Using (5), and the fact that $x' < x$, we have $\frac{dI_i}{dy}(y|x',\bar{u}') > \frac{dI_i}{dy}(y|x,\bar{u}) > 0$. A standard argument then establishes the remainder of the proof.

(ii) Using $x' - I_i(0|x',\bar{u}')\left((\tau+f)\,x'+F\right) = x' - \tau x - I_i(x|x',\bar{u}')F$, we have $I_i(x|x',\bar{u}') = \frac{I_i(0|x',\bar{u}')\left((\tau+f)x'+F\right)-\tau x}{F}$. Similarly, we have $I_i(x|x,\bar{u}) = \frac{I_i(0|x,\bar{u})((\tau+f)x+F)-\tau x}{F}$. Since $I_i(0|x',\bar{u}') = I_i(0|x,\bar{u})$ and $x' < x$, we may conclude that $I_i(x|x',\bar{u}') < I_i(x|x,\bar{u})$. *Q.E.D.*

Figure 4 illustrates the possible configurations of the indifference curve of types x and x' for this case. The indifference curves of types x' and x then either do not cross, cross once, cross twice, or coincide over the entire interval $[x,\infty)$ and do not cross elsewhere.

[Insert Figure 4 here]

Lemma 4 *In any D1 equilibrium, every taxpayer i with income $x = 0$ reports the income $y = 0$ with probability one.*

Proof : Suppose to the contrary that there exists a taxpayer i with income $x = 0$ that reports $y^* > 0$ with positive probability. Let $\pi_i(y^*)$ denote the equilibrium probability of auditing i's report y^*. Also let $u_i^*(x)$ denote taxpayer i's equilibrium utility when her income equals x.

We now claim that for any income $x' > 0$, it must be that $I_i(0|x',u_i^*(x')) < I_i(0|0,u_i^*(0))$. To see this, note that since reporting y^* is always feasible, any income type's equilibrium indifference curve lies on or below the point $(y^*, \pi_i(y^*))$. From Lemma 1 we have $\frac{dI_i}{dy}(y|0,u_i^*(0)) = -\frac{\tau}{F}$ for all y, and $\frac{dI_i}{dy}(y|x',u_i^*(x')) \geq -\frac{\tau}{F}$, with strict inequality for $y < x'$. Therefore, we may conclude that $I_i(0|x',u_i^*(x')) < I_i(0|0,u_i^*(0))$.

Regardless of whether 0 is an equilibrium report, it then follows that in any $D1$ equilibrium we must also have $\mu_i(0|0) = 1$. Since $c > 0$, with such beliefs the IRS will not audit $y = 0$. Hence type $x = 0$ can profitably deviate to reporting $y = 0$, contradicting equilibrium. *Q.E.D.*

Lemma 5 *In any D1 equilibrium, for every taxpayer i there must exist some type $x > 0$ that reports $y = 0$ with positive probability.*

Proof : Suppose that there exists a taxpayer i such that all types above 0 do not report $y = 0$ with positive probability. Then by the previous lemma we would have $\mu_i(0|0) = 1$. With such beliefs, the IRS will not audit $y = 0$, in which case all types x uniquely maximize $u_i\left(y, \pi_i(y)|x\right)$ at $y = 0$, a contradiction. *Q.E.D.*

Lemma 6 *In any D1 equilibrium, we have $I_i(0|x,u_i^*(x)) < \frac{\tau}{\tau+f}$ for all $x \geq 0$ and all $i \in [0,1]$.*

Proof : We claim that $\pi_i(0) < \frac{\tau}{\tau+f}$. Since reporting $y = 0$ is a feasible choice for every type x, this implies $I_i(0|x,u_i^*(x)) < \frac{\tau}{\tau+f}$ for all $x \geq 0$.

Suppose that contrary to the claim we had $\pi_i(0) \geq \frac{\tau}{\tau+f}$. Let $x_M = \sup\{x : u_i(0, \pi_i(0)|x) = u_i^*(x)\}$; by Lemma 5 x_M is well defined and satisfies $x_M > 0$. We now claim that for any taxpayer i with income $x \neq x_M$, it must be that $I_i(x_M|x, u_i^*(x)) < I_i(x_M|x_M, u_i^*(x_M))$.

Consider first any $x < x_M$. Then we have $I_i(x_M|x, u_i(0, \pi_i(0)|x)) < I_i(x_M|x_M, u_i^*(x_M))$. If $\pi_i(0) > \frac{\tau}{\tau+f}$ this follows from Lemma 3(ii); if $\pi_i(0) = \frac{\tau}{\tau+f}$ this follows from Lemma 1. Since $y = 0$ is a feasible report for type x, we have $I_i(x_M|x, u_i^*(x)) \leq I_i(x_M|x, u_i(0, \pi_i(0)|x))$. Consequently, $I_i(x_M|x, u_i^*(x)) < I_i(x_M|x_M, u_i^*(x_M))$ for all $x < x_M$.

Next, consider any $x > x_M$. If $\pi_i(0) > \frac{\tau}{\tau+f}$ then by Lemma 3(i) we have $I_i(x_M|x, u_i(0, \pi_i(0)|x) < I_i(x_M|x_M, u_i^*(x_M))$, and so again we may conclude that $I_i(x_M|x, u_i^*(x)) < I_i(x_M|x, u_i(0, \pi_i(0)|x)$. If $\pi_i(0) = \frac{\tau}{\tau+f}$ then by the definition of x_M we have $I_i(0|x, u_i^*(x)) < \frac{\tau}{\tau+f}$. It then follows from Lemma 1 that $I_i(x_M|x, u_i^*(x)) < I_i(x_M|x_M, u_i^*(x_M)) = \frac{\tau}{\tau+f}$.

Regardless of whether x_M is an equilibrium report, it then follows that $\mu_i(x_M|x_M) = 1$. Since the IRS stands nothing to gain from auditing x_M, and since $c > 0$, we must have $\pi_i(x_M) = 0$. Since $\pi_i(0) \geq \frac{\tau}{\tau+f}$, this contradicts the definition of x_M. *Q.E.D.*

Lemma 6 and Lemma 2 imply that if $x' < x$ then the equilibrium indifference curves of types x' and x either do not cross, cross once, or coincide over the interval $[x, \bar{x}]$. To rule out the latter case, we shall now prove that it is never optimal for any type x to report $y > x$. More precisely, define

$$Y_i^*(x) = \arg\max_y \; u_i(y, \pi_i(y)|x).$$

Then we have:

Lemma 7 *Suppose $\pi_i(0) > 0$. Then for any $x > 0$, $y \in Y_i^*(x)$ implies $y \leq x$.*

Proof : Suppose that contrary to the statement of the lemma, there existed x and $y \in Y_i^*(x)$ such that $y > x$. Let $x' = \inf\{z : \text{there exists } y \in Y_i^*(z) \text{ s.t. } y > z\}$.

By definition of x' there exists a sequence $x_n \downarrow x'$ and $y_n \in Y_i^*(x_n)$ such that $y_n \geq x_n$. By taking subsequences, if necessary, we can assure that the sequence y_n converges to a limit y_∞. By u.h.c. of the equilibrium correspondence, it follows that $y_\infty \in Y_i^*(x')$.

We claim that regardless of the value of y_∞, we have $I_i(x'|x, u_i^*(x)) < I_i(x'|x', u_i^*(x'))$ for all $x > x'$. Suppose first that $y_\infty > x'$. Then if $x > y_\infty$, it follows from Lemma 2(i) that $I_i(x'|x, u_i(y_\infty, \pi_i(y_\infty)|x)) < I_i(x'|x', u_i^*(x'))$. If $y_\infty \geq x$, the same conclusion follows from Lemma 2(ii). Since $u_i^*(x) \geq u_i(y_\infty, \pi_i(y_\infty)|x)$, the claim follows. Next, suppose that $y_\infty = x'$. Let n

be sufficiently large that $x > y_n$. It follows from Lemma 2(i) that $I_i(x'|x, u_i(y_n, \pi_i(y_n)|x)) < I_i(x'|x', u_i^*(x'))$. Since $u_i^*(x) \geq u_i(y_n, \pi_i(y_n)|x)$, the claim again follows.

We also claim that $I_i(x'|x, u_i^*(x)) < I_i(x'|x', u_i^*(x'))$ for all $x < x'$. Indeed, it follows from the definition of x' that $u_i^*(x) > u_i(y_\infty, \pi_i(y_\infty)|x)$, implying $I_i(x'|x, u_i^*(x)) < I_i(x'|x, u_i(y_\infty, \pi_i(y_\infty)|x))$. By Lemma 2(ii) we have $I_i(x'|x, u_i(y_\infty, \pi_i(y_\infty)|x)) = I_i(x'|x', u_i(y_\infty, \pi_i(y_\infty)|x')) = I_i(x'|x', u_i^*(x'))$, establishing the claim.

The two claims imply that in any D1 equilibrium, we must have $\mu_i(x'|x') = 1$. As the IRS gains nothing from auditing x', it must then be that $\pi_i(x') = 0$. This contradicts the definition of x', establishing the statement of the Lemma. *Q.E.D.*

Proof of Proposition 2 **:** Note that the equilibrium indifference curves must cross, for otherwise the type with the higher indifference curve would wish to deviate to a signal sent by the type with the lower indifference curve. Let y be such that $I_i(y|x', u^*(x')) = I_i(y|x, u^*(x))$ for some $x' < x$. By Lemma 6, Lemma 2 applies. If we had $y \geq x$, then any equilibrium report γ of type x must be no less than x, otherwise type x' could mimic type x and receive higher utility than $u^*(x')$. Since $I_i(y|x', u^*(x')) = I_i(y|x, u^*(x))$ for $y \geq x$, it would follow that $\gamma \in Y^*(x')$, contradicting Lemma 7. The result then follows from Lemma 2(i). *Q.E.D.*

Appendix B

Proof of Proposition 3 : Since all types of every taxpayer report $y = 0$, the IRS's expected revenue from auditing $y = 0$ from any taxpayer equals $(\tau + f) \int_0^{\bar{x}} x dH(x) > c$. Hence the IRS will exhaust its audit budget, i.e. $B = c \int_0^1 \pi_i(0)\mathrm{d}i$.

We claim that the IRS will not audit any income level $y > \bar{y} = \bar{x} - \int_0^{\bar{x}} x dH(x)$. Indeed, since $\bar{x}$ is an upper bound to the support of $\mu_i(x|y)$, it follows that the most the IRS could expect to receive in revenue from auditing a report y from taxpayer i equals $(\tau+f)(\bar{x}-y) < (\tau+f)(\bar{x}-\bar{y}) = (\tau + f) \int_0^{\bar{x}} x dH(x)$, which is less than the IRS's expected revenue from auditing $y = 0$. Therefore, optimality of the IRS's strategy implies $\pi_i(y) = 0$ for all $y > \bar{y}$. Thus, by reporting some $y > \bar{y}$, type $\bar{x}$ can obtain utility arbitrarily close to $\bar{x} - \tau y_{\max}$. Since type $\bar{x}$'s utility from reporting $y = 0$ is $\bar{x} - \pi_i(0)((\tau+f)\bar{x}+F)$, for a riot equilibrium to exist we must have $\pi_i(0)((\tau+f)\bar{x}+F) \leq \tau y_{\max}$. Rewriting the latter inequality, we have $\pi_i(0) \leq \frac{\bar{B}}{c}$ and $B = c\int_0^1 \pi_i(0)\mathrm{d}i \leq c\int_0^1 \frac{\bar{B}}{c}\mathrm{d}i = \bar{B}$. *Q.E.D.*

In any D1 equilibrium, given a taxpayer i define

$$\Omega_i = \{x \in (0, x_{\max}] : \text{type } x \text{ reports } y = 0 \text{ with positive probability}\}$$

and let

$$\widetilde{x}_i = \sup \Omega_i.$$

By Lemma 5 we have $\widetilde{x}_i > 0$.

Proof of Proposition 4 : Let us start by assuming that H is a continuous function. At the end of the proof, we indicate how to handle masspoints in the distribution.

First, we claim that for every i the correspondence $Y_i^*(x) = \{y : u_i(y, \pi_i(y)|x) = u_i^*(x)\}$ must be nondecreasing, i.e. $y \in Y_i^*(x)$, $y' \in Y_i^*(x')$, and $x > x'$ implies $y \geq y'$. It follows from Proposition 2 that there exists a point y_C such that $I_i(z|x', u_i^*(x')) > I_i(z|x, u_i^*(x))$ for all $z < y_C$ and $I_i(z|x', u_i^*(x')) > I_i(z|x, u_i^*(x))$ for all $z > y_C$. Thus if $y \in Y_i^*(x)$ we must have $y \geq y_C$, and if $y' \in Y^*(x')$ we must have $y' \leq y_C$, establishing the claim.

Next, we claim that any type $x' < \widetilde{x}_i$ must report $y = 0$ with probability one. Suppose to the contrary that there exists $x' < \widetilde{x}_i$ that reports $y' > 0$ with positive probability. Since Y_i^* is nondecreasing, $0 \notin Y_i^*(x)$ for all $x > x'$, contradicting the definition of $\widetilde{x}_i$.

Next, we argue that any D1 equilibrium must be separating on $[\widetilde{x}_i, \bar{x}]$. Suppose to the contrary that y is reported with positive probability by some type $x' \in [\widetilde{x}_i, \bar{x})$ and that some type $x'' > x$

also reports y with positive probability. It follows from the definition of $\widetilde{x}_i$ that $y > 0$. Since $Y_i^*(x)$ is nondecreasing, it must be that all types $x \in (x', x'')$ report y with probability one. Hence the IRS's expected revenue from auditing y equals at least

$$(\tau + f)\frac{\int_{x'}^{x''}(x - y)\,\mathrm{d}H(x)}{\int_{x'}^{x''}\mathrm{d}H(x)}$$

Now consider any $y' < y$. Proposition 2 implies that the equilibrium indifference curve of type x crosses the equilibrium indifference curve of type x' from below at some point $y^*(x) \geq y$. Consequently, regardless of whether or not y' is an equilibrium report, in any D1 equilibrium we must have $\mu_i(x|y') = 0$ for all $x > x'$. Thus the IRS's expected revenue from auditing y' is then bounded above by

$$(\tau + f)(x' - y')$$

Since for y' sufficiently close to y we have

$$x' - y' < \frac{\int_{x'}^{x''}(x - y)\,\mathrm{d}H(x)}{\int_{x'}^{x''}\mathrm{d}H(x)},$$

it follows that the IRS will not audit such y'. This contradicts the optimality of reporting y, thereby establishing that any D1 equilibrium must be separating on $[\widetilde{x}_i, \bar{x}]$.

Let

$$\lambda_i = \frac{\int_0^{\widetilde{x}_i} x\mathrm{d}H(x)}{H(\widetilde{x}_i)}$$

We now claim that any type $x > \widetilde{x}_i$ must report

$$y_i(x) = x - \lambda_i \tag{6}$$

with probability one. Let $y_i^*(x)$ denote any equilibrium report of such a type x. Since separation requires that any equilibrium report from a type $x < \bar{x}$ must be audited with strictly positive and distinct probability, it must be that the IRS is indifferent between auditing any such reports. Thus the IRS's expected revenue from auditing $y_i^*(x)$, $x \in [\widetilde{x}_i, \bar{x})$ must equal its expected revenue from auditing the report $y = 0$. The latter revenue equals $(\tau + f)\lambda_i$. Thus we must thus have

$$(\tau + f)\lambda_i = (\tau + f)(x - y_i^*(x)),$$

from which we deduce $y_i^*(x) = x - \lambda_i$, establishing the claim for every $x \in [\widetilde{x}_i, \bar{x})$. It remains to be shown that type $\bar{x}$ reports

$$\bar{y} = \bar{x} - \lambda_i$$

with probability one.

To establish this, we first show that the IRS does not audit any income level above $\bar{y}$, i.e. that $\pi_i(y) = 0$ for all $y > \bar{y}$. Indeed, since $\bar{x}$ is an upper bound to the support of $\mu_i(x|y)$, the most the IRS could expect to receive in revenue from auditing y equals

$$(\tau + f)(\bar{x} - y) < (\tau + f)(\bar{x} - \bar{y}) = (\tau + f)\lambda_i,$$

so we must have $\pi_i(y) = 0$ for all $y > \bar{y}$. Next, since $Y_i^*(x)$ is nondecreasing, and since the equilibrium is separating, we must have $y_i^*(\bar{x}) \geq \bar{y}$. But since $\pi_i(y) = 0$ for all $y > \bar{y}$, it would never be optimal for type $\bar{x}$ to report $y_i^*(\bar{x}) > \bar{y}$, so $y_i^*(\bar{x}) = \bar{y}$. Equilibrium then requires that $\pi_i(\bar{y}) = 0$.

Finally, we determine the equilibrium audit probability $\pi_i(y)$ for $y \leq \bar{y}$. For $y \in (\widetilde{x}_i - \lambda_i, \bar{y})$, note that in order for the taxpayer of type $x \in (\widetilde{x}_i, \bar{x}]$ to optimally report $y_i\left(x\right)$, we must have

$$\frac{\partial u_i}{\partial y}\left(y_i\left(x\right), \pi_i\left(y_i\left(x\right)\right)|x\right) = 0$$

Performing this differentiation explicitly yields the differential equation

$$-\tau + (\tau + f)\pi_i - \frac{d\pi_i}{dy}\left[(\tau + f)\left(x - y_i\left(x\right)\right) + F\right] = 0 \tag{7}$$

We now set out to solve this differential equation. Substituting (6) into (7) yields

$$-\tau + (\tau + f)\pi_i - \frac{d\pi_i}{dy}\left[(\tau + f)\lambda_i + F\right] = 0$$

This is a standard linear differential equation, whose solution is given by so we may finally conclude that

$$\pi_i\left(y\right) = \frac{\tau}{\tau + f} - ke^{\frac{(\tau+f)y}{(\tau+f)\lambda_i+F}} \tag{8}$$

Note that in equilibrium $\pi_i(y)$ needs to be strictly decreasing in y, so $k > 0$. To determine the constant k, we use the boundary condition $\pi_i(\bar{x}) = 0$. From (6) we have $y_i(\bar{x}) = \bar{x} - \frac{\lambda_i}{\tau+f}$, and so

$$0 = \frac{\tau}{\tau + f} - ke^{\frac{(\tau+f)\bar{x}-\lambda_i}{\lambda_i+F}}.$$

Substituting this into (8) yields our final expression for $\pi_i(y)$:

$$\pi_i\left(y\right) = \frac{\tau}{\tau + f}\left(1 - e^{\frac{(y-\bar{x})+\lambda_i}{\lambda_i+\alpha}}\right), \text{ for } y \in (\widetilde{x}_i - \lambda_i, \bar{y}].$$

To determine $\pi_i(0)$, note that by u.h.c. of $Y_i^*(x)$, type $\widetilde{x}_i$ must be indifferent between reporting $y = 0$ and $y = \widetilde{x}_i - \lambda_i$:

$$\pi_i\left(0\right)\left[(\tau + f)\widetilde{x}_i + F\right] = \tau\left(\widetilde{x}_i - \lambda_i\right) + \pi_i\left(y(\widetilde{x}_i)\right)\left[(\tau + f)\lambda_i + F\right]$$

which may be solved to yield the required expression for $\pi_i(0)$.

To determine $\pi_i(y)$ for $y \in (0, \widetilde{x}_i - \lambda_i)$, note that by the monotonicity of $Y_i^*(\cdot)$ and Proposition 2 the equilibrium indifference curve of type $\widetilde{x}_i$ lies strictly above the equilibrium indifference curve of any type $x \neq \widetilde{x}_i$ for all $y \in (0, \widetilde{x}_i - \lambda_i)$. Application of D1 then yields $\mu_i(\widetilde{x}_i|y) = 1$ for all $y \in (0, \widetilde{x}_i - \lambda_i)$. Since $\mu_i(\widetilde{x}_i|y_i(\widetilde{x}_i)) = 1$, and since the IRS is indifferent between auditing and not auditing $y_i(\widetilde{x}_i)$, it follows that $\pi_i(y) = 1$ for all $y \in (0, \widetilde{x}_i - \lambda_i)$.

We now claim that $\lambda_i = \lambda$ for all $i \in [0,1]$. Suppose to the contrary that there exists k and l such that $\lambda_k > \lambda_l \geq 0$. IRS optimization then requires that either $\pi_k(0) = 1$ or $\pi_l(0) = 0$. If $\pi_k(0) = 1$, then types $x > 0$ of taxpayer k will never choose to report 0, contradicting $\pi_k(0) = 1$. If $\pi_l(0) = 0$, then all types of taxpayer l will report 0, so $\widetilde{x}_l = \bar{x}$. Since λ_i is strictly increasing in $\widetilde{x}_i$, we must have $\lambda_l \geq \lambda_k$, a contradiction. It follows that in equilibrium $\lambda_i = \lambda$, which also implies $\widetilde{x}_i = \widetilde{x}$.

By assumption, there exists j, $x < \bar{x}$, and $y > 0$ such that $y \in Y_j^*(x)$. Monotonicity of Y_j^* then implies that $0 \notin Y_j^*(x')$ for any $x' > x$, so $\widetilde{x} = \widetilde{x}_j \leq x < \bar{x}$. The condition $\widetilde{x} < \bar{x}$ implies $\pi_i(0) > 0$, so the IRS must be willing to audit, i.e. $(\tau + f)\lambda \geq c$. If $(\tau + f)\lambda = c$, then $\widetilde{x}$ must satisfy the equation

$$\frac{\int_0^{\widetilde{x}} x \mathrm{d}H(x)}{H(\widetilde{x})} = \frac{c}{\tau + f}. \tag{9}$$

Finally, if $(\tau + f)\lambda > c$, the IRS must exhaust its audit budget, i.e. we must have

$$\frac{B}{c} = \int_0^1 \left\{ \pi_i(0) H(\widetilde{x}) + \int_{\widetilde{x}}^{\bar{x}} \pi_i(y_i(x)) \mathrm{d}H(x) \right\} \mathrm{d}i,$$

which yields (3).

Finally, let H be arbitrary, and let M denote its set of masspoints. The previous proof then holds verbatim, except that when $\widetilde{x}$ is a masspoint, the definitions of (2) and (3) need to be adjusted. This is because type $\widetilde{x}$ may randomize between the reports 0 and $\widetilde{x} - \lambda$, and this randomization affects both the IRS's expected revenue from auditing the report 0, and its audit expenses. If $\widetilde{x} \in M$, let $\rho(\widetilde{x}) = H(\widetilde{x}) - \lim_{x \uparrow \widetilde{x}} H(x)$ denote the size of the masspoint. Also let η denote the probability with which $\widetilde{x}$ selects the report 0. Then we have

$$\lambda(\widetilde{x}, \eta) = \frac{\int_0^{\widetilde{x}} x \mathrm{d}H(x) - (1-\eta)\widetilde{x}\rho(\widetilde{x})}{H(\widetilde{x}) - (1-\eta)\rho(\widetilde{x})} \tag{10}$$

Also, (3) becomes $E(\widetilde{x}, \eta) = B$, where

$$\frac{E(\widetilde{x}, \eta)}{c} \equiv \pi_i(0)(H(\widetilde{x}) - (1-\eta)\rho(\widetilde{x})) + \frac{\tau}{\tau + f} \left\{ \int_{\widetilde{x}}^{\bar{x}} \left(1 - e^{-\frac{\bar{x} - x}{\lambda(\widetilde{x}, \eta) + \alpha}}\right) \mathrm{d}H(x) + (1-\eta)\rho(\widetilde{x})(1 - e^{-\frac{\bar{x} - \widetilde{x}}{\lambda(\widetilde{x}, \eta) + \alpha}}) \right\}. \tag{11}$$

Q.E.D.

***Proof of Theorem 1*:** Let us start by assuming that H is continuous.

We first argue that in any equilibrium in which the IRS does not exhaust its budget, the cutoff $\widetilde{x}$ is uniquely determined. From Proposition 4, $\widetilde{x}$ must then satisfy (9). Because the l.h.s. of this equation is strictly increasing in $\widetilde{x}$, its solution $\widetilde{x}_c$ is unique. Furthermore, by assumption 1 $\widetilde{x}_c < \bar{x}$.

Next, consider any equilibrium in which the IRS exhausts its budget. From Proposition 4, we must have $\lambda(\widetilde{x}) \geq c/(\tau+f)$. Since λ is strictly increasing in $\widetilde{x}$, we have $\widetilde{x} \geq \widetilde{x}_c$. We shall establish that over the range $[\widetilde{x}_c, \bar{x}]$ the IRS's aggregate audit expense,

$$E(\widetilde{x}) = c\left\{\pi_i(0)\, H(\widetilde{x}) + \frac{\tau}{\tau+f}\int_{\widetilde{x}}^{\bar{x}}\left(1 - e^{-\frac{\bar{x}-x}{\lambda+\alpha}}\right)\mathrm{d}H(x)\right\},$$

is a strictly decreasing function of $\widetilde{x}$.

This implies that with every given level of B there is associated a unique equilibrium value of $\widetilde{x}$, and hence a unique D1 equilibrium. To see why, note that $E(\bar{x}) = \bar{B} < E(\widetilde{x}_c)$. If $B \leq \bar{B}$, there therefore does not exist any equilibrium covered by Proposition 4. In this case, the equilibrium is necessarily a riot equilibrium. If $B \in (\bar{B}, E(\widetilde{x}_c))$, we necessarily have $B = E(\widetilde{x})$, and hence $\widetilde{x}$ is uniquely determined. If $B \geq E(\widetilde{x}_c)$, then there exists an equilibrium in which $\widetilde{x} = \widetilde{x}_c$. There cannot be any equilibrium in which the IRS does exhaust its budget and $\widetilde{x} > \widetilde{x}_c$, for this would imply $E(\widetilde{x}) < E(\widetilde{x}_c) \leq B$.

It remains to show that $E(\cdot)$ is strictly decreasing on $[\widetilde{x}_c, \bar{x}]$. It follows from Proposition 4 that $E(\widetilde{x}) = \frac{\tau c}{\tau+f}(1 - h(\widetilde{x}))$, where

$$h(\widetilde{x}) = \frac{\lambda+\alpha}{\widetilde{x}+\alpha}H(\widetilde{x})e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}} + \int_{\widetilde{x}}^{\bar{x}} e^{-\frac{\bar{x}-x}{\lambda+\alpha}}\mathrm{d}H(x)$$

Observe that $h(\widetilde{x}) = k(\widetilde{x}, \lambda(\widetilde{x}))$, where

$$k(\widetilde{x}, \lambda) = \frac{e^{-\frac{\bar{x}-x}{\lambda+\alpha}}}{\widetilde{x}+\alpha}\left(\alpha H(\widetilde{x}) + \int_0^{\widetilde{x}} x\mathrm{d}H(x)\right) + \int_{\widetilde{x}}^{\bar{x}} e^{-\frac{\bar{x}-x}{\lambda+\alpha}}\mathrm{d}H(x) \tag{12}$$

$$= \frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{\widetilde{x}+\alpha}\left((\widetilde{x}+\alpha)H(\widetilde{x}) - \int_0^{\widetilde{x}} H(x)\mathrm{d}x\right) + 1 - e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}H(\widetilde{x}) - \frac{1}{\lambda+\alpha}\int_{\widetilde{x}}^{\bar{x}} H(x)e^{-\frac{\bar{x}-x}{\lambda+\alpha}}\mathrm{d}x \tag{13}$$

$$= 1 - \frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{\widetilde{x}+\alpha}\left(\int_0^{\widetilde{x}} H(x)\mathrm{d}x\right) - \frac{1}{\lambda+\alpha}\int_{\widetilde{x}}^{\bar{x}} H(x)e^{-\frac{\bar{x}-x}{\lambda+\alpha}}\mathrm{d}x \tag{14}$$

where (12) follows from (2), and (13) from integration by parts.

It follows from (12) that k is increasing in λ. From (14) we have:

$$\begin{aligned}
\frac{\partial k}{\partial \widetilde{x}} &= -\left(\int_0^{\widetilde{x}} H(x)\mathrm{d}x\right)\left(-\frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{(\widetilde{x}+\alpha)^2} + \frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{\widetilde{x}+\alpha}\frac{1}{\lambda+\alpha}\right) + \frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{\lambda+\alpha}H(\widetilde{x})\left(1-\frac{\lambda+\alpha}{\widetilde{x}+\alpha}\right) \\
&= \left(\frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{\lambda+\alpha}\right)\left\{-\left(\int_0^{\widetilde{x}} H(x)\mathrm{d}x\right)\frac{1}{\widetilde{x}+\alpha}\left(1-\frac{\lambda+\alpha}{\widetilde{x}+\alpha}\right) + H(\widetilde{x})\left(1-\frac{\lambda+\alpha}{\widetilde{x}+\alpha}\right)\right\} \\
&= \left(\frac{e^{-\frac{\bar{x}-\widetilde{x}}{\lambda+\alpha}}}{(\lambda+\alpha)(\widetilde{x}+\alpha)}\right)\left(1-\frac{\lambda+\alpha}{\widetilde{x}+\alpha}\right)\left\{-\left(\int_0^{\widetilde{x}} H(x)\mathrm{d}x\right) + (\widetilde{x}+\alpha)H(\widetilde{x})\right\} > 0
\end{aligned}$$

showing that k is also increasing in $\widetilde{x}$, and hence that h is strictly increasing. We conclude that E is strictly decreasing.

Now consider any arbitrary H. Note that the function $\lambda(\widetilde{x}, \eta)$ is strictly increasing in η, and $E(\widetilde{x}, \eta)$ is strictly increasing in η, for any $\widetilde{x} \in M$. Define the correspondences $\widehat{\lambda}$ and $\widehat{E}$:

$$\begin{aligned}
\widehat{\lambda}(\widetilde{x}) &= \{\lambda(\widetilde{x}, \eta) : \eta \in [0,1]\} \\
\widehat{E}(\widetilde{x}) &= \{E(\widetilde{x}, \eta) : \eta \in [0,1]\}
\end{aligned}$$

Note that $\widehat{\lambda}$ and $\widehat{E}$ are convex-valued. Furthermore, since they coincide with λ and E for all $x \notin M$, they are monotone correspondences. Hence we may define x_c as the unique solution to $c \in \widehat{\lambda}(\widetilde{x})$. Furthermore, if $x_c \in M$, there exists a unique value η_c such that $\lambda(x_c, \eta_c) = c$. It follows from the monotonicity and convex-valuedness of $\widehat{E}$ that for every $B < E(x_c, \eta_c)$ there exists a unique value of $\widetilde{x}$ such that $B \in \widehat{E}(\widetilde{x})$, and if $\widetilde{x} \in M$, a unique value of η such that $E(\widetilde{x}, \eta) = B$, demonstrating uniqueness for the general case. *Q.E.D.*

Proof of Proposition 5 : The proof of Theorem 4 applies, regardless of whether or not there is a masspoint at $\bar{x}$. The proof of Theorem 1 also applies, except that when $\widetilde{x} = \bar{x}$, the function $E(\bar{x}, \eta) = \bar{B}$ for all $\eta \in [0,1]$. To see this, note that at $\widetilde{x} = \bar{x}$ (11) becomes

$$E(\bar{x}, \eta) = c\pi_i(0)(1-(1-\eta)\rho(\bar{x}))$$

Substituting (10) into $\pi_i(0)$ yields

$$\begin{aligned}
\pi_i(0) &= \frac{\tau\left[(1-(1-\eta)\rho(\bar{x}))\,\bar{x} - \left(\int_0^{\bar{x}} x\mathrm{d}H(x) - (1-\eta)\rho(\bar{x})\bar{x}\right)\right]}{((\tau+f)\,\bar{x}+F)\,(1-(1-\eta)\rho(\bar{x}))} \\
&= \frac{\tau(\bar{x} - \int_0^{\bar{x}} x\mathrm{d}H(x))}{((\tau+f)\,\bar{x}+F)\,(1-(1-\eta)\rho)}.
\end{aligned}$$

It follows that

$$E(\bar{x}, \eta) = \frac{\tau c(\bar{x} - \int_0^{\bar{x}} x\mathrm{d}H(x))}{(\tau+f)\,\bar{x}+F} = \bar{B}.$$

We conclude that for $B \neq \bar{B}$, there is a unique equilibrium, and that at $B = \bar{B}$ type $\bar{x}$ can select any $\eta \in [0,1]$. *Q.E.D.*

Proof of Proposition 6 : If every taxpayer but i reports the truth, then the reported income distribution will be H, and hence taxpayer i will face audit probabilities $\pi_i(\cdot|H) = \pi_i^*(\cdot)$. By reporting $y(i)$ taxpayer i therefore would receive expected utility $u(y(i), \pi_i^*(y(i))|x(i)) \leq u(x(i), \pi_i^*(x(i))|x(i))$. Hence reporting $x(i)$ is optimal for taxpayer i.

At the same time, if all taxpayers report truthfully, the expected revenue from auditing any tax report of any taxpayer equals zero, so the IRS is indifferent about how to select audit probabilities. Since $\int_0^1 \pi_i^*(x(i))\mathrm{d}i \leq K^*$, selecting $\pi_i(\cdot|H) = \pi_i^*(\cdot)$ is both feasible and optimal. *Q.E.D.*

Proof of Theorem 2 : Let $L(x) = 1$ for all $x \in [0, \bar{x}]$. For any $G \neq L, H$ let $\pi_i(0|G) = 0$. Also let $\pi_i(0|L) = 1$ for $i \in [0, K^*]$ and $\pi_i(0|L) = 0$ for $i \in (K^*, 1]$. Then for any $G \neq L, H$, any $i \in [0,1]$, any $x \in X$, and any $m \neq 0$ we have $u_i(0, \pi_i(0|G)|x) = x > u_i(m, \pi_i(m|G)|x)$, so G cannot be an equilibrium reported income distribution. Furthermore, if $G = L$, then for $i \in [0, K^*]$ we have $u_i(0, \pi_i(0|L)|x) < u_i(x, \pi_i(x|L)|x)$, so L cannot be an equilibrium reported income distribution.

If $\pi_i(0|L)$ must be symmetric, then let $\pi_i(x'|L) = 0$ for all i. Also let $\pi_i(0|L) = K^*$. It follows that $u_i(x', \pi_i(x'|L)|\bar{x}) = \bar{x} - \tau x' > \bar{x} - (\tau + f)\bar{x}K^* > u_i(0, \pi_i(0|L)|\bar{x})$. It follows that a positive measure of types can profitably deviate to reporting x', showing again that L cannot be an equilibrium reported income distribution. *Q.E.D.*

Proof of Proposition 7 : First, consider any critical equilibrium of the delegation game, inducing a reported income distribution G. Denote the symmetric strategies of the delegation game by $(g^D(y), \pi^D(y|G))$. We will prove that in any symmetric credible direct mechanism, it is an equilibrium for taxpayers to use the reporting strategy g^D. Note that if taxpayers do so, the reported income distribution in the mechanism will equal G.

Because the equilibrium of the delegation game is critical, there are at most two equilibrium reports, $y = 0$ and possibly some $y^* > 0$. Since the IRS strictly prefers auditing 0 to y^*, we must have $\pi^D(0|G) = K^*/G(0)$ and $\pi^D(y^*|G) = 0$. Credibility then requires that in any symmetric mechanism we have $\pi^c(0|G) = K^*/G(0)$ and $\pi^c(y^*|G) = 0$. Next, we show that no type of any taxpayer can gain by deviating to an off equilibrium report.

Consider first the case where 0 is the only equilibrium report. We then claim that $\bar{y} \in X$ implies $(\bar{y}, 0) \precsim_{\bar{x}} (0, \pi(0|G))$. Indeed, if we had $\bar{y} \in X$ then by part (ii) of Definition 1 we would have $\pi^D(\bar{y}) = 0$. But then $(\bar{y}, 0) \succ_{\bar{x}} (0, \pi(0|G))$ and $\pi(0|G) = \pi^D(0|G)$ would imply that type $\bar{x}$ of any taxpayer could profitably deviate in the delegation game, contradicting that $(g^D(y), \pi^D(y|G))$ is a critical equilibrium.

Now consider any off equilibrium report $y > 0$ s.t. $y \in X$. If y is such that $(y, 0) \precsim_{\bar{x}} (0, \pi(0|G))$, then regardless of the value of $\pi(y|G)$, no type of any taxpayer can gain by deviating to y. If y is such that $(y, 0) \succ_{\bar{x}} (0, \pi(0|G))$, then criterion D1 implies that the beliefs following any report $y > 0$ of any taxpayer i are $\mu(\bar{x}|y) = 1$. Because we cannot have $y = \bar{y}$, it follows that $\pi(y|G) = \pi^D(y|G) = 1$. Consequently, no type of any taxpayer can gain by any deviation.

Next, consider the case where 0 and $y^* > 0$ are both equilibrium reports. If $y > y^*$ then since $\pi(y^*|G) = 0$, no type of any taxpayer can gain by deviating to y, regardless of the value of $\pi(y|G)$. If $y \in (0, y^*)$, then since $\pi^D(y^*|G) = 0$ and $y < y^*$, for y^* to be an equilibrium report requires that we must have $\pi^D(y|G) > 0$. Part (ii) of Definition 1 then implies that the IRS must strictly prefer auditing y. Hence, we must have $\pi(0|G) = 1$. Consequently, no type of any taxpayer can gain by any deviation.

Next, suppose the delegation game at K^* has no critical equilibria. For any symmetric credible direct mechanism $\pi(\cdot|\cdot)$, let $\Gamma(\pi)$ denote the set of reported income distributions that are induced by some nontruthful equilibrium to the reporting stage. We will construct a symmetric credible direct mechanism $\pi^c(\cdot|\cdot)$ such that $\Gamma(\pi^c) = \phi$. The construction will modify an arbitrary symmetric credible direct mechanism $\pi(\cdot|\cdot)$ with $\pi_i(\cdot|H) = \pi_i^*(\cdot)$.

Consider any distribution G. If $G \notin \Gamma(\pi)$, then let $\pi^c(\cdot|G) = \pi(\cdot|G)$. Thus $G \notin \Gamma(\pi^c)$. If $G \in \Gamma(\pi)$, then we will design a symmetric credible $\pi^c(\cdot|G)$ such that some type of some taxpayer wants to deviate from her equilibrium strategy. Hence, $G \notin \Gamma(\pi^c)$. Therefore, we will be able to conclude that $\Gamma(\pi^c) = \phi$.

For any $G \in \Gamma(\pi)$, let $g(\cdot, \cdot)$ be the equilibrium strategy profile that induces G and supports the D1 belief that makes $\pi(\cdot|G)$ credible. Then $(g(\cdot, \cdot), \pi(\cdot|G))$ must be a D1 equilibrium for the delegation game.

First, suppose that in this equilibrium the IRS cannot make any profit from auditing an equilibrium report. If 0 is an off equilibrium report for some taxpayer i, then the proof of Lemma 4 implies $\mu_i(0|0) = 1$. Therefore, regardless of whether 0 is an equilibrium report, the IRS cannot make any profit from auditing a report 0 from any taxpayer. Hence, we can design π^c such that $\pi^c(y|G) = \pi(y|G)$ for all $y \neq 0$ and $\pi^c(0|G) = 0$. All taxpayers would then deviate to reporting 0, so $G \notin \Gamma(\pi^c)$.

Second, suppose that the IRS makes positive profit from auditing some equilibrium report y' from some taxpayer i. We claim that 0 must then be an equilibrium report for all taxpayers and $\pi(0|G) \in (0,1)$. First, suppose contrary to the claim that there exists a taxpayer j that never reports 0. From the argument in Lemma 4, D1 requires that $\mu_j(0|0) = 1$. Hence, the IRS cannot make any revenue from auditing a report 0 of taxpayer j. Since π is a symmetric equilibrium of the delegation game, we must have $y' \neq 0$. IRS optimization then requires that either $\pi(0|G) = 0$ or $\pi(y'|G) = 1$. If $\pi(0|G) = 0$, then all types of taxpayer will report 0, yielding a contradiction. If $\pi(y'|G) = 1$, then all types of taxpayer i except type y' will strictly prefer telling the truth to reporting y', which contradicts that the IRS makes positive revenue from auditing y' of taxpayer i. Next suppose contrary to the claim that $\pi(0|G) = 0$. Then all types of all taxpayers will report 0. By Assumption 1, the IRS should select $\pi(0|G) > 0$, a contradiction. Finally, suppose that contrary to the claim we had $\pi(0|G) = 1$. Then the IRS makes 0 revenue from auditing 0. Since the IRS makes positive revenue from auditing y' from taxpayer i, we must have $\pi(y'|G) = 1$, which again yields a contradiction.

We consider three possible cases. First, suppose that in addition to the equilibrium report 0, there are two additional equilibrium reports, y^* and y', with $0 < y^* < y'$. Note that because the IRS's equilibrium expected revenue from auditing is positive, Proposition 2 still holds. Thus equilibrium requires that $\pi(0|G) > \pi(y^*|G) > \pi(y'|G) \geq 0$. Hence, $\pi(y^*) \in (0,1)$. From the above claim we also have $\pi(0|G) \in (0,1)$. Hence, the IRS must be indifferent between auditing 0 and y^*. We can then design $\pi^c(\cdot|G)$ such that $\pi^c(y|G) = \pi(y|G)$ for all $y \notin \{0, y^*\}$, and such that the IRS audits y^* before auditing 0. Hence, we have $\pi^c(0|G) < \pi^c(y^*|G)$. Any type that reported y^* in the equilibrium of the delegation game will then be strictly better off reporting 0 in the reporting stage of the mechanism π^c. Thus $G \notin \Gamma(\pi^c)$.

Suppose next that there are two equilibrium reports, 0 and y^*. Since the delegation game has no critical equilibrium, either (i) the IRS must be indifferent between auditing 0 and y^*, or (ii) the IRS strictly prefers auditing 0 to y^*, and there exists an off equilibrium report y', and a D1 belief s.t. the IRS is indifferent between auditing 0 and y', such that $\pi(y'|G) > 0$. If (i) holds, then we can repeat the argument from the previous paragraph, and construct $\pi^c(\cdot|G)$ such that $G \notin \Gamma(\pi^c)$. If (ii) holds, then we must have $y' < y^*$. Otherwise, since $\pi(y^*|G) = 0$, we could set $\pi(y|G) = 0$ for $y > y^*$, and obtain a critical equilibrium. But if $y' < y^*$ we can select $\pi^c(\cdot|G)$ such that $\pi^c(y|G) = \pi(y|G)$ for all $y \neq y'$ and $\pi^c(y'|G) = 0$. Then reporting y^* is dominated by reporting y' for all taxpayers. Hence, $G \notin \Gamma(\pi^c)$.

Finally suppose the only equilibrium report under $(g, \pi(\cdot|G))$ is 0. Since the delegation game has no critical equilibrium, $\bar{y}$ must be an off equilibrium report, and $(\bar{y}, 0) \succ_{\bar{x}} (0, \pi(0|G))$. We may then select $\pi^c(\cdot|G)$ such that $\pi^c(y|G) = \pi(y|G)$ for all $y \neq \bar{y}$ and $\pi^c(\bar{y}|G) = 0$. Then type $\bar{x}$ can gain by deviating to reporting $\bar{y}$. Therefore, $G \notin \Gamma(\pi^c)$. *Q.E.D.*

***Proof of Proposition 9* :** Consider the equilibria of the delegation game in which the IRS has audit capacity K. The equilibria of this model coincide with the equilibria characterized in Proposition 4, when we set $B = cK$ and allow c to converge to zero. Let

$$\bar{K} = \frac{\tau\left(\bar{x} - \int_0^{\bar{x}} x \mathrm{d}H(x)\right)}{(\tau + f)\bar{x} + F}$$

It follows from the proofs of Theorem 1 and Proposition 5 that with each $\widetilde{x} \in [0, \bar{x})$ there is associated a unique $K > \bar{K}$, and that $\widetilde{x} = \bar{x}$ implies $K \leq \bar{K}$. Since truthtelling requires $\widetilde{x} = 0$, and since rioting requires $\widetilde{x} = \bar{x}$, we conclude that $K^* > \bar{K}$, so the delegation game with $K = K^*$ does not have a riot equilibrium. The result then follows from Proposition 7. *Q.E.D.*

Figure 1: “The Congestion Effect in Tax Compliance and Enforcement” (Deneckere and Liang)

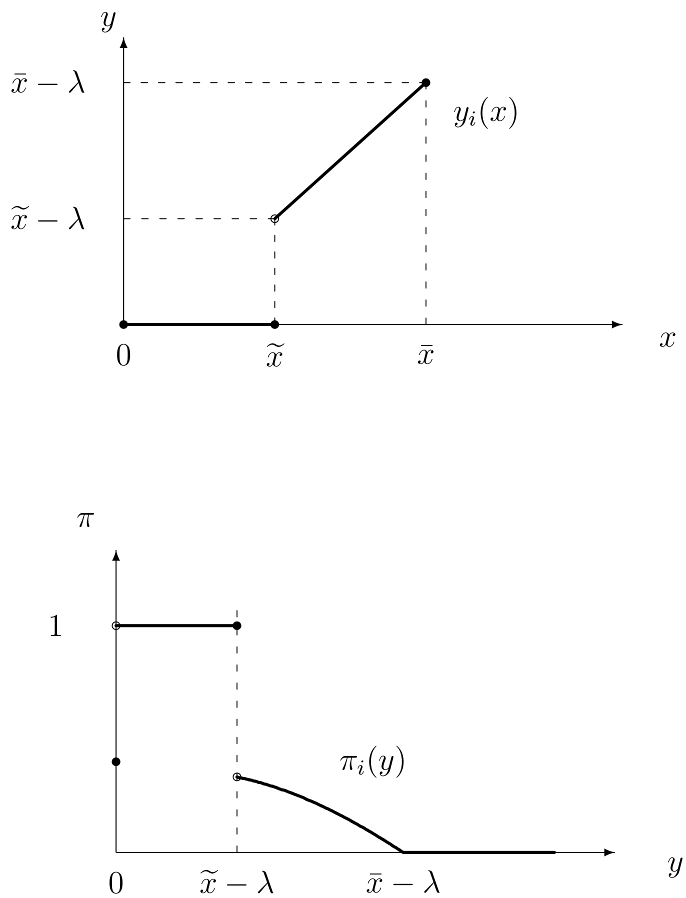

Figure 1: Taxpayer reporting strategy as a function of income, and probability of audit as a function of reported income.

Figure 2: “The Congestion Effect in Tax Compliance and Enforcement” (Deneckere and Liang)

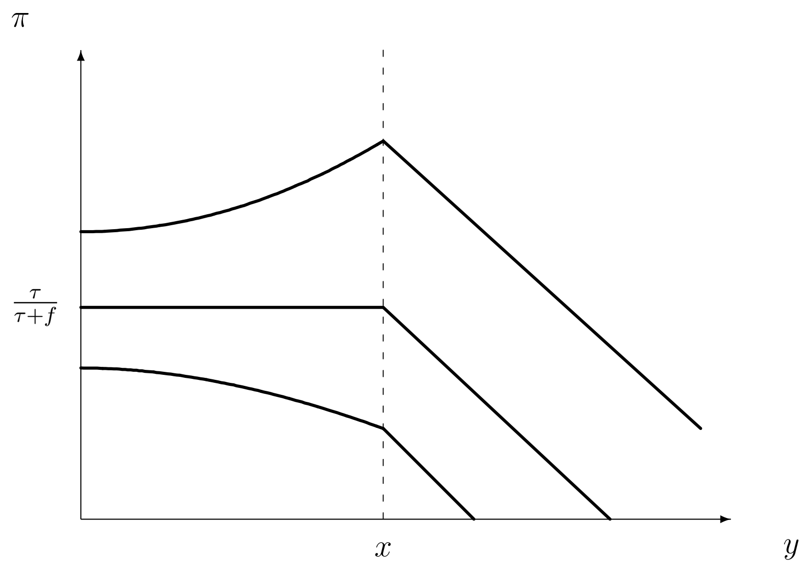

Figure 2: Shape of taxpayer i's indifference curves in (y, π) space

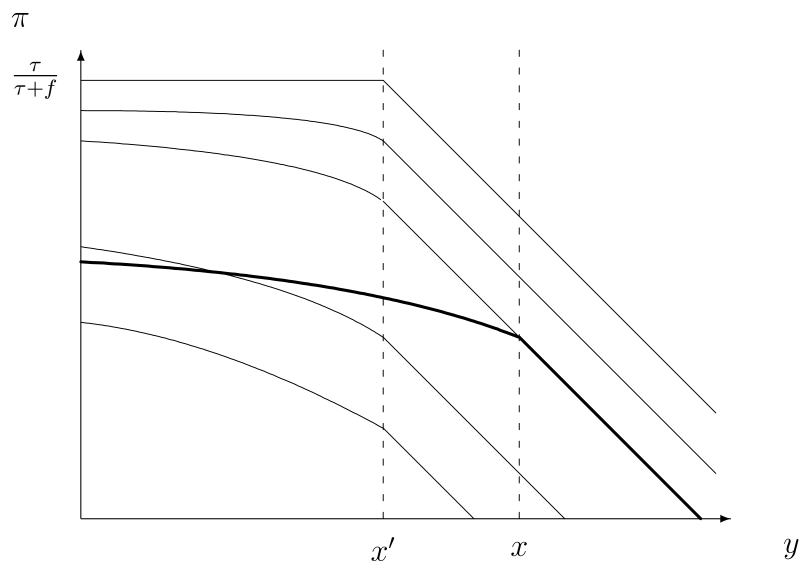

Figure 3: Crossing of indifference curves of types x and x' of taxpayer i, when $x > x'$ and $\pi_i(0) < \frac{\tau}{\tau+f}$

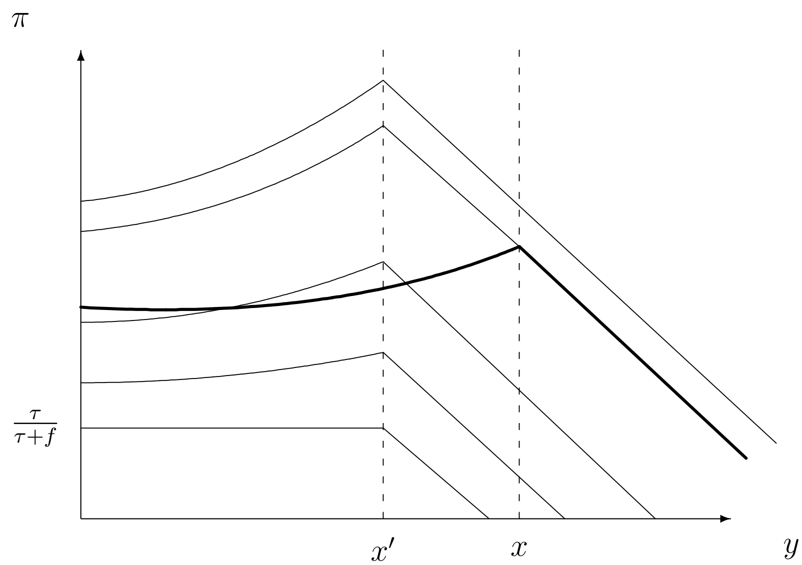

Figure 4: Crossing of indifference curves of types x and x' of taxpayer i, when $x > x'$ and $\pi_i(0) > \frac{\tau}{\tau+f}$