

Multiproduct firms and the endogenous choice of variety

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Abstract

In this paper I show that managerial talent endowments can be a source of selection effects: when the talent endowment becomes larger, productive firms expand their product portfolio and put more pressure on less productive firms. The productivity cut-off is therefore higher. In open economies with labor mobility, we have an integrated world and free trade enhances the welfare of the trading countries. When labor cannot move, however, firms in the country with more talent will have a greater scope but a smaller intensive margin than those firms with the same productivity level in another country. The paper calls for governments to improve both the quantity and the quality of talent in their countries in order to enhance welfare.

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1 Introduction

The emergence of multiproduct firms has been well documented both in advanced and in developing countries. Bernard, Redding and Schott (2010) found that 39 percent of American firms produce more than one product; these accounted for 87 percent of output in 1997. In Brazil, according to Arkolakis and Muendler (2009), 25 percent of exporters ship more than 10 products at the HS-6 digit level; they accounted for 75 percent of total exports in 2000.

This fact explains the recent and growing literature about multiproduct firms. This literature focuses mostly on the consequences of competition effects: the degree of competition intensity determines the number of firms and their scope. On the demand side, bigger markets (less competitive) host more firms, each produces more products. On the production side, an influx of foreign firms, thanks to international trade, induces more competition causing more firms to exit the market and making the surviving firms reduce their scope. In general, these competition effects depend on whether the firms are identical in their productivity or not. Feenstra and Ma (2008) show that in the former case, an increase in market size leads to more producing firms, whereas in the latter case, their number is independent of the market size. Allanson and Montagna (2005) use a nested CES preference utility function to prove that in the long run, a firm's scope is independent of the market size. Using the core-product approach Mayer, Melitz and Ottaviano (2009) claim, however, that firms respond to more intense competition by focusing on a few products and discontinuing the ones that are far from their core product.

In this paper I take a rather different approach. The source of the competition effect does not come from the size of the market or from trade opening, but from the multiproduct feature of the model. Managing multiple brands is deemed to be costly because it requires organizational and managerial skills (Nocke and Yeaple 2006). When this management cost is lower though, the more productive firms can expand their scope by introducing new varieties. This creates two effects: on the production side, new varieties absorb input resources, leaving the unproductive firms with less input for their production; on the demand side, the introduction of the new varieties reduces the market

share of existing ones varieties (the cannibalization effect). Both effects result in the unproductive firms producing and selling less, even to the extent that some of them do not make enough profit to cover their fixed costs.

The cost of introducing varieties can take many forms, as explained in the literature. It can be in the form of the cannibalization effect: when a firm introduces new varieties that drive down demand for existing varieties, including those of the firm itself (Feenstra and Ma 2008). Another form is flexible manufacturing or core competence, which is introduced in Eckel and Neary (2010), and Mayer, Melitz and Ottaviano (2009): every firm has a product that they can produce the best, and the farther a product is from this core product, the higher the marginal cost it incurs.

Based on the finding of Schoar (2002) that firms see their total factor productivity diminish when introducing new varieties, one can say that the more varieties a firm has, the more difficult they are to manage, which raises costs (or lowers productivity). Nocke and Yeaple (2006) assume that the marginal cost of a firm increases with its scope. The implication of their assumption is that bigger firms, when everything else is controlled, are likely to be less productive, which is inconsistent with many empirical findings. For example Lee and Tang (2001) report that in Canada firms with more than 500 employees and firms with between 100 and 500 employees are 17 percent and 15 percent respectively more productive than firms with less than 100 employees. The results they found regarding the U.S. were similar. Other evidence can be found in Van Ark and Monnikhof (1996) and Baldwin, Jarmin and Tang (2002). This paper takes a different approach, assuming that the marginal costs of a firm are independent of its scope. However, a firm has to pay a management cost that increases with the number of varieties it develops. Unlike in Arkolakis and Muendler (2009), in which this cost increases linearly with the number of brands, I assume a diminishing marginal return; this allows me to create the competition effect I discuss above.

In my model a firm needs to hire talent to work in its headquarters to manage its product lines. With the diminishing marginal return of talent, we have a similar implication as in the flexible manufacturing literature: it is better, at least for the unproductive firms, to focus on a few products that they can produce best. In relation to this, each country too, is endowed with a number of talented people. When total talent increases, due to education or

immigration policies, management costs are lowered, thus favoring the more productive firms. Indeed, they can expand their scope. The market also becomes more competitive, the productivity of marginal firms rises, and the number of incumbent firms falls. As a result, the price index falls and social welfare is enhanced. This gain depends on the degree of firm heterogeneity and the managerial technology of the country. When firms are heterogeneous, new varieties have a large cost-advantage over the other varieties, resulting in a significant fall in the price index. In addition, when firms can manage their brands efficiently (for instance through more skilled managers), there will be more new varieties introduced, making consumers better off via their love of variety. A country with advanced managerial skills or high firm heterogeneity can benefit from education or immigration policies that improve the quantity or quality of the national talent endowment.

When two countries, identical in everything but their talent endowments, decide to engage in trade, the productive firms in those countries have the opportunity to sell abroad. This enables them to expand their production, both the intensive and extensive margins. They therefore use more labor and talent, which puts pressure on the unproductive firms. When labor is allowed to move between countries, for example with regional integration, the firms in the country with more talent that would have to exit under autarky now compete with the less productive firms in the other country for the same pool of workers (note that when under autarky, the country with more talent has a higher productivity cutoff for survival). As a result, the cutoff in the former country is no longer higher than that of the other country: they are in fact equal. When labor is not allowed to move, however, the unproductive firms in the country with more talent are not competing with the unproductive firms in the other country, and in fact they are in competition for labor and talent with the more productive firms in their country. The result is that the country with more talent has a higher cutoff than the other country, as in autarky.

With labor mobility, the two countries form an integrated economy with factor price equalization. Firms in both countries with the same productivity level will have identical intensive and extensive margins. Consumers are better off thanks to a higher number of varieties and a lower price index. With labor immobility, firms in the country with more talent will have a greater scope but lower sales per brand. Consumers in the country with less

talent can be worse off if their wages fall more than the price index does.

Besides opening a new channel of competition as discussed above, this paper also provides a picture of the extensive margin of the firms. In the models with single product firms, when there are less incumbent firms due to more competition, we have a loss of varieties. However, in my model, when there is more competition, productive firms expand their product portfolio. As a result, knowing the number of products per firm can help to correct any bias when quantifying the variety gain. Moreover, productivity measurement can be biased in the case of single product firms (De Loecker 2010). Knowing how the firm chooses its scope is one way to correct this bias (Bernard, Redding and Schott 2009).

The paper is organized as follows: in the next section, I will discuss the case of a closed economy; the case of an open economy will be considered in Section 3; while Section 4 has the conclusion.

2 Closed economy

I assume there are L workers and S managers in an economy. They have different skills that are used in different positions, as will be clarified later. The markets for workers and managers are competitive. All the workers and managers are, however, consumers with the same preferences. There are an infinite number of potential firms in a single industry. The firms compete in a monopolistic market. Each of them, if they decide to produce, can choose how many products and how much of each product they want to produce and sell.

2.1 Consumer's preferences and demand

Our consumers' preferences follow the Dixit-Stiglitz framework. There is a certain number M of producing firms, which has to be determined endogenously, in the economy, each of which is associated with a certain productivity level. A firm with productivity level θ decides to produce a certain number of products $n(\theta)$. The utility of a representative consumer is given by:

$$U = \left(\int c(\theta)^\rho Mn(\theta) f(\theta) d\theta \right)^{\frac{1}{\rho}} \quad 0 < \rho < 1.$$

The function $f(\theta)$ is the probability density function of the firms' productivity. All of the brands within a firm are identical; therefore, all will have the same price. All the varieties in the economy are symmetric to the consumers. Assume that a representative consumer has disposable income E . His consumption equation will be:

$$\max_{\{c(\theta)\}} U \text{ s.t. } \int p(\theta) q(\theta) Mn(\theta) f(\theta) d\theta = E,$$

This provides the following:

$$q(\theta) = \left(\frac{p(\theta)}{P} \right)^{\frac{-1}{1-\rho}} \frac{E}{P}$$

where P is considered as the price index in the industry:

$$P = \left(\int p(\theta)^{\frac{-\rho}{1-\rho}} Mn(\theta) f(\theta) d\theta \right)^{-\frac{1-\rho}{\rho}}. \quad (1)$$

Since all the consumers have the same preferences, total demand for each variety is given by

$$q(\theta) = \left(\frac{p(\theta)}{P} \right)^{\frac{-1}{1-\rho}} \frac{R}{P}, \quad (2)$$

where R is the total revenue (or total spending) in the industry.

2.2 Production of multiproduct firms

The game for the producers is now detailed. To enter the industry each firm has to pay a fixed entry cost f_e in units of unskilled labor, which is used

as the numeraire. Denote M_1 the number of firms that pay that entry cost. Those firms then draw their productivity from an exponential distribution with the following probability distribution function:

$$f(\theta) = \gamma e^{-\gamma\theta}.$$

The lower γ is, the more heterogeneous the firms are in their productivity level.

After discovering its productivity level, a firm decides whether to stay or exit the industry. If it stays, it has to pay a fixed cost F . Let M_2 denote the number of firms that stay in the industry. These producing firms decide how many products they want to develop, and how much of each they want to produce. I assume that managing the brands requires skilled labor, which I call talent. Production is assumed to be simpler and can be done by unskilled workers¹ Managing is assumed to be characterized by the diminishing marginal return. More specifically, the number of managers required for a given number of brands is

$$F_m(n) = n^m, m > 1.$$

The parameter m represents the managerial "technology" in this country. A low m indicates the firms are more efficient in managerial use. It also indicates the quality of the managers. The more skilled they are, the lower m is.

Together with choosing the number of brands to develop, the firm decides how much to produce of each brand. The unit cost of production, in units of unskilled labor, depends on its productivity draw:

$$c(\theta) = e^{-\theta}.$$

Hence the problem of choosing the quantity for each brand is as follows:

¹This management cost can be considered as a type of fixed cost. Here I divide the fixed cost into high level and low level costs. The former consists of administrating and coordinating the many products by the managers or the skilled labor. The latter are the "simpler" activities such as advertising, customer service, etc. that can be done by less skilled labor.

$$\max_p pq - cq.$$

With the demand given from (2), we have the pricing strategy

$$p(\theta) = \frac{c(\theta)}{\rho}, \quad (3)$$

which yields the profit per brand:

$$\pi_1(\theta) = (1 - \rho)P^{\frac{\rho}{1-\rho}} Rc(\theta)^{-\frac{\rho}{1-\rho}}. \quad (4)$$

I want to clarify the notation for the profits of the firms. Each producing firm pays three different types of cost. First, it pays its workers, then it pays its managers, and finally, it pays its fixed costs. I will use the subscripts 1, 2, 3 to denote the profit of the firms after paying each of the different types of cost. The total profit of the industry after paying the variable cost (payment to workers) is given by:

$$\Pi_1 = \int_{\hat{\theta}}^{\infty} M_1 n(\theta) \pi_1(\theta) \gamma e^{-\gamma\theta} d\theta$$

Besides choosing the intensive margin and paying its workers, the firm also has to decide on its optimal scope. If it chooses to develop n brands, it needs to hire n^m managers, each of whom receiving a wage w . Their management cost is then wn^m , which leads to the equation for choosing the optimal scope as:

$$\max_n n\pi_1(\theta) - wn^m$$

This equation yields the following solution:

$$\pi_1(\theta) = wmn^{m-1}(\theta) \quad (5)$$

2.3 Equilibrium

There are S managers in the country, so the total management cost is wS . After net management costs, the total profit in the industry is then:

$$\Pi_2 = \Pi_1 - wS.$$

Finally, each firm has to pay a fixed cost F in units of labor. From (5) we see that the profit of a firm with productivity θ after paying the management is $\frac{m-1}{m}n(\theta)\pi_1(\theta)$.

The marginal firm is defined as a firm that produces one product ($n(\hat{\theta}) = 1$) and whose net profit after management costs is just enough to cover the fixed cost F . Therefore the zero-profit cutoff condition is

$$\frac{m-1}{m}\pi_1(\hat{\theta}) = F. \quad (6)$$

Here $\hat{\theta}$ denotes the cutoff productivity level. The probability that an entrant is active is then $e^{-\gamma\hat{\theta}}$, which yields the relationship between M_1 and M_2 :

$$M_2 = M_1 e^{-\gamma\hat{\theta}}. \quad (7)$$

Since only incumbent firms pay the fixed costs, the total net profit in the industry is given by:

$$\Pi_3 = \Pi_2 - M_2 F.$$

There are an infinite number of entrants who are free to enter the industry if they pay the entry cost, the ex-ante expected profit they would receive after entry has to be at least equal to the entry cost. The expected profit

they receive is $\int_{\hat{\theta}}^{\infty} n(\theta)\pi_3(\theta)\gamma e^{-\gamma\theta} d\theta$ where $\pi_3(\theta)$ is the net profit per brand. We then have the free entry condition:

$$f_e = \int_{\hat{\theta}}^{\infty} n(\theta)\pi_3(\theta)\gamma e^{-\gamma\theta} d\theta. \quad (8)$$

In the monopolistic framework, the firms have a constant markup, as shown in the pricing strategy (3). This feature implies that the variable costs (which are the payment to the workers) are proportional to the total revenue in this industry. As a result, the total net profit of the whole industry after paying variable costs is also proportional to total revenue

$$\Pi_1 = (1 - \rho) R. \quad (9)$$

The management cost can be calculated by replacing the number of products per firm by (5) in the talent market clearing condition:

$$wS = \frac{\Pi_1}{m}. \quad (10)$$

From (8) we can show that the total net profit of the active firms is equal to the entry cost that the new entrants have to pay:

$$\Pi_3 = M_1 \int_{\hat{\theta}}^{\infty} n(\theta)\pi_3(\theta)\gamma e^{-\gamma\theta} d\theta = M_1 f_e$$

Labor in this economy is used to pay for the entry cost, the variable production cost, and the fixed cost. With the entry cost equal to Π_3 and the management cost proportional to total revenue (see equations 9 and 10), we have:

$$R = \frac{L}{1 - \frac{1-\rho}{m}}. \quad (11)$$

Lemma 1 *Total income R is proportional to the number of unskilled workers and is unchanged when the talent endowment increases.*

Then from (10) and (11) I calculate the salaries of the managers:

$$w = \frac{(1 - \rho) R}{mS} = \frac{(1 - \rho) L}{(m - 1 + \rho) S}. \quad (12)$$

An increase in talent endowment implies lower salaries for the managers. Indeed, a larger talent endowment means more managers, therefore making the talent market more competitive, reducing the price of talent. Having more managers also induces more competition in the good market leading to the following result:

Proposition 2 *Higher talent endowment makes the competition among firms more intense: the productivity cut-off level is higher.*

Proof. See Appendix. ■

The intuition is as follows. Higher talent endowment reduces wage for talent, as in (12). As a result, from (5) we see that the productive firms are able to introduce more varieties. This has two effects on the unproductive firms. The first effect is on the demand side where the revenues of the unproductive firms shrink due to the arrival of the new varieties with lower prices. This effect is similar to a decrease in the market size. The difference here is that the consequences are not the same for every firm as the least productive firms take the hardest hit because their price is far less an advantage than the others. From the demand function (2) we see that the extent of the change on the whole economy depends on the degree of firm heterogeneity: when the

firms are more heterogeneous (low γ), the new varieties have a far greater cost advantage over existing varieties, and the impact on the unproductive firms is more severe. The second effect is on the production side, which is similar to what is described in Melitz (2003) when the firms are allowed to export. We can think of an export variety as a new variety introduced by the more productive firms in my model. They now can expand their production, hence using more labor. The unproductive firms as a consequence now have less labor to produce with. The extent of this effect depends on how many new varieties are introduced into the market, which depends on the management technology of the country, as in (5). When the parameter m is low, the firms are efficient in managing their brands, and more varieties will be introduced. The competition in the labor market is also more intense. Both the effects on the demand side and those on the production side have the same consequence: the unproductive firms have fewer sales. Some of them (the least productive ones) will have to exit the market, as they can not pay their fixed costs. As the result, the cutoff productivity level is raised.

From (20) since the cutoff level is higher and the total income is a constant, we can see that the price index is lower. From (7) and (21) we see that after an increase in the number of talent, the number of entrants rises but the number of incumbents falls. This encourages more firms to enter the industry, but the competition is so fierce that only the more productive firms can survive. This result does not depend on the assumption that managers are only employed in the management and the fixed cost F describes unskilled labor. What does matter is that when the talent endowment is larger, there are more people who can manage the products. As I discussed above, this results in more intense competition as the productive firms can increase their number of products.

More competition in the market implies a lower price index (see the proof in the appendix). With the wage taken as the numeraire, the workers benefits from an increase in talent endowment since their purchasing power is higher. The managers, however, may be worse off because their wage is lower than before. Indeed, their welfare depends on how many of them in the market: if there are not many of them then they are better off when there is additional talent; but when there are already many managers in the market, a few more of them means a reduction in their welfare (the proof is given in the appendix). This result is intuitive: if there are many managers in the

market, it becomes more difficult for the firms to recruit the new managers as they bring little profit to the firms. This lowers the compensation of the current managers, making them worse off.

I summarize those results by the following proposition:

Proposition 3 *An increase in the talent endowment lowers the price index, which provides welfare gains for workers. Managers are better off when there is not much talent and are worse off otherwise.*

Proof. See Appendix. ■

3 Open economy

In this section I consider two identical countries that engage in free trade. These two countries will be denoted as Home (H) and Foreign (F). They differ only in their talent endowment: the Home country is assumed to have more talent than the Foreign country. A firm's nationality is defined as the location of its headquarters. This model also assumes that managers cannot move across borders; labor, however, may or may not move across the border². The two possibilities will be discussed below.

3.1 Regional integration with labor mobility

In this first case, labor can move freely across borders. An example of this case is the European Union, in which people are free to work in the country of their choice³. With a competitive labor market, the wages for workers have to be the same across countries. I take this wage level as the numeraire.

²We will see later that the immobility example leads to factor price equalization, therefore the assumption of labor mobility can be made without loss of generality.

³It can be argued that there might be some moving costs that restrict people from relocating their house. However, one can think that the low skill workers, in particular from Eastern Europe, might have low moving costs and are willing to move.

I will use the subscripts H and F to distinguish the variables in the Home and in the Foreign country respectively. However, the subscripts will be dropped if there is no confusion. Due to the trade balance, the total revenues of the firms and the total incomes of the workers and managers in each country have to be equal:

$$R = L + wS$$

Similar to a case when both countries are under autarky, it can be shown that total income of each country is proportional to the number of workers in that country:

$$R = \frac{L}{1 - \frac{1-\rho}{m}},$$

As a result, the wages of managers in each country are given by:

$$w_H = \frac{(1 - \rho) L_H}{(m - 1 + \rho) S_H} \quad (13)$$

$$w_F = \frac{(1 - \rho) L_F}{(m - 1 + \rho) S_F}. \quad (14)$$

Because of labor mobility, the wages of workers are equal across countries. Without loss of generality, assume that the salaries of managers in the Home country are lower than in the Foreign country. Since the wages of workers are the same, the marginal costs of two firms with equal productivity⁴, one in each country, are the same. Therefore they have the same sales, and the same profit per product. As the salaries of managers in the Home country are lower, the Home firm develops more products than the Foreign firm.

This is, however, not a stable equilibrium. In fact, for any firm in the Foreign country, there is at least one firm in the Home country that can use the one additional unit of labor more efficiently. Indeed, with one additional

⁴From now on, I will compare the firms in the Home country with the firms in the Foreign country with the same productivity level. They will be called comparable firms.

unit of labor, the Home firm can increase its output per product by $\frac{e^\theta}{n_H(\theta)}$. Its total product then increases by e^θ . This increase does not depend on where the firm is located; a foreign firm with the same productivity level also increases its product by the same level. The difference appears in the extensive margin, which has an impact on the price the firm charges. Indeed, since the Home firm has more products, its intensive margin increases by less, which leads to a lower fall in its price. As a result, its total sales are higher than the Foreign firm's. Consequently, the Home firm can pay more to one additional unit of labor than the Foreign firm can, which results in a movement of labor from the Foreign country to the Home country. This labor movement stops when we have factor price equalization.

The two countries are identical in all aspects other than their talent endowments. In particular, the consumers in both countries have the same preferences over the same basket of goods, so there is the common price index

$$P = \left[\int_{\hat{\theta}_H} M_{1,H} p^{-\frac{\rho}{1-\rho}}(\theta) n_H(\theta) \gamma e^{-\gamma\theta} d\theta + \int_{\hat{\theta}_F} M_{1,F} p^{-\frac{\rho}{1-\rho}}(\theta) n_F(\theta) \gamma e^{-\gamma\theta} d\theta \right]^{-\frac{\rho}{1-\rho}},$$

where $\hat{\theta}_H$ and $\hat{\theta}_F$ denote the Home and Foreign cutoffs. With this same price index and the same pricing strategy due to the monopolistic competition, two firms with the same productivity level in the Home and Foreign countries will have the same gross profit (after paying the labor production cost) per variety:

$$\pi_{1,H}(\theta) = \pi_{1,F}(\theta).$$

The formula that determines the cut-off level (6) then implies that the cutoffs in both countries are the same. As workers are paid equally, if the

firms have the same productivity level in both countries, they will have the same marginal cost, or the same unit price as in (3). From the demand function (2), for each brand they develop they have the same sales, the same

revenues and the same profit π_1 . Since management costs are equal due to factor price equalization, from the optimal scope condition (5) the firms in the Home country have the same number of brands as the comparable ones in the Foreign country. As a result, we have an integrated economy.

Proposition 4 *In both countries, the price index is lower than in autarky.*

Proof. See Appendix. ■

This lower price index implies that both countries gain from trade (note that the total revenues are proportional to the number of workers). However, the gains are not distributed equally among consumers. The workers in both countries are the clear winners because the price index, in terms of their wages, decreases. The welfare effect on managers is less clear-cut. Talent in the Home country is the winner with the largest gains because not only is the price index lower, their salaries also increase since the relative factor endowment favors them in free trade ($\frac{L}{S_H} < \frac{2L}{S_H+S_F}$) according to equations 12 and 13. Talent in the Foreign country, however, can find themselves disadvantaged compared to in an autarky. Indeed, their salaries are lower since the relative factor endowment does not favor them in free trade ($\frac{L}{S_F} > \frac{2L}{S_H+S_F}$). Therefore, if the decrease in their salaries is more than that of the price index, they are worse-off. This happens when the talent endowments are sufficiently different as this reduces significantly the salaries of Foreign managers; and when the firms are less heterogeneous, which implies the price index does not decrease much. This result is in line with Krugman (1994) when he shows that the scarce factor (which is talent in our case) loses when the two countries are dissimilar in their factor endowment and when the products are less differentiated.

To summarize the above results, we have the following proposition:

Proposition 5 *With labor mobility, the productivity cutoffs in both countries are the same. The country with the higher talent endowment will see an influx of workers from the other country, which imports the management services. The two countries form an integrated world. Almost all consumers are better off, except for the managers in the Foreign country who could be worse off when talent endowments are greatly different and firm heterogeneity is small.*

3.2 Free trade with labor immobility

We have seen in the previous section that where labor is free to move across borders the two countries form an integrated economy where the wages are the same. As a result, the firms in both countries behave the same way: with the same productivity level, they develop the same number of products and sell the same amount of units per product. In this section, labor is not allowed to move. We will see that the production costs, in particular the salaries of workers and managers are different, which implies that the comparable firms will not produce the same amount of goods. First, I will prove the following result:

Lemma 6 *If the firms in one country produce more products than the comparable firms in the other country, they will produce less units per product than the comparable firms in the other country. In other words, if $n_H(\theta) > n_F(\theta)$ then we must have $q_H(\theta) < q_F(\theta)$ and vice versa.*

Proof. Without loss of generality, I assume that the Home firms have higher intensive and extensive margins than the comparable firms in the Foreign country. Since the intensive margin of a Home firm is higher, its marginal cost has to be lower than that of a foreign firm with the same productivity level, which implies that the wage of workers in the Home country is lower.

$$\begin{aligned}
 n_H(\theta) \pi_{3,H}(\theta) &= n_H(\theta) \pi_{2,H}(\theta) - F \\
 &= \frac{m-1}{m} n_H(\theta) \pi_{1,H}(\theta) - F \\
 &> \frac{m-1}{m} n_F(\theta) \pi_{1,F}(\theta) - F w_o \\
 &= n_F(\theta) \pi_{3,F}(\theta).
 \end{aligned}$$

The zero cutoff profit conditions in the Home country and in the Foreign country are

$$\frac{m-1}{m} \pi_{1,H}(\hat{\theta}_H) = F = \frac{m-1}{m} \frac{\pi_{1,F}(\hat{\theta}_F)}{w_o}.$$

We then have $\pi_{1,H}(\widehat{\theta}_H) < \pi_{1,F}(\widehat{\theta}_F)$ because $w_o > 1$. Hence $\pi_{1,H}(\widehat{\theta}_H) < \pi_{1,H}(\widehat{\theta}_F) < \pi_{1,F}(\widehat{\theta}_F)$, which implies $\widehat{\theta}_H < \widehat{\theta}_F$. Because the cutoff is lower, and $n_H(\theta)\pi_{3,H}(\theta) > n_F(\theta)\pi_{3,F}(\theta)$, the average profit in the Home country is higher:

$$\int_{\widehat{\theta}_H}^{\infty} n_H(\theta)\pi_{3,H}(\theta)\gamma e^{-\gamma\theta} d\theta > \int_{\widehat{\theta}_F}^{\infty} n_F(\theta)\pi_{3,F}(\theta)\gamma e^{-\gamma\theta} d\theta.$$

The entry cost in the Home country f_e is, however, lower than in the Foreign country (because the wages are higher in the Foreign country). We thus have a contradiction. ■

Since managers are more numerous in the Home country, it can be shown that the salaries of the Home managers, relative to the wages of the workers, are lower than in the Foreign country. Indeed, with the same argument as in the case of mobile labor (the actual proof can be found in the appendix), the relative salaries for managers in the Home and Foreign countries are given by:

$$w_H = \frac{1 - \rho}{m} \frac{L/S_H}{\frac{m-1}{m}(1 - \rho) + \rho}. \quad (15)$$

$$\frac{w_F}{w_o} = \frac{1 - \rho}{m} \frac{L/S_F}{\frac{m-1}{m}(1 - \rho) + \rho}. \quad (16)$$

These two formulas yield:

$$w_H < \frac{w_F}{w_o}.$$

The lemma below shows that that the wages of workers in the Foreign country have to be lower than in the Home country.

Lemma 7 *The country with more talent has a higher wage for workers.*

Proof. By contradiction, let me assume the inverse. This implies that the marginal cost is higher in the Foreign country, and hence the unit price is higher:

$$p_F(\theta) = \frac{w_o e^{-\theta}}{\rho} > \frac{e^{-\theta}}{\rho} = p_H(\theta).$$

The monopolistic demand (2) implies that the sale per product is therefore higher in the Home country. As a result, Home country firms have more profit per brand than those in the Foreign country:

$$\pi_{1,H}(\theta) > \pi_{1,F}(\theta).$$

The number of brands per firm in the Home country is given by

$$n_H(\theta) = \left(\frac{1}{w_H} \frac{\pi_{1,H}(\theta)}{m} \right)^{\frac{1}{m-1}}. \quad (17)$$

Similarly, the number of brands per firm in the Foreign country is

$$n^F(\theta) = \left(\frac{w_o^F}{w^F} \frac{\pi_1^F(\theta)}{m} \right)^{\frac{1}{m-1}}. \quad (18)$$

Since $\pi_{1,H}(\theta) > \pi_{1,F}(\theta)$ and $\frac{1}{w_H} > \frac{w_o}{w^F}$ then we have $n_H(\theta) > n^F(\theta)$. This is a contradiction, according to lemma 6, because the Home firms can not develop more products and sell more units per product than the comparable firms in the Foreign country. ■

The two lemmas above lead us to the following result:

Proposition 8 *The firms in the country with more talent develop more products but sell less units per product.*

Proof. From lemma 7, the pricing rule (3) and the demand function (2), it

is clear that the Home firms produce less per product than the comparable firms in the Foreign country. From (15),(16),(17) and (18) we have

$$\begin{aligned} \left(\frac{n_H(\theta)}{n_F(\theta)} \right)^{m-1} &= \frac{\frac{1}{w_H} \pi_{1,H}(\theta)}{\frac{w_o}{w_F} \pi_{1,F}(\theta)} \\ &= \frac{S_H}{S_F} (w_o)^{\frac{\rho}{1-\rho}}. \end{aligned}$$

The ratio of the extensive margin of two comparable firms in the Home and in the Foreign country does not depend on their productivity. Therefore, either the Home firms have higher extensive margins, or the Foreign firms have higher extensive margins. The second scenario is ruled out by Lemma 7. Therefore the Home firms have higher extensive margins than the Foreign firms. ■

Having a larger talent endowment makes the Home firms focus on developing more products and also creates a more competitive environment where only the good firms can survive as we see in the following proposition.

Proposition 9 *In the country with a larger talent endowment, the competition is fiercer, represented by a higher productivity cut-off.*

Proof. Since the wages of workers in the Home country are higher, it is clear that $\pi_{1,F}(\theta) > \pi_{1,H}(\theta)$. From (6) we have

$$\frac{m-1}{m} \pi_{1,F}(\hat{\theta}_F) = w_o F < F = \frac{m-1}{m} \pi_{1,H}(\hat{\theta}_H).$$

We can rewrite this as

$$\pi_{1,H}(\hat{\theta}_F) < \pi_{1,F}(\hat{\theta}_F) < \pi_{1,H}(\hat{\theta}_H).$$

The formula above implies that the cutoff in the Home country is higher than that in the Foreign country. ■

Note that in the previous case, in which labor is mobile, the cutoffs in both countries are the same. As discussed above, the opportunity to sell abroad allows the more productive firms in both countries to expand their production, both the intensive margin (the market size effect, see equation 4) and the extensive margin (more profit per brand enables the firms to pay for the managers to develop more brands, as in equation 5). Increases in the intensive margin imply more demand for labor, whereas increases in the extensive margin lead to higher demand for talent. These two effects put pressure on the unproductive firms. The difference between the case of labor mobility and the case of labor immobility is that in the former case, the unproductive firms in the Home country compete with all the other firms, in particular the unproductive firms in the Foreign country, for the same pool of labor. In the latter case, however, their competitors are the more productive firms in the Home country. Compared to the Foreign country, the high talent endowment enables the more productive firms in the Home country to produce more varieties, and therefore, acquire more labor than the firms with the same productivity level in the Foreign country. Consequently, the unproductive firms in the Home country face more competition than their comparable firms in the Foreign country, which explains why the cut-off in the Home country is higher.

The last two propositions show the trade pattern. Since the cut-off level in the Foreign country is lower, this country exports the varieties produced by the unproductive firms, which have high prices. With more talent in the Home country, the productive firms in this country develop more varieties than the comparable firms in the Foreign country. As a result, the Home country exports the varieties with low prices. These varieties use more labor and less talent relatively to the varieties exported by the Home firms. Therefore the factor content of trade is that the Home country exports talent and the Foreign country exports labor.

It can be shown (in the appendix) that the wages of the managers, relative to the wages of the workers, do not change from autarky to free trade in both countries. Therefore it is possible that free trade worsen welfare for one of the countries. Indeed, while free trade enhances welfare for the Home country, the Foreign country can be worse off. This is because with less talent, competition in the Foreign country is less fierce than in the Home country (the cut-off is lower). As a result, there are more unproductive firms

in the Foreign country. When the Foreign country opens up to trade with the Home country, the Foreign firms have to lower their prices in order to compete with the more productive firms in the Home country. The result is that wages in the Foreign country are lower than in autarky.

To illustrate this possibility, I run a numerical case in which I choose $L = 1$, $S_H = .1$, $S_F = .09$, $f_e = 10$, $F = 1$, $m = 5$, $\gamma = 5$, $\rho = .75$. (This ρ was chosen to be consistent with other studies, in particular Broda and Weinstein 2006.) In this case, the price index in the Home country in autarky is 3.34 and the wage of the managers is .53. In the Foreign country, the corresponding values are 3.37 and .59. When the two countries engage in free trade, the price index is 2.65. Since the wages of workers and salaries of managers in the Home country do not change, they are all better off. However, the wages of workers and managers in the Foreign country are now .30 and .17 respectively. They are all worse off as their wages fall more than the change in the price index.

Unlike the previous case, the workers in the Foreign country can not move to the Home country where labor is more needed. In this case, compared to the case with factor mobility, the Home managers and the Foreign workers lose: while the Home managers are no longer the winners with the largest gains in free trade, the Foreign workers now may become the losers.

4 Conclusion

In this paper I extended Melitz's (2003) model so as to allow firms to produce more than one product. This proves to be a new source of competition and a new channel for welfare gains. With more talent available, the more productive firms can develop more products. This has two effects on the unproductive firms: not only do they have to sell less due to the arrival of new varieties with lower prices, they also face more pressure on the labor market. A higher survival cut-off is the result of this more competitive market. Moreover, the economy enjoys more new varieties from the productive firms, with lower prices. A higher talent endowment enhances welfare across the whole economy, although talented workers could be worse off if their wage decreases more than the price index. I also show that the effects are more

pronounced (the gains are bigger) if the firms are more heterogeneous or management is more efficient. One way to make management more efficient is to improve the quality of the managers. The talent endowment can be increased via education and immigration policies, but policy makers should also pay attention to the quality of new talent.

When two identical countries, differing only in their talent endowment, engage in free trade, and labor is allowed to move between these countries, I show that we can replicate an integrated economy with factor price equalization. The price index is lower than in autarky, and both countries hence gain from trade, although the managers in the country with less talent may be worse off.

When labor is not allowed to move, we do not have factor price equalization: the country with more talent has higher wages for labor and lower wages for managers than in the other country. Competition in the more talented Home country is more intense, resulting in a higher survival cut-off than in the Foreign country. The Home firms develop more products but produce less of each product than Foreign firms with the same productivity level. The larger talent endowment presents advantages to the Home country: their consumers are guaranteed to be the winners in free trade.

5 Appendix

5.1 Proof of proposition 2

From (1) we have:

$$\begin{aligned}
P^{-\frac{\rho}{1-\rho}} &= \int_{\widehat{\theta}} M_1 p^{-\frac{\rho}{1-\rho}}(\theta) n(\theta) df(\theta) \\
&= \int_{\widehat{\theta}} M_1 p^{-\frac{\rho}{1-\rho}}(\theta) \left(\frac{\pi_1(\theta)}{wm} \right)^{\frac{1}{m-1}} \gamma e^{-\gamma\theta} d\theta \\
&= \rho^{\frac{\rho m}{(1-\rho)(m-1)}} (1-\rho)^{\frac{1}{m-1}} R^{\frac{1}{m-1}} (mw)^{-\frac{1}{m-1}} P^{\frac{\rho}{(1-\rho)(m-1)}} M_1 \int_{\widehat{\theta}} \gamma e^{(\frac{\rho m}{(1-\rho)(m-1)} - \gamma)\theta} d\theta \\
&= \frac{\rho^{\frac{\rho m}{(1-\rho)(m-1)}} (1-\rho)^{\frac{1}{m-1}} R^{\frac{1}{m-1}} P^{\frac{\rho}{(1-\rho)(m-1)}} M_1 e^{(\frac{\rho m}{(1-\rho)(m-1)} - \gamma)\widehat{\theta}}}{(mw)^{\frac{1}{m-1}} \left(1 - \frac{\rho m}{\gamma(1-\rho)(m-1)} \right)},
\end{aligned}$$

or

$$P = \frac{(mw)^{\frac{1-\rho}{\rho m}} \left(1 - \frac{\rho m}{\gamma(1-\rho)(m-1)} \right)^{\frac{(1-\rho)(m-1)}{\rho m}}}{\rho (1-\rho)^{\frac{1-\rho}{\rho m}} R^{\frac{1-\rho}{\rho m}} M_1^{\frac{(1-\rho)(m-1)}{\rho m}}} e^{(\frac{\gamma(1-\rho)(m-1)}{\rho m} - 1)\widehat{\theta}}. \quad (19)$$

We can make use of the zero-profit cutoff condition (6):

$$\begin{aligned}
F &= \frac{m-1}{m} \pi_1(\widehat{\theta}) = \frac{m-1}{m} (1-\rho) P^{\frac{\rho}{1-\rho}} R c(\widehat{\theta})^{-\frac{\rho}{1-\rho}} \\
&= \frac{m-1}{m} \frac{(1-\rho) \rho^{\frac{\rho}{1-\rho}} (mw)^{\frac{1}{m}} \left(1 - \frac{\rho m}{\gamma(1-\rho)(m-1)} \right)^{\frac{m-1}{m}} R}{\rho (1-\rho)^{\frac{1}{m}} R^{\frac{1}{m}} M_1^{\frac{m-1}{m}}} e^{\frac{\gamma(m-1)}{m}\widehat{\theta}}.
\end{aligned} \quad (20)$$

Finally from the free-entry condition (8) we have

$$f_e = \frac{\Pi_3}{M_1} = \frac{\Pi_1 - wS - M_2 F}{M_1} = \frac{(1-\rho) \left(1 - \frac{1}{m} \right) R - e^{-\gamma\widehat{\theta}} M_1 F}{M_1},$$

or

$$M_1 = \frac{(1 - \rho) \left(1 - \frac{1}{m}\right) R}{f_e + F e^{-\gamma \hat{\theta}}}. \quad (21)$$

Substituting M_1 by (21) into (20) we have

$$\begin{aligned} F &\propto w^{\frac{1}{m}} e^{\frac{\gamma(m-1)\hat{\theta}}{m}} \left(f_e + F e^{-\gamma \hat{\theta}}\right)^{\frac{m-1}{m}} \\ &\propto w^{\frac{1}{m}} \left(f_e e^{\gamma \hat{\theta}} + F\right)^{\frac{m-1}{m}}, \end{aligned} \quad (22)$$

The formula above shows that when the wage of managers w is lower (due to an increase in the talent endowment), the cut-off $\hat{\theta}$ is higher.

5.2 Proof of proposition 3

First I will prove that the price index falls when there is more talent. From (20), we have:

$$F \propto w^{\frac{1}{m}} e^{\frac{\gamma(m-1)\hat{\theta}}{m}} \left(\frac{R}{M_1}\right)^{\frac{m-1}{m}}.$$

Using (19) we have:

$$\begin{aligned} F^{\frac{1-\rho}{\rho}} &\propto w^{\frac{1-\rho}{\rho m}} e^{\frac{\gamma(1-\rho)(m-1)\hat{\theta}}{\rho m}} \left(\frac{R}{M_1}\right)^{\frac{(m-1)(1-\rho)}{m\rho}} \\ &\propto R^{\frac{1-\rho}{\rho}} P e^{\hat{\theta}}. \end{aligned}$$

From (11), (22) and (12) we have:

$$P \propto L^{-\frac{1-\rho}{\rho}} \left[\left(\frac{L}{S}\right)^{-\frac{1}{m-1}} - B \right]^{-\frac{1}{\gamma}} \quad (23)$$

B here is a constant. This formula shows an increase in the talent endowment S reduces the price index. Therefore, the workers are the clear winner. The managers, however, may be worse-off because their salaries are lower. Indeed, from (12) and (23) the real salaries of the managers are proportional to $L^{\frac{1-\rho}{\rho}} \left[\left(\frac{L}{S} \right)^{\gamma - \frac{1}{m-1}} - B \left(\frac{L}{S} \right)^{\gamma} \right]^{\frac{1}{\gamma}}$. Since $\gamma - \frac{1}{m-1} < \gamma$, this real salary increases with S (or decreases with $\frac{L}{S}$) when S is small enough (or $\frac{L}{S}$ big enough).

5.3 Proof of proposition 4

From (6) we have:

$$F = (1 - \rho) P^{\frac{\rho}{1-\rho}} R_C \left(\hat{\theta} \right)^{-\frac{\rho}{1-\rho}},$$

or

$$\begin{aligned} \left(\frac{F}{1 - \rho} \right)^{\frac{1-\rho}{\rho}} &= P R^{\frac{1-\rho}{\rho}} e^{\alpha \hat{\theta}} \\ &= P \left(\frac{2L}{1 - \frac{1-\rho}{m}} \right)^{\frac{1-\rho}{\rho}} \left(B w^{-\frac{1}{m-1}} - F \right)^{\frac{1}{\gamma}} \\ &\propto P (2L)^{\frac{1-\rho}{\rho}} \left(\left(\frac{S_H + S_F}{2L} \right)^{\frac{1}{m-1}} - C \right)^{\frac{1}{\gamma}} \\ &\propto P L^{\frac{1-\rho}{\rho}} \left(\left(\frac{S_H + S_F}{L} 2^{\frac{(1-\rho)(m-1)\gamma}{\rho} - 1} \right)^{\frac{1}{m-1}} - C \right)^{\frac{1}{\gamma}}. \end{aligned}$$

C here is a constant. The price index in the integrated economy is lower than that of the Foreign country in autarky because $\frac{S_H + S_F}{2L} > \frac{S_F}{L}$. It is also lower than that of the Home country because $\frac{S_H + S_F}{L} > \frac{S_H}{L}$ and $2^{\frac{(1-\rho)(m-1)\gamma}{\rho} - 1} > 1$.

5.4 The wage for managers in free trade with labor immobility is proportional to the number of workers

Similarly to the case in autarky, the managerial cost is proportional to Π_1 :

$$\begin{aligned}
 S &= \int_{\hat{\theta}}^{\infty} M_1 n^m(\theta) \gamma e^{-\gamma\theta} d\theta \\
 &= \int_{\hat{\theta}}^{\infty} M_1 \frac{\pi_1(\theta)}{mw} n(\theta) \gamma e^{-\gamma\theta} d\theta \\
 &= \frac{\Pi_1}{mw}.
 \end{aligned}$$

From the pricing strategy (3), the production labor cost is always proportional to the sales of the firm. Therefore, we have

$$\Pi_1 = (1 - \rho) R.$$

The free-entry condition in the Home country is given by

$$\int_{\hat{\theta}}^{\infty} n(\theta) \pi_3(\theta) \gamma e^{-\gamma\theta} d\theta = f_e,$$

which yields

$$\Pi_3 = M_1 f_e.$$

Total demand for labor in the Home country is

$$\begin{aligned}
L^D &= M_1 f_e + M_2 F + \rho R \\
&= \Pi_3 + M_2 F + \rho R \\
&= \Pi_1 - wS + \rho R \\
&= \frac{m-1}{m} \Pi_1 + \rho R \\
&= \left[\frac{m-1}{m} (1-\rho) + \rho \right] R.
\end{aligned}$$

Then,

$$R = \frac{L}{\frac{m-1}{m} (1-\rho) + \rho}.$$

From the formulas above, the wage for managers are then proportional to the number of workers.

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