Affiliation and Entry in First-Price Auctions with Heterogeneous Bidders*

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Abstract

In this paper we study the timber sales auctions in Oregon. We propose an entry and bidding model within the affiliated private value (APV) framework and with heterogeneous bidders, and establish existence of the entry equilibrium and existence and uniqueness of the bidding equilibrium with the joint distribution of private values belonging to the class of Archimedean copulas. We then estimate the resulting structural model, and find that the hauling distance plays a significant role in bidders' entry and bidding decisions. We quantify the extent to which the potential bidders' private values and entry costs are affiliated. The structural estimates are then used to conduct counterfactual analyses to address policy related issues. In particular, we quantify the effects of reserve price, affiliation, and merger on the end auction outcomes.

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1 Introduction

Auctions have long been used as a means for price determination under a competitive setting and an incomplete information environment. Auction theory developed within the game-theoretic framework with incomplete information (Harsanyi (1967/1968)) not only helps us understand how auctions work, but also offers insight in analyzing many other economic problems. A celebrated result in auction theory is Vickrey's (1961) revenue equivalence theorem, which postulates that all the four auction formats (first-price sealed-bid, second-price sealed bid, English, and Dutch auctions) generate the same average revenue for the seller with symmetric, independent, and risk-neutral bidders.

The revenue equivalence theorem is a powerful result that offers insight into how auction mechanisms work, and also raises important questions as to how this powerful result can be affected when the standard assumptions are relaxed. A large part of auction theory has focused on answering these questions. Milgrom and Weber (1982) give revenue ranking with symmetric and affiliated bidders in which the English auction generates highest revenue among the four formats and the second-price auction ranks next; they also establish that with symmetric, affiliated, and risk-averse bidders who have constant absolute risk aversion, the English auction can generate at least as high revenue as the second-price auction. Myerson (1981) derives the optimal auctions with asymmetric bidders, and Maskin and Riley (1984) consider the case with risk-averse bidders. Levin and Smith (1994) extend the revenue equivalence and ranking results from Vickrey (1961) and Milgrom and Weber (1982) to the case with symmetric bidders (independent or affiliated) using mixed entry strategies.

Using timber sale auctions organized by the Oregon Department of Forestry (ODF), this paper attempts to address a set of questions that include with heterogeneous bidders and when entry is taken into account, how the seller's revenue could change with the extent to which the bidders' private values are affiliated, and whether the reserve price currently set by the ODF is optimal with respect to maximizing the seller's revenue/profit. Moreover, merger and bidder coalition have been an important issue to economists interested in competition policy, yet no empirical work has studied this issue taking into account participation from potential bidders. That we consider heterogeneous bidders is motivated by the evidence from the previous work studying the timber auctions in Oregon (e.g. Brannman and Froeb (2000) using data consisting of oral auctions, and Li and Zhang (2008) using the same data used in this paper comprising first-price sealed-bid auctions), that hauling distance plays an important role in bidders' bidding (Brannman and Froeb (2000)) and entry (Li and Zhang (2008)) decisions. This means that bidders are asymmetric and heterogeneous. Furthermore, Li and Zhang (2008) find a small but strongly significant level of affiliation among potential bidders' private information (either private values or entry costs). Lastly, recent empirical work in auctions in general and in timber auctions in particular (e.g. Athey, Levin and Seira (2004), Bajari and Hortaçsu (2003), Kransnokutskaya and Seim (2006), Li and Zheng (2007, 2009)) has demonstrated that bidders' participation and entry decision is an integrated part of the decision making process that has to be taken into account when studying auctions. In view of these, in this paper we attempt to study the timber auctions organized by the ODF within a general framework in which potential bidders are affiliated and heterogeneous, and they make entry decisions before submitting bids.

Auction theory offers little guidance in answering these questions for auctions with entry and asymmetric potential bidders with affiliated private values. On the other hand, to gain insight on these questions from an empirical perspective, one needs to observe two states of world, such as pre and post the change of the affiliation level, or pre and post merger. Usually in auction data, as is the case in our data, however, one cannot observe these two states of world. Therefore we adopt the structural approach in our empirical analysis.

We develop an entry and bidding model for asymmetric bidders within the APV paradigm. Establishing existence and uniqueness of the entry and bidding equilibria within the APV model with entry and with heterogeneous bidders is a challenging problem. We extend the results by Lebrun (1999, 2006) for the IPV case with asymmetric bidders and without entry to our case and establish the existence and uniqueness of the bidding equilibrium and existence of the entry equilibrium for a general class of joint (affiliated) distribution of private values. This can be viewed as a contribution of the paper to the theoretical literature.

Because of the general framework we adopt, the answers to the aforementioned questions of our interest depend on the interactions of affiliation, entry, and asymmetry, as well as competition. As is well known, the optimal reserve price in a symmetric independent private value (IPV) model without entry does not depend on the number of potential bidders. This result can change if entry is introduced (see, e.g., Levin and Smith (1994), Samuelson (1985), Li and Zheng (2007)), or if bidders have affiliated private values (Levin and Smith (1996), Li, Perrigne, and Vuong (2003)). In our case, on the other hand, assessing the optimal reserve price is complicated further by the APV framework with entry and asymmetric bidders. Therefore we can only address this issue through a counterfactual analysis using the structural estimates. Furthermore, while the effect of the number of potential bidders on winning bids and seller's revenue is clear in an IPV model with symmetric bidders and without entry, it becomes less clear in a more general setting, such as the IPV model with entry and symmetric bidders (Li and Zheng (2007, 2009)), and the APV model without entry (Pinkse and Tan (2005)). In particular, Li and Zheng (2007, 2009) show that in terms of the relationship between the number of potential bidders and the expected seller's revenue, in addition to the usual "competition effect," there is an opposite effect due to the entry which they term as the "entry effect." On the other hand, Pinkse and Tan (2005) postulate that in a conditionally independent private value model, a special case of the APV paradigm, in addition to the "competition effect," there is an opposite effect caused by affiliation they term as the "affiliation effect." Zhang (2008) shows that in the APV model with entry and symmetric bidders, these three effects, namely, the "competition effect," the "entry effect," and the "affiliation effect" are at work. While we expect these three effects to remain in the APV framework with entry and asymmetric bidders, it becomes challenging to pinpoint them with asymmetric bidders. Since the effect of merger is closely related to how the seller's revenue changes with the set of potential bidders, i.e, not only the number of potential bidders, but also the identity of potential bidders when they are heterogeneous, and at the same time, theory does not yield good predictions, we rely on the structural analysis to gain insight on this issue.

We then develop a structural framework to estimate the entry and bidding model we propose. We use the estimated structural parameters to conduct counterfactual analyses of our interest. We find that for a representative auction, the optimal reserve price should be much larger than the current one. In evaluating the merger effects we find that the merger from two non-participating bidders is likely to promote the entry of the merged bidder. But the merger has little impact on non-merging bidders' entry behaviors. While the overall merger effect on the seller's revenue is not theoretically clear in our model, we find that at the current reserve prices and dependence levels, merger is beneficial to the seller, but it could mean a loss for the seller for some values of reserve price or some dependence levels. The merger from two least competitive bidders is preferred to the merger from two most competitive bidders to the seller.

Asymmetry is an indispensable element of the model given the asymmetric feature of the data. The analysis of the model, however, is complicated from both theoretical and econometric viewpoints due to the introduction of asymmetry. Because of the complexity of the model, and in particular, because that there is no closed form solution for the bidding function, we have to rely on some numerical approximation procedure. Moreover, while the structural analysis of auctions with asymmetric bidders has focused on the case with two types of bidders (Athey, Levin, and Seira (2004), Campo, Perrigne and Vuong (2003), and Kransnokutskaya and Seim (2006)), our model allows for all potential bidders to be different from each other, motivated by the fact that in our data, asymmetry is driven by the difference among bidders' hauling distances.

This paper makes contribution to the growing literature of the structural analysis of auction data since Paarsch (1992).[1] While the structural approach has been extended to the APV paradigm by Li, Perrigne and Vuong (2000, 2002), Campo, Perrigne and Vuong (2003), and Hubbard, Li and Paarsch (2009), this paper is the first one in estimating a structural model within the APV paradigm and taking into account entry. On the other hand, while the recent work has started to pay attention to the problem of participation and entry, all the work has focused on the IPV framework with Bajari and Hortaçsu (2003) being an exception as they consider a common value (CV) model. In contrast, this paper considers the entry problem within the APV paradigm, a more general framework.

Our empirical analysis of the timber auctions and the resulting findings offer new insight on timber sale auctions and policy related issues. While most of the empirical analysis of timber sale auctions is based on the IPV model without entry (e.g. Paarsch (1997), Baldwin, Marshall and Richard (1997), Haile (2001), Haile and Tamer (2003), Li and Perrigne (2003)) or the IPV model with entry (Athey, Levin and Seira (2004), Li and Zheng (2007)), ours is based on the APV model with entry and heterogeneous bidders. As a result, our findings can be more robust, and also can be

[1] Another strand of the literature is to test implications from game-theoretic auction models poineered by Hendricks and Porter, e.g., Hendricks and Porter (1988). See Porter (1995) for a survey.

more useful for addressing the policy-related issues as our analysis takes into account the affiliation effect, the entry effect, and the asymmetry effect. Moreover and probably more interestingly, we study the merger effect within the asymmetric APV framework with entry, and offer new insight into how merger as well as other issues related to competition policy can be affected by complications arising from affiliation, entry, and asymmetry, and how they can be addressed within a unified framework as adopted in this paper.[2]

This paper is organized as follows. Section 2 describes the data we analyze in the paper. In Section 3 we propose the asymmetric APV model with entry. Section 4 is devoted to the structural analysis of the data, and Section 5 conducts a set of counterfactual analyses studying the effects of reserve prices, affiliation levels, and mergers. Section 6 concludes.

2 Data

The data we study in this paper are from the timber auctions organized by the ODF between January 2002 and June 2007. Before an auction is advertised, the ODF "cruises" the selected tract of timber and obtains information of the tract, such as the composition of the species, the quality grade of the timber and so on. Based on the information it obtains, the ODF sets its appraised price for the tract, which serves also as the reserve price. After the "cruise," a detailed bid notice is usually released 4-6 weeks prior to the sale date, which provides information about the auction, including the date and location of the sale, species volume, quality grade of the timber, appraised price as well as other related information. Potential bidders acquire their own information or private values through different ways and decide whether and how much to bid. Bids are submitted in sealed envelopes that are opened at a bid opening session at the ODF district office offering the sale. The sale is awarded to the bidder with the highest bid. All the sales are therefore first price sealed bid scale auctions.

The original data contain 415 sales in total. Among them, some sales have more than one bid species, which are deleted from our sample because of the "skewed bidding" issue discussed in Athey and Levin (2001). We focus on the sales in which Douglas-fir is the only bid species and drop the sales with other than Douglas-fir as bid species, because Douglas-fir is a majority species. Considering the time that our estimation program takes, we focus on the auctions with at most 8 potential bidders. The resulting final sample has 203 sales and 1074 observed bids.

For each sale, we directly observe some sale-specific variables including the location and the region of the sale, appraised price, appraised volume measured in thousand board feet or MBF, length of the contract, and diameter at breast height (DBH) as well. Noting that the bid species is often a combination of a mixture of several grades of quality, we use number 1, 2, $\cdots$, up to 18 to denote the letter-grades used by ODF so that the final grade of a sale is the weighted average of grades with volumes of grades as the weight. In addition to sale-specific variables, as shown in

[2] It is worth noting that to the best of our knowledge, Brannman and Froeb (2000), considering oral timber auctions within an IPV paradigm without entry, is the only paper assessing the merger effect in auctions using the structural approach.

Brannman and Froeb (2000) and Li and Zhang (2008), hauling distance is an important bidder-specific variable that affects bidders' bidding and entry decisions. However, hauling distance is not observed directly. We use the hauling distance variable constructed in Li and Zhang (2008) who transfer the location of a tract into latitude and longitude through the Oregon Latitude and Longitude Locator[3] and find the distances between the tract and the mills of firms by using Google Map.

The key information related to entry is the identities of potential bidders, which are not observed. Unlike some procurement auctions, where information on bidders who have requested bidding proposal is available and can be used as a proxy for potential bidders (Li and Zheng (2009)), we do not have such information in our case, as is usual for timber sale auctions. Therefore we follow Athey, Levin, and Seira (2004) and Li and Zheng (2007) to construct potential bidders. Specifically, we first divide all sales in the original data set into 146 groups each of which contains all sales held in the same district in the same quarter of the same year. The potential bidders of a sale are then all bidders who submit at least one bid in the sales of the group that the sale belongs to. In other words, all auctions in the same group have the same set of potential bidders. Note that in constructing the potential bidders we use the original data set including all auctions removed from our final sample. Summary statistics of the data are given in Table 1. Notably, the entry proportion, which is calculated as the ratio of the number of actual bidders and the number of potential bidders, is about 0.66 on average, meaning that while there is strong evidence of entry pattern from the potential bidders, on average more than half of the potential bidders would participate in the auction.

3 The Model

In this section we propose a theoretical two-stage model to characterize the timber sales, extending the models in Athey, Levin, and Seira (2004) and Krasnokutskaya and Seim (2006) with two groups of bidders within an IPV paradigm to the APV paradigm that allows potential bidders to be different from each other. Specifically, motivated by the finding of Brannman and Froeb (2000) that the hauling distance plays a significant role in bidders' bidding decision in oral timber auctions in Oregon, and the finding of Li and Zhang (2008) using the same data studied in this paper that the hauling distance is important in potential bidders' entry decision and potential bidders are affiliated through their private information (either private values or entry costs), we consider a first-price sealed-bid auction within the APV paradigm with a public reserve price, entry, and asymmetric bidders.

In the model, a single object is auctioned off to N heterogenous and risk-neutral potential bidders, who are affiliated in their private information. For each auction, a reserve price, r, is announced prior to the letting. Bidder i has private entry cost k_i, including the cost of obtaining private information and bid preparation, and does not obtain his private value v_i until he partici-

[3]It is available at http://salemgis.odf.state.or.us/scripts/esrimap.dll?name=locate&cmd=start

pates in the auction. We allow both private values and entry costs to be affiliated across bidders, that is $V_1, \ldots, V_N$ and $K_1, \ldots, K_N$ jointly follow a distribution $F(\cdot, \ldots, \cdot)$ with support $[\underline{v}, \overline{v}]^N$, and a distribution $G(\cdot, \ldots, \cdot)$ with support $\left[\underline{k}, \overline{k}\right]^N$, respectively. Affiliation is a terminology describing the positive dependence among random variables, which was first introduced into the study of auctions by Milgrom and Weber (1982). It is equivalent to the concept called multivariate total positivity of order 2 (MTP_2) in the multivariate statistics literature. Following Milgrom and Weber (1982), affiliation has the following formal definition.

Definition. *Let z and z' be any two values of a vector of random variables $Z \subseteq \mathbb{R}^n$ with a density $f(\cdot)$. It is said that all elements of Z are affiliated if $f(z \vee z') f(z \wedge z') \geq f(z) f(z')$, where $z \vee z'$ denotes the component-wise maximum of z and z', and $z \wedge z'$ denotes the component-wise minimum of z and z'.*

Intuitively, affiliation means that large values for some of the components in Z make other components more likely to be large than small. We also denote the marginal distribution and density of bidder i's private value by $F_i(\cdot)$ and $f_i(\cdot)$ and marginal distribution and density of bidder i's entry cost by $G_i(\cdot)$ and $g_i(\cdot)$, respectively, and assume that $f_i(\cdot)$ is continuously differentiable and bounded away from zero on $[\underline{v}, \overline{v}]$. The subscript of distribution function implies that all potential bidders are of different types due to the different hauling distances.

3.1 Bidding Strategy

Because the entry decision is based on the pre-entry expected profit, which depends on the bidding strategy of bidder i, we first describe the bidding strategy of bidder i. We assume that bidder i knows the number of the actual competitors in the bidding stage,[4] and thus bidder i's bidding strategy is determined by the first order condition of the following maximization problem,

$$\pi_i(v_i|a_{-i}) = \max (v_i - b_i) \Pr(B_j < b_i|v_i; a_{-i}),$$

where B_j denotes the maximum bid among other actual bidders and

$$a_{-i} \in A_{-i} = \{(a_1, \ldots, a_N) | a_j = 0 \text{ or } 1, j = 1, \ldots N, j \neq i\}$$

is one possibility of the 2^{N-1} combinations of entry behavior of $N-1$ other potential bidders, where $a_j = 1$ if bidder j participates. Denote the number of actual bidders of the combination a_{-i} by $n_{a_{-i}}$. As usual we consider a continuously differentiable and strictly increasing bidding strategy, $b_i = s_i(v_i)$, therefore the first order condition is

$$-F_{V_{-i}|v_i}\left(s_j^{-1}(b_i), j \neq i|v_i\right) + (v_i - b_i) \sum_{j \neq i}^{n_{a_{-i}}} \frac{\partial F_{V_{-i}|v_i}\left(s_j^{-1}(b_i), j \neq i|v_i\right)}{\partial v_j} \frac{\partial s_j^{-1}(b_i)}{\partial b_i} = 0, \tag{1}$$

[4] When the lower support of private value is below the reserve price, bidder i only knows the active bidders who participate in the auction but not actual bidders who submit bids. In our case, the number of active bidders is equal to the number of actual bidders, since the lower support of private value is assumed to be just the reserve price.

where $F_{V_{-i}|v_i}$ denotes the joint distribution of $V_j, j \neq i$ conditional on $V_i = v_i$ and $s_i^{-1}(\cdot)$ is the inverse function of the bidding function of bidder i. A set of equation (1) with boundary conditions $s_i^{-1}(\underline{v}) = \underline{v}$ for $i = 1, \ldots, n$ form a system of differential equations characterizing the equilibrium bids for all n actual bidders.

3.2 Entry Decision

In the initial participation stage, each potential bidder i only knows his own entry cost, joint distributions of entry costs and private values. Therefore the entry decision of bidder i is determined by his pre-entry expected profit from participation, Π_i. Specifically, he participates in the auction only if his entry cost is less then Π_i. Let p_i denote the entry probability of bidder i, respectively. The ex ante expected profit Π_i is given by

$$\Pi_i = \sum_{a_{-i} \in A_{-i}} \int_{\underline{v}}^{\overline{v}} \pi_i(v_i|a_{-i}) \, dF_i(v_i) \Pr(a_{-i}|a_i = 1), \tag{2}$$

where $\Pr(a_{-i}|a_i = 1)$ is a function of $p_i, i = 1, \ldots, N$, which can be denoted by $\Pr(a_{-i}; p_1, \ldots, p_N|a_i = 1)$. As a result, the pre-entry expected profit is the sum of 2^{N-1} products of the post-entry profits and corresponding probabilities with the unknown private value integrated out. On the other hand, in equilibrium probabilities of entry are given by $p_i = \Pr(K_i < \Pi_i) = G_i(\Pi_i)$, for all i.

Note that although the number of potential bidders does not directly affect the bidding strategy in the bidding stage, it affects the number and the identities of actual bidders, which in turn have impact on the bidding strategy.

3.3 Characterization of the Equilibrium

Existence and uniqueness of the Bayesian Nash equilibrium with asymmetric bidders has been a challenging problem studied in the recent auction theory literature. See, e.g. Lebrun (1999, 2006) and Maskin and Riley (2000, 2003) within the IPV framework, Lizzeri and Persico (2000) within the APV framework and two types of bidders. The analysis of our model is further complicated by the introduction of affiliation and entry, as well as that we allow all potential bidders to be different from each other. To address the issue of existence and uniqueness in our case, we look at the case where the joint distribution of bidders' private values is characterized by the family of Archimedean copulas. For the copula concept and the characterization of the Archimedean copulas, see Nelsen (1999). Copula can provide a flexible way of modeling joint dependence of multivariate variables using the marginal distributions.

Specifically, by Sklar's theorem (Sklar (1973)), for a joint distribution $F(x_1, \ldots, x_N)$, there is a unique copula C, such that $C(F_1(x_1), \ldots, F_N(x_N)) = F(x_1, \ldots, x_N)$. For the Archimedean copulas, the copula C can be expressed as $C(u_1, \ldots, u_n) = \phi^{[-1]}(\phi(u_1) + \cdots + \phi(u_n))$, where ϕ is a generator of the copula and is a decreasing and convex function, and $\phi^{[-1]}$ denotes the pseudoinverse of ϕ[5]. The family of Archimedean copulas include a wide range of copulas. For example, the generators $\phi(u) = \frac{1}{q}(u^{-q} - 1)$, $\phi(u) = (-\ln(u))^q$, and $\phi(u) = \ln\left(\frac{\exp(qu)-1}{\exp(q)-1}\right)$ correspond to the widely used Clayton copula, Gumbel copula, and Frank copula, respectively. Since we consider a differentiable bidding strategy, we have to confine ourself to the strict generator, that is $\phi^{[-1]} = \phi^{-1}$. Since $C_i(F_1(x_1), \ldots, F_N(x_N)) = F_{X_{-i}|x_i}(x_1, \ldots, x_N)$ (e.g. Hubbard, Li and Paarsch (2009)), the first order condition (1) determining the equilibrium bids can be written as follows

$$\frac{ds_i^{-1}(b)}{db} = \frac{\phi^{-1\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}{\left(n_{a_{-i}} - 1\right)\phi'\left(F_i\left(s_i^{-1}(b)\right)\right) f_i\left(s_i^{-1}(b)\right)\phi^{-1\prime\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}\left[\sum_{k\neq i}\frac{1}{s_k^{-1}(b) - b} - \frac{n_{a_{-i}} - 2}{s_i^{-1}(b) - b}\right]. \tag{3}$$

Note that with the copula specification for the joint entry cost distribution, the entry probabilities in (2) can be expressed in terms of the joint entry cost distribution. For example, $\Pr(a_{-i}; p_1, \ldots, p_N | a_i = 1)$, for the case that given the participation of bidder i, bidder 1 up to bidder $i-1$ participate in the auction while bidder $i+1$ up to bidder N do not, can be expressed as

$$\begin{aligned} & \Pr(a_1 = \cdots a_{i-1} = 1, a_{i+1} = \cdots a_N = 0 | a_i = 1) \\ = & \frac{\Pr(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0)}{\Pr(a_i = 1)} \end{aligned} \tag{4}$$

where

$$\begin{aligned} & \Pr(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0) \\ = & C(p_1, \ldots, p_i, 1, \ldots, 1; q_k) - \sum_{i+1 \le j \le N} C(p_1, \ldots, p_i, p_j, 1, \ldots, 1; q_k) \\ & \cdots + (-1)^{N-i} C(p_1, \ldots, p_N; q_k), \end{aligned}$$

and $\Pr(a_i = 1) = C(1, \ldots, 1, p_i, 1, \ldots, 1; q_k)$.

Equilibrium of the model consists of two parts, entry equilibrium and bidding equilibrium. Based on the choice of Archimedean copulas for the joint distribution of private values, the existence of the equilibrium is guaranteed. Moreover, with some additional conditions, the bidding equilibrium is unique. The next proposition describes the equilibrium formally.

Proposition (Characterization of Equilibrium)**.** *Define* $lv = \max(\underline{v}, r)$. *Assume (a) the marginal distribution of entry cost of bidder i,* G_i *is continuous over* $\left[\underline{k}, \overline{k}\right]$ *for all i; (b) marginal distribution of private value of bidder i is differentiable over* $(lv, \overline{v}]$ *with a derivative* f_i *locally bounded away from zero over this interval for all i; (c) joint distribution of private values follows an Archimedean*

[5] ϕ is a decreasing convex function from $[0,1]$ to $(0,\infty]$ with $\phi(1) = 0$. $\phi^{[-1]}$ is defined as

$$\phi^{[-1]}(u) = \begin{cases} \phi^{-1}(u), & 0 \le u \le \phi(0), \\ 0, & \phi(0) \le u \le \infty. \end{cases}$$

Copula.

i. Bidding Equilibrium *In the bidding equilibrium, bidder i adopts a continuously differentiable and strictly increasing bidding function $b_i = s_i(v)$ over $(lv, \overline{v}]$. The inverse functions of s_i for all i, $s_1^{-1}, \ldots, s_n^{-1}$ are the solution of the system of differential equations (3) with boundary conditions (5) and (6) :*

$$s_i^{-1}(lv) = lv \tag{5}$$
$$s_i^{-1}(\eta) = \overline{v}. \tag{6}$$

for some η.

ii. Uniqueness of Bidding Equilibrium *Moreover, if $F_i(lv) > 0$ and $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u, then the bidding equilibrium is unique.*

iii. Entry Equilibrium *In the entry equilibrium, bidder i chooses to participate in the auction if his entry cost is less than the threshold $\Pi_i(p)$ and stay out otherwise, where $p = (p_1, \ldots p_N)$ and p_i is the entry probability of bidder i and is determined by*

$$p_i = G_i(\Pi_i(p)). \tag{7}$$

As is seen here, the existence of the entry equilibrium is equivalent to the existence of the entry probability p_i, given by the equation (7). Since Π_i is continuous in p_i and thus G_i is continuous over $[0, 1]$, there exists a solution p_i of equation (7), according to Kakutani's fixed point theorem (Kakutani (1941)). To show the uniqueness of the bidding equilibrium is to show that there is a unique η such that $s_i^{-1}(\eta) = \overline{v}$. Then starting from η, according to Lipschitz uniqueness theorem, s_i^{-1} is unique over $(\underline{v}, \eta]$. Note that a Clayton copula satisfies the condition for uniqueness that $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. The formal proofs are provided in Appendix A.

4 The Structural Analysis

We estimate the model proposed in the last section using the timber sales data. Our objective is to recover the underlying joint distributions of private values and entry costs using observed bids and the number of actual bidders. The structural inference in our case is complicated because of the generality of our model that accounts for affiliation, asymmetry, and entry. Our approach circumvents the complications arising from the estimation of our model and makes the structural inference tractable. First, to model the affiliation in a flexible way, we adopt the copula approach in modeling the joint distribution of private values and the joint distribution of entry costs.[6] Second, since we allow bidders to be asymmetric, the system of differential equations consisting of equation (3) that characterizes bidders' Bayesian Nash equilibrium strategies does not yield closed-form solutions. To address this problem we adopt a numerical method based on Marshall, Meurer,

[6] Hubbard, Li and Paarsch (2009) use the copula approach to model affiliation within the symmetric APV framework without entry and propose a semiparametric estimation method.

Richard and Stromquist (1994) and Gayle (2004). Third, because of the various covariates we try to control for and the relatively small size of the data set, the nonparametric method does not work well here. Therefore, we adopt a fully parametric approach.

4.1 Specifications

We adopt the Clayton copula to model the joint distributions of both private values and entry costs. With the generator of Clayton copula given above, the joint distribution of private value is specified as $F(v_1, \ldots v_n) = \left(\sum_i F_i(v_i)^{-q_v} - n + 1\right)^{-1/q_v}$, and the joint distribution of entry costs is specified as $G(k_1, \ldots, k_n) = \left(\sum_i G_i(k_i)^{-q_k} - n + 1\right)^{-1/q_k}$, where q_v and q_k are dependence parameters and F_i and G_i are the marginal distributions of private value and entry cost. The F_i is specified as a truncated exponential distribution $F_{V_{\ell i}}(v|\mathbf{x}_{\ell i}; \beta) = \frac{\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \exp\left(-\frac{1}{\lambda_{v_{\ell i}}}v\right)}{\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\overline{v}\right)}$, and G_i is also assumed to be exponential but without truncation $G_{K_{\ell i}}(k|\mathbf{x}_{\ell i}; \beta) = 1 - \exp\left(-\frac{1}{\lambda_{k_{\ell i}}}k\right)$ for bidder i of the ℓ-th auction, $\ell = 1, \ldots, L$, where L is the number of auctions, $\lambda_{v_{\ell i}}$ and $\lambda_{k_{\ell i}}$ are the parameters in both exponential distribution and equal $\exp(\beta \mathbf{x}_{\ell i})$ and $\exp(\alpha \mathbf{x}_{\ell i})$, respectively, and $\mathbf{x}_{\ell i}$ is a vector of covariates that are auction specific or bidder specific, and in our case includes variables such as hauling distance, volume, duration, grade, and DBH.[7] Auction specific covariates are used to control heterogeneity of auctions and the hauling distance is used to capture asymmetry among bidders. The truncated specification of the marginal distribution of private value makes the numerical method adopted to solve the equilibrium bids possible.[8] In practice, $\underline{v}$ is equal to the reserve price of ℓ-th auction minus a small number, that is, $\underline{v} = r_\ell - \varepsilon$, $\overline{v}$ is equal to \$1000/MBF. We set the lower support of private value below the reserve price in order to make $F(r)$ strictly positive and then guarantee the uniqueness of the bidding equilibrium. We then model the joint distributions of private values and entry costs in auction ℓ as Clayton copula with different dependence parameters q_v and q_k. The use of the Clayton copula offers several advantages. First, it guarantees the existence and uniqueness of the equilibrium as discussed in Section 3.3. Second, it preserves the same dependence structure when the number of potential bidders changes. Third, it is relatively easy to draw dependent data from the Clayton copula, as it has a closed form that can be used to draw data recursively. Lastly, since q is the only parameter that measures the dependence, we can easily evaluate the impact of the dependence level on the end outcomes of an auction by changing the value of q.

Note that in these specifications, the asymmetry across potential bidders is captured by the inclusion of the hauling distance variable in $\mathbf{x}_{\ell i}$, while both α and β are kept constant across different bidders. This enables us to estimate a relatively parsimonious structural model and at the same time control for the asymmetry.

[7] Here we do not introduce unobserved auction heterogeneity into the model, as Li and Zhang (2008) show that it does not have a significant effect in bidders' entry behaviors.

[8] Here we need an upper bound for the private value to make the algorithm of finding the equilibrium bids possible. See Appendix B for the detail of the algorithm.

4.2 Estimation Method

Because of the complexity of our structural model, we employ the indirect inference method to estimate the model. Initially proposed in the nonlinear time series context by Smith (1993) and developed further by Gourieroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996), the indirect inference method is simulation based and obtains the estimates of parameters by minimizing a measure of distance between the estimates for the auxiliary parameters of an auxiliary model using the original data and simulated data. More specifically, let θ denote the vector of parameters of interest, γ be the parameters of the auxiliary model, $\widehat{\gamma}_T$ and $\widehat{\gamma}_{ST}^{(p)}(\theta)$ be the estimates of the auxiliary model using the original data and the p-th simulated data out of P sets of simulated data from the model given a specific θ, respectively. Then the estimator of θ, denoted by $\widehat{\theta}_{ST}$, is defined as

$$\widehat{\theta}_{ST} = \arg\min_{\theta} \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{p=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right]' \Omega \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{P=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right], \tag{8}$$

where Ω is a symmetric semi-positive definite matrix. Therefore to implement the indirect inference method, we have to draw data from the model for a given θ, which involves calculating the equilibrium bids and the thresholds of the entry costs. Basically we use numerical approximation method, which is an extension of but in similar spirit to the methods in Marshall, Meurer, Richard and Stromquist (1994) and Gayle (2004) to find the equilibrium bids, and iteration to find the equilibrium entry probabilities. The algorithm to find the equilibrium bids is illustrated in detail in Appendix B.

Specifically, we adopt a two-step indirect inference method. In the first step we apply the indirect inference method to the observed bids. The assumptions that the entry cost and private values are independent and that the actual bidders know the number of the actual bidders at the time of bidding enable us to recover the distribution of private values with only the observed bids. With the estimated distribution of private values, we apply the indirect inference method again to the observed entry behavior in the second step and estimate the distribution of entry costs. One practical issue in the second step estimation is that we need to compute the equilibrium entry probabilities determined by equation (7) in order to evaluate the objective function in the indirect inference method. Therefore the second step estimation should include two loops, namely, the inner loop which is the one solving for the equilibrium entry probabilities, and the outer loop which is the one solving the optimization problem that is computationally intensive. To address this issue, we change the order of loops. Specifically, we first estimate the distribution of entry costs using the indirect inference together with any given entry probabilities.[9] With the estimated distribution, we then update the entry probabilities and estimate the distribution again. We repeat these two steps until the estimates and entry probabilities converge.

The auxiliary model, which is usually simpler than the original model and easier to estimate as

[9] Actually, the initial entry probabilities we use are determined by a reduced-form probit model, which should be close to the equilibrium if the observed entry behavior is the result of our game-theoretical model.

well, plays an important role in the indirect inference method. In this paper, following Li (2005) we employ a relatively simple and easy-to-estimate auxiliary model to make the implementation tractable and the inference feasible. Specifically, since we use the number of actual bidders and bids in the estimation of entry and bidding model, respectively, our auxiliary model includes two separate regressions: a linear regression of the observed bids and a Poisson regression of the number of actual bidders, which have the following specifications,

$$\begin{aligned} b_\ell &= \gamma_{10} + X_\ell'\gamma_{11} + \left(X_\ell'\right)^2\gamma_{12} + \ldots + \left(X_\ell'\right)^m\gamma_{1m} + \varepsilon_{1\ell}, \\ \Pr\left(n_\ell = k\right) &= \frac{\exp\left(-\lambda_\ell\right)\lambda_\ell^k}{k!}, \lambda_\ell = \exp\left(\gamma_{20} + X_\ell'\gamma_{21} + \left(X_\ell'\right)^2\gamma_{22} + \ldots + \left(X_\ell'\right)^m\gamma_{2m}\right), \end{aligned}$$

where b_ℓ is the average bid of auction ℓ, and X_ℓ denote the vector of auction-specific covariates of auction ℓ and the average of bidder-specific covariates. In practice, X_ℓ is a 6×1 vector including hauling distance, volume, duration of a contract, timber grade, DBH, and the number of potential bidders, and m is chosen to be 2 which makes our model over-identified.

An issue arising from the implementation of the second step indirect inference method is the discontinuity of the objective function of equation (8) because of the discrete dependent variable (the number of actual bidders) in the auxiliary model that makes gradient-based optimization algorithm invalid. We address this issue by using simplex, a nongradient-based algorithm. Alternatively, one can follow Keane and Smith (1993) to smooth the objective function using a logistic kernel.

4.3 Estimation Results

Table 2 reports the estimation results. For the (marginal) private value distribution, all the estimated parameters have the expected signs. Of particular interest is the parameter of the hauling distance variable, which is used to control for heterogeneity across bidders. It has a significantly negative coefficient, meaning that bidders are asymmetric and that the longer the hauling distance is, the less is the private value mean. Furthermore, the average marginal effect of the hauling distance variable is about -0.512, meaning that one mile increase in the distance would reduce the private value mean by \$0.512/MBF while everything else is fixed. Another parameter of particular interest is the dependence parameter q_v in private values, which turns out to be relatively small ($q_v = 0.127$) but significant. To get some idea of how large the dependence is with $q_v = 0.127$, we use a measure called Kendall's τ (Nelsen (1999)), which lies between $[-1, 1]$ and is used to measure the concordance of two random variables. Concordance is not really the same concept as affiliation, but measures the positive dependence in a similar way. Kendall's τ is defined as the probability of concordance minus the probability of discordance:

$$\tau_{X,Y} = \Pr\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) > 0\right] - \Pr\left[\left(X_1 - X_2\right)\left(Y_1 - Y_2\right) < 0\right].$$

For the Clayton copula with two random variables, $\tau = q/(q+2) = 0.060$. Therefore $q_v = 0.127$ implies that the event of any two bidders' private values being concordant is about 6.0% more likely than the event of being discordant.

Two points regarding the estimates in the distribution of entry costs are worth noting. First, the hauling distance variable is significant and positive in the entry cost distribution and its marginal effect is 0.699. Second, the dependence level among the entry costs is 0.534, implying a Kendall's τ of 0.21 in the two bidders case, much higher than implied in the distribution of private values. This indicates that the affiliation among the entry behaviors is mainly driven by the affiliation among the entry costs.

5 Counterfactual Analyses

With the estimated structural parameters we are now ready to answer the questions put forward in the introduction section empirically. We focus on both end outcomes, namely, the number of actual bidders, and winning bids (or seller's unit revenue). We conduct counterfactual analyses on the 99th auction of our data. We use this auction as a representative auction, as the values of covariates of this auction are close to the average values of all auctions in our data set. In particular, the number of potential bidders in this auction is 7, about the same as the average number of potential bidders in the original data.

The seller's expected unit revenue is given as follows

$$\begin{aligned} E(w) &= E(w|w>0)\Pr(w>0) + E(w|w<0)\Pr(w<0) \\ &= E(w|w>0)\Pr(w>0) + v_0 \Pr(w<0), \end{aligned}$$

where w denotes the winning bid and v_0 is the valuation of the timber to the seller, and the second equality follows the assumption that if the timber is not sold successfully then the seller gets his own value. In the following analyses we assume $v_0 = 0$, thus the expected revenue is equivalent to the expected profit.

5.1 Effects of Reserve Price and Dependence Level

In this subsection, we quantify the effects of reserve price and dependence levels not only in the cases with asymmetric bidders we study in this paper but also in two hypothetical symmetric cases where every bidder has the same hauling distance, which is set to be the shortest and longest distance, respectively and in the independent case, where the dependence parameters are equal to zero. In each panel of Figures 1, 2, and 3, there are three lines representing both asymmetric and symmetric cases, where the solid line represents the asymmetric case and dash and dash-dot lines represent the symmetric cases with the shortest and longest distance, respectively. In Figure 1 the additional black dot line represents the independent case.

Intuitively the effect of the reserve price can be seen from two aspects. On one hand, a higher reserve price is associated with a lower ex ante expected profit, i.e., a lower cut-off entry cost according to equation (2) as it narrows the integration range, and thus fewer participating bidders and lower probability of being sold, which may lower bids in our APV model with asymmetric bidders. On the other hand, a higher reserve price raises the lowest acceptable bids and of course makes bidders bid higher. Our counterfactuals shown in Figure 1 confirm such trade-off. The solid lines in three panels in Figure 1 show how the reserve price affects the number of actual bidders, the probability of being sold, and the seller's revenue. The number of actual bidders is decreasing in the reserve price as is shown in the first panel. The average number of participating bidders drops dramatically from 5.25 to 1.39 when the reserve price is raised from \$293.42/MBF to \$880/MBF. The probability of being sold is also negatively related to the reserve price. The change in the winning bid is the final result of all effects associated with change in the reserve price. As is seen in the last panel, the optimal reserve price is around \$498/MBF, which is about \$200 more than the current reserve price. This implies that when the reserve price is below \$498/MBF the positive effect on the winning bid outweighs the negative effect associated with the lower probability of being sold.

The APV model we estimated also enables us to quantify the effects of the dependence level among bidders. To this end, we change the values of the dependence parameters of both private values and entry costs while keeping other parameters fixed. We are able to conduct such analysis as the change of the dependence parameter does not affect the marginal distributions of private values and entry costs. As in the analysis of the effects of the reserve price, we are interested in the three effects of the dependence parameters. Results are provided in Figure 2 and Figure 3. As is seen from the solid lines in two figures, the dependence level has little impact on the probability of being sold, which is almost 100 percent all the time. In Figure 2, it can be seen that the number of participating bidders is slightly negatively related to the dependence level of private values. Since the dependence level of private values does not affect bidders' entry costs, it must be true that if bidders become more correlated with each other through their private values, their pre-entry expected profits become less, which leads to a decrease in the number of participating bidders. As can be seen from the solid line in Figure 2, the seller's revenue drops from \$512 to \$443 as the dependence level of private values changes from almost zero to 0.36 in terms of Kendall's τ, and increases back to \$459 when the Kendall's τ is raised to 0.64.

The effects of the dependence level of entry costs are different than the dependence level of private values. The first panel in Figure 3 presents a slightly positive relationship between the dependence level of entry costs and the number of participating bidders at least for the dependence levels that are less than 0.32 (kendall's τ). It is intuitive because compared with the independence case, the affiliation among the entry costs should lead to similar entry decisions among potential bidders, which is to participate in the auction in this case. This is also the idea of the affiliation test in Li and Zhang (2008). Note that the relationship between the dependence level of entry costs and the revenue has the same pattern as the relationship between the dependence level of entry costs and the number of participating bidders, which can be seen from the comparison of the first panel and the last panel of Figure 3, but the magnitudes of both effects are quite small.

5.1.1 Symmetry v.s. Asymmetry

We also examine the effects in the two symmetric cases. As mentioned previously, in the two symmetric cases each bidder in one auction is assumed to have the same hauling distance, which is the shortest and longest distances observed in that auction, respectively. Such setting provides useful insight on the difference between symmetric and asymmetric scenarios. It is clear that for both symmetric and asymmetric cases, the interactions between the effects of reserve prices and dependence levels have almost the same patterns but are different in terms of magnitude. A noticeable feature is that the symmetric case with the shortest hauling distance leads to more participation and more revenue to the seller, while the other symmetric case has less participation and revenue. In particular, as the reserve price changes, the symmetric case with the shortest hauling distance causes about 1.15 more actual bidders on average compared with the asymmetric case, which in turn has about 0.96 more actual bidders on average than the symmetric case with the longest hauling distance. The corresponding average change in the revenue effects are \$52.82 and \$89.36, respectively. We observe a similar difference when the dependence levels are changed, as is shown in Figure 2 and Figure 3. Note such difference cannot solely be attributed to the difference between symmetry and asymmetry, as in the three cases, the hauling distances are different as well. Due to the nature of asymmetry in the data, we are not able to compare symmetric and asymmetric cases by keeping the hauling distance fixed, but two extreme symmetric cases considered here should provide some ideas on how the asymmetry affects the auction results.

5.1.2 Independence v.s. Affiliation

In the independent case, both dependence parameters are equal to zero. A noticeable feature is that the number of bidders is less in the independent case represented by the black dot line than in the affiliated case represented by the blue solid line and the difference is becoming larger as the reserve price increases. This is consistent with what is observed in Figure 3. As discussed above, bidders have similar entry decisions with being affiliated among their entry costs. The effects on the revenue seem similar in both cases, except that the optimal reserve price in the independent case is \$322/MBF. It is interesting to point out that this figure, while far below the optimal reserve price \$498/MBF we find in the affiliated case, it is indeed very close to the actual reserve price used in the auction, an indication that affiliation may be ignored among others when the ODF sets the reserve prices.

5.2 Effect of Bidding Coalition or Merger

Our asymmetric model is ideal for evaluating the merger effects on the end outcomes of auctions, because asymmetry is intrinsically involved in the merger. The merged bidder will be different from other bidders even if they are symmetric pre-merger. For the purpose of measuring the effects of bidding coalition or merger, we conduct two hypothetical mergers, the “best” and “worst” mergers. In the “best” merger two least competitive bidders are merged, which means that two bidders with the longest hauling distances are merged in our case. On the contrary, in the "worst" merger two bidders with the shortest hauling distances are merged into one entity. On the other hand, according to the pre-merger entry behavior of the merging bidders, mergers can be divided into three groups: mergers between two participating bidders, who participated pre-merger, mergers between two non-participating bidders, who did not participate pre-merger, and mergers between one participating bidder and one non-participating bidder. It is obvious that most "worst" mergers belong to the first group while most "best" mergers belong to the second group, because strong bidders are more likely to participate in the auction than weak bidders do. We therefore focus on the merger effects of the first group "worst" mergers and the second group "best" mergers. These two polar cases should shed light on other mergers.

The private value V_m and the entry cost K_m of the merged bidder are defined as $V_m = \max(V_1, V_2)$ and $K_m = \min(K_1, K_2)$, respectively, assuming that bidder 1 and bidder 2 are merged without loss of generality where m denotes the merged bidder. Therefore the marginal distributions of private values and entry costs of the merged bidder are defined as $F_m(v_m) = C(F_1(v_m), F_2(v_m); q_v)$, and $G_m(k_m) = \widetilde{C}(1 - G_1(k_m), 1 - G_2(k_m); q_k)$ in terms of copula, where $\widetilde{C}$ is the survival copula associated with C. In practice we simulate 1000 auctions based on covariates of the representative auction and conduct "best" and "worst" mergers for these 1000 auctions and compare the pre-merger and post-merger end outcomes.

When it comes to the merge effects on entry, we are not only interested in the merged bidder's entry behavior, but also concerned with whether a merger induces non-participating bidders into the auction or crowds participating bidders out of the auction. The first panels of Figure 4 to Figure 9 demonstrate the interactions between entry behavior of bidders and the reserve price and two dependence levels in both "worst" and "best" mergers. The blue solid lines represent the entry probabilities of the merged bidders. The red dot lines are the probabilities that participating bidders would stay out of the auction post-merger, and the probabilities of participation of non-participating bidders are represented by the green dash lines. As is seen from Figure 4, Figure 5, and Figure 6, in the "worst" mergers the merged bidder would always participate in the auction, which is not surprising, as two merging bidders participated in the auction pre-merger. In the "best" mergers, although the two merging bidders did not participate in the auction before the merger, merger increases the chance of participation of the merged bidder, as the merged bidder is "stronger" than the two merging bidders. And this chance varies as the reserve price or the dependence parameters change. Specifically, the entry probability of the merged bidder is 0.5 on average for different values of reserve price and dependence levels, except that when the reserve price becomes larger (greater than \$580), it decreases gradually, which is due to the fact that high reserve price leads to low expected profit. On the contrary the effects of both types of mergers on entry behavior of non-merging bidders are almost negligible. Considering all effects on entry, it is more likely that the auction loses about one participating bidder in the "worst" merger on average and gains one participating bidder in the "best" merge with a probability of 0.5.

The changes in the number and identities of participating bidders affect the final bids and thus the winning bids through several channels. The first channel is called the "competition effect." The increase or decrease in the number of participating bidders makes bidders more or less aggressive. To the merged bidder in the "worst" merger, the competition between two merging bidders is removed due to the merger, which causes the merged bidder bid less. Second, the merger may affect the bids and winning bids through affiliation. Within the APV framework, a bidder would think that he may overestimate the common factor which affects all bidders' private values when he wins the auction. By taking this into account and trying to alleviate this effect, the bidder reduces his bid. This effect is called the "affiliation effect" in Pinkse and Tan (2005) and can make bidders bid more as the number of potential bidders decreases. Lastly, the merger yields a stronger bidder meaning a smaller marginal distribution of private value, through which the merger affects bidders' bids and possibly the winning bids. Intuitively this should lower the winning bid in the "worst" merger and raise the winning bid in the "best" merger, because a stronger winner is undesired while a stronger competitor is desired in terms of the degree of competition. Note that the first two channels are essentially effective through the change in the number of participating bidders and the last one is effective through the change in identities of participating bidders. How merger affects the seller's revenue depends on the interactions of these effects. Because of the complexity of the model we consider, however, analytically we cannot quantify the extent to which each effect impacts on the seller's revenue. Therefore we can only rely on the counterfactual analysis to quantify the overall effect of merger on the seller's revenue as we do here.

The effects on the revenue are presented in the second panels of Figure 4 to Figure 9. The first thing we note is that both types of mergers are beneficial to the seller at the current levels of reserve price and dependence levels. The "worst" merger generates \$5.08/MBF more to the seller and the "best" merger yields \$37.87/MBF more. This implies that the "competition effect" might dominate the other effects in the "best" merger, while the "affiliation effect" might dominate other effects in the "worst" merger. This is, however, not always the case. At some reserve prices (e.g. \$446~\$616) or some dependence levels of private values (e.g. $\tau = 0.6$), the "worst" merger could be harmful to the seller, i.e. it lowers the revenue. It is interesting to note that in Figure 4 when the reserve price is above \$700/MBF, the change in revenue climbs up to about \$250/MBF as the reserve price is raised to \$880/MBF. The reason is that as the reserve price becomes higher and higher in the "worst" merger, the probability that bidders enter the auction but do not bid due to the high reserve price is higher pre-merger than post-merger. As a result, the seller would experience the dramatic revenue change from zero to a high number which must be greater than the reserve price with some probability. It implies that if he is aware of the "worst" merger, the seller could obtain large average benefit from it by raising the reserve price. The effects on the revenue also interact with dependence levels, which means that as the dependence levels vary, the benefits to the seller from mergers are different. In most interactions there is no clear relationship, but some points are still worth noting. In the "worst" merger, shown in Figure 5, the interaction with the dependence level of private values forms an arch. When Kendall's τ reaches 0.25, the seller has the largest gain which is about \$12/MBF. In the "best" merger, the change in the revenue is negatively related to the dependence level of private values. Although the interactions are quite different, one common thing is that the "best" merger always generate more revenue than the "worst" merger does and it would never mean a loss to the seller.

To summarize, we find the following results regarding the merger effects.

i. The "best" merger promotes entry of the merged bidder.

ii. The merger has little impact on non-merging bidders' entry behavior.

iii. Both types of mergers are beneficial to the seller at the current levels of reserve price and dependence levels.

iv. The "best" merger is preferred to the "worst" merger in terms of the change in the seller's revenue.

6 Conclusion

In this paper we study how affiliation and entry can affect bidders' bidding behavior and the seller's revenue using the timber sales auction data from the ODF. We develop an entry and bidding model with heterogeneous bidders within the APV framework, and establish the existence and uniqueness of the Bayesian-Nash equilibrium. We adopt the structural approach to obtain the estimates for the structural parameters in the bidders' private values distribution. We are able to quantify the extent to which the potential bidders' private values and entry costs are affiliated, respectively, and find that the affiliation among bidders' private information found in Li and Zhang (2008) is mainly driven by the affiliation among bidders' entry costs. We then use the structural estimates to conduct counterfactual analysis to address the policy-related issues. In particular, we quantify how the seller's revenue could change with the changes in the reserve price or the dependence level. Moreover, we quantify the merger effect and evaluate how it changes with the changes in the reserve price or the dependence level.

Since we allow bidders to be heterogeneous and have affiliated private values, and also take entry into account, our approach is general and closer to the real timber auction environment. On the other hand, the analysis of the end auction outcomes and welfare implications is complicated by the interactions of affiliation, asymmetry, and entry. The structural approach we propose offers a promising way to disentangle these effects through the counterfactual analysis in addressing policy-related issues such as the merger effect.

Appendix A: Proof of Proposition

The proof of Proposition adapts Lebrun (1999, 2006). We first need the following lemma.

Lemma. *Consider a continuously differentiable and strictly increasing bidding strategy. Assume* $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ *is decreasing in* u. *If* $\widetilde{\eta} > \eta$ *and* $\widetilde{s}_i^{-1}(b)$ *and* $s_i^{-1}(b)$ *for all* i *are two solutions of the system of differential equations* (3) *with boundary condition* (6) *over* $(\widetilde{\gamma}, \widetilde{\eta}]$ *and* $(\gamma, \eta]$, *respectively, then the inverse bidding functions satisfy the following condition:* $\widetilde{s}_i^{-1}(b) < s_i^{-1}(b)$ *for all* b *in* $(\max(\gamma, \widetilde{\gamma}), \eta]$, *where* $\gamma > \underline{v}$.

Proof. Since we know that s_i^{-1} is strictly increasing over $(\gamma, \eta]$, we have $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta) = \overline{v}$. Define g in $[\max(\gamma, \widetilde{\gamma}), \eta]$ as follows:

$$g = \inf\left\{b \in [\max(\gamma, \widetilde{\gamma}), \eta] \left| \widetilde{s}_i^{-1}(b') < s_i^{-1}(b'),\text{ for all } i \text{ and all } b' \in (b, \eta]\right.\right\}.$$

We want to prove that $g = \max(\gamma, \widetilde{\gamma})$. According to the definition of g, $\eta > g$. Suppose that $g > \max(\gamma, \widetilde{\gamma})$. By continuity, there exists i such that $\widetilde{s}_i^{-1}(g) = s_i^{-1}(g)$. From the definition of g, we also have $\widetilde{s}_j^{-1}(g) \leq s_j^{-1}(g)$ for all j. Moreover, there exists $j \neq i$ such that $\widetilde{s}_j^{-1}(g) < s_j^{-1}(g)$, because all the solutions coincide at the point g and therefore coincide in $(g, \eta]$ due to the fact that the right hand side of equation (3) is locally Lipschitz at $b = g$, which contradicts the fact that at point η $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta)$.

From equation (3), we know $ds_i^{-1}(b)/db$ is a strictly decreasing function of $s_j^{-1}(b)$, for all $j \neq i$, since $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. Consequently, $d\widetilde{s}_i^{-1}(g)/db > ds_i^{-1}(g)/db$. Therefore there exists $\delta > 0$ such that $\widetilde{s}_i^{-1}(b) > s_i^{-1}(b)$, for all b in $(g, g + \delta)$. This contradicts the definition of g. □

Proof of Proposition

Proof. First we prove the first part of the proposition by showing that there exist an η, such that $s_i^{-1}(\eta) = \overline{v}$.

(i) Bidding Equilibrium

Let $i, 1 \leq i \leq n$ denote bidders who have the highest bids, denoted by η', at the upper bound of private value $\overline{v}$ and $j, 1 \leq j \leq n$ denote bidders who has the second highest bid, denoted by η, at the upper bound of private value $\overline{v}$. So $\eta' \geq \eta$.

For bidder i, we know that

$$(\overline{v} - \eta') \Pr(B_{-i} < \eta' | \overline{v}) \geq (\overline{v} - \eta) \Pr(B_{-i} < \eta | \overline{v}).$$

It is obvious that $\Pr(B_{-i} < \eta' | \overline{v}) = 1$

$\Pr(B_{-i} < \eta | \overline{v}) = \Pr(b_j < \eta, b_k < \eta, k \neq i, j | v_i = \overline{v}) = \Pr(b_k < \eta, k \neq i, j | v_i = \overline{v})$, since b_j is not larger than η.

$\Pr(B_{-j} < \eta | \overline{v}) = \Pr(b_i < \eta, b_k < \eta, k \neq i, j | v_j = \overline{v})$.

Since the joint distribution of private values follows Archimedean copulas, we have

$$\begin{aligned}
\Pr\left(B_{-i}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}\left(\eta\right)\right)\right)+\phi\left(F_j\left(s_j^{-1}\left(\eta\right)\right)\right)+\phi\left(F_i\left(\overline{v}\right)\right)\right)\phi'\left(F_i\left(\overline{v}\right)\right) \\
&= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}\left(\eta\right)\right)\right)+\phi\left(1\right)+\phi\left(1\right)\right)\phi'\left(1\right)
\end{aligned}$$

and

$$\begin{aligned}
\Pr\left(B_{-j}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}\left(\eta\right)\right)\right)+\phi\left(F_j\left(\overline{v}\right)\right)+\phi\left(F_i\left(s_i^{-1}\left(\eta\right)\right)\right)\right)\phi'\left(F_j\left(\overline{v}\right)\right) \\
&= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}\left(\eta\right)\right)\right)+\phi\left(1\right)+\phi\left(F_i\left(s_i^{-1}\left(\eta\right)\right)\right)\right)\phi'\left(1\right)
\end{aligned}$$

If $F_i\left(s_i^{-1}\left(\eta\right)\right)<1$, then $\phi\left(F_i\left(s_i^{-1}\left(\eta\right)\right)\right)>\phi\left(1\right)$ and $\Pr\left(B_{-i}<\eta|\overline{v}\right)>\Pr\left(B_{-j}<\eta|\overline{v}\right)$ since $\phi'\left(1\right)<0$ and $\phi^{-1\prime}\left(x\right)$ is increasing in x. Therefore

$$\left(\overline{v}-\eta'\right)\Pr\left(B_{-j}<\eta'|\overline{v}\right)>\left(\overline{v}-\eta\right)\Pr\left(B_{-j}<\eta|\overline{v}\right)$$

since $\Pr\left(B_{-j}<\eta'|\overline{v}\right)=1$. But this is impossible because the optimal bid of bidder j at $\overline{v}$ is η, therefore we have $F_i\left(s_i^{-1}\left(\eta\right)\right)=1$ and $\eta'=\eta$.

(ii) Uniqueness of Bidding Equilibrium

Suppose that there exist two equilibria and thus two different values η and $\widetilde{\eta}$ such that the respective solutions $s_i^{-1}\left(b\right)$ and $\widetilde{s}_i^{-1}\left(b\right)$ are also solutions of the system of differential equations for all i. Without loss of generality, we can assume that $\eta<\widetilde{\eta}$. The value of $\ln\left(\Pr\left(v_j<s_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)$ at $b_i=\eta$ is strictly larger than the value of $\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)$ at the same point. We have shown that $\widetilde{s}_i^{-1}\left(b\right)<s_i^{-1}\left(b\right)$ for all b in $(\underline{v},\eta]$. When b converges to $\underline{v}$, $s_i^{-1}\left(\underline{v}\right)=\underline{v}$.

On the other hand, the first order condition can be written as follows

$$\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)}{db}=\frac{1}{s_i^{-1}\left(b_i\right)-b_i}.$$

Therefore $\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}\left(b\right),j\neq i|v_i\right)\right)}{db}<\frac{d\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}\left(b\right),j\neq i|v_i\right)\right)}{db}$. Therefore, the difference between these two logarithms increases as b decreases towards $\underline{v}$. On the other hand, $\ln\left(\Pr\left(v_j<\underline{v},j\neq i|v_i\right)\right)$ is a finite value since $F_j\left(\underline{v}\right)>0$. Therefore for two solutions, $\ln\left(\Pr\left(v_j<s_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)$ cannot both converge to the same finite value as b decreases towards $\underline{v}$. Therefore η and $\widetilde{\eta}$ coincide and the equilibrium is unique.

(iii) Entry Equilibrium

The entry probability p_i is determined by equation (7). Let $p=\left(p_1,\ldots,p_n\right)\in\left[0,1\right]^n$ and

$G_p = (G_1 \circ \Pi_1(p), \ldots, G_n \circ \Pi_n(p))$. Since $s_i(v)$ and G_i is continuous, the pre-entry expected profit Π_i and $G_i \circ \Pi_i$ is continuous in p. So $G_p : [0,1]^n \to [0,1]^n$ and is continuous in p. A fixed point of p follows Kakutani's fixed point theorem (Kakutani (1941)). □

Appendix B: Solving Equilibrium Bids

Equilibrium Bids

Note that with the choice of the Clayton copula, the first order condition given in equation (3) can be written as follows,

$$(1+q)(v_i - b)\sum_{j\neq i} \frac{dF_j^{-q}\left(s_j^{-1}(b)\right)}{db} = -q\left(\sum_{k=1}^{n} F_k^{-q}\left(s_k^{-1}(b)\right) - n + 1\right).$$

Define $F_i^{-q}\left(s_i^{-1}(b)\right) = l_i(b)$, then $v_i = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right)$, and F.O.C. becomes

$$(1+q)\left(F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b\right)\sum_{j\neq i} l_j'(b) = -q\left(\sum_{k=1}^{n} l_k(b) - n + 1\right)$$

Rewriting all terms in the equation as polynomials

$$\begin{aligned}
l_i(b) &= \sum_{j=0}^{\infty} a_{i,j}(b-b_0)^j, \\
l_i'(b) &= \sum_{j=0}^{\infty} (j+1)\, a_{i,j+1}(b-b_0)^j, \\
l_i^{-\frac{1}{q}}(b) &= \sum_{j=0}^{\infty} g_{i,j}(b-b_0)^j, \\
F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) &= \sum_{j=0}^{\infty} p_{i,j}(b-b_0)^j, \\
F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b &= \sum_{j=0}^{\infty} \widetilde{p}_{i,j}(b-b_0)^j, \\
F_i^{-1}(x) &= \sum_{j=0}^{\infty} d_{i,j}(x-x_0)^j, \\
x_i^{-\frac{1}{q}} &= \sum_{j=0}^{\infty} c_{i,j}(x-x_0)^j,
\end{aligned}$$

where $\widetilde{p}_{i,0} = p_{i,0} - b_0$, $\widetilde{p}_{i,1} = p_{i,1} - 1$, and $\widetilde{p}_{i,j} = p_{i,j}$ for $j > 1$.

Computation of $p_{i,j}, g_{i,j}$: following the lemma in Appendix C in Marshall, Meurer, Richard and Stromquist (1994), we have

$$p_{i,J} = \sum_{r=1}^{J} d_{i,r}\theta_{i,r,J} - z_J,\ p_{i,0} = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b_0)\right) \quad (10a)$$

$$\theta_{i,r,J} = \sum_{s=1}^{J-r+1} g_{i,s}\theta_{i,r-1,J-s},\ \theta_{i,0,0} = 1, \quad (10b)$$

$$g_{i,J} = \sum_{r=1}^{J} c_{i,r}\varphi_{i,r,J}, \quad (10c)$$

$$\varphi_{i,r,J} = \sum_{s=1}^{J-r+1} a_{i,s}\varphi_{i,r-1,J-s}, \varphi_{i,0,0} = 1. \quad (10d)$$

Computation of $a_{i,j}$: from the FOC, we have

$$(1+q)\left(\sum_{j=0}^{\infty}\widetilde{p}_{i,j}(b-b_0)^j\right)\sum_{j\neq i}\sum_{s=0}^{\infty}(s+1)\,a_{j,s+1}(b-b_0)^s = -q\left(\sum_{k=1}^{n}\sum_{s=0}^{\infty}a_{k,s}(b-b_0)^s - n + 1\right)$$

$$(1+q)\left(\sum_{j=0}^{\infty}\widetilde{p}_{i,j}(b-b_0)^j\right)\sum_{s=0}^{\infty}(s+1)\left(\sum_{j\neq i}a_{j,s+1}\right)(b-b_0)^s = -q\left(\sum_{s=0}^{\infty}\left(\sum_{k=1}^{n}a_{k,s}\right)(b-b_0)^s - n + 1\right)$$

$$(1+q)\sum_{s=0}^{\infty}(s+1)\left(\sum_{r=0}^{s}\widetilde{p}_{i,s}\sum_{j\neq i}a_{j,s+1-r}\right)(b-b_0)^s = -q\left(\sum_{s=0}^{\infty}\left(\sum_{k=1}^{n}a_{k,s}\right)(b-b_0)^s - n + 1\right)$$

Comparing the coefficients of $(b-b_0)^s$ we have

$$(1+q)(s+1)\left(\sum_{r=0}^{s}\widetilde{p}_{i,s}\sum_{j\neq i}a_{j,s+1-r}\right) = -q\left(\sum_{k=1}^{n}a_{k,s}\right),\ \text{for } s>0 \quad (11a)$$

$$(1+q)\,p_{i,0}\sum_{j\neq i}a_{j,1} = -q\left(\sum_{k=1}^{n}a_{k,0} - n + 1\right),\ \text{for } s=0. \quad (11b)$$

Algorithm:

1. $d_{i,j}, c_{i,j}$ for $j = 1, \ldots, J$, can be computed by Taylor expansion. In practice, $J = 3$.
2. Decide $a_{i,0}, \widetilde{p}_{i,0}, \theta_{i,0,0}$,and $\varphi_{i,0,0}$ by the boundary conditions.
3. Calculate $\widetilde{p}_{i,1}$ from equations (10) given $a_{i,0}, \widetilde{p}_{i,0}, \theta_{i,0,0}$,and $\varphi_{i,0,0}$.
4. Calculate $a_{i,1}$ from equations (11) given $\widetilde{p}_{i,1}$.
5. Repeat step 3 and 4 until $a_{i,j}, j = 1, \ldots, J$ are calculated.

Now since we have found the coefficients of the Taylor expansion of the inverse bidding function up to the J-th order, we are able to find the equilibrium bid for a given private value for bidder i through the obtained Taylor expansion at a appropriate point. One issue regarding the algorithm is the boundary conditions. From the Proposition we know that there are two boundary conditions associated with the equilibrium. Note that it cannot be used here although the boundary condition at the lower bound of bids is known to us, since it causes the problem of singularity. Therefore we have to use the condition at the upper bound, which is unfortunately unknown to us. To address this problem we follow the method described in Marshall, Meurer, Richard and Stromquist (1994) and Gayle (2004) to find the common η first. Roughly speaking, it is to find an η which generates the best equilibrium bids at point $\underline{v}$ according to the algorithm described above. For details see Marshall, Meurer, Richard and Stromquist (1994) and Gayle (2004).

References

[1] Athey, S., and J. Levin (2001): "Information and Competition in U.S. Forest Service Timber Auctions," *Journal of Political Economy*, 109(2), 375-417.

[2] Athey, S., J. Levin, and E. Seira (2004): "Comparing Open and Sealed Bid Auctions: Theory and Evidence from Timber Auctions," Working Paper, Stanford University.

[3] Bajari, P., and A. Hortaçsu (2003): "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions," *RAND Journal of Economics*, 34(2), 329-355.

[4] Baldwin, L. H., R. C. Marshall, and J.-F. Richard (1997): "Bidder Collusion at Forest Service Timber Sales," *Journal of Political Economy*, 105(4), 657-699.

[5] Brannman, L., and L. M. Froeb (2000): "Mergers, Cartels, Set-Asides and Bidding Preferences in Asymmetric Oral Auctions," *The Review of Economics and Statistics*, 82(2), 283-290.

[6] Campo, S., I. Perrigne, and Q. Vuong (2003): "Asymmetry in First-Price Auctions with Affiliated Private Values," *Journal of Applied Econometrics*, 18(2), 179-207.

[7] Gallant, A. R., and G. Tauchen (1996): "Which Moments to Match?," *Econometric Theory*, 12(2), 657-681.

[8] Gayle, W.-R. (2004): "Numerical Analysis of Asymmetric First Price Auctions with Reserve Prices," Working paper, University of Pittsburgh.

[9] Gourieroux, C., A. Monfort, and E. Renault (1993): "Indirect Inference," *Journal of Applied Econometrics*, 8(S), S85-118.

[10] Haile, P. A. (2001): "Auctions with Resale Markets: An Application to U.S. Forest Service Timber Sales," *American Economic Review,* 91(3), 399-427.

[11] Haile, P. A., and B. Tamer (2003): "Inference with an Incomplete Model of English Auctions," *Journal of Political Economy,* 111, 1-51.

[12] Harsanyi, J. C. (1967-1968): "Games with Incomplete Information Played by "Bayesian" Players," *Management Science*, 3,5, 159-182,320-344.

[13] Hendricks, K., and R.H. Porter (1988): "An Empirical Study of an Auction with Asymmetric Information," *American Economic Review*, 78, 403-426.

[14] Hubbard, T. P., T. Li and H. J. Paarsch (2009): "Semiparametric Estimation in Models of First-Price, Sealed-Bid Auctions with Affiliation," Working Paper, Vanderbilt University.

[15] Kakutani, S. (1941): "A Generalization of Brouwer's Fixed Point Theorem," *Duke Mathematical Journal*, 8(3), 457-459.

[16] Keane, M., and A. A. Smith, Jr (2003): "Generalized Indirect Inference for Discrete Choice Models," Working paper, Yale University.

[17] Krasnokutskaya, E., and K. Seim (2006): "Preferential Treatment Program and Participation Decisions in Highway Procurement," Working paper, University of Pennsylvania.

[18] Lebrun, B. (1999): "First Price Auctions in the Asymmetric N Bidder Case," *International Economic Review*, 40(1), 125-142.

[19] Lebrun, B. (2006): "Uniqueness of the Equilibrium in First-Price Auctions," *Games and Economic Behavior*, 55(1), 131-151.

[20] Levin, D., and J. L. Smith (1994): "Equilibrium in Auctions with Entry," *American Economic Review*, 84(3), 585-599.

[21] Levin, D., and J. L. Smith (1996): "Optimal Reservation Prices in Auctions," *Economic Journal*, 106(438), 1271-1283

[22] Li, T. (2005): "Indirect Inference in Structural Econometric Models," *Journal of Econometrics*, forthcoming.

[23] Li, T., and I. Perrigne (2003): "Timber Sale Auctions with Random Reserve Prices," *The Review of Economics and Statistics*, 85, 189-200.

[24] Li, T., I. Perrigne, and Q. Vuong (2000): "Conditionally Independent Private Information in OCS Wildcat Auctions," *Journal of Econometrics*, 98(1), 129-161.

[25] Li, T., I. Perrigne, and Q. Vuong (2002): "Structural Estimation of the Affiliated Private Value Auction Model," *RAND Journal of Economics*, 33(2), 171-193.

[26] Li, T., I. Perrigne, and Q. Vuong (2003): "Semiparametric Estimation of the Optimal Reserve Price in First-Price Auctions," *Journal of Business & Economic Statistics*, 21, 53-64.

[27] Li, T., and B. Zhang (2008): "Testing for Affiliation in First-Price Auctions Using Entry Behavior," *International Economic Review*, forthcoming.

[28] Li, T., and X. Zheng (2007): "Information Acquisition or/and Bid Preparation: A Structural Analysis of Entry and Bidding in Timber Sale Auctions," Working Paper, Vanderbilt University.

[29] Li, T., and X. Zheng (2009): "Entry and Competition Effects In First-Price Auctions: Theory and Evidence from Procurement Auctions," *Review of Economic Studies*, 76, 1397-1429.

[30] Lizzeri, A., and N. Persico (2000): "Uniqueness and Existence of Equilibrium in Auctions with a Reserve Price," *Games and Economic Behavior*, 30, 83-114.

[31] Marshall, R. C., M. J. Meurer, J.-F. Richard, and W. Stromquist (1994): "Numerical Analysis of Asymmetric First Price Auctions," *Games and Economic Behavior*, 7(2), 193-220.

[32] Maskin, E. S., and J. G. Riley (1984): "Optimal Auctions with Risk Averse Buyers," *Econometrica*, 52(6), 1473-1518.

[33] Maskin, E. S., and J. G. Riley (2000): "Asymmetric Auctions," *Review of Economic Studies*, 67(3), 413-438.

[34] Maskin, E. S., and J. G. Riley (2003): "Uniqueness of Equilibrium in Sealed High-Bid Auctions," *Games and Economic Behavior*, 45(2), 395-409.

[35] Milgrom, P. R., and R. J. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50(5), 1089-1122.

[36] Myerson, R. B. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6(1), 58-73.

[37] Nelsen, R. B. (1999): *An Introduction to Copulas*. Springer.

[38] Paarsch, H. J. (1992): "Deciding Between the Common and Private Value Paradigms in Empirical Models of Auctions," *Journal of Econometrics*, 51(1-2), 191-215.

[39] Paarsch, H. J. (1997): "Deriving an Estimate of the Optimal Reserve Price: An Application to British Columbian Timber Sales," *Journal of Econometrics*, 78(2), 333-357.

[40] Pinkse, J., and G. Tan (2005): "The Affiliation Effect in First-Price Auctions," *Econometrica*, 73(1), 263-277.

[41] Porter, R.H. (1995): "The Role of Information in U.S. Offshore Oil and Gas Lease Auctions," *Econometrica*, 63, 1-27.

[42] Samuelson, W. F. (1985): "Competitive Bidding with Entry Costs," *Economics Letters*, 17(1-2), 53-57.

[43] Sklar, A. (1973): "Random Variables: Joint Distribution Functions and Copulas," *Kybernetika*, 9, 449-460.

[44] Smith, A. A., Jr (1993): "Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregressions," *Journal of Applied Econometrics*, 8(S), S63-84.

[45] Vickrey, W. (1961): "Counterspeculation, Auctions, and Competitive Sealed Tenders," *Journal of Finance*, 16(1), 8-37.

[46] Zhang, B. (2008):"Affiliation Effect, Competition Effect, and Entry Effect in First-Price Auctions," Manuscript, Vanderbilt University.

Table 1: Summary Statistics of Bidder- and Auction-specific Covariates

	Observation	**Mean**	**Std. Dev.**
Bid	1074	384.5844	103.7889
# of Potential Bidders	203	5.8276	1.5690
# of Actual Bidders	203	3.6946	1.7250
Entry Proportion	203	.6550	.2826
Appraised Price	203	331.291	94.322
Distance	1183	75.2779	45.6976
Volume	203	3318.468	2674.112
Duration	203	780.010	199.965
Grade	203	10.326	.461
DBH	203	16.722	4.812

Table 2: Estimation Results

	Private Value distribution		**Entry Cost distribution**	
	Coefficient	**Std. Error**	**Coefficient**	**Std. Error**
Hauling Distance	-0.473	0.044	1.261	0.231
Volume	0.103	0.045	0.071	0.079
Duration	-0.028	0.085	0.030	0.122
Grade	1.365	0.051	1.208	0.482
DBH	0.187	0.118	-0.086	0.311
Dependence Parameter	0.127	0.051	0.534	0.116

Figure 1: Effect of Reserve Price

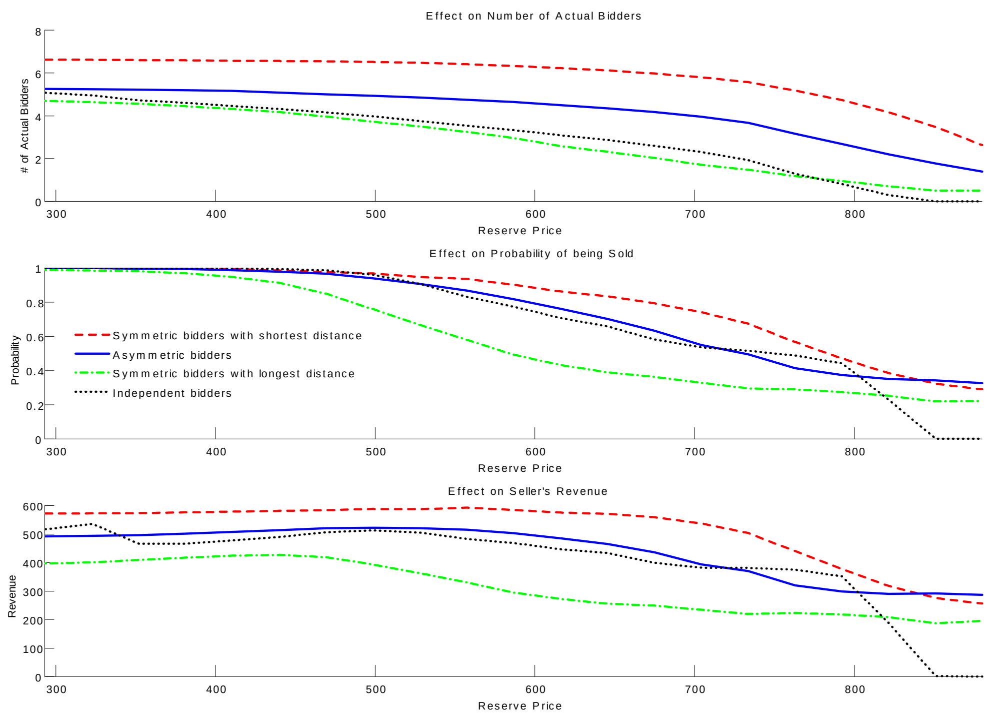

Figure 2: Effect of Dependence Level of Private Values

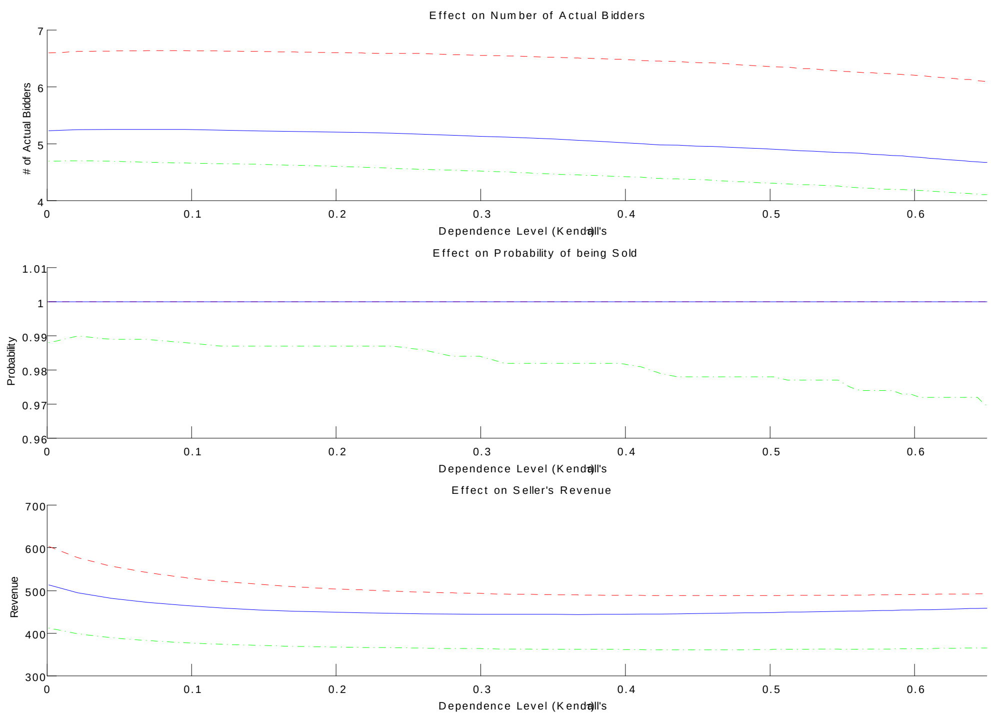

Figure 3: Effect of Dependence Level of Entry Costs

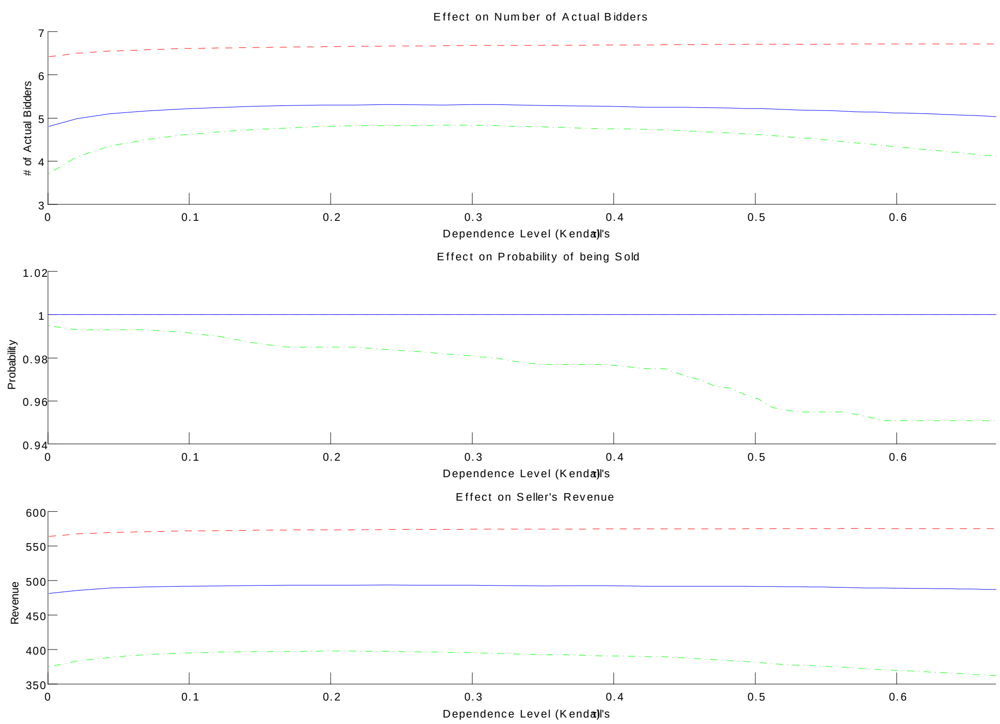

Figure 4: Interaction between "Worst" Merger and Reserve Price

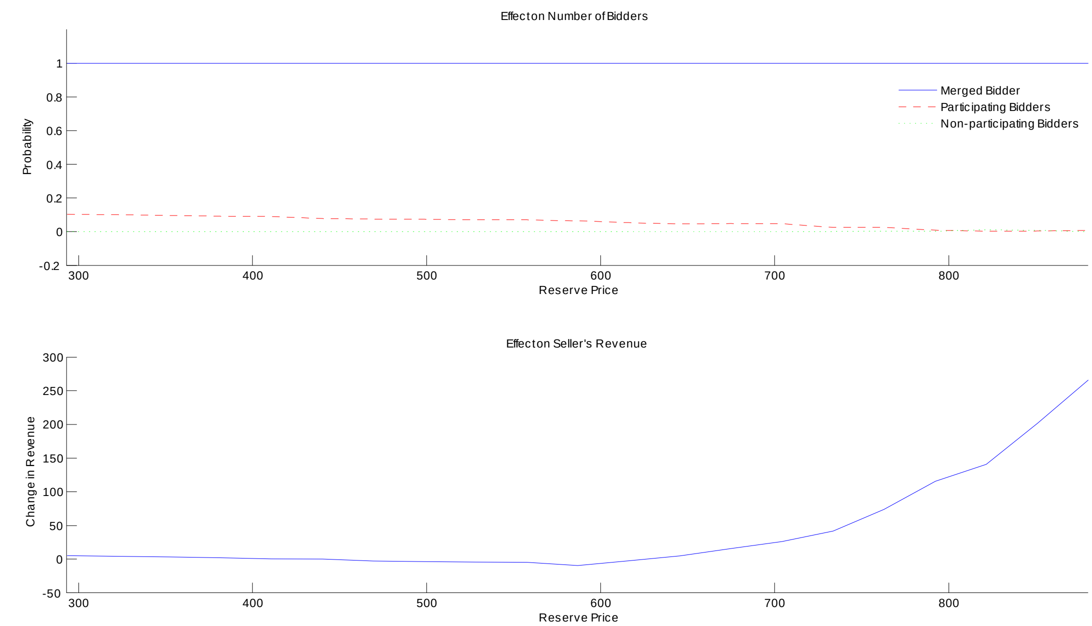

Figure 5: Interaction between "Worst" Merger and Dependence Level of Private Values

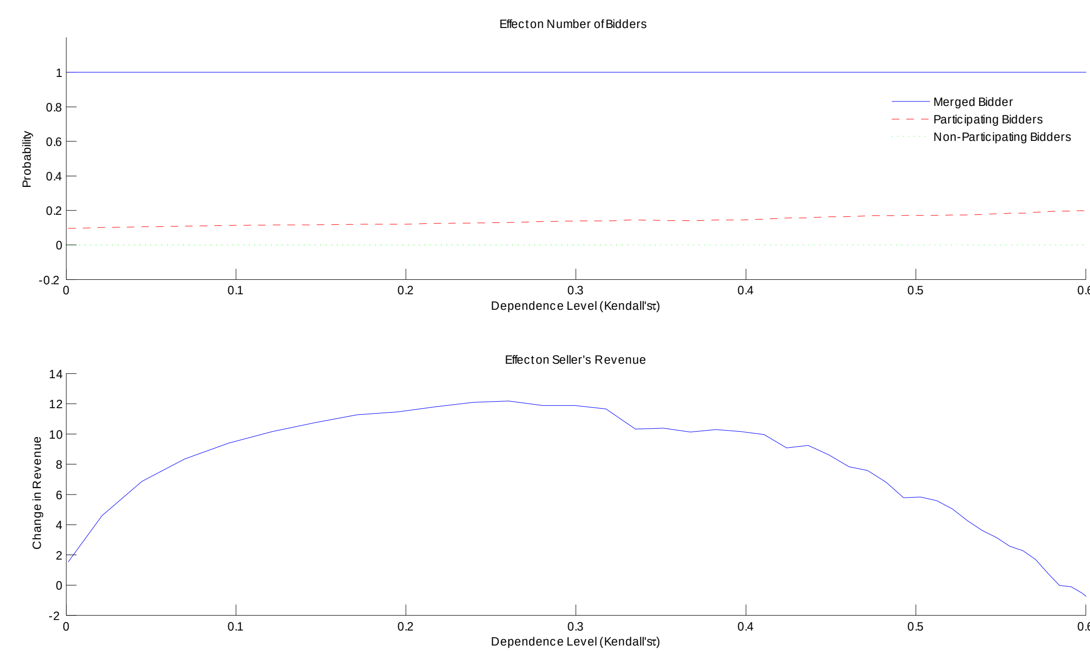

Figure 6: Interaction between “Worst” Merger and Dependence Level of Entry Costs

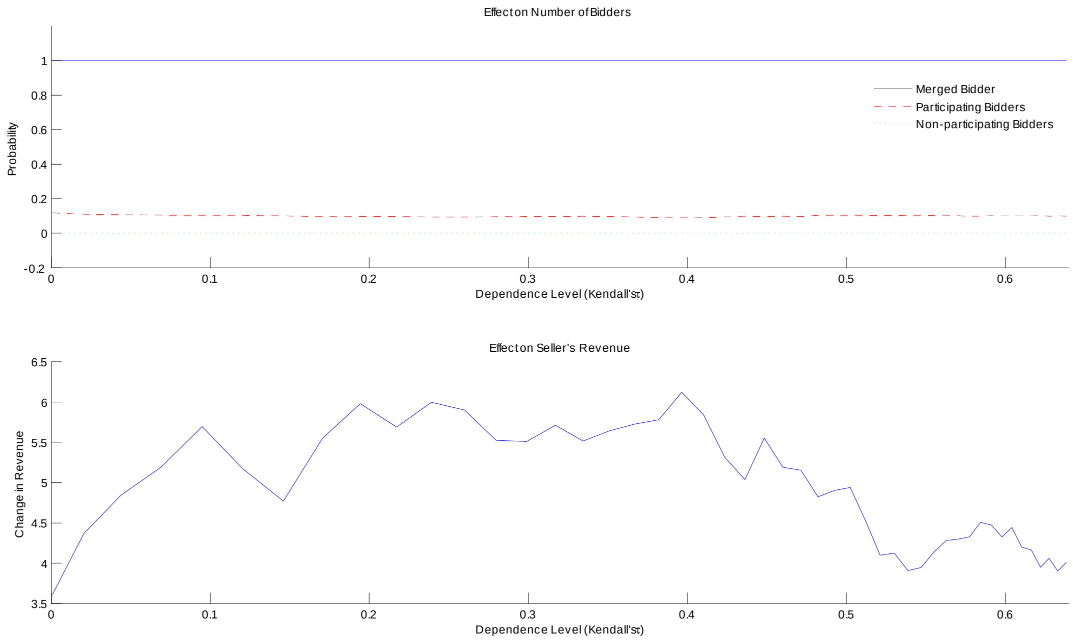

Figure 7: Interaction between “Best” Merger and Reserve Price

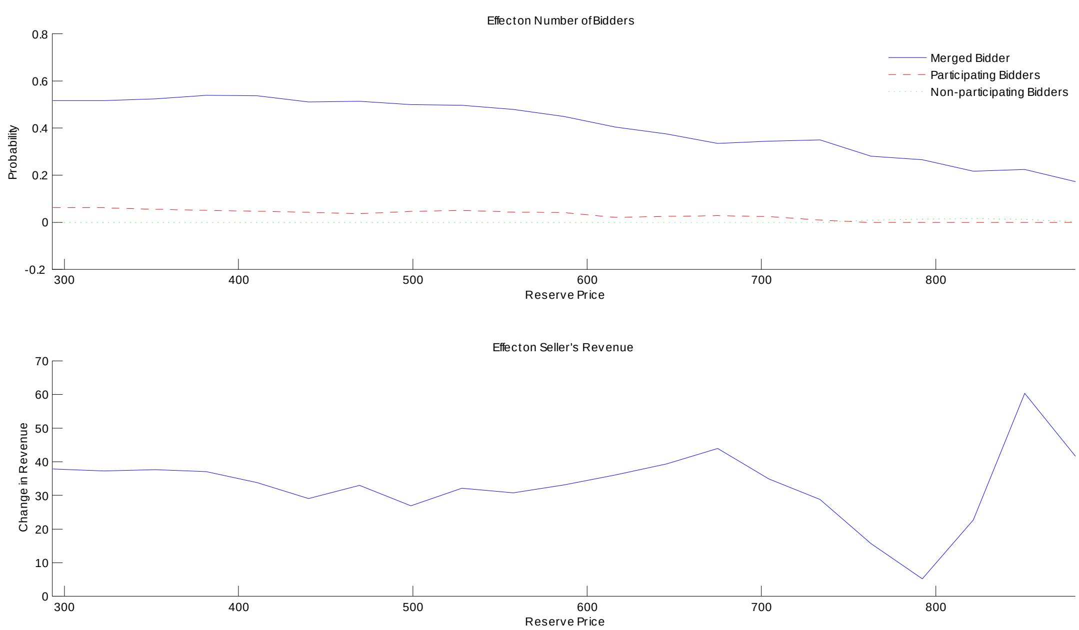

Figure 8: Interaction between “Best” Merger and Dependence Level of Private Values

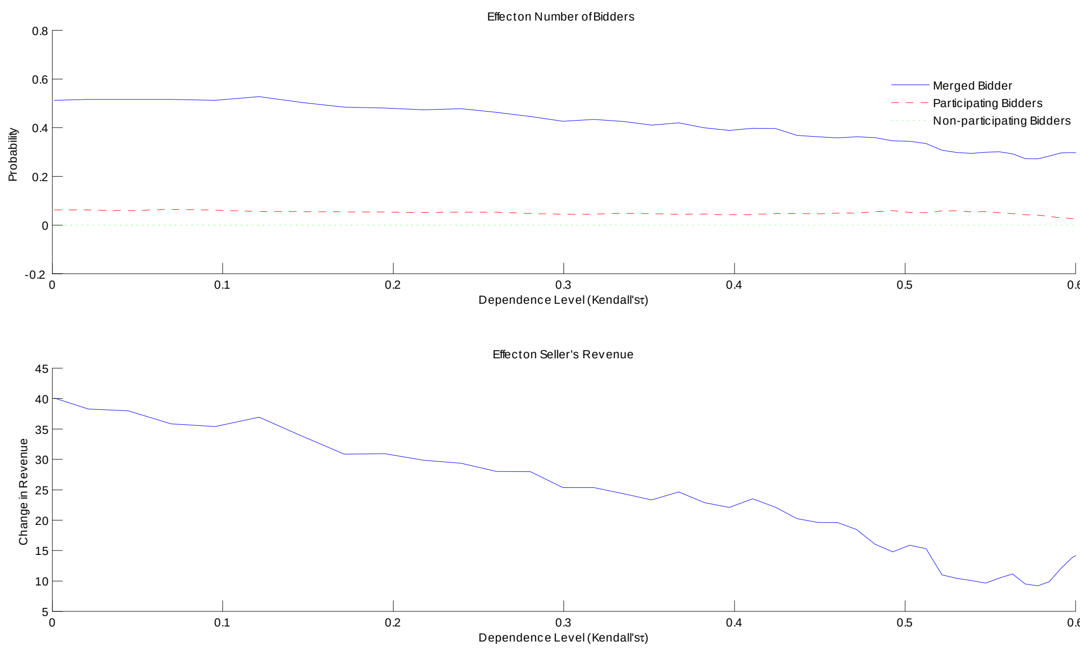

Figure 9: Interaction between “Best” Merger and Dependence Level of Entry Costs